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Letter to Alan Turing

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Dear Alan,

It is with great joy that I have accepted the invitation to write you a personal letter. Even in your scientific writings your presence as a person is very strong, which is unusual for a mathematician. The traces of your personal life and your dramas extend beyond the limits of your own personal circumstances to concern us all – primarily because of the work that you did in World War II, but also because of the sufferings of a free young man, a homosexual, in a context that was totally hostile. I shall try to offer my own readings of some of your fundamental texts, placing them in relation to others that were equally revolutionary; and then I shall discuss with you some of the problems that we face, more than half a century later, including some that follow in the wake of your inventions. Science today is very different from the science that you knew, and it presents other challenges. Alongside the many possibilities that are offered to us – alongside the potentials (and actualities) for a global interaction of networks of what we could call modern *Turing machines* that connect us all, and for a debate of everyone with everyone on the face of this planet, and for a memory of humanity made available to humanity – I shall also tell you how new forms of techno-science are deforming and emptying out the "meaning'" of the object of analysis, making it difficult to arrive at an inventiveness of scientific thought of the kind that you were so well able to express.

Knowing how to be within phenomena

First, you knew how to immerse yourself in phenomena, in the scientific object, to give it 'meaning': initially, to make yourself as a machine (1936)[1], then to live your dramatic question about being man-woman-machine (1950)[2], finally enabling us to see and almost to touch the continuous reshaping and the genesis of forms in a "hardware" without "software" (1952)[3].

1 – The Machine

Your first article reminded me of Archimedes, when he sees himself as a body in water and thus

1 Turing, A. 1936, 'On computable numbers, with an application to the Entscheidungsproblem', *Proc. London Maths. Soc.* (Series 2), 42: 230–265; also in Davis 1965 and Gandy and Yates 2001

2 Turing, A. 1950, 'Computing machinery and intelligence', *Mind*, 50: 433–460; also in Boden 1990, *Collected Works* (Volume 1) online at: https://www.csee.umbc.edu/courses/471/papers/turing.pdf

3 Turing, A. 1952, 'The chemical basis of morphogenesis', *Phil. Trans. R. Soc. London B* 237: 37–72; also in *The Collected Works of A. M. Turing: Morphogenesis*, P. T. Saunders (ed.), Amsterdam: North-Holland, 1992.

comes to give us the principle of symmetrical thrusts. Or Einstein when he sees himself as photon "surfing" on a light wave to grasp the amazing invariance of the speed of light – just as, when we surf on a wave, the surrounding waves seem to be relatively unmoving. In the same way, you see yourself as "a man in the process of computing a real number": the "Logical Computing Machine" that you invent is effectively a "human computer". "Computing is normally done by writing certain symbols on paper. We may suppose this paper is divided into squares like a child's arithmetic book." "The behaviour of the computer at any moment is determined by the symbols which *he* is observing. and *his* 'state of mind' at that moment "(my emphasis). So its action is perfectly "stepwise", machinic, devoid of meaning, but ... human: you read 0 or 1 on the notebook / tape, you write 1 or 0 by moving to right or left, according to the internal state (of "mind") among a possible number of states q0, ..., qn. This is not a machine that is becoming human, but a man who is making himself into a machine. Did Hilbert want an absolute certainty of deduction in the "potential mechanisability of mathematics"[4]; did he claim the deductive completeness of systems of formal calculus? Alright, so here is a man who reduces himself to a machine to effect logical-formal deduction: you, as mathematician, turn yourself into a formal deductive machine, a "computer [who] works by such a desultory manner that he never does more than one step at a sitting". And then you define a function that is not computable, which thus escapes deductive mechanics: like Gödel, you destroy the formalist programme from within. Then, as the stated purpose of your article, you give the first definition of computable real number, thus also of non-computable ones. I shall attempt to reconstruct more closely the most significant ideas in your writings.

To show the limits of the formal system, which computes without meaning, you do not resort to the necessity of understanding what is being done; you do, however, show that the machine is incapable of calculating a function defined by a simple, purely formal diagonal procedure. In this you are imitating Gödel, but you do not follow the technique invented by Gödel – rather, you propose your formalism, which is much simpler and more human, too human to be machine, and yet *machinic*. And you accompany the invention with long explanatory discourses – you guide the reader step by step to your intuition, you are present in a human sense in the construction of your Logical Computing Machine, albeit so formal. This is the opposite of Gödel, whose 1931[5] article is a masterpiece of perfect formal rigour and closure, also in the writing, which is totally formal and "self-contained", almost unhuman, without any attempt at explanation, or an evocation of meaning, of the mathematical gesture proper to the construction carried out, apart from a few lines in the introduction. Gödel's article is an untouchable formal diamond: in no way can its proof be simplified, or take shortcuts, except a possible weakening in the assumption of consistency and by rewriting the heavy notation – it is essential and perfect. Your article, on the other hand, is a pleasant chat; you take the reader by the hand, and discuss with us, and reason with us, all together. You are present with your humanity, as you will always continue to be later. Today, we are able to describe your formal machine in just a few lines, but in doing this we lose the originality of your inventive journey, the daringness of your turning yourself into a machine. That's why I'm so glad to be writing to you: in reading your mathematical articles, we get to know you as a person. Perhaps it's a bit like reading "Récoltes et Sémailles" by Alexander Grothendieck[6], the great mathematician with no nationality, who was educated and lived in France, and was always out of place – as you were yourself, by reason of your character and your homosexuality. Alexander began his activity at the time of your death and, like you, was to do mathematics for only 20 years before then choosing scientific, if not physical, suicide. I shall

4 See David Hilbert, Mathematical Problems, trans. Maby Winton Newson *Bulletin of the American Mathematical Society* 8 (1902), 437-479. Online at: https://mathcs.clarku.edu/~djoyce/hilbert/problems.html/

5 Gödel, K. Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, I", *Monatshefte für Mathematik und Physik*, 38: 173–198. Reprinted in Gödel 1986, pp. 144–195.

6 Récoltes et Sémailles, Réflexions et témoignage sur un passé de mathématicien, is an unpublished manuscript. The introduction is available in English at: https://www.babelio.com/livres/Grothendieck-Recoltes-et-semailles/529740/

return to this parallel.

