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Information at the Threshold of Interpretation
Science as Human Construction of Sense

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Abstract. This text will first discuss the great and productive "linguistic turn" that originated in Logic, at the end of the XIX century. After Turing's work on the Logical Computing Machine (1936-1950) and Shannon's theory (1948), Brillouin attributed information content to Maxwell's demon's action (1956), with a scientific invention and, in my opinion, a touch of humor, thus granting to inert matter an intelligent activity capable of inverting entropy. Moreover, since the 1950s, many identified the digital elaboration of data with human intelligence and some projected the processing of information onto inert matter. In the same years, DNA was identified as the complete information carrier of phylogenesis and ontogenesis. In these approaches, an ambiguity has been arbitrarily played out, and a threshold has been sometimes crossed, sometimes not; first, in Logic and Computing, between "formal principles of proof" or of computation and their "semantics" (their mathematical meaning); then in Biology, between elaboration of information and biological dynamics in their context. Today, well beyond classical Artificial Intelligence on discrete data types, Brillouin's invention and Deep Learning on continuous mathematical structures are the new loci of confusion between knowledge construction and transmission or processing of information. Yet, the construction of human knowledge is even more radically by-passed and science dehumanized when information is directly embedded in a world that is viewed as a massive information processor. The remarkable scientific and economic productivity of our new observable, information, must instead be seen in the context of our human, ecosystemic and historical activities, in order to subordinate it to interpretation and meaningful action that enrich and do not strip the sense from our forms of knowledge and life. To this purpose, we must strictly distinguish between information, as formal elaboration and transmission of signs, and information as production of "meaning" in our active friction on reality.

1 - Introduction: the origin of sense

We may dream of the origin of the human construction of sense in the early gestures by which our ancestors, while inventing human communicating communities, tried to interpret the totally meaningless spots of light in the night's sky. They probably pointed out to each other configurations of stars by interpolating them with lines, a founding mathematical gesture. Language allowed to name them: here is a lion, there a bison, or whatever. This surely had also some use, in orientation, in recognizing seasons… but the shared meaningful myth was probably at the origin of this early invention of configurations of sense in a meaningless dotted sky.

Structuring, organizing reality, which in turn resists and canalizes these actions of ours, these are the primary gestures of human knowledge. We single out and qualify fragments of reality by setting contours, by giving names and associating properties with them, and correlating unrelated fragments.

These cognitive gestures are not to be reduced to, but have their condition of possibility in our biological activity. Recall that Darwin's first principle of heredity in evolution is "reproduction with modification". But also allopatric speciation, by individual or populations' movement, and, in general, hybridizing migrations, are at the core of his earliest analysis: the finches of Galapagos he observed are the result of moving populations. Thus "motility" must be integrated as the other founding principle for the understanding of biological evolution (Soto et al. 2016). In other words, *agency*, in the broadest sense, is essential for life, from motility and action in space to cognitive phenomena, since "motility is the original intentionality" (Merleau-Ponty, 1945). Constraints interactively impose limits and canalize motility and organismal dynamics, from cells and organisms (Montévil, Mossio, 2015) to human activities.

Agency is at the origin of significance, since a signal, a disturbance, a collision that impacts a moving amoeba is "meaningful" for the amoeba according to the way it affects this original intentionality – it may favor or oppose the ongoing protensive action, the amoeba's anticipations (Saigusa et al 2008) - a thesis hinted in (Bailly, Longo 2006).

It is always an organism, as a unity, that acts and moves, that attributes meaning to an incoming signal or perturbation. The "correlated changes" induced by a hit, a paraphrase of Darwin's "correlated variations" within organisms and within an environment, are made possible only by the organismal unity and its history in an ecosystem. The formation of sense is thus historical, beginning with the phylogenetic and then the ontogenetic history, including embryogenesis for multicellular organisms.

As for comparing organisms and machines, including computers, note that embryogenesis is obtained by *reproduction with differentiation* from one organism, the zygote. At each step of embryogenesis, cells' reproduction preserves the organism's unity. Differentiation has nothing to do with the assemblage of parts, elementary and simple ones, by which we construct any artifact. A child is not obtained by attaching a leg, gluing a nose, introducing an eye in a hole, but by reproductive differentiation of individual cells. Thus, at each step, the unity of the organism is preserved and is a subject of sense: sensations and reactions to sensation are meaningful at the proper level of the forming individual, they are interpreted. This is radically different from the piecewise assembly of any machine whatsoever, which precludes the constitutive formation of meaning as interpretation of the "deformation" of a self-constructed, differentiating and self-maintained biological unity and of its correlation to an environment. An "interpretation" is always the result of a historical and contextual construction of sense: different histories, such as embryogenesis and assemblage, produce different meanings.

