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The dialectic between nature and culture in the interplay between discrete and continuous

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Abstract: In an effort to contribute to the debate between nature and culture, we explore a specific cultural sphere, hoping to enrich the many often profound yet abstract philosophical reflections on the subject. We examine the interplay between discrete and continuous mathematics as a site for analysing a dialectic within scientific debate, beginning with the debate between Einstein and Bohr in the 1930s. This inquiry will help us understand how we, as animals, act in the world and within the space-time continuum, and how we might better distinguish ourselves from machines. Mathematics, in fact, is written in a "natural language" — a notion whose possible meanings we will gradually explore. It is not, as Galileo famously suggested, nature that is written in mathematical language. By viewing mathematics as a symbolic and cultural practice, we will examine how it is rooted in the basic gestures of animals like us: gestures shaped by, and made possible through, our biological bodies and specified by the expressiveness of language. We will emphasize how mathematics continually returns to seek "meaning" in spatial and temporal activities. At the heart of this inquiry is alphabetic writing, another fundamental rupture with — and link to — nature. It discretizes the continuous song of language and reframes, in different terms, the nature-culture dialectic we seek to identify in this text.

Key words: perspectives on nature, discrete, continua, mathematical cultures, intelligibility

Causality, between continuous and discrete descriptions

From Aristotle to Einstein, every event possesses a "cause." The same could be said for Darwin, who looks for the "causes" of phylogenetic (evolutionary) changes in the permanent production of diversity proper to living reproduction and eco-systems' dynamics. Our entire scientific culture, at least the Western one, has been based on an intelligibility of nature largely built on the "search for causes." These were then framed, in physics, by "symmetry principles", as we will argue. A "then," however, that goes back as far as the Greek world: for Aristotle, the Earth, formed by the aggregation of heavy

bodies around the center of the universe toward which they converge, is round "for reasons of symmetry"; for Archimedes, a balance with equal weights is in equilibrium, and a body in water receives a push from bottom to top equal to the weight of the displaced water, "for reasons of symmetry." This is what "causes" the sphericity of the Earth, the balance of a scale and the stability of a body in water. Hermann Weyl will explain that "every fundamental principle of physics is a principle of symmetry." But what are these symmetries, that happen to be *continuous*? Aristotle and Archimedes think of rotational or reflection (mirror) symmetries; Euclid's axioms are continuous gestures on the plane ("draw" a straight line... "produce" a circle...), that maximize symmetries in the construction (Longo, 2025); Euclid proofs are based on translations and rotations of the plane, on diagrams (Panza, 2012). And this is the crux: continuous hand movements that draw justify and ground Euclidean geometry in terms of symmetries.

Two thousand years later, Noether's theorems show that the conservation principles of physics (of momentum or inertia, of energy...) are *continuous symmetries* in the equations (Kosmann-Schwarzbach, 2010; Longo and Montévil, 2014), in all existing physical theories, even those that are incompatible with each other (such as those of the relativistic and quantum field) (Aspect, Grangier and Roger, 1982).¹ These theorems explain why conservation principles are so fundamental in physics: they are not arbitrary rules, but consequences of the underlying symmetries of space-time (Kosmann-Schwarzbach, 2010).² Thus they unify various physical phenomena and provide a deep conceptual basis for conservation laws in classical mechanics, quantum mechanics and field theory. We will correlate this enormous and beautiful cultural construction to the primary gestures of our animal body in space and time, showing the unity between the two, as our symbolic constructions in turn modify our also-bodily being in the world.

Let us return first to the scientific problem of causality. In the 1930s, Einstein and Bohr knew well and shared the role of continuous symmetries in physics (and greatly appreciated the work of Noether and Weyl). On this basis, Einstein even made it possible to understand gravitation — as we mentioned in a note below — as the cause of the fall of a heavy body (and the turning of planets), in terms of continuous symmetry: it is an inertial motion (a conservation principle, thus a "symmetry principle") in curved spaces. Thus, when confronted with the phenomena of quantum entanglement and the spin-up or down of an electron, he thinks that they must have "causes," like any event in physics, but that these may be "hidden" (Bohm, 1952; Kochen and Specker, 1967; Abbott, 2015).³ The former otherwise could not take place in the space-time continuum with the topological property essential to relativistic and classical physics: given two distinct points in space, each point and each of its observable properties must be "separable," that is, contained in different "neighborhoods" — two small circles/spheres, say, that do not intersect. "Natural" topologies have this property, for Euclid and Riemann and ... for us animals who act in space and are able to "well separate" places and objects. The same applies to the phenomenon that produces a discrete observable in the act of measurement — the spin of an electron, a kind of 0 or 1, for Einstein, Bohm and others: against Bohr's view, its (hidden) causes, parameterized in the continuum, must be discovered and well framed by Noether's theorems on continuous symmetries.

Bohr instead assumes the peculiarity of micro-physics, far from our senses, accessible only through complex measuring instruments that co-constitute the event in the act of accessing the quantum level: the intrinsic contingency (a-causality) of some of its phenomena, such as the spin of a quanton.