Your result is a negative result, as were those of Poincaré (1892)[7] and Gödel (1931)[8], who opened new ways by saying "no, that is not how it is": this or that programme and way of seeing things, which was dominant at the time, does not work. Poincaré demonstrates the possible predictive incompleteness of systems of non-linear equations: it is perfectly possible to determine a physical process with a system of equations, but its dynamics can be non-predictable ("et nous avons des phénomènes aléatoires" [and we have random phenomena], as Poincaré put it in 1902[9]). Gödel, within any system that contains arithmetic, builds a formal assertion that is undecidable (the first theorem of incompleteness) and which, moreover, implies the formal non-demonstrability of coherence (of the non-contradictoriness of arithmetic, second theorem). Your proof does not allow this latter virtuousity, the very fine gameplay of Gödel's second theorem, but, in compensation, it is much simpler and, as in the cases of Poincaré and Gödel, it paves the way for a new scientific construction[10]. Following Kleene[11], you then go on to prove that your Logic Machine is equally as expressive (it defines the same class of functions) as the formal systems of Herbrand, Gödel, Kleene and the lambda calculus of Church[12], all of which are of the 1930s. All very different logical-mathematical systems: these proofs of equivalence, between 1936 and 1940, will thus show that you all have invented a fundamental mathematical invariant, the class of formally computable functions. An invariant of Hilbertian writing and rewriting systems, i.e. of systems of transformation of signs into signs: any formal system in the style of Hilbert defines (at most) such a class. The mystics, as always, will take such an invariant for an absolute.

Lambda calculus, a very elegant paradigm for systems of (re-)writing, will be the "means" for your (difficult) proofs of equivalence. It is the system that is richest in "proper theorems"; it will provide the basis for logics with and without Types, from Church (1932[13], 1940[14]) and Gödel (1958)[15] to Girard (1971)[16] and Martin-Löf (1980)[17] [18]. It is a means of great mathematical expressivity, at the heart of Mathematical Logic and of many of the programming styles that

7Poincare H. 1892, Les méthodes nouvelles de la mécanique célestes, Tome 1, Paris: Gauthier-Villars,
8Gödel, op cit.

9Poincare H. 1902. *Science and Hypothesis*, London and Newcastle-on-Tyne, The Walter Scott publishing Co. Ltd. or in French, La Science et l'Hypothese, Paris, Flammarion.

10Longo, G 2018 "Interfaces of Incompleteness" in Minati, G, Abram, M & Pessa, E (Eds.) *Systemics of Incompleteness and Quasi-systems,* Springer, New York, NY, to appear (preliminary version: *"*Incompletezza*"* per *La Matematica*, vol. 4, Einaudi (both *downloadable*, as all Longo's papers: http://www.di.ens.fr/users/longo/))

11Kleene, S. C. 1936 "Lambda-definability and recursiveness". *Duke Mathematical J.*, vol. 2, pp. 340-353.

12Church A. 1941, *The Calculi of Lambda-Conversion*. Series: Annals of Mathematics Studies, Volume 6, Princeton: Princeton University Press.

13Church, A., 1932, "A Set of Postulates for the Foundation of Logic", *Annals of Mathematics* (Second Series), 33: 346–366.

14Church A,1940, "A Formulation of the Simple Theory of Types", *Journal of Symbolic Logic*, 5: 56–68; reprinted in Benzmüller *et al*. 2008b.

15Gödel K., 1990 Über eine bisher noch nicht benützte Erweiterung des finiten Standpunktes", *Dialectica*, 12: 280–287. Reprinted in Gödel, pp. 240–25

16Girard, J-Y. 1971 "Une Extension de l'Interpretation de Gödel à l'Analyse, et son Application à l'Élimination des Coupures dans l'Analyse et la Théorie des Types". *Proceedings of the Second Scandinavian Logic Symposium*. Amsterdam. pp. 63–92

17Martin-Löf P. 1984 *Intuitionistic Type theory*, Napoli: Bibliopolis

18For surveys see: Barendregt H. (1984). *The Lambda Calculus. Its Syntax and Semantics*. Studies in Logic and Found. Mathematics 103, Amsterdam: North-Holland. Longo G. (1988). "The Lambda-Calculus: connections to higher type Recursion Theory, Proof-Theory, Category Theory" (downloadable). *Revised version of* "On Church's Formal Theory of Functions and Functionals", in *Annals Pure Appl. Logic,* 40: 93-133, 1988. Girard J.Y., Lafont Y., Taylor P. *Proofs and Types,* Cambridge U. Press, 1990.

created modern computer science. In the 1970s to 1990s, the results of many people will make it possible to immerse lambda calculus and its Types into mathematical structures that also derive from a central notion in Grothendieck, that of "topos"[19] (categories of great structural and logical richness). These constructions gave a "geometric" meaning, possibly in *continua*, to your alpha-numeric *Discrete-State* machine (what an honour for me to see my photo in the 1984 reprint of Henk Barendregt's classic book on lambda calculus, a few chapters after yours). Thus, again in 2008, new categories, inspired by Church's lambda calculus and the work of some of us between 1984 and 1990[20], will be soundly described as "Turing categories"[21]. In summary, on the one hand, your Logic Computing Machine, for its simplicity and the separation between instructions and physical structure, inspired, after the war, the practical distinction between software and hardware, as well as the construction of operating systems and compilers – which are an implementation of your "Universal Machine". On the other hand, your work also opened the way to so much mathematics, of 30 and even 70 years later, thanks to both the bridging results inaugurated by yourself, with lambda calculus, and subsequently between this calculus and the mathematical theory of categories (and Gorthendieck's topoi).

I stress the point, however: all your results of the 1930s were born as "negative results". And so it was that your demonstrations, in placing the scaffolding for, and the limits to, an approach to knowledge, laid the foundations of new scientific universes: Poincaré had done this as for the geometry of dynamic systems, Gödel, yourself, and the others I mentioned as for computability. To say "no" you had to refine an existing approach to the point of inventing new notions, which then showed themselves to be very fertile: Poincaré – bifurcations, homocline trajectories, the geometry of dynamical systems; Gödel and you - computable functions and machines. Who would dare today to propose a multi-million-dollar research project, the only way to have jobs, postgraduate researchers, collaborations, and activities guaranteed for three to five years, to show that "no, it cannot be done ... this or that process is unpredictable, this or that system is incomplete, this or that function is not computable"?

2 – Man / woman / machine / Universe

The war interrupts your scientific work and, as always in the twentieth century, it puts a freeze on all science. At best, dozens of great scientists like yourself will be set to work to find technical solutions to urgent problems in already established theoretical frameworks, from the chemistry of poison gases, and aviation, during World War I, and nuclear fission for military purposes, or your computer with mechanical gears for decoding German codes, in World War II. Of course, in order to build it, you had to use your "arithmetical" talent, your ability to work on discrete combinations of integers and alphabetical letters, in the interplay between the two; but you will be obliged to build a machine that is traditional, a device of wheels and gears, such as had been in existence for decades, in order to fight the German machine with its cogs that continually generated different codes. The urgency of war did not allow you to develop your scientific idea, the Logical Computing Machine with a software that was separate from the hardware, and your article of 1936 remained ignored for more than ten years – even by yourself – until, eventually, in peacetime, you and von Neumann could think about what would later become the "von Neumann architecture"[22] for modern electronic computers.

