The difference between generalization and conceptual transfer, based on Georg Kreisel's reflections on computability

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Four Letters by Georg Kreisel, 1982-84

http://www.di.ens.fr/users/longo/files/FourLettersKreisel.pdf

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Conceptual **Transfers** (''plaquage'')

(G. Kreisel (1923-2015), Cambridge, IAS (Gödel), Stanford)

Two ways towards Generalization in Mathematics

1 – G. Kreisel (1971) **Some Reasons for Generalizing Recursion Theory**, *Studies in Logic and the Foundations of Mathematics*, (Gandy, Yates, eds) *vol. 61*, pp. *139-198*,

G. Kreisel (1987) **Church's Thesis and the Ideal of "Informal Rigour"**, *Notre Dame Journal of Formal Logic,* Vol. 28, 4.

A "Set Theoretic" frame: *Computability as "relativized" term-rewriting*

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Asperti A., Longo, G. (1991) **Categories, Types and Structures: Category Theory for the working computer scientist.** M.I.T. Press.

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Concepts and transformations (diagrams) → **Grothendieck Topos**

Diagrams for … computability (in higher types)

Origin of the debate with Kreisel:

ETH, Zürich a 1982 *draft with* ***two main*** **diagrammatic** ***definitions and conjectures*** *for typed and type-free structures*, a dialogue ...

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G. Longo, E. Moggi. *The Hereditary Partial Effective Functionals and Recursion Theory in Higher Types*. **The Journal of Symbolic Logic** Vol. 49, No. 4 , pp. 1319-1332, Dec., 1984

G. Longo, S. Martini. *Computability in higher types, P-omega and the completeness of type assignment*. **Theoretical Computer Science**, 2-3(46):197–218, 1986

Diagrams for … Categories

J.R. Cockett, P.J. Hofstra, *Introduction to Turing categories*, **Annals of Pure and Applied Logic** 12; 156(2-3):183-209, ***2008***

> where “a convenient setting for the categorical study of abstract notions of computability is presented”,
>
> “Longo et al., in [30,29], ... formulated the appropriate categorical concepts corresponding to Gödel numberings and parametrization.”

Back to *two ways* towards Generalization

- **Kreisel's** focus on Computability (Recursion Theory) as ***term re-writing systems*** (Turing Machines and Church's lambda-calculus as the key systems):

Bezem, M, Klop, JW, Roelde Vrijer, R **(**2013) **Term Rewriting Systems.** Cambridge: Cambridge U. Press

Barendregt H. (1984). **The Lambda Calculus. Its Syntax and Semantics.** Studies in Logic and Found. Mathematics 103, Amsterdam: North-Holland.

- "It is sometimes said that the axiomatization problem is to generate the set of valid statements. But this is **a logician's parody** of the role in mathematics of genuine axiomatic theories" (**Kreisel**, 1971)

Extracting what is relevant:

Back to *two ways* towards Generalization

1 – *Generalizing* **decidability, finiteness** and **induction** (*Logic-Linguistic*, Gentzen-Turing-Kreisel):

- Oracles (generalization by *relativizing*, Turing, 1938)
- Finiteness and ordinals (Lω, ordinal extension and *relative* definability and *transfinite induction*)

2 – *Constitution of invariants* and **invariance** *preserving transformations* within a perspective (Geometric-Categorical, Weyl-Eilenberg-MacLane):

- Diagrams
- Categories
- **Toposes** (Grothendieck; in Logic: Lawvere, for references see (Longo, 2015))

More on two ways towards Generalization: **1**

1 – *Generalizing* **decidability, finiteness** and **induction**:

- **Turing's oracles** as Relative ***Decidability***:

  $A \leq B$ or $A = T(B)$, cf. Enumeration Operators etc

  [H. Barendregt, G. Longo, Recursion theoretic operators and morphisms of numbered sets. *Fundamenta Mathematicae*, 1982]

- **Turing's generalization** of "***finite***" in PA: "relative *definability*" in T and/or by adding *consistency statements* up to limit ordinals $L\omega$:

  "By repeating the process we get a sequence L, $L1 = L'$, $L2 = L1'$, $L3 = L2'$,... of logics" (Turing, PhD, 1938)