Moreover, entanglement is due to an inherent holistic nature of quantum events, made objective only in and by measurement, thanks to the "artificial" (cultural) interface between us and the micro-physical reality: the measuring instrument. Since the measurement result is a co-constituent of the "friction" between (classical) instrument and micro-physics, at its scale, this value preserves the trace of correlation at the quantum level. The spin-up or down, in turn, a discrete (classical) result of measurement, a 0/1, is inherently "causeless," for Bohr; it is pure contingency, period — useless to look for any cause at the micro-physical level. Indeterminacy is thus put back into the theoretical heart of Quantum Physics, as Heisenberg had proposed it in his analysis of measurement: even more radically, thus, the spin of an electron is not only unpredictable, it has no causes. But then, if Bohr and Heisenberg are right, is the world discrete and a-causal? It depends on the scale and state of the object: unlike the bound electron, the energy spectrum of the free electron is continuous. In general, space-time is described in the continuous but, so far, there is no unity between the spaces of microphysics and classical or relativistic spaces. (Connes, 1994) provides a very elegant mathematical bridge, by a "non-commutative geometry", whose construction is grounded on quantum (non-commutative) measurement. In general, measurement, which stands at the interface between "nature" and a theoretical-cultural construction, including that of the classical measuring instrument in Quantum Physics, produces a new entity, the measured micro-object. That is, knowledge of the "natural" quantum object presupposes the "cultural" construction of a measuring instrument, since measurement is the only form of access we have to nature, in science.

In summary, Quantum Physics proposes to us an original and difficult intelligibility of the world, intricate, discrete and continuous at the same time, or, perhaps, neither — for example, conservation laws, as continuous symmetries, hold in the very abstract Hilbert spaces, possibly of infinite dimension, where Schrödinger's equation unfold the dynamics, resulting at measurement in a discrete probability value. Thus, the epistemological teaching of Quantum Physics is of extraordinary richness (Bitbol, 1998). In particular, it helps us to understand that nature is not continuous nor discrete, it is what it is. We culturally organize it differently by the tools of continuous or of discrete mathematics, both grounded in different forms of human activities in space, time and society.

What everyone agrees on, from Einstein and Bohm to Bohr and Heisenberg, is that when we move to a complete description of the world with discrete mathematics, causality disappears. In Quantum Physics, we said, the construction and use of the measuring instrument provides access to a scale far from our senses and shows the separation between classical/relativistic physics and microphysics, as theories: it adds a form of randomness intrinsic to quantum theory, to the determination that frames classical causality and that justifies it in terms of continuous symmetries, i.e., Quantum Physics highlights phenomena that can "have no cause" (such as the spin up or down of an electron) or that can be correlated without even information passing between them (instantaneous entanglement prohibits the passage of any signal, whose speed is limited by that of light). But the two physics separate scales of analysis and, for a century, attempts have been made to unify their theoretical foundations and thus propose an intelligibility of the inert that would become even richer, with partial successes, such as Connes' Non-commutative Geometry, in continua. The culture, of which we shall speak, that emerges from the separation of alpha-numeric, thus discrete, algorithmic knowledge from bodily gestures in continuous space and time, retroacts quite differently on our relationship to nature: it subordinates it to techno-cultural artifices whose design is now all political and aims predominantly at the control of

nature and the social, without caring for intelligibility (Anderson, 2008; Calude and Longo, 2017; Garapon and Lassègue, 2021). As we shall argue below, all operating systems in our computers, or Discrete State Machines, as Turing calls them in 1950, are written in *imperative languages*: they work with and follow orders. The point is that, once causality is gone, dynamics either unfold '*at random*' or *follow orders*. Computer networks are today a fundamental tool for human interaction. How does their causal structure shape knowledge and the social, which image of nature do they suggest or impose?

The Greek alphabet, between democracy and techno-feudalism

Among our many symbolic-cultural constructions, the radical rupture between sign and meaning, which originated in the invention of the alphabet, has been particularly fruitful and rich in spin-offs, such as the current alpha-numeric machines that are changing the world. Symbolic inventions have set themselves up as a barrier and link to nature, because at the same time they remove us from it and allow us to construct knowledge, taking a distance from reality, as well as to intervene in it by science and techniques. To this end, perhaps the invention of the alphabet is one of the most powerful — we see it today. It is obtained by discretizing the continuous song of language — a song produced by modulating the flow from the lungs as a bagpipe. Thanks to alphabetic writing, finite sequences, words and phrases, of discrete signs (letters), belonging to a finite list, the alphabet, make it possible to reconstruct meaning through phonemes, voice, were it even produced "in the head," in silence. But it also becomes possible to manipulate these sequences automatically, without any reference to their meaning in the world.

In the book with Jean Lassègue (Lassègue and Longo, 2025), we propose a conceptual history of the invention of the current "electronic reticular alpha-numeric writing" at the heart of the computer networks that blanket the world. Developing a lead identified by Herrenschildt (2007), from the alphabet to modern computing, we recall that the Greeks are the first to make the alphabetic script inherited from the Phoenicians "phonetically complete." In short, they add enough vowels to it to remove the ambiguities proper to languages written only with the use of consonants, called "abjad," such as Phoenician, classical Hebrew and Arabic. To interpret the latter, in order to understand when reading, it is necessary to complete words, composed of consonants only, by adding, for example, dots that specify the phoneme and, thus, the meaning (Lassègue and Longo, 2025).⁴ This drastic reduction of ambiguities is at the heart of the invention of Greek democracy, where every literate man was able to read and interpret the written law, without the need for scribes and priests to interpret it. It is precisely this drastic reduction of interpretive ambiguities that makes it possible to read "mechanically" and arrive, distracted, at the bottom of a page and realize that we... have understood nothing. This is impossible in abjad languages, even less so in Chinese or Egyptian: one has to interpret to read — and in Egypt, it probably took two or three priest-scribes who, discussing, proposed a possible understanding of the texts, sacred rebuses.