Of course, there is a huge distance, actually several "critical transitions", between this biological formation of sense, as action and reaction of an organism in an ecosystem, and the meaningful constructions of human knowledge. Yet, the evolutionary and historical dynamics of knowledge formation have their roots, and their *conditions of possibility*, in these primary aspects of life: the moving eukaryote cell, the embryogenesis of a multicellular organism, whose active unity and motility allow to interpret, actually force to attribute meaning to its deforming "frictions" within the ecosystems. By a long evolutionary and historical path, this original formation of sense, as *deformation of an active unity*, becomes knowledge construction in the human communicating community, with its proper levels of interaction, thus its proper unity.

The challenge we have today concerns a newly invented observable, information, and its relation to sense and knowledge construction. Of course, "information" exists since long, or as soon as an animal brain forms the invariants of action: the retention of what matters in an ongoing action and its selective recalling for a protensive activity jointly single out what may be considered the "relevant information" to be retained and later used for further action. This relative independence from the context of the retained action underlies the construction of the invariant, a trajectory, a sound … and may be analyzed as a primary form of information. That is, the constitution of cognitive invariants w.r.to a context may be viewed as the founding practice of the subsequent notion of information that only language and, even more so, writing allows to properly single-out and define. Yet, only the modern invention of machines for transmission and elaboration of information, independently of meaning, as strings of dots and lines in Morse alphabet, 0s and 1s in today's computers and in Shannon's theory, fully detached information from other observables and, in particular, from meaning. In conclusion, once the invariants resulting from animal and human cognitive activities were made independent from action and stabilized as (electronic) signs[1], transmission and elaboration of information became a mathematical science, actually at least two very relevant sciences (Turing's and Shannon's). Then, information could be formally analyzed, elaborated and transmitted, independently of any interpretation.

2 - The modern origin of elaboration of information as formal deduction: productivity and limits of "nonsense" in the foundational debate in mathematics

It may be fair to say the powerful promotion of meaningless formalisms as information carriers has its modern origin in the formal-axiomatic approach to the Foundations of Mathematics, at the end of the XIX century. Historians may better specify the predecessors of this "linguistic turn" in Logic, such as Leibniz's "lingua characteristica", as well as whether this turn was just a consequence or a symptom or an actual promoter of the new vision of human knowledge that will mark the XX century, as formal elaboration of signs. In any case, the foundational issue in Mathematics brought to the limelight the formal-linguistic perspective in the clearest way. This issue is at the origin, in the '30s, of the various mathematical notions of computation, such as Church's lambda-calculus (1932) and Turing's "Logical Computing Machine", as he called it (1936). In short, whatever was the general cultural role of the debate internal to Logic, computers, as mechanical devices for elaborating information, and their networks that are changing our lives were invented within that debate on the foundations of Mathematics[2]. In order to criticize the abuses of the computational approaches to the notion of information, which we can find in many, vague, un-scientific, commonsensical references in biology and even in other sciences, let's briefly survey its robust origins within Mathematics.

The issue at stake at the end of the XIX century was to save Mathematics from the major foundational crisis induced by the invention of non-euclidean geometries: the reliability and certainty of the mathematical edifice, guaranteed by the perfect correspondence between physical space and Euclid's geometry in Cartesian spaces, was suddenly lost. Hilbert (1898), with extreme rigor, proposed formal axiomatic approaches for the various existing geometries, where certainty was no longer based on "meaning", as reference of the geometric properties to actual physical space, but only on the exact axiomatic presentation and on the internal coherence of the theories so defined. The information had to be entirely formalized in the axioms and deduction developed

1 We use "sign", in reference to a priori meaningless strokes, letters, 0s and 1s …, thus we use the expression "sign pushing" instead of "symbol pushing" widely used in Computer Science. "Symbol" retains the Greek etymology of "sym-ballein" or "to bring together" or to unify different acts of experience or synthesize meaning. Cassirer's notion of "symbolic form" is a reference for this (Lassègue 2016).