19 J. L. Verdier, B. Saint-Donat, A. Grothendieck, (1972) *Théorie des Topos et Cohomologie Etale des Schémas*, (2 vols.) Springer Lecture Notes in Mathematics, Springer, Berlin & Heidelberg

20Kreisel G (1982-84) Four letters on computability, http://www.di.ens.fr/users/longo/files/FourLettersKreisel.pdf

21Cockett J.B.R. and J.W. Hofstra (2008) Introduction to Turing categories in *Annals of Pure and Applied Logic* Vol.156, 2-3 Dec., pp 183-209 link: https://www.sciencedirect.com/science/article/pii/S0168007208000948

22Von Neumann J. 1945 First Draft of a Report on the EDVAC in *IEEE Annals of the History Of Computer* Vol. 15 N.4 1993. Online at: https://www.di.ens.fr/~pouzet/cours/systeme/bib/edvac.pdf

After the war, however, the resumption of scientific thought will make you change your point of view on the Machine. You would first grasp the practical importance of your mathematical invention, building a physical prototype of it, as I recalled, and then you understood that it could constitute a turning point in the representation of the world. Moreover, in your 1950[23] article you were to rename your Logical Computing Machine the Discrete-State Machine, thus highlighting the physical structure of its hardware, with discrete states, and the discrete-writable, alphabetical nature of its software.

I have to admit that I hesitated to read your 1950 article on "The Imitation Game". Is it possible, I asked myself, that a great man such as yourself would take up so uninspiringly a discovery of the 1920s, when the electrical discharges of neurons, the “spikes”, were measured as beeps on a galvanometer? And the electrostatic and material deformations of the neurons, of enormous complexity, sort of "critical transitions" in a continuous dynamic – the electric-chemical-physical activity of a neuron – were then interpreted in simplistic terms as steps from a 0 (inactivity) to a 1 (discharge), like in the drawings in your children's machine notebook. I was amazed that your article could contain such a commonplace, that you might still think that an animal (or human) brain could be a Discrete-State Machine, 0 and 1 on a tape, written or deleted according to predetermined rules .... But no, actually you say exactly the opposite: "The nervous system is certainly not a discrete-state machine. A small error in the information about the size of a nervous impulse impinging on a neuron, may make a large difference to the size of the outgoing impulse." (p. 451)

In fact, what you are proposing is only an "imitation game", as you say on several occasions: the project is to fool a person who, by asking questions via a teleprinter, seeks to establish whether the respondent is a woman or a machine. In no way are you trying to find out how a human brain works, or to make a mathematical model of it; rather you are living the drama of a possible imitation. I say drama, because, in my opinion, you already know that the police might, at any moment, ask you the same question: but, are you a man or a woman? And so you make your machine answer this dull and insolent question, opening the way to a very rich symbolic game, but always a game – you, who knew from very young how to turn yourself into a machine, in the logical game of 1936, and maybe you were continually asking yourself that question: man / machine / woman?

On the one hand, I must admit that I am not particularly interested in the psychological considerations that you adduce to convince the reader of the plausibility of the game of imitation and the possibility of tricking the reader: you do not go much beyond banalities such as "Do you have long hair? Can you write a poem? " ... "Add 34957 to 70764, (Pause about 30 seconds and then give as answer) 105621" That's wrong: the machine has to imitate a woman and, as we well know, women, in mathematics.... Yet, Jean Lassègue claims that even in this case you lightly but deeply get into a profound semeiotic issue, concerning gender differences … maybe he is right[24]. Then, with great modesty, you dare to suggest that by 2000, in a game of not more than five minutes, machines will have a 30% chance of passing themselves off as a woman, in a dialogue via teleprinter (p. 442). This was very far from the fantasies of perfect humanoid robots that were being promised to us a thousand times over by Classical AI. Today, the networks of artificial neurons of the new AI, based on *continuous variations* of connectivity and capable of learning – an idea to which you refer – are able gradually to establish invariants of images, and, after a lot

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23 Turing, A. 1950, ‘Computing machinery and intelligence’, *Mind*, 50: 433–460; also in Boden 1990, *Collected Works* (Volume 1) online at: https://www.csee.umbc.edu/courses/471/papers/turing.pdf

24 Lassègue, J. 1996. “What Kind of Turing Test did Turing have in Mind ?”, *Tekhnema; Journal of Philosophy and Technology* (ISBN : 2-9509944-0-7) (3) : 37-58.

of computing, are able to distinguish a cat from a flat-iron (which, however, does not seek to imitate the cat) – no mean undertaking for a machine. These formally continuous neural nets are mathematically very different from your Discrete-State Machine, yet, so far, they still need to be implemented on the latter.

On the other hand, I was excited by the many observations you make of a physical-mathematical nature. They open the way to the other major article you were writing, the one about morphogenesis, which appeared in 1952[25]. Here, in contrast to your discrete-state machine, you grasp the role of the continuum, of the interplay between non-linear dynamics and physical measurement, which is always an interval, always approximate. Thus, on page 440 you say: "The system of the 'universe as a whole' is such that quite small errors in the initial conditions can have an overwhelming effect at a later time. The displacement of a single electron by a billionth of a centimetre at one moment could make the difference between a man being killed by an avalanche a year later or escaping." In other words, a disturbance, a fluctuation below the best measurement possible for a measuring device, at the appropriate scale – the man killed vs. the movement of an electron – can be amplified over time into a phenomenon that is both observable and unpredictable. This is the crux of the deterministic unpredictability of phenomena that we represent with non-linear systems, sensitive to the initial conditions, those that Poincaré[26] had thoroughly analysed 60 years earlier and that very few had developed further: in them the role of measurement as an interval in the continuum (the in-principle completeness of all converging series, or of all convergent measurements) is crucial. Classical unpredictability, or randomness, arises at the interface between measurement and non-linear determination (equations, evolution function).
Poincaré, Hadamard and some Russian mathematicians had worked on such systems in celestial mechanics, or in the classical physics of large systems. Perhaps these latter mathematicians, such as Pontryagin, were not even known to you (Kolmogorov[27] would only write in 1953 the first version of what would then become, in 1963, the Kolmogorov-Arnold-Moser theorem, a pillar of those systems). Alone, therefore, or at most following on Poincaré, you grasp the importance of non-linearity, of its non-Laplacian character (determination does not imply predictability) far beyond the rational mechanics that was still being taught through to the 1960s and 1970s (including to myself, unfortunately, a very bad mathematical physics, alongside extraordinary courses in mathematics).