The automaticity of the Greek writing of which we are the heirs, already intuited by Turing (Lassègue and Longo, 2012), a pioneer of algorithm theory, is the basis of today's numerical machines-systems for *re-writing writing*: algorithms are sets of instructions, written as associations of letter sequences to letter sequences, that tell how to re-write letter sequences: the rule (aa → ab) orders to re-write aacd as abcd.⁵ All formal and numerical computation is just based on "sequence-matching" and "sequence-

replacement" (checking identity of sequences, replacing (sub-)sequences) (Bezem, Klop and de Vrijer, 2013). One can thus write and rewrite — without understanding anything, mechanically, following the written rules — so works the digital machine. It is in fact easy to encode, with sequences of 0s and 1s, switches of an electronic machine, sequences of alphabetic letters. The separation of writing from meaning is then radical: to manipulate it, we no longer need the phoneme that evokes, by sound, meaning — a machine can do it. We, heirs of the Greek alphabet, and only we, have thus invented an alpha-numeric machine for writing and re-writing, without meaning. And we move even further away from the animal cry, rich in meaning, a cry of call, alarm, pain, joy — at the origin of human language.

Dualism and complexity as the stacking of the simple

This separation of (alphabetical) sign and sense is found in another, recent form of dualism: the sharp distinction between software and hardware, fundamental to doing computing today — a dualism to put Descartes to shame and which makes some people dream of transferring the soul, well encoded in 0s and 1s, into software (Bamford, 2012; Sabry, 2022; Gilbert, 1992).⁶ Observe instead that the "computer" of the Chinese is a very different, *geometric* machine: the abacus. The unity of sign (ideogram) and meaning is found both in the inseparability of soul and body proper to classical Chinese culture and in the relationship between writing and space: houses and gardens may be built in the form of ideograms, or these are drawn following lines that organize space (Li, 2022). A synthetic, systemic worldview dominates (Nisbett, 2003).

Instead, alphabetical writing is the basis of an analytical way of thinking: like language with the letters of the alphabet, so the world is described and constructed by stacking *elementary* and *simple* components (Democritus annotated with letters the atoms and Euclid the points, "semeia," signs). A vision that would later prove most useful in making machines, all of which can be constructed by assembling elementary and simple components. A pioneer in this distinction, Leonardo da Vinci had a "holistic" and dynamic (and often "turbulent," dramatic) view of nature and clearly distinguished his hundreds of naturalistic drawings from technical drawings, his own invention, obtained by breaking down and re-composing, analytically, whole machines, piece-by-piece, drawing plane, then three-dimensional sections (Longo and Longo, 2021). A little later, forgetting Leonardo's turbulent motions, "ephemeral phenomena" he himself acknowledged, the scientific revolution begins with "celestial mechanics," an analysis of perfect and eternal motions, planetary clockwork (Descartes, Kepler), and is primarily a revolution of machines (Rossi, 1962). Along these lines, the early 19th century analytical approach of Laplace and Fourier gave us a great tool of mathematical analysis: the decomposition of systems of equations into linear *series*: like in the construction of machines, complexity is obtained by composing elementary and simple parts — this is linearity.

The alphabetic discretization of the world, from the atomistic views to the linear composition of the elementary and the simple to reach the complex, marked our powerful analyses of nature. These were intended as a complete description of the world, as Laplace well explicated: the equations that determine a physical dynamic allow us to predict it, up to an approximation that at most grows linearly. What is more, linearization allows a discretization of nonlinear processes: as we said, every process is supposed to be (mathematically, linearly) decomposable into elementary and simple components. Such

a view, still common today, has delayed the understanding and development of systemic views, such as Poincaré's "geometry of non-linear dynamical systems" (Poincaré, 1892; Charpentier et al., 2006),⁷ or Prigogine's far-from-equilibrium thermodynamics (Nicolis and Prigogine, 1977) in continua - an analysis of "ephemeral phenomena," he was reproached. These have long remained, and still are, minor canons, although almost all "world systems" are nonlinear and far from equilibrium. Their unpredictability thus results from the conjunction of the approximation of (classical) measurement and the non-linearity: in the presence of bifurcations (a notion invented by Poincaré), for example, in a non-linear trajectory, a fluctuation below the best measurement does not allow us to predict which path will be followed by the continuous dynamics. These notions are better understood in continua – again, this is not nature "per se," but a fruitful understanding or mathematical organization of relevant phenomena, in particular of movement, of change. As a matter of fact, on a static screen the discrete pixels may approximate an analogue (continuous) image ever more closely as their number increases. In non-linear dynamics, by contrast, the discrete space trajectory does not approximate the continuous one: the two diverge after a very short time. Only under highly restrictive conditions can one find, at best, a continuous trajectory that approximates (shadows) a given discrete dynamics in space and time (Pilyugin, 1999) – the opposite of the myth of "discrete approximation". To be sure, there is no better place to analyse a highly turbulent hurricane than a good numerical simulation on a computer screen — provided one knows what is actually happening. In particular, by pressing "restart" on identical initial and boundary pixels — an operation made possible by their exact, discrete nature — one may observe the same dynamics unfolding identically, iteration after iteration: a physical nonsense.

The social consequences of discretizing nature

The identification of thought and nature with its alphabetical discrete "writing," more precisely of the "Greek" type (in fact in English today), is not a bridge between culture and nature, but crushes nature, hides it and proposes to manipulate it thanks to a specific cultural construction that is highly effective, but carries with it the "bias" of a very specific artifice, imbued with mechanicality, deeply dualistic and analytical: everything thinkable and existing can be broken down into the elemental and simple of meaningless letters, atoms of knowledge and of the real. The alphabetic writing of a program makes everything intelligible. Even more deeply, against the whole scientific mountain built on the role of continuous symmetries in physics, alluded to above, illustrious computer scientists (Turing Awards!) such as J. Pearl and L. Valiant, tell us that a stone falls because it is "programmed to fall": it just follows instructions. The laws of nature would indeed be algorithms, they claim, enriched by statistical methods (Pearl and Mackenzie, 2018) or interacting (echo-rhythms) (Valiant, 2013). This erases two thousand years of a very rich scientific history, that of symmetries and mathematics of continua, from Euclid and Galileo (inertial movement) to Poincaré and Einstein, but above all the rootedness of scientific culture in bodily gestures in space and time. The algorithmic approach to nature, replaces them with a very specific form of alphabetical writing, algorithms, far removed from meaning and the world, and only grounded on "counting" well separated entities. As Riemann had already observed, while in the continuous, measurement, hence metric, is associated with the structure of space (its curvature, in geometry) and, he conjectures, with the "cohesive forces between bodies" (Einstein will unify gravitation and inertia just so), in the discrete "one can only count," says Riemann (Riemann,