- **Gentzen's** analysis of **Transfinite *Induction*,** 1934:

  "another ordinal logic, of a very different type … " (Turing, 1938)

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Analogy: "**extend the notion of area** to a wide class of sets by use of Borel or Lebesgue-measure'' (Kreisel, 1971)

More on two ways towards Generalization: **2**

2 – *Constitution of invariants* and **invariance** *preserving transformations,* within a *perspective*

- **Diagrams** (since Euclid: symmetries)
- **Categories** (*from* natural transformations *to* functors, morphisms)

  S. MacLane … with **Logic**: *W. Lawvere* (Logic);

  with **Types**: [Asperti A., Longo, G. (1991) *Categories, Types and Structures*. M.I.T. Press]
- **Toposes** (“sheaves on a site”, Grothendieck; “transversality”)

Grothendieck: “topological groups”, “metrics on vectorial spaces” (1956), *applications aux* C∗-algèbres, espaces d'opérateurs, inégalités de Bell et leur "violation" en MQ, problème P=NP et à la théorie des graphes.

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An application: Logical **unpredicativity** as “small completeness” in a topos, i.e. closure under generalized indexed products [Longo, Moggi, 1991].

Common Features

Both Generalizations *relativize, extend applicability,* ***unify*** …

1 - which are the invariant properties of being ‘‘**decidable, finite**’’? by relative decidability, ordinals, cardinals (finiteness) … Turing’s oracles, Definibility, Gentzen’s ordinals …

(“*Gentzen's work makes clear, beyond a shadow of doubt, that* ***proof theory begins where recursion theory ends***” (Kreisel, 1971)

2 – by (new) **invariant** properties under transformations of Theories: Algebraic Geometry, Categorical Logic, Differentiability in Geometric Toposes (applications to Physics) ...

Mathematics is …

abstract, symbolic, rigorous

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1 - **Abstract**: constitution of **invariants** w.r.to **transformations**

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Some possible cognitive grounds:

- basic **counting**: a "*practical invariant"* independent from the intended objects (early formation of meaning), beyond Brouwer's « twoness of time »
- *memory*, e.g. of a **trajectory** ... of a gesture, forgetting details (memory: ***forgetful***, re-constructing, intentional)
- Edelman's analysis of **memory** (*forgetting* details, interpreted abstraction, recovered state)

All: **conditions of possibilities**, as this cognitive early practice of invariance *is not the concept*. This requires **intersubjectivity** and language.

Abstract, symbolic, rigorous

Mathematics is

2 - **Symbolic:**

- Symbols "bring together": complex, synthetic, not *signs,* but *evoked gestures*, by language; meaningful signs in resonance with the world
- numbers: "symbols" for an *action* (counting);

Numbers: the common invariant of *ordering* and passing *time* (Brouwer).

Relevance of mathematical symbols:

the symbols for the concept of **infinity**, ω, allow *further action* (operating on infinities, since Cantor), the invention of a further praxis

Abstract, symbolic, rigorous

Mathematics is

3 – **Rigorous**: Kreisel's *"Church Thesis and the Ideal of Informal Rigor":*

- Definition of terms (the right level of invariance)
- *Two* key examples: the use of **Church Thesis,** CT (cf. Davis vs. Rogers books)

1 - M. Davis, **Computability & Unsolvability**, 1958:

Computable? Give the TM, *thus* any other system … by **CT**

2 - H. Rogers, **Theory of Recursive Functions and Effective Computability**, 1967

Computable? *First,* informal-rigorous construction of the basic steps for describing "effectively" the function, in a "informal-formalism" (a "recipe" to formalize), *then* use **CT**

"*Informal Rigor is involved in the 2000-year-old tradition*" **(Kreisel)**

Abstract, symbolic, rigorous

Euclid:

- ***Axioms*** "**maximize symmetries**" *over lines with no thickness*

G. Longo. Theorems as Constructive Visions. *Invited Lecture*, Proceedings of **ICMI 19 conference on Proof and Proving,** Taipei, Taiwan, May 10 - 15, 2009, (Hanna ed.) Springer, 2010

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"The primary evidence should not be interchanged with the evidence of the "axioms"; since the axioms are the result already of an original formation of meaning (*Sinnbildung*) and they already have this formation itself always *behind them*"

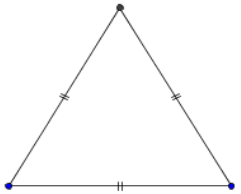
[**Husserl**, The origin of Geometry, 1933].