And so, in contrast to the sensitivity to initial conditions, or to the unpredictability of nonlinear dynamics in the continuum, you explain: "It is an essential property of the mechanical systems which we have called 'discrete-state machines' that this phenomenon does not occur. Even when we consider the actual physical machines instead of the idealised machines, reasonably accurate knowledge of the state at one moment yields reasonably accurate knowledge any number of steps later."[28] And you stress: "It will seem that given the initial state of the machine and the input signals it is always possible to predict all future states. This is reminiscent of Laplace's view that from the complete state of the universe at one moment of time, as described by the positions and velocities of all particles, it should be possible to predict all future states."[29] That is, maybe it can imitate a woman, but you stress – and still not many people understand it today – that your

25Turing A. 1952, 'The chemical basis of morphogenesis', *Phil. Trans. R. Soc. London B* 237: 37–72; also in *The Collected Works of A. M. Turing: Morphogenesis*, P. T. Saunders (ed.), Amsterdam: North-Holland, 1992.

26Since his work on the "Three Body Problem", mentioned above, beginning with: Poincaré, H., 1884, "Sur certaines solutions particulières du probléme des trois corps," *Bulletin Astronomique, Serie I,* 1 pp. 65-74.

27A. N. Kolmogorov, "On the Conservation of Conditionally Periodic Motions under Small Perturbation of the Hamiltonian [О сохранении условнопериодических движений при малом изменении функции Гамильтона]," *Dokl. Akad. Nauk SSR* **9**8 (1954).

28Turing, A. 1950, 'Computing machinery and intelligence', p.440

29Ibid.

discrete-state machine is Laplacian, since its access to data is exact, its dynamic develops in a discrete space, it is a system of alphabetical writing and rewriting, as you explain clearly in a brief observation in which you compare alphabetical writing to ideogrammatic writing[30]. This is the key invariant property of the "knowledge of the state", in digital measurement, which is exact and develops exactly along the calculation process, something which does not happen in the physical Universe to which we "access" only by approximate or indeterminate measurement. Your "electron effect" (its displacement by a billionth of a centimetre with later macroscopic effects) anticipates by ten years the well-known "butterfly effect" of Lorentz (1962)[31], which only came to be so called in 1972. As I have already written elsewhere, we should rather have been speaking in terms of Turing's "electron effect", a much earlier example of unpredictable (chaotic) dynamic, sensitive to initial conditions. But your 1950 article had been read principally in relation to Artificial Intelligence; this did not allow people to grasp the tragic and symbolic theatre that you play, between a man, a woman and a machine, and even less your insight as a great mathematician who was working on morphogenesis, as a physical dynamic in the continuum.

So, decades after your very clear exposition, people are still saying (with Wolfram for example), that the Universe "as a whole" is a (very large!) Turing Machine, of which obviously living organisms, and the brains contained in them, are seen as "emergent" computational aspects, inasmuch as they are "computationally irreducible". And this, without even being able to give a rigorous mathematical definition of computational irreducibility for a discrete process in two dimensions, see Wolfram, *Cellular Automata*, or *A New Kind of Science,* analysed in the margins of the doctoral thesis of Alastair Abbott, supervised by Calude and Longo, 2015.[32] The technical point is that computational irreducibility is not an invariant of "coarse graining" (changes of graining or scale), as shown since long[33], but ignored by the ideologues of reducing nature to your machine, against your views. In addition, I have been vainly asking these computationalists of the Universe to tell us whether the fundamental constants of physics, which appear in all the relevant equations, namely G, c, h and the non-dimensional α, are computable real numbers, in the sense defined by yourself: two may be set equal to 1 … and the others? Theory and measurement will always give an interval or indeterminate values, producing the unpredictability of both classical and quantum dynamics – far away from Discrete-States Cellular Automata. In biology, these computationalist views of organisms have severely distorting research[34], far away from your work on continuous morphogenesis, I will recall next.

There is something of the common to be understood in this computational folly that surrounds us – everything becomes discrete computation, numerics, digital programmes, from the Universe to the brain, from DNA to economics. Indeed, the whole of knowledge can be replaced by correlations evidenced computationally on huge discrete data bases – Big Data, which supposedly is going to make science obsolete. I am raising this with you in this letter because you are the right man to understand all of this – you who, with Gödel and Church, invented the theory of computability and you, the Discrete-State Machine by showing their limits w.r.to

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30 Lassègue, J., Longo, G. 2012 "What is Turing's Comparison between Mechanism and Writing Worth?" Invited lecture, *The Turing Centenary Conference (CiE 2012)*, Cambridge, June 18 - 23, 2012; in *Computability in Europe*, LNCS 7318 (S.B. Cooper, A. Dawar, and B. Löwe, Eds.), pp. 451–462, Springer.

31 Lorentz, G. 1963 Deterministic nonperiodic flow. *Journal of Atmospheric Sciences*. Vol.20 : 130—141

32 Abbott A. 2015. *Value Indefiniteness, Randomness and Unpredictability in Quantum Foundations*, University of Auckland and Ecole Politechnique, Paris,.

33 Israeli, N, Goldenfeld, N 2004 Computational irreducibility and the predictability of complex physical systems, *Phys Rev Lett.*, Feb 20;92(7):074105.

34 Longo, G. 2018 Information and causality: Mathematical Reflections on Cancer Biology. *Organisms. A journal in Biological Sciences* (to appear).

knowledge construction - producing an undecidable sentence, an incomputable function …. But you were also able to invent other scientific paths, you had many strings to your bow, and you were always open-minded and curious about the world.

3 – The genesis of forms

About fifteen years ago, a biologist colleague told me that he was working on various issues of embryonic morphogenesis by following the work of an English mathematician, Alan Turing, and he asked if I knew him .... He was unaware of your work in Logic and Calculability and I knew nothing about your 1952 work, apart from the title! In fact, your work moves in a completely other universe, that is conceptual and mathematical. As a great thinker, you address another phenomenality by inventing other original tools for dealing with it. Except that then, in the end, you reflect on correlations, and on a possible critical use of the pre-existing tools, the ones you had invented.

From the outset you stress: "The purpose of this paper is to discuss a possible mechanism by which the genes of a zygote may determine the anatomical structure of the resulting organism. The theory does not make any new hypotheses; it merely suggests that certain well-known physical laws are sufficient to account for many of the facts."[35] You then write a classical non-linear system in the continuum: the model, as you call it, does not need "new hypotheses", especially not for the role of the " genes", to which I shall return. It is based on a system of equations, and is very simple. As usual, your genius way of working: first, the almost childlike simplicity of the Logic Computing Machine of 1936, with which, however, one can reconstruct everything that is computable in the discrete; now, the elementary nature of this original idea of a chemical action that causes a reaction, and therefore a diffusion with waves that are both regular and irregular, that propagate in a continuum and that generate forms. A *model* that seeks to understand a process, which could be wrong ... but which, as you say, can "falsify" – an unusual term. So, this is not an *imitation* whose purpose is solely to "deceive a questioner", and which does not help to understand, let alone to falsify, but ... what could be falsified by your model of morphogenesis? I shall try to understand this...