1854). In the discrete one thus loses the fundamental form of access to the real, the classical and relativistic measure, approximated, given in an interval, associated with a metric: it is reduced to counting. That is, in the discrete, measurement is just counting one bit, one pixel after another: access to the real is exact, as with the integers, and we forget Poincaré and return to Laplace, with the addition of quantum randomness, recognizes Pearl — who explicitly claims that this is how nature goes: it is Laplacian or quantum (Pearl, 2000). In our short synthesis, from the remarks of the previous section to the alphabetic decomposition of nature, causes, which are framed by continuous symmetries, must be replaced, in the discrete, by "instructions/orders": that's how the world moves. In a view of biology as "following the instructions" written with the four letters of DNA, Monod (1970) claims that what does not obey orders is "noise".⁸ One finds again a vision so dear to the politics to which today's top numerical leaders adhere: the real (and the social) must obey commands ("executive orders") and woe to those who make noise.

In summary, the laws of physics and of the living are programs, alphabetical writings of orders, in particular genetic instructions to be executed (Jacob, 1974; Monod, 1970);⁹ a living therefore to be controlled, by re-programming it (Doudna and Sternberg, 2017; Longo, 2021a).¹⁰ This is hardly a new unity between nature and culture; rather, it erases much of the intelligibility of the physical dynamics and justifies the growing massacre of the living — a programmed machine to be manipulated at will before understanding the organism and its preferred "space," the ecosystem, with their interactions (Giraud and Amelier, 2024; Diamanti-Kandarakis et al., 2009; Noble, 2024; Ball, 2024; Longo, 2023). The development of the few functioning bio-technologies based on the notion of a "genetic program" has by no means "dissolved the traditional boundary between nature and culture and between biology and society" (Rabinow and Rose, 2006), nor made cancer disappear within a decade after the DNA sequencing, as promised (von Eschenbach, 2003). Instead, by a mechanistic, monochromatic and authoritarian reading of the living, in content and management (it follows the orders/instructions written in the DNA and we can modify them), it has prevented the growth of a dialectical, theoretical and practical link between culture and nature. An analysis of the organism at the crossing of an historical (evolutionary) path and an ecosystemic (relational) structure may propose a new understanding of life and of our active, cultural shaping of it (Marinucci et al, 2027).

It is difficult today to distinguish research orientations in AI as well, from authoritarian political choices, those of the new techno-feudalism (Morozov, 2022): the funding of the huge oligopolistic structures that hold the reins is largely oriented toward surveillance or control (Zuboff, 2019), from the commercial one, for now and in our countries (what do you buy and how can they sell your personal data? (Alombert and Giraud, 2024)), to the political one in China, to war (de Roucy-Rochegonde, 2024), with its drones and automatically controlled weapons, whose "collateral damage" is part of the inevitable. AI-related techno-feudalism, made possible by alpha-numeric machines derived from "democratic" Greek writing, again relies on a caste of scribes, the programmers of lines of code, indecipherable to the layman, almost always proprietary and secret. Between us and meaning stands the barrier of a script that has become unreadable: the computer code.

This orientation of the digital was not necessary. In the 1980s and early 1990s, the first computer networks accompanied a strong growth of exchange, of new correlations between different researchers scattered over the surface of the Earth, which outweighed the risks of uniformity, a formidable tool

developed far from the AI of the time. The gradual formation of huge oligopolies profoundly changed the operation and control of networks. The myth of the "singularity" (the imminent replacement of the human) and the computational reading of cognition, of the living and even of physics (Longo, 2020)¹¹ have thus changed the ends and methods of research, driven by a culture aimed at crushing the human and all of nature into a single imperative paradigm.

Intelligence, movement, body gestures

It should not be forgotten that the brain was formed to manage the movement in space of certain multicellular animals. And that this is what it has mainly served for about five hundreds millions of years. And even today, it allows us to drive a car as we did when we were hunting: protensive saccades (eye movements) precede anything that moves, thanks to the retention (memory or neuronal trace), at the level of the retina and primary visual cortex, of a perceived trajectory. What is more, this "sticking" or "gluing" of the retention and protention of a trajectory also occurs in peripheral vision, where we are unable to recognize objects, but appreciate movement (Berthoz, 1997). Self-driving cars, which, for now, advance by "recognizing all possible traffic configurations," are a paradigm of artificial stupidity: they move with great difficulty and immense energy costs on the same roads as we animals, who move as when hunting or evading a predator, that is anticipating everything that moves. Moreover, the "gluing" operation that is done in these anticipations (the retentions of a movement extending into protention, via an anticipatory movement, an ocular saccade) is a primary, visual and bodily practice of continuity, made rigorous by differential geometry (Longo, 2021b), and allows mathematical physics to make causality intelligible. This is not "naive physics" as many among the artificially intelligent say, but the complex cognitive entrenchment of a "geometric judgment," related to movement, visual and gestural practice of a trajectory: "this line is continuous." "Trace, produce ... a line, a circle ..." says Euclid and in language further specifies: the line is without thickness (definition beta) and he invents by this the difficult notion of border. Only in the interplay between gesture and language, drawing and saying, or rather, in the writing that stabilizes the concept, does mathematics develop. It originates in a "mathematical pulse" which "becomes the very core of this activity's capacity, the specific way in which mathematical thinking ventures" and "stumbles upon a piece of writing based on an intuition" (Guitart, 2000). It is the result of "our human activity in the spaces of our humanity" (Husserl, 1933). But it is a humanity in all its diversity.