2 Note that computability and its machines were invented in order to prove the *limits* of the formal-linguistic approach. As hinted above, in order to prove undecidability and incomputability, Gödel, Church and Turing had first to give precise definitions of "computable function" and "formal derivation", thus of formal computing/arithmetic machines, the mathematical foundation of modern digital computers - see (Longo, 2010) for the relevance of these and other *negative results*.

independently of meaning, in particular as reference to a "meaningful" geometric structure of space[3]. The absence of logico-formal contradiction, that is of a purely formal deduction of a proposition A and of its negation, $notA$, was the only quest and guarantee for certainty. By an analytical encoding of all the mathematical theories he had axiomatized, into Arithmetic, that is in the Formal Theory of Integer Numbers, Hilbert transferred the burden of coherence into Arithmetic: a formal, ''potentially mechanisable" proof of its coherence could re-assure the "absolute certainty" of existing Mathematics, non-euclidean Geometries first – this was one of the famous 23 open problems he posed in 1900. Gödel, Church, and Turing's negative answers required first … the positive definitions of "formal deduction or computation", for example by the construction of the Logical Computing (Arithmetic) Machine by Turing, that is of a rule based formal system. By their negative results they did not set a limit to thinking, as the analytic view of mind as a "sign pushing" and information-carrier device makes us believe, since the actual limit is elsewhere and cannot be *thought*, surely not from outside. Yet, they set a limit only to the formal-linguistic manipulation of signs, and this from *inside* – as often forgotten: by smart diagonal techniques on strings of signs, they formally showed that certain strings of signs, as formal definitions of formulae or of functions, cannot be proved nor negated. As it was later shown, even in mathematics, actually in Arithmetic, we do think (and prove) beyond that limit set to formal axioms and deductions, so that we can now look at them from outside – see below and (Longo 2011).

In spite of the limitations thus proved (the existence of undecidable sentences, the formal unprovability of coherence, the construction of incomputable functions), the mechanical notion of deduction is still now setting a paradigm for knowledge, well beyond its role as a remarkable (yet incomplete) mechanical *tool* for knowledge. And coded "sign pushing" has been even extended to an understanding of biology (DNA as "an encoded program") and, by some, of physics. Moreover, even when considered just as a tool for knowledge, it should be clear that no tool is neutral: it organizes, it shapes the object of investigation.

2.1 - Reconquering meaning

As for human cognition, the formal, digital elaboration of information was soon to be identified with "knowledge construction" and more generally "intelligence". This excludes what Alessandro Sarti calls "imagination of configurations of sense": those primary gestures of knowledge that give meaning even to nonsense, by tracing, interpolating, structuring the world, from connecting the bright dots in the sky to Euclid's notion of the "line with no thickness". The invention of this founding mathematical structure, spelled out in Euclid's books definition beta, is at the origin of western mathematics (Longo, 2015). This line is a pure length, a constructed border of a figure, a trajectory - all organizing features of action in space.

As a further answer to the formalists myths of mathematics as sign pushing, note that any relevant proof requires the invention of new mathematical structures, from Euclid's lines to … Grothendieck's toposes, or even the insight into provably non-formalizable properties, as we will hint. A mathematician goes nowhere without the imagination of new concepts and structures, often "deformations" of existing ideas or invention of new correlations. These may be, sometimes, but not always, only a posteriori fully formalizable. But, what does *a posteriori* mean? Proving a theorem, in general, in Mathematics, is not proving an already given formula, the formalist parody – this is very exceptional, such as the quest for "Fermat's last theorem", i.e. the proof of a simple formula that required four centuries to be given[4]. Normally, proving a theorem is answering a

3 H. Weyl, Hilbert's "best student", recalls his supervisor's philosophy by quoting: "a typically Hilbertian manner: "It must be possible to replace in all geometric statements the words point, line, plane by table, chair, mug." In [Hilbert's] deductive system of geometry the evidence, even the truth of axioms, is irrelevant ..." (Weyl 1953).