The idea, very simple, but which nobody had posed in these terms before you, is therefore that a chemical action / reaction / diffusion can generate forms. In precisely those years Belusov, in Russia, had observed the phenomenon empirically, in a chemical reaction whose description, not believed at the time, was to be published only after a long delay[36], and would only be understood after Zhabotinsky's[37] experiments in the 1960s. Thus, you intuit mathematically something that had not yet been observed experimentally, and had not been reported by others, but something that was possible: an equilibrium that is macroscopically homogeneous, but unstable, that a fluctuation, below the level of the observable, transforms into a dynamic of forms. Breakings of symmetry, catastrophic instability... these are the terms that you use. Here again, Poincaré and a few others had opened the way to the analysis of non-linear systems at equilibrium, but nobody had applied that vision of physical continuous dynamics to such areas, which you analyse in order to understand forms in biology.

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35 Turing, A. 1952, 'The chemical basis of morphogenesis',

36Belousov B. P. 1959 "Периодически действующая реакция и ее механизм"*Сборник рефератов по радиационной медицине*. 147: 145. "Periodically acting reaction and its mechanism", Collected Reports on Radiation Medicine, 147: 145.

37Zhabotinsky M. 1964 Периодический процесс окисления малоновой кислоты растворе (исследование кинетики реакции Белоусова). [Periodic processes of malonic acid oxidation in a liquid phase.] Биофизика [Biofizika], 9:306–311,.

Obviously, you work at a linear approach to the solution of the system of equations that you propose, but you go into a long discussion about the properties of non-linearity (the presence of second-order terms, as you put it). You do not propose a mathematics that is original or difficult, as you say several times in your affectionate dialogue with the reader: you assure him that the equations and the calculations are elementary. But you invent an extremely original way of dealing with the problem of generating organic forms: the idea is, so to say, "very pure". As with your Machine, which computes like a child, on a squared notepad, now, you identify the terms of a physical dynamics that is minimal, but extremely expressive. As usual, you immerse yourself in the phenomenon, without pre-judgement, completely setting aside your previous experience, your own invention, the Logical Discrete-State Machine. You move beyond what Thom defines as the fundamental aporia of mathematics: the discrete vs. the continuum. And your 1950 article, on imitation, with your observations on deterministic unpredictability and the electron effect, had already brought the aporia to light, the step that you were preparing yourself to take. Thus, you assign a central role to measurement, to access to phenomena: fluctuations below the best possible measurement are at the core of your analysis of continuous “symmetry breaking” and “catastrophic dynamics” (a wording by which you seem to refer to critical transitions, whose theory was yet to be invented). These notions make little sense in the discrete, or may at most be “imitated”. But, at the end of the article, you pose the problem of implementing these dynamics on your Machines: you conclude that it will only enable the treatment of special cases and you promise yourself to do more work on this. The trial for homosexuality that will begin later that year will tear you away from that, and will lead to your suicide.

So, a model, invented in order to understand, and not an imitation. And to falsify ... what? In conversations reported by Gandy[38], you say that you didn't like Huxley's approach to Darwinian evolution. This was all centred on chromosomes and was to open the way to a new molecular biology that would see in DNA the *complete programme* of ontogenesis. For you, as you write in the article, genes are at most the producers of enzymes which are involved in the reactions that interest you, and it is the speed of this production, you say, that contributes to a process that is global, interactive, and based on physical continua, not "computational", even less “programmed”. The idea of the descriptive completeness of the chromosomes, a finite sequence of code-letters, did not sit well with you, who had shown, in your own way, the incompleteness of the axioms of arithmetic, also a finite sequence of signs.

Schrödinger's little book of 1944,[39] the first part of which proposed the idea of a "code" of biological forms inscribed in chromosomes, was already well-known. And von Neumann had already published, in 1951, an article on cellular automata[40] and had attributed to chromosomes the role of "programming of reproduction and ontogenesis". You do not cite them. What you propose is, however, compatible with the alternative identified by Schrödinger in the second part of the book: ontogenesis as a dynamic based on the absorption of negative entropy, described in Gibbs's free energy terms[41] (together with Francis Bailly I have written about this, calling it anti-entropy, but the shadow of your work is in there too)[42]. Instead you cite only three biologists, all

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38Gandy, R. O. and C. E. M. Yates (eds.), 2001, *The Collected Works of A M. Turing: Mathematical Logic*, vol.4, Amsterdam: North-Holland.

39 Schrödinger, E. 1944, *What is Life? The Physical Aspect of the Living Cell*. Based on Lectures delivered under the auspices of the Institute at Trinity College, Dublin, in February 1943, Cambridge: University Press. 1944.

40Von Neumann, J., 1951, “The General and Logical Theory of Automata”, in *Cerebral Mechanisms in Behavior: The Hixon Symposium*, New York: John Wiley & Sons.

41 See, for the fullest explication of this, Gibbs, J. W. 1876. "On the Equilibrium of Heterogeneous Substances". *Transactions of the Connecticut Academy of Arts and Sciences. vol.* 3, 1874-1878, pp.108–248.

42Bailly F., Longo G. *Biological Organization and Anti-Entropy*, in J. of Biological Systems, Vol. 17, n. 1, 2009.

of them very original: Child, D'Arcy Thompson, Waddington.[43] All of them were outside of the growing fixation on the idea of the completeness of the analysis of the chromosome as a way of understanding phylogeny and ontogeny (the latter was an embryologist-geneticist, but he too interpreted the action of chromosomes always within the interactions between the organism and the ecosystem). You, as the inventor of the notion of the "computer programme", of software that came to be defined independently of hardware, were against its use in biology. That's what your model demonstrates as false, as you hint at the beginning of your paper. In order to create forms (also biological) there is no need for a "predefined design", for a programme (you also say that your theory "does not make any new hypotheses; it merely suggests that certain well-known physical laws are sufficient to account for many of the facts."[44]). The homunculus coded in DNA, a programme of ontogenesis and even of behaviour, which will become definitively fashionable precisely in those years, is the opposite of the purely physical dynamics that you describe and that *falsifies* the need for a programme code for morphogenesis.