Different mathematical cultures

In the great synthesis of Chinese mathematics and its commentary (Liu Hui, 2004),¹² Liu Hui calculates, too, by approximation with inner and outer polygons the length of the circumference, but he says: when "you can no longer see the difference," the calculation stops. So for the square root of 2. We are a long way from the Greek anguish for the a-logos, the irrational, due to the line without thickness: only in this way do we catch a glimpse of the apeiron, the infinite in the finite, the a-logos in the logos, the irrational in the rational ... and upstream, Zeno's paradox, that sinks into the infinite. Liu Hui does engineering, mathematics in the service of building and trading: he does not argue in a dialogue of symmetries with the gods. But these dialogues with the infinite allowed us to invent the spaces of the

scientific revolution, which originated in a theological-pictorial gesture in painting, the Italian perspective (Longo and Longo, 2020), and, then, to invent infinitesimal calculus. On the one hand, we thus produced much knowledge and more machines and cannons, thanks in part to the "alphabetic" view of machines and the world and, on the other hand, better analyses of trajectories, thanks to differential/infinitesimal calculus. By contrast, we, the Greeks, developed for centuries a poorly annotated number theory, without 0 or negative numbers, imported only in the 13th century by the Arabs, but appearing much earlier in China and India: they were essential to keep merchants' accounts well, not concerned with the essence of the world, like the Greeks.

In the approach hinted in this paper, we build cultural tools from manifold practices of and about nature and reorganize nature with culture. But in different ways. The example of the continuous/discrete play, the infinite and the finite shows this. Yet, the practice of the discrete, of counting, is also part of the evolutionary story. We share with many animals the "small-numerosity discrimination" that is, counting small amounts of objects well separated in space. However, only in language, and then in number writing, do we know how to specify and stabilize the invariant, the concept of integer number, common to the same quantity of objects that even the newborn or many mammals know how to compare (Starkey, Spelke and Gelman, 1983).¹³ Actions therefore in the world and about the world are the "episteme" of two symbolic structures, the continuous and the discrete mathematics, whose epistemological diversity we have emphasized: from physical causality to the diversity in the analysis of the constitution of scientific meaning. In other words, conceptual constructions both rooted in natural practices, diverge in the ways of organizing the real, falling back on it in different ways. On the one side, our gestures in space and the geodesics of physics, optimal paths in continuous spaces, on the other, counting a few objects that we share with many animals and the discretization of language by the alphabet. This late, peculiar invention lead us to the trajectories made of pixels, mimicking on the screen of our numerical machines the falling of a stone, by changes of state, from one color to another, following orders. As we mentioned, nothing moves on a computer screen, there are only changes of state of the pixels. This is a formidable technology, a catastrophe of knowledge and action on nature, if identified with nature. Like Quantum Physics, but differently, Biology requires a complex blend of insights. The discrete structure of macromolecules is embedded in a cellular, organismal and eco-systemic context whose active deformations give them function and scientific meaning. There is no privileged causal level (Soto et al, 2016): constraints canalize all dynamics at all levels of organization and enable changes of the very "phase spaces" – we will return to this.

We have tried to relate, in a very cursory way, this enormous and beautiful cultural construction, mathematics, to the primary gestures of our animal bodies in space and time, showing the unity between the two, as our symbolic constructions in turn modify our also-bodily being in the world. But the cognitive grounding of knowledge constructions can lead to very different outcomes, in which the choice between one or the other turns out, perhaps always, certainly today, to be eminently political. In short, the dialectic between nature and culture is not neutral. The alleged orders given to plants and animals whose DNA is claimed to be re-programmed, GMOs, are an important example of this: they are changing nature and society, from the disappearance of insects, pollinators and naturally fertile soil (Giraud and Amelier, 2024), to the abuse of pesticides with endocrine disrupting roles (Stoker et al., 2010; Diamanti-Kandarakis et al., 2009), to new social suffering associated with the sale of seeds to be

renewed annually (Bhatnagar, 2021). On the one hand, imperative linguistic structures are far from allowing us to control nature: they do not drive biological dynamics, but change their "constraints" (Montévil and Mossio, 2015), including such important ones as DNA, with largely unpredictable outcomes — but surely a decline in bio-diversity and a growth of pathological consequences (see Diamanti-Kandarakis et al., 2009). On the other hand, while they make us lose the intelligibility of nature, from the physical to the living, they make it possible to control societies, through AI, as never before.

Making the invisible visible

Let's return to the topic discussed above, the invention of writing and alphabetic writing in particular: Jaynes's (1976) theses in this regard are fascinating, though questionable to many. On the one hand, as Herrenchmidt also writes, cited above, writing made visible the invisible of language, of thought. Being able to say, "there, there is written what I say, what I think," humans have objectified speech and, perhaps, constructed "consciousness" itself differently or made it possible in new ways. In addition, the shift from pictographic or syllabic writing systems to the phonetic alphabet profoundly changed cognitive processes and ways of thinking. Ideogrammatic or hieroglyphic writing systems are more visual and engage both hemispheres of the brain, particularly the right hemisphere, which seems more associated with holistic, image-based thinking. The alphabet, phonetic rather than pictorial, shifts the burden of reading and writing to the left hemisphere, which is more oriented toward sequential, linear, and analytical processing. The brain's plasticity hints at its possible structural changes in historical times: brain connectivity seems to change rapidly, for example in motorists and those who practice virtual realities for a long time, forgetting, thanks to GPS, city maps and movement in space (Fajnerová et al., 2018; Dahmani and Bohbot, 2020). And the brain is by far the most plastic organ among the components of an organism, while all evolution is shaped primarily by the plasticity of development (West-Eberhard, 2003). Genetic expression itself is modified by the ecosystem context.¹⁴ And we, with culture, have been changing ecosystems ever since we started carving stone and lighting fires with our free hands. Evolution has gone along in all dimensions of the living, and these are intertwined with each other (Jablonka and Lamb, 2008). Analyses of the production of novelty, proper to all "histories," the biological, evolutionary one, and that of our symbolic cultures, must be distinguished, but grasping a dialectic, in the similarities and differences between the two, helps to enrich both.