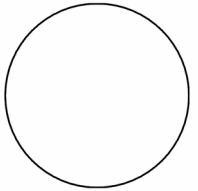
- ***Proofs*** by logic and … symmetries: rotations, translations %

"Informal Rigor is involved in the 2000-year-old tradition" (Kreisel):

1 - Euclid's axioms and definitions (definition beta):

- **the line is a length with no thickness** (*for us:* an axis of rotations)





– The invention of **borders**: the a-logos $\sqrt{2}$, the a-peiron π …

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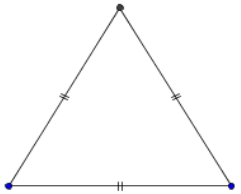

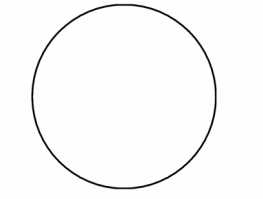

– The invention of **borders**: the a-logos $\sqrt{2}$, the a-peiron π …

2 - Lu Hui, III century, Chine *(negative numbers*… soon *0):*

– a different metaphysics, a different praxis:
approximate the circle inside/outside: stop when the difference is *no longer visible*

– The larger the triangle, the more compute $\sqrt{2}$….

Liu Hui (III siècle) *Les Neuf Chapitres,* traduit en français per K. Chemla et Guo Shuchun (2004)

3 - Grothendieck's notions (sheaves, sites, topos …)

Zalamea F. (2012)*, Synthetic Philosophy of Contemporary Mathematics*, Urbanomic (UK), Sequence Press (USA)

Longo G. (2015) Synthetic Philosophy of Mathematics and Natural Sciences, Conceptual analyses from a Grothendieckian Perspective

Unify by setting bridges: groups… manifolds ...

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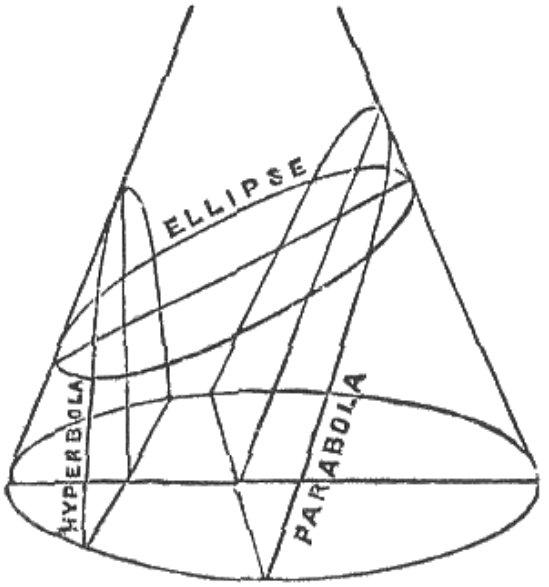
Unify by setting bridges: groups… manifolds ...

An analogy: **Apollonius** (Perga, Gr, III c.)

inventing the **cone**, as a "*Topos*",

unifying different "*sites*":

circle, ellipse, parabola, hyperbola:



Cone—with Sections.