So, you, who distinguished software from hardware for us, and who thus invented the science of programming on discrete data types, and of software, describe now a dynamic that is purely hardware, without software, physical-chemical shaping and reshapings of forms in the continuum. This is the style of a great mind in addressing problems, always knowing how to renew ways of thinking, to enrich the gaze with new perspectives, and to invent or use a variety of tools, driven only by the desire to understand: the opposite of the flat transfer of just one conceptual tool, so much in fashion today – everything is information and digital computation. And in this way you opened a new pathway in science: after a delay of some decades, your analysis of morphogenesis was widely taken up and developed.

Intermezzo: Alexander Grothendieck (1928-2014)

I would like to tell you about another great mathematician who shares with you the drama of an early and deliberately chosen cessation of all activity, you with death, and him by extracting himself into the solitude of an agrarian life away from the world. But above all, I would like to emphasise your shared pursuit of "purity" of method, of an "ingenuousness" of the gaze, always innovative, which both of you devote to the object of your interest. You set about looking in a way that is different, original, "naive" (the ingenuousness of the child and his squared notepad, in your article of 1936), at areas that had already been studied by others, in computability, and so you open up new pathways; then, with just as much simplicity and originality, you invent a relatively simple, mathematical approach to morphogenesis. Alexander, like you, re-invents, with a new and very original and synthesising way of looking, huge and deep areas of mathematics. His mathematical notions are very "pure": they focus on invariance and maximal conceptual stability; they are very general without ever being empty. In this way, Grothendieck brought together previously distant constructions, offering surprising invariants, "bridging" notions and structures, shared, for example, by groups, topological spaces, manifolds of various kinds (differential, geometric, etc). Thus, he was able to bring together different areas of mathematics, transferring methods and correlating techniques[45]. For example, the "sheaves on a site", a notion of his that is difficult and profound and motivated Grothendieck's notion of topos[46], make it

43 Turing, A. 1952, 'The chemical basis of morphogenesis', op cit.

44 Turing, A. 1952, 'The chemical basis of morphogenesis', op cit.

45« Pour trouver un cadre commun permettant d'englober ces théories et d'autres » p. 119 *in* Grothendieck, A. (1957) "Sur quelques points d'algèbre homologique", *Tohoku Mathematical Journal*, vol. 9, nº 2, p. 119-221.

46Mac Lane S., Moerdijk I. (1992) *Sheaves in Geometry and Logic: A First Introduction to Topos Theory,* Berlin, New York: Springer-Verlag. For a philosophical perspective: Zalamea F. (2012) *Synthetic Philosophy of Contemporary Mathematics*, New York: Urbanomic & Sequence Press.

possible to move between discrete and continuous structures, beyond the foundamental aporia of mathematics – the interplay between the discrete and the continuum, which divides your activity into two distinct parts. Grothendieck's topoi produced a very interesting bridging between logic and geometry, also thanks to the works of Lawvere, since the '60s[47], to the point of giving a "geometrical" meaning even to "impredicative definitions", that terrible circular bogey for the logicians, from Russell to the present. Impredicativity becomes instead a very interesting property of logical closure, in the lambda calculus of Girard[48], and a structural closure, in the categories between sets and topoi in the articles published by Hyland, Moggi and myself[49]. The basis for this understanding was set by Dana Scott, in the late '60s and 70s, when he proposed the most successful approach linking computability, programming and categories[50]. The ideas of Grothendieck, together with those of Serre and various others in France, in some sense touched on all fields of mathematics and were the main theoretical revolution of the post-war period, perhaps only comparable to the four or five of the seventy years that preceded World War II (including the geometry of dynamic systems, relativistic physics, quantum physics ... and mathematical logic, to which you yourself contributed), but with even greater merits of unifying various and diverse fields.

After 20 years of activity, in the early 1970s, the scientific purity of Alexander revealed its true nature, which was also ethical – as it was for you: you could not accept that you were at fault for being homosexual; you believed in the integrity of your person, as also of your way of thinking, you were perfectly ingenuous, but lucid, in the rigour with which you presented yourself, almost denouncing yourself, to the police, on the occasion of a petty theft suffered by an occasional companion. An ethically even more radical attitude was to lead Grothendieck to sever all relations with the academic world and the world of research: he felt that they were compromised by military funding, and not attentive to problems of the ecosystem. Pierre Lochack [51], in a recent book, shows the continuity between his very difficult life as a stateless person immigrating as a child into France, and the "asceticism" of his approach to mathematics and social life. The "purity" of Alexander's scientific and moral vision joins the uncompromising political commitment of his parents, who were anarcho-syndicalists. His father in particular had been present in all the European revolutions, starting from 1905 in Russia. Jailed for ten years by the Tsar, released as a hero in 1917, he became hostile to Lenin, was persecuted by the Bolsheviks and ended up going into exile in Germany in 1921. As a result of his commitment against the rise of Nazism, when the Nazis came to power he took refuge in France, in 1933, and then, together with Alexander's mother, he went to fight in Spain in 1936. He died in 1942 in a German concentration camp. Alexander was to continue, in his own way, their uncompromising attitude – even in mathematics, with the absolute rigour of a conceptual and ethical purity that gave no quarter, until the dramatic moment when he decided to drop everything and resign his position at

47Lawvere F.W. (1966) "The Category of categories as a Foundation for Mathematics", *in* Proc. Conf. on *Categorical Algebra*, S. Eilemberg et al. (eds.), La Jolla, 1965, Springer-Verlag. Lawvere F.W. (1976) "Variable quantities and variable structures in topoi", *in Algebra Topology and Category Theory: a collection of papers in honor of Samuel Eilenberg*, A. Heller and M. Tierney (eds.), Academic Press, (101-131).

48 Girard J.Y. (1971) "Une Extension de l'Interpretation de Gödel à l'Analyse, et son Application à l'Élimination des Coupures dans l'Analyse et la Théorie des Types". *Proceedings of the Second Scandinavian Logic Symposium*. Amsterdam. pp. 63–92. Girard J.Y., Lafont Y., Taylor P. (1990) *Proofs and Types,* Cambridge U. Press.

49 M. Hyland (1988) A small complete category, in *Ann. Pure Appl. Logic*, Vol. 40, Amsterdam: North-Holland, (pp. 93-133). G. Longo, E. Moggi (1988) Constructive natural deduction and its modest interpretation in Meseguer (Ed.), *Workshop, Semantics of Natural and Programming Languages*, Stanford, March 1987, Cambridge: MIT Press. Asperti A., Longo, G. (1991) *Categories, Types and Structures; Category Theory for the working computer scientist*. M.I.T. Press.

50 Scott D. (1970) "Outline of a mathematical theory of computation" 4th Ann. *Princeton Conf. on Info. Syst. Sci.* (pp. 169-176). Scott D. (1980) "Lambda-calculus, some models, some philosophy," *The Kleene Symposium* (Barwise et al. eds.) North-Holland.