Thus, one misplaces the problem by asking whether (continuous) symmetries, our powerful symbolic construction, are "intrinsic" to nature, as so many "Platonist" mathematicians say. The real is very rich; we grasp fragments of it by very particular, contingent, historical perspectives. Our animal bodies are shaped by symmetries related to movement, not a plant, which at most has growth symmetries. In movement we practice an isomorphism between (we identify) the line detected by the vestibular system and the visual line, the one constructed by the saccade that precedes the path to be taken, the Poincaré-Berthoz isomorphism (Teissier, 2006). This imaginary line and the vertical axis of gravitation identify a fundamental plane of symmetry, that of bilateral symmetry common to (almost) all moving organisms. These symmetries, these invariants, are not in the world, but they allow us to organize the world and our action in it. Thus, we project them onto nature, building houses and stable structures thanks to perfectly flat walls, right angles... but where are flat surfaces and right angles in a forest? Yet these

symmetries allow us to unify so much physics. So we extend the practice of counting to negative numbers, invented very early in Asia, it was said — today we can see them as a symmetry of positive numbers with respect to 0, but they did not originate that way, it is not inherent in them. Numbers, particularly negative numbers, are not "already there": one can long make splendid mathematics of borders and symmetries, like Euclid, without making much use of numbers. In general, numbers, integers, rationals, real numbers..., are not already in nature, but result from the constitution of invariants of counting and from measuring operations in the continuum of action, of gestures that relate us to the real, down to the difficult physical, relativistic and quantum measurements. The real resists, channels our measurements, our gestures of knowledge: the symbolic production that follows is not arbitrary. But different ones can be given: we have mentioned here two that differ in the constituent paths of mathematical knowledge, the discrete and the continuous — as well as the Chinese variant, with its scriptural traces of evocative spatial diagrams. Not only do they offer a different dialectic between culture and nature, but, within mathematics itself, the interplay between the discrete and the continuous is productive of new conceptual structures: from those of algebraic geometry to analytic number theory, to recent theories of Topos (Grothendieck) that allow for original bridges between discrete and continuous (Tamme, 1994; Caramello, 2025; Zalamea, 2012; Longo, 2015). Such correlations make it possible to move from one structure to another, without undoing the differences, indeed allowing us to highlight their different roles in applications, in physics for example. Let's try to explain this by an analogy. Apollonius (3rd c. BC), faced with different plane curves, circles, ellipses, parabolas, hyperbolas, invented the three-dimensional cone and observed, in Grothendieck's style: those curves are different sections of the same cone, one can move from one to another by tilting the section-plane — and thus we can see them all at once, in his new "topos" that unifies different "sites." The cone allows us to look at the plane curves, from another mathematical dimension, "from above," from a new "topos," a new unifying perspective. Yet, in physics, if an astronaut is happy to travel on an ellipse around the Earth, he/she would change mood if, from the control centre, they inform him/her that he/she is actually traveling on a hyperbola... even though they are "just" different sections of the same cone. Mathematics may elegantly correlate structures with highly different physical meaning.

In sum, we make symbolic inventions anchored in natural practices, in actions on the world, which are conditions of their possibility, do not determine them, and which may diverge in the accumulation of historical paths. Some go so far as to propose principled caesuras with nature. These become most serious if they come to assume the role of a monochromatic political government of nature and societies, which seeks to (and succeeds in) preventing debate. Authoritarianism is the current political counterpart of this programmed and re-programmable view of nature, which thus obeys orders. Scientism, of which Grothendieck explicitly accuses the theoretical foundations of geno-centric biology (Grothendieck, 1971),¹⁵ is its counterpart. The rupture with nature, and its slaughter due to a governance without intelligibility, follows. Alternative proposals primarily seek a unity between an organism's development and its ecosystem, its evolution. On the one hand, evolution is the result of the plasticity of development (West-Eberhard, 2003; Jablonka and Lamb, 2008); on the other hand, the analysis of development (ontogeny) can be based on the same principles that make evolution (phylogeny) intelligible (Soto, Longo and Noble, 2016).

The analysis of culture then also requires other tools, those that make different human symbolic and technical cultures intelligible: the tools of history, economics... These tools have in common with the

analyses of the living the appreciation of the historicity of processes: the living and culture permanently construct new observables and objects of knowledge, this is at the heart of their historicity, far from the stability of the "space of phases" (observables and relevant parameters) proper to existing theories of the inert (Kauffman, 2002; Koppl, Kauffman, Felin and Longo, 2014; Longo, 2017). The analysis of the natural rootedness and, then, constitutive autonomy of the notions of both the mathematical continuous and discrete, of their possible "genesis," as well as of their internal dialectic and, then, with nature, is the attempt made here.¹⁶