The **formalist** philosophical **parody**, *but* **mathematically effective**:

Abstract = symbolic = rigorous = **formal**

Focusing only on **formal proofs**, thus where

computers' programs are proofs

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An alternative view:

An **epistemological** analysis instead includes an analysis of the **constitution of the mathematical concepts and structures**

An unification by constructing **conceptual bridges** and stressing **differences** (e.g; in applications, in physics, in biology)

An homage to the joint work with "firm formalists"

R. Hindley (H.B. Curry) and **H. Barendregt** (G. Kreisel)

Project:

From the foundation (and the philosophy) of Mathematics as **an annex of a Philosophy of Language,**

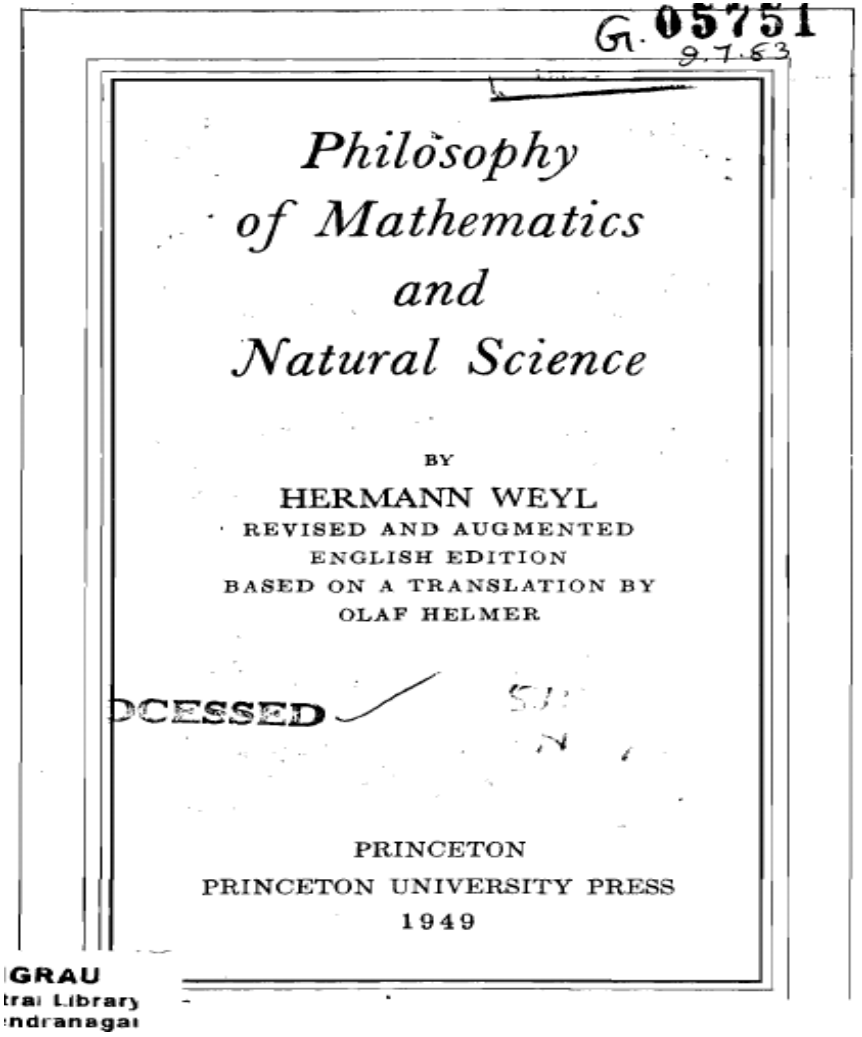
Towards an essential component of **a Philosophy of Nature**

Aim:

- a constructive **epistemology of Mathematical Structures** (objects, categories, invariants, transformations …)
- a fundamental tool for "**le découpage**" and "**qualification**" of the real (*physics* vs *biology)*
- a tool for **unity** in the sciences of nature

Opening to theoretical foundations in Physics and Biology ...