51 Lochack P. (2015), *Mathématique et finitude*. Paris: Éditions Kimé

the IHES in Paris, five years after having received the Fields Medal, at the height of his scientific glory, and at the same age as yourself when you committed suicide.

This way of working, this conceptual intransigence is often found in mathematics, and is perhaps one of its properties: in mathematical invention, there is always a revolutionary radicality and purity – a profound concept or a proof cannot be "patched", they cannot be the outcome of a compromise, even with the real. I have had teachers and friends, since my student years, and then collaborators and colleagues, who share these characteristics, people in whom "innocence" and a purity of ways of looking combine with a permanent re-invention and a depth of vision. Perhaps, at the best of times, every mathematician knows how to be equally uncompromising with concepts. They expect the maximum from them, they propose the purity of the simplest notions, like yourself, or the most general, like Grothendieck, or limit notions, as in Euclid's geometry, where "the line has no thickness" (definition β) – the core invention of Western geometry, measures and organises the world with figures made of pure edges, solely of lines without thickness.

4 – Networks and Big Data

In computing, over the last 30 years, there has been a major development, which you had not foreseen: the creation of networks. Networks of computers, all of which are individually extremely powerful, to an extent inconceivable to you (and thanks also to physics), are changing science and the world. A "symbolic" turning point, the third great writing revolution, says Clarisse Herrenschmidt,[52] after the invention of writing, and of alphabetic writing in particular, which she has studied as an archaeologist, and that of minted money, as a writing, as a symbolon, of value. Like every deep revolution in symbolism, and thus in human interaction, the present situation offers original challenges, that we still do not understand well, let alone control. These networks bring us all closer together, and offer us unprecedented opportunities to appreciate human diversity, and thus to enrich the experience of all, to provide the inspiration for new inventions, as a result of hybridisations and new syntheses.

However, the fact of having so many neighbours, as is suggested by the physics of "mean field", can also imply that we all become "average", all the same, or nearly so. The challenge is open. The management of science is its a first victim: bibliometrics, on which I have written a little article that is downloadable[53] from my web page (how lovely it is to have a web page accessible by anyone, where you can publish your own writings!), obliges us to work in dominant strands, where even a small advance can be quoted by many. Inventions like yours took ten, twenty, thirty years to be appreciated: the impact factor of journals is now computed by machines based on the numbers of citations of articles in the *two* years following publication. In mathematics and in physics ... it takes ten years just to understand a difficult result on an original track, which is then ignored for a long time, unless it is a difficult answer to problems that have been open for decades. Networked machines that compute the numbers of citations kill from the outset any attempt to venture, as you did, onto paths that are entirely new.

To this we have to add the madness, as I mentioned, of the "everything is computational", starting from the analysis of the living, the opposite of what you were able to offer, and creating the myth of the Universe as a Turing Machine, against your very precise observations. These colleagues, who are using the only technique that they know, and applying it to every possible

52Herrenschmit C. (2007) *Les Trois Écritures, Langue, nombre, code*. Paris: Gallimard

53Longo, G. (2014) Science, Problem solving and bibliometrics, in *"Use and Abuse of Bibliometrics",* Stockholm, May 2013. Proceedings, Wim Blockmans et al. (eds), Portland Press.

phenomenon, flattening it onto a universe that is without meaning and made only of formal calculations, act as if yours is the last machine that man will be capable of inventing: it is co-terminous with the world! I am convinced that we shall invent others, but these prophesies risk becoming self-affirming: piling computational techniques onto computational techniques, always in the same theoretical universe, to grasp the complexity of the Universe (of the brain, of DNA ...), in increasingly and abstrusively difficult ways without the very simple purity and depth that mathematical invention requires – all of this prevents us from being able to think about the... next machine, which humankind will certainly find ways of inventing.

Discrete mathematical structures play a central role in all of this. Discrete databases are exact, and you can access them with precision. The great challenge of physical measurement is forgotten. The early 1900s, as I said, had brought this challenge to the limelight. Poincaré had grasped the role of the interface between the non-linearity of mathematical dynamics and physical processes, given by measurement; you yourself addressed this very well: fluctuations below the level of the measurable are amplified into observable phenomena that turn out to be unpredictable. Quantum physics also begins, in 1900, from the question of the intrinsic indeterminacy of measurement, and the surprising discrete measure of the energy spectrum, in the continuum of space-time. All of this is set aside by computational dynamics, as alpha-numeric systems of re-writing, as they are defined in generality. Starting with exact values, they always iterate in the same way: this is the correctness of the programmes. Then computer networks (Internet, typically) introduced the randomness proper to fluctuations in the space-time continuum, to perturbations in the operations of a node .... But the colleagues who work in networks and on concurrency call this form of randomness "do not care": everything possible is done to render it negligible. And they succeed – the networks operate with exactitude, with rare exceptions, thanks to the exactness of discrete data bases, with no nuances and without uncertainties in access to data. If discrete database networks become one with the world, if this is done without understanding the method which is thus imposed (the implicit grid of reading), one loses the meaning of the variation, which is "averaged out" by the average behaviours of networks, of the nuance, the approximation and the perturbation that contribute to novelty production, like in your analysis of morphogenesis. In particular, one loses the sense of the interface between our mathematical/theoretical description and the world: namely measurement. This is precisely what you were able to achieve, attributing a key role to fluctuations, from your "electron effect", below the level of measurability, that leads to a man being killed by an avalanche a year later, to the ones that, as you say, "trigger" morphogenesis. This helps us to understand the point at which your change of gaze, from the discrete to the continuum, enables us to talk about the world in another way: I shall return to this.
The writing of equations, or of a function of evolution, of a dynamic, from Newton to Schrödinger, is not the "same thing" as the process of which they claim to be the "model", in the sense of your model of morphogenesis. Some out of this world Platonists still say that "a planet integrates a differential equation", forgetting, first and foremost, that just two are sufficient, around the sun, for the system not to be integrable (and yet the planets move...). The equations, the function, propose or derive from a proposal of a causal structure, as in your analysis of morphogenesis, and are instruments of intelligibility and, in some rare cases, forecasting, mainly qualitative (here an attractor, there a singularity ... a certain type of forms ...). In more general terms, the equations can be derived from laws of conservation (energy, momentum...), and thus from symmetries, which structure them (the equations of motion, typically). Then man, or the machine if we are able to programme it properly, can apply algorithms to arrive at a solution, if they exist, or produce an approximate computation that "follows" the dynamic. We know that just a minimum of non-linearity is sufficient, namely the description of interactions (more bodies or agents), for the computation to diverge quickly from the physical dynamics. And this is easily shown, even without comparing the mathematical calculation with the physical process. In other

words, it is not necessary to measure the process at the initial instant and at a later instant: it is sufficient to observe that a very small difference in the preferred (decimal) approximation (the 15th decimal, say, for the very simple logistic function) gives rise to radical differences after a few iterations of the calculation (50 in that case, and it occupies the entire space of the phases). Since physical measurement (classical) is always an interval, this difference shows that a fluctuation below the best measurement possible makes the physical process unpredictable by a mathematical calculation. In quantum physics, measurement produces probability values that are real numbers, while calculation (the Schrödinger equation) is performed on complex numbers. In short, in either case, physical measurement is a fundamental and complex interface between our theoretical attempts, as far as possible mathematical, and phenomena; it shows the gap between them, and make human science possible, in the interplay between us and the world. The mathematical model (equations, typically) and the algorithmic calculations based on it are quite different from the physical process: measurement both connects and separates them, radically.