Extended criticality, anti-entropy and heterogenesis

Is the inertial continuum of physics, the first and fundamental principle of conservation (Galileo), a continuous symmetry in space¹⁷, adequate to say anything about the living? Almost nothing, except that a dog falls with the same acceleration as a stone — without denying the role of gravitation in morphogenetic processes, a constraint on their unfolding. But the "primary cause" of the generation of biological forms, morphogenesis, is the "reproduction with variation and motility" of cells. Of course, under massive "constraints." Primarily, one internal to the cells themselves, DNA, which provides the "pattern" of macromolecules, in largely stochastic interactions (Elowitz et al., 2002). Thus, the many fractal structures (lungs, vascular system...) do not have the crystalline regularity of physical optimality, only shaped by the forces that impose its fractality, but are irregular, diverse (Brown et al., 2002). And this is functional: the diversity of lungs, for example, within an individual, among individuals in a population, contributes to the resilience of the individual, of the population, in the face of environmental changes. And every cell reproduction is a "critical transition": a bifurcation that re-organizes the overall structure (the tissue) into a new coherence structure, in which even the two daughter cells are different from each other and from the mother cell. We called this criticality "extended" (Bailly and Longo, 2008; Longo and Montévil, 2011): transitions occur in cascades, are "dense," that is, present in every relevant time interval — metabolic cycles, heartbeats, respirations. Instability is channeled, first by DNA, an internal constraint, a very important chemical-physical trace of evolution, then by all external conditions, from pre-existing tissue structure, to the hormonal system, great regulator of cell reproduction in an organism, to eco-systemic conditions (from gravitation and air pressure, to temperature... to other organisms). The "smooth" (differentiable) continuity of classical physical processes is continuously broken, in the living, by non-differentiable transitions, the cascading, critical transitions: the continuum is not discretized, but "punctuated" by structural changes in local organization (the two new cells, the re-organized tissue...).

Instability, stabilized by constraints, which are also changing in time, is the engine of novelty production in biology, which we have called anti-entropy (Bailly and Longo, 2009; Longo and Montévil, 2012). This does not oppose entropy with a negative sign (negentropy), but has its own dimension — it coexists with the production of entropy in an organism, indeed it feeds on it: the processes of chemical diffusion, in a cell, in an organism, are entropic and are essential to life (Chollat-Namy and Longo, 2023). Anti-entropy is the permanent production of biological novelty, of new organization therefore: to forms it associates biological functions. So energy is certainly globally conserved, but an organism absorbs and transforms energy, using it to produce and rebuild organization (anti-entropy), always different, also using entropic processes.

All this breaks yet another fundamental continuous symmetry of all existing physical theories: the conservation of phase space. The primary task of the theoretical physicist, when faced with an original problem or experience, is to invent a good space of observables and relevant parameters (a "phase space"), even an entirely new one, in which, possibly, to treat the problem mathematically. But this space then remains fixed for the theory; at most it can change the number of parameters. In historical theories, on the other hand, what changes, it was said, is first and foremost the space of the possible, which is not pre-given (Longo, 2017). The evolution of the living produces new observable phenotypes, which combine new forms with new functions, which we may need new parameters to be analyzed. As the space of phases (of the possible) changes, the laws change. Consider when, as a result of the invagination of a bacterium into an archaeon, mitochondria, as organelles of the eukaryotic cell, are formed (Margulis, 1996): everything changes. Metabolism, its laws, change profoundly. The DNA of the cell and of its organelles changes. Macromolecular interaction networks are built whose existence is made possible only by the cell with its new biological structure, impossible to derive from fundamental chemical properties. Of course, those molecular interaction networks may be formally *described* and are compatible with existing chemical theories, but they are *not derivable* from them: it is a form of incompleteness as it has been rigorously described within mathematics (Longo, 2011). Only evolution and the new ecosystem of the complex eukaryotic cell make the chemistry of the cell intelligible (Sharma et al., 2023).¹⁸ In short, for the purpose of knowledge, biology is not reducible to chemistry, not least because one needs biological, evolutionary historicity to understand the chemistry of the cell itself.

Thus, new functions are historical, unnecessary novelties, like those of all the organs of multicellular organisms that will follow in the course of evolution; they are not already given in a list of all possible functions. This changing of the laws themselves, in space and time, is now mathematically treated as "heterogenesis" (Sarti, Citti and Piotrowski, 2022): in short, the "laws," the equations, the differential constraints, change from point to point. A culture of change makes the historical nature of biological and human processes better intelligible, enriching classical determination, but also quantum indeterminacy, with a new, more radical, form of unpredictability: that of the very space of the possible (phase space), in the analysis of life and of culture, both the result of a history. Indeed, of histories, intertwined, mutually shaping each other.

Notes

1. Quantum entanglement is a phenomenon in which two or more particles are related in such a way that the state of one particle instantaneously affects the state of the other(s), regardless of their distance. That is, measuring the property of one particle (such as spin or polarization) gives instantaneous information about the corresponding property of the other, defying classical notions of locality and separability (Aspect, Grangier and Roger, 1982).

2. Noether's theorems specifically link conservation laws to continuous symmetries because the proofs rely on the ability to perform infinitesimal transformations, a property of continuous groups (Lie groups). These small transformations are used to derive the conserved properties

through the Euler-Lagrange equations of motion. And this subsists even when moving from Euclidean geometry to the various Riemannian geometries (elliptic, hyperbolic), spaces of intelligibility for Einstein's Relativity. In the latter, the symmetry expressed by Euclid's fifth axiom, that of parallels, does not hold: it is equivalent to saying that the sum of the interior angles of triangles of any magnitude is 180° . As is well known, this is false in triangulations between stars. On the one hand, a fundamental continuous symmetry is thus lost, by enlargements and reductions (homotheties of the plane); on the other hand, Einstein allows us to say that "a body falls for reasons of symmetry," since, in the "curved" spaces of Relativity, inertia and gravitation are equivalent and the former is a principle of (continuous) symmetry — a splendid unification of cultures, physics and mathematics, contemporary and Greek. In biology, we need to go beyond symmetries — knowledge is grounded on "non-conservation principles", beginning with Darwin's first principle: "descent with modification" (Longo and Montévil, 2014).