Mathématiques et philo de la nature



Philosophy of Mathematics and Natural Science

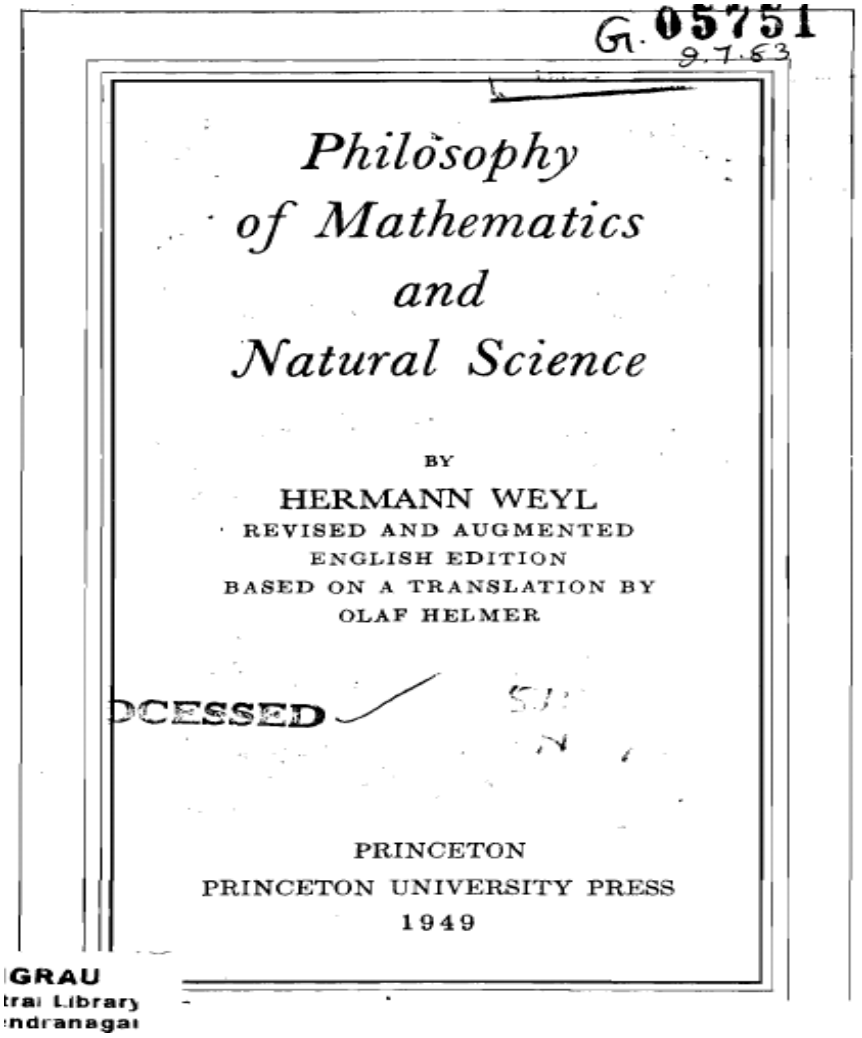
BY

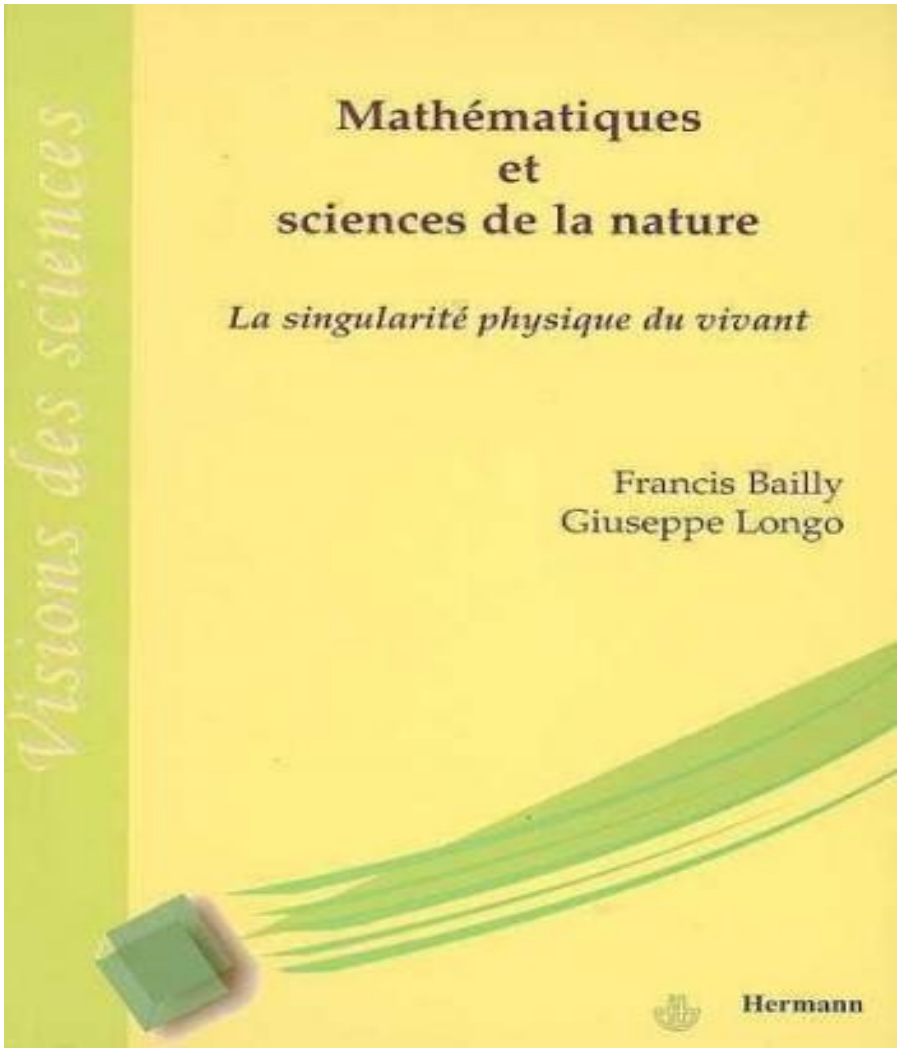
HERMANN WEYL

REVISED AND AUGMENTED ENGLISH EDITION
BASED ON A TRANSLATION BY OLAF HELMER

PRINCETON
PRINCETON UNIVERSITY PRESS
1949

Mathématiques et philo de la nature





More work with physicists T. Paul and M. Mugur-Schachther

Diversity and Unity in Physical Theories

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- **Physics:** *Different (incompatible) theories and Phase Space*s:
  Classical Mechanics (position and momentum)
  Thermodynamics (time; energy; entropy (not conserved))
  Relativity (Space***x***Time *add* Energy***x***Momentum Tensor)
  Quantum Mechanics (Hilbert Spaces, bounded operators)

  - Each of these theory uses a pre-defined **Phase Space**
  - Fundamental Unity: **conservation laws**

  - ***An issue of symmetries*** (Noether's Theorems, 1920)

    Noether → Hamilton → Newton → Kepler ...

    Noether → Hamilton → Schrödinger …

[Bailly, Longo, 2006; Longo, Montévil, 2014]

Search for Unity and Generalization

In physics:

- **No reduction**
- **Unification:**

  Newton, Maxwell, Boltzmann, Einstein

Chibbaro, S, Rondoni, L & Vulpiani, A, 2015 *Reductionism, Emergence and Levels of Reality: The Importance of Being Borderline*, Springer, Berlin

(CR: Longo, 2016; cf. O. Rey *Une question de taille,* 2014)

Unity: *conservation* laws as symmetries: "the notions of group theory help one analyze concepts of symmetry" (Kreisel, 1971)

Weyl's work on ***symmetries*** (Gauge Theory) and *foundations:*