What, on the other hand, is done by these "computationalists" of the physical and biological world? They identify the Universe as a discrete structure, or rather as a discrete, alpha-numeric *writing* and they say, along with Wolfram[54]: "We can certainly imagine a universe that operates like some behaviour of a Turing machine." The systems for computability, as for instance Church's lambda calculus, and also your machine, are systems of re-writing – finite strings of signs are transformed into other strings, following rules / instructions. In this way, without the problem of measurement, of the interface, as I said, the "physical" process, simulated and implemented in the machine, takes place *exactly like* the dynamics of re-writing of signs, in the discrete, and is identified / identifiable to symbolic dynamics: once the interface disappears, i.e. without the problem (which is huge, in physics) of measurement, the correspondence between mathematical writing and the world is exact. The world *is* a discrete-state machine, a machine for re-writing: the transformation of alpha-numeric strings into alpha-numeric strings.

This is how Chaitin describes also biological dynamics, in *Proving Darwin*: "Life as randomly evolving software, software that describes a random walk of Increasing fitness in program space." In the discrete, without measure, DNA is *made equivalent to* a software and its physical materiality is not important: "we shall ignore bodies and metabolism and energy and consider purely software organisms".[55]

In this approach, in physics, in biology, phenomena are "alienated" into a alphabetic formalism. They are no longer observed, because they are no longer measured. The computational world goes its own way, outside of the world, far from its physical and biological materiality, because in the actual world *there are no written or numerical characters out-there*. It is *we* who associate numbers (or letters) to phenomena and processes, through the difficult challenge of measurement. The discrete replaces measurement and enumeration of acts of measurement, proper to continuous manifolds, with solely enumeration - a beautiful remark by Riemann[56]: in the discrete, one can only count.

Chaitin and Wolfram developed their theses, in both physics and biology, in many articles, and summarised them in two articles that appeared in a book published in your honour, where they seem to suggest: it is a pity that Turing did not understand this, but his machine is like the Universe as a whole, is like the dynamics of biology[57]. A real insult to yourself, you who were

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54 Wolfram, S (2013) "The importance of Universal Computation", *in A. Turing, his work and impact*, Cooper ed., Elsevier.

55 Chaitin, G. (2012), *Proving Darwin. Making biology mathematical*. New York: Vintage.

56 Riemann B. "On the hypothesis which lie at the basis of Geometry", 1854 (Engl. by W. Clifford, **Nature**, 1973)

57 Cooper, S. B. & Van Leuween, J. (eds.) (2013) *Alan Turing. His work and impact*. London: Elsevier

able to “immerse” yourself so deeply in phenomena, playing in the interface, getting a sense of the interplay between the discrete and the continuum, the role of measurement.

Perhaps the greatest catastrophe of anti-scientific computationalism can be seen in the recent theory of "The End of Theories". In a series of widely quoted articles, informaticians or managers of very large databases explain that: "Correlation supersedes causation, and science can advance even without coherent models, unified theories"[58]. In short, networked computers, bringing to light very extended correlations in huge databases, make it possible to predict and act, without the need to "understand": scientific intelligibility is an uncertain luxury that is subjective and outdated, and theories are fallible proposals. Data, especially in large quantities – tera-terabytes, Big Data – is objective, is a new form of the absolute, is individually exact, expressed in digits. Thus, they argue, the larger that databases become, the more that statistical regularities, brought to light by computers, can govern us without the need to understand the meaning of the correlations, to interpret them, and without the need for theories about them, for interpretations.

Fortunately, mathematics allows us to demonstrate the absurdity of these claims: Cristian Calude and I have written an article about this[59]. Precisely the immensity of data involved has allowed us to apply the theorems of Ramsey and Van der Waerden. These make it possible to show that, given any "regularity", or any correlation between sets of numbers, you can find a number p large enough, such that *every* set with at least p elements contains a regularity (or a correlation between numbers) with the same structure. Now, since this applies to every sufficiently large set (with at least p elements), this also applies when it is generated … by a random process. Indeed, we observe, almost all sets of fairly large numbers are algorithmically random (one can give a mathematical definition of them, in terms of incompressibility), i.e., the percentage of non-random tends to 0 as p goes to infinity. So, if you observe regularities in increasingly large databases, it is increasingly likely that the inserted data are due to chance, in other words are perfectly without meaning and do not allow prediction nor action.

Which brings us to Frank Ramsey. You were not able to have known Ramsey personally. He too was a precocious mathematician at Cambridge. He died in 1930, at the age of 27. He was a translator and a friend of Wittgenstein, with whom you too were later to have an intensive exchange.[60] John Maynard Keynes was the strong, stable bond between you, an amazing group of attenders of each others lectures and friends (but, in your opinion, did Wittgenstein actually have friends ...?). They will certainly have talked to you about Ramsey, and I am sure that you will have liked his very fine result of finite combinatorics[61]. Perhaps you might also have liked our simple application that demolishes the "Theory of the End of Theories", you who were always proposing increasingly original theories and mathematical frameworks, and putting forward different points of view, and turning yourself into a discrete-state machine, and inventing the software for it, and immersing yourself in continuous material shaping of forms, without software to programme them, and interpreting the real and your own invention of the real. And in the process profoundly changing our reality.

Translation: Ed Emery – 4.xi.2017

58Anderson, C. (2008) “The end of theory: The data deluge makes the scientific method obsolete”, *WIRED*.

59Calude, C. & Longo, G. (2017) “The deluge of Spurious Correlations in Big Data*”* in *Foundations of Science* Vol. 22, Issue 3, pp 595–612. Online at: http://www.di.ens.fr/users/longo/files/BigData-Calude-LongoAug21.pdf

60 See, for remarks on the discussions between Wittgenstein and Turing, Monk, R., 1991, *Wittgenstein, the duty of genius*, London: Vintage, pp.417-422

61 Ramsey F.P., 1929, "On a problem in formal logic," *Proc. London Math. Soc.*, Vol. 30, pp. 264–286

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