3. Hidden variables are parameters that vary in the continuum — their necessary "non-locality" makes their use ineffective as an alternative to the interpretation defended by Bohr (Kochen and Specker, 1967; Abbott, 2015).

4. For example, in an abjad, one can write "ktb" to mean "book", "library", "librarian": only the context allows one to understand what the writer is referring to. Even more complex and demanding is the interpretation of hieroglyphic or ideogrammatic writings: hieroglyphs even have to be read as "rebuses," the interpretation of which was entirely in the hands of a caste of scribes (Lassègue and Longo, 2025).

5. "Programs" are algorithms implemented in specific programming languages. All programming languages, even "functional", "declarative" or "Object-Oriented" languages, which are widely used in AI, are based on an underlying imperative structure (by the Church-Rosser Theorem, Barendregt, 1984; and Normalization, Girard, Taylor and Lafont, 1990). Moreover, most compilers and all operating systems are written in imperative languages: the deep structure of the machine can only execute orders.

6. Symmetrically, in the book by a much-funded biologist, Gilbert (1992), it is argued that once the human genome was sequenced (2001), one could transfer the genetic code to a digital memory and say, "Here is a human being, this is me!". Who knows, perhaps in time, the two compact-disks, one for the mind and the other for the body (as coded in the DNA), for now well separated, might even be reunited in just one disk...

After proving that simple nonlinear systems (three bodies in their gravitational fields) do not admit of solutions (integrals) and that, almost everywhere, they cannot even be approximated linearly — the end therefore of Laplace's project — Poincaré proposes a qualitative (geometric) analysis of such deterministic but unpredictable systems. . Widespread interest in Poincaré's approach did not occur until after the 1960-70s, with the various theories of "deterministic chaos."

7. "Evolution is due to noise," says Monod (1970)... how annoying noise is... In physics, a fine and important distinction can be made between randomness and noise (Calude and Longo, 2016; Bravi and Longo, 2015).

8. "La surprise, c'est que la spécificité génétique soit écrite, non avec des idéogrammes comme en chinois, mais avec un alphabet comme en français," F. Jacob, inaugural lecture at the Collège de France, 1965. The most prominent theoretical text describing the living as a "Cartesian machine programmed by DNA" is that of Monod (1970), co-recipient of the Nobel Prize in biology with J. Jacob, in 1965. J. Doudna's text cited below is still and explicitly within this framework of thought, enriched by smashing promises, advertising the "start-ups" in which she is involved.

9. For a critical account of Nobel Laureate (2020) J. Doudna's book, see Longo (2021a).

10. We may say "even" because physics has two thousand years of a very solid history; therefore, it takes a strong ideological commitment to forget it — but it is easy: just cut off funding to theoretical physics, fund only theses and projects that are AI-oriented or use it massively, such as recent Nobel Prize winners. Instead, this author thinks that we should go in the direction of a "naturalization" of physics, immersing it, as a singular part of theoretical biology, in the context of the relationship between living and inert, in continuity with the history of the former (Longo, 2020).

11. Liu Hui, 3rd century, the major "commentator" of "Les Neuf Chapitres", the summa of Chinese mathematics, translated into French by Karine Chemla and Guo Shuchun (2004).

12. The establishment of an invariant, linguistic, conceptual, graphic... is always the result of a plurality of "practices," of "acts of experience" (Weyl, 1949): for example, counting in space and the perception of a temporal rhythm, a counting in time (Brouwer, 1948) precede the concept of number, which will be the linguistic invariant of these two practices, conditions of possibility for the concept.

13. Hybridization of blind cavefish from different populations may partially restore vision in their offspring: it is possible that, in some species, the DNA necessary for producing functional eyes is present and silenced (perhaps "methylated") or that some gene expression networks have changed, including through mutations. That is, different types of mutations or control of gene expression in separate populations lead to the loss of eyes and, when re-combined through hybridization, can form new expression networks, leading to the development of functional eyes in hybrids (Borowsky, 2008; Sifuentes-Romero et al., 2020). As West-Eberhard (2003) says, "genes are mostly followers" of ecosystem changes (the cave, interspecies encounters etc.). The formation of the eukaryotic cell, by the penetration of a bacterium, which becomes the mitochondria, into an archaeon, is one of the most important examples of phenotypic transformation, in the very particular ecosystem of a billion years ago, which induced subsequent radical genetic changes (Margulis, 1970).

14. With surprising lucidity, Grothendieck (1971) describes Monod's 1970 book, quoted above, as "scientism," in the sense, in my view, that having captured a fragment of the real (the control

of genetic expression by genes themselves — an interesting phenomenon, but one that led to later building epicycles upon epicycles of "control" in order to avoid epigenetics, i.e., organismic and ecosystemic control of gene expression), Monod claims that he can understand everything with this knowledge, that DNA is a complete description of the organism and its development.

15. "The powerful dogma of the fundamental break between epistemological clarification and historical explanation, as well as the psychological explication in the frame of sciences of mind, the dogma of the break between epistemological origin and genetic origin, this dogma, as long as the concepts of "history," of "historical analysis" and of "genesis" are not limited in an inadmissible way, as they usually are, this dogma must be reversed from top to bottom" (Husserl, 1933).

16. Continuous and differentiable: it possesses not only the first derivative, but all the other, continuous ones, which are themselves differentiable.

17. The phenomenon is analogous to some propositions that may be stated in Number Theory, but cannot be proved within it (Longo, 2011). In this mathematical sense, biology is a "nonconservative extension" of chemistry (and, a fortiori, of physics): it allows to justify also some chemical processes that are (statistically) impossible to obtain in the inert (and to derive from their theories).

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