"Unification" by Symmetries

H. Weyl, Symmetries**:** “All a priori statements in physics have their origin in symmetry” (p. 65)

“Whenever you have to do with a structure-endowed entity Σ, try to determine its group of automorphisms, i.e. those element-wise transformations which leave all structural relations invariant.”

Gauge Theory is a search for unity by common symmetries

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In Weyl's vision of **symmetries**, physics moves

from "causal lawfulness" to the structural organization of space and time (structural lawfulness),

or even from causal laws to the "legality/normativity" of mathematical (geometric) structures.

(Correspondence between Husserl and Weyl (reproduced in [Tonietti, 1988], [Mancosu, Ryckman, 2002] with reference also to Becker, see [Bailly, Longo, 2006])

Genericity of Mathematical and Physical Objects

Relativity: A body falls for … symmetry reasons …

QM: "We distinguish the n electrons by attaching the **labels** 1, 2, . . . , n to them … The symmetry prevailing is two-fold:

> First we must have invariance with respect to transition from one Cartesian co-ordinate system to another … a rotational symmetry
>
> Secondly, **all electrons are alike**: … permutation consists of a re-arrangement of the labels" (Weyl, Symmetry, p. 69)

The key role in Physics of:

Genericity of objects (symmetries: theory/experiment)

Specificity of trajectories (symmetries: conservation)

Principles of Construction *vs.* Principles of Proof

Conceptual Construction Principles:

symmetries, Euclid's continuity of lines, (well) ordering ...