4 Even Wiles' 1993 proof of this easy to state arithmetic property, Fermat's last theorem, required the invention of new deep mathematical ideas, a complex blend of advanced algebraic geometry, the

question, which may later lead to “writing a formula”. For example, the reader should try to tell which is the sum of the first *n* integers, for *n* generic integer. If you know the formula, a computer may easily prove it by induction, but … where is the formula? Its invention requires an insightful game of symmetries, i.e. the imagination of a geometric configuration of numbers, (Longo, 2011)[5].

The “vision” here concerns an order in space and its inversion, that is, young Gauss’ proof in the footnote, a typical mathematical construction, is the “imagination of a configuration of sense”, a sense given by the human *aim* of the proof and the g*eometrical structuring* of numbers, as an order. Interpolating, ordering stars and giving them names of animals or gods is a similar invention. None of these constructions is an “elaboration of (pre-given?) information”, surely not in the formalist or mechanical sense. There is no miracle in this, but *active* insight and conceptual bridges based on the *meaning* of mathematical structures – geometric/algebraic meaning as ways to organize scattered points in space or concepts in mathematics. The Greek word “theorem” has the same root of “theater”. In “acting out” a theorem, though, a mathematician moves possibly original pieces in the game, (s)he makes organizing “*gestures*”. The notion of gesture is a hard to define; it is at the core of Chatelet’s close analysis of the foundations of the XIX century’s physics and mathematics, when mathematical gestures invented a new physics and motivated new mathematics (Chatelet 1993). More recently, Grothendieck’s ideas and geometric “insights”, probably one of the most original and broad mathematical work of the last 70 years, reinvented algebraic geometry; a philosophical account of his organizing mathematical gestures may be found in (Zalamea 2012), see (Longo 2015z) for a review.

In summary, the formalist myth of pure calculi of signs (formal handling of information) was born or enhanced by the analysis of the foundations of mathematics. This is why a reflection on these foundations may greatly help in the discussion, in particular in setting the limits and specifying the alternatives to formal rule-based thinking, viewed as the only access to rationality and even to … biological dynamics. This view was promoted by the same formalist trend that enabled the invention of digital computers, from Turing Machines to von Neuman or Crutchfield automata, that we will mention, and today’s computers i.e. the possibility of moving signs according to formal rules prescribing how to write and re-write (manipulate) them without any reference to meaning. These are all Discrete State Machines, implementing a dynamics of information as strings of signs, handled by rules, both signs and rules encoded by 0s and 1s. How can this rule based computation replace knowledge construction, if it cannot invent the formula that computes the sum of the first *n* integers or, even worse, fully encode the proofs of some recent interesting *Arithmetic* statements, well beyond Gödel’s fantastic diagonal trick - the Liar’s inspired “this formula is not provable”? As a matter of fact, some “concrete” (meaningful) propositions of Arithmetic are provably *unprovable* within Arithmetic – this is much stronger than Gödel’s diagonalization. As a matter of fact, different sorts of “geometric judgments” unavoidably step into their proofs, see (Longo, 2011) for a technical and philosophical introduction. In these cases, even a posteriori proof-checking is badly defined. The “everything is information” fans should then clarify

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“vision” of a path that linked homology to the analysis of elliptic curves, leading to the totally unrelated new ideas and techniques of the “modularity lifting theorem” (Cornell et al., 2013). Checking formally the proof, a posteriori – once it has been given, may be very useful, but it is a different matter, (Hesselink, 2008).

5 In short, following a proof allegedly given by Gauss at the age of 7, place

1 2 … n on a row

n (n-1) … 1 on the next row (an audacious mirror symmetry of the usual order of number writing), then obtain

(n+1) … (n+1) by adding the columns. Thus, the sum of the first *n* integers is $(n+1)n/2$, which is easy now to *check* by induction; yet, we had first to produce the formula, by this or other similar constructions. The rule-based formalist approach confuses *proving*, in mathematics, which includes inventions like here, with a posteriori *proof checking*, a remarkable techniques in Computer Science, that we will mention below.

whether their notion of information does or does not include sense construction, gestures, and frictions on reality, often of a pre-linguistic nature and at the origin of interpretation.