They are the conceptual invariants constituted in **action** and **language**, in writing, by shared praxes.

They ***found mathematics*** in the epistemological sense, by the analysis of a "genealogy of concepts (and structures)", before proofs.

Symmetries (*groups*, *space*), **order** (*semi-groups*, *time*) … are ***shared*** with Theorizing in Physics: **Noether's** Theorem and **Geodetic** or Least Action Principle, see [Bailly, Longo, 2006, in English, 2011)]

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Proof Principles (formal induction, formal logical rules…) are very important, yet, ***provably incomplete*** (since Gödel, better PH, KF)

PH, KF, Normal F.: in the gap between principles of **construction** *vs* **proof** (invariance, ordering, [Longo, 2012])

Conceptual dualities Math-Physics/Biologie

Physics:

Genericity of objects *(invariants of theory and experiments)*

Specificity of trajectories *(geodesics – conservation:*

Noether&Weyl → Hamilton → Newton&Schrödinger)

Biology:

Specificity of objects *(organisms: individualized, historical)*

Genericity of trajectories *(phylogenetic: possibilities)*

The very different sense of "generalizing"

Ongoing work, since (Bailly, Longo, 2006; Longo, Montévil, 2014):

- Soto AM, Longo G (guest editors) "From the century of the genome to the century of the organism: New theoretical approaches". **Prog Biophys Mol Biol**. 2016;122(1).

From “generalizing” to the mechanical conceptual transfer (le “plaquage”)

“What one has to guard against is to imitate mechanically

the basic developments of recursion theory” (Kreisel, 1971)

Le plaquage sur le cerveaux (sur la cognition): Classic AI

1 - The Mind/Brain **is** a Turing Logical Computing Machine (TM, 1936; Discrete State Machine 1950: *strong AI*)

2 - The Mind/Brain can be **imitated** by a TM (*weak AI:* in 30% of cases ...)

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Against Turing: "The nervous system is **surely not a** DSM… a small error in the information about the size of the nervous impulse…" [Turing 1950, p. 57]

"In a Discrete State Machine (DSM)... it is always possible to predict all future state … This is reminiscent of Laplace's view ... The prediction follows"... from formal determination [Turing, 1950; p. 47]

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"... in fifty years' time ... an average interrogator will not have more than 70 per cent. chance of making the right identification after five minutes of questioning" [Turing, 1950; sect.6]

Turing 1952: Morphogenesis **model**: a continuous dynamics of forms … ***hardware***, **no software** ("design": Child, Waddington, D'Arcy Tompson)

Le plaquage sur l'Univers

“The Turing machine … a complete means of describing everything that can exist in our universe … the universe that operates like some behaviour of a Turing machine." [Wolfram, 2012; *in honor of Turing*]

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“The system of the 'universe as a whole' is such that quite **small errors in the initial conditions** can have an overwhelming effect at a later time. The displacement of a single electron by a billionth of a centimetre at one moment might make the difference between a man being killed by an avalanche a year later, or escaping. It is an essential property of the mechanical systems which we have called **‘discrete state machines' that this phenomenon does not occur**. Even when we consider the actual physical machines instead of the idealised machines … ” [Turing, 1950]

(see also [Turing, 1952], morphogenesis: catastrophic instability, continuous sym. break)

Le plaquage sur l'Univers

« the universe may be seen as a large Turing Machine » (Wolfram)

That is, a **body falls** because it is programmed to fall

Sort of “vertu tombative” …

cf. falling by “symmetries”, Relativity Theory

… what about the **fundamental constants** (in equations): G, c, h … alpha??

Are they rational, computable numbers ???

Gabriele **Veneziano**: fix two ...

Some references (*papers downloadable:* Google: Giuseppe Longo)

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