A close mathematical analysis, that we only hinted here, can help to better specify the vague references to "insight" and "practices" in human reasoning used by many authors (see the survey in (Nickles, 2018)). In mathematics, one may "see" the gestures that correlate, organize possibly new structures by interpolating points, constructing borders, by symmetries, by ordering …. As we will stress, a human body in a history, beginning with embryogenesis, is needed for this, embedded in an evolutionary, linguistic and historical community. In particular, the largely pre-linguistic shared action in space enables the human universality of mathematics: we all share the monkey's practice of trajectories and surfaces, symmetries and ordering, that, once made explicit by human language, founded mathematics, before any formalization (see (Longo, 2011v) for an analysis of the symmetries that underlie Euclid's axioms). Symmetries are one of our active organizational interpretations of the world; they precede and give meaning to geometric constructions, from Euclid's axioms to the founding diagrams of Category Theory, e.g. the "Natural Transformations" that motivated Eilenberg and MacLane's work, (Mac Lane 1970), (Asperti, Longo, 1991) and Grohendieck Toposes (Zalamea 2012).

In the development of "intelligent" machines', though, a major turn happened recently, by the invention of multi layered neural nets, better known as Deep Learning. This is based on ideas that go well beyond the rule-based logical formalisms of classical Artificial Intelligence and it will be discussed below. The debacle of classical AI's view of the brain as a formal-deductive 0-1 machine (see below) parallels a paradigm shift in biology and, perhaps, also in (analytic) philosophy. In the latter case, "information" seems to stand, now and at least for a few, for the "construction of a perspective", of a theoretical frame or *interpretation* of natural phenomena. Two more "ways out" from the debacle of the formalist philosophy and the linguistic turns, from Logic to Biology, will be worth mentioning next. A discussion about them may further help in clarifying what one means by the fashionable reference to information: is this dehumanized, formal sign pushing, like in digital machines, or interpreted information, that is the explicit proposal of an interpreting perspective for knowledge construction?

3 - The role of ''interpretation" in programming, as elaboration of information

The religion of life and intelligence, and even of physical dynamics, as elaboration and transmission of information, as moving strings of bits and bytes, has its own limits within Programming Language Theory, first. Do programs, our invention, "stand alone"? Do they work correctly when piled up in complicated interactions or do they run into bugs and inconsistencies? A simple consequence of Gödel's and Turing's negative results shows that there is no general algorithmic way to prove the correctness of programs (Rice Theorem, (Rogers, 1967)). Decades of Software Engineering, Programing Language Theory etc have been dedicated to produce rigorous programming frames, in order to avoid bugs and inconsistencies and to invent methods for proving partial correctness - see the broad literature on Model Checking (Sifakis 2011), on the Abstract Interpretation (Cousot 2016), on Type Theory (Girard et al. 1989) and a lot more. Two of the many amazing successes of the Science of Programming are … one, the Internet, whose functioning is also made possible by geometric tools in Concurrency (Aceto et al. 2003) and in networks (Baccelli, 2016), and, the other, Embedded Computing and their Nets, such as Aircraft and Flight Control, following, among others, the approach in (Sifakis 2011).

In short, also in order to design sufficiently robust programming languages and analyze the correctness of formal computations, we human scientists have to *interpret* programs for elaborating information in "meaningful contexts" of mathematics, often of a topological nature, (Scott 1982) - for a survey see (Longo 2004)). Otherwise, sufficiently complicated or long programs produce bugs, inconsistencies, loopholes etc… Formal computations do not stand alone: geometric meaning frames their design, helps checking their correctness. Recall, say, the dramatic improvement of abstract algebra since Argand and Gauss (end of XVIII century) interpreted the imaginary number/letter *i*, a purely algebraic-formal sign, on the Cartesian plane: its meaning in space renewed analysis and geometry and provided a paradigm for the enriching interplay of formal syntax and semantics.

Programming is our modern and even more abstract alphabetic invention, as formal writing of signs that are modified according to rules written as signs, that is by replacement rules: current elaboration of information is a formal "term writing and re-writing system" (Bezem et al. 2013). We, the people of the alphabet, invented it, together with cryptography – a way to encode letters by numbers and/or other letters. Letters and numbers are not in *nature*, though. Ideograms or hieroglyphics are completely different inventions for writing, thus stabilizing and making visible the invisible sounds of language in their own way: none of these human techniques is ''already there", in the world.

Moreover, in order to associate a number or a letter to a physical process, we need to make the difficult operation of measurement, i.e. chose an observable, a metrics, construct an instrument etc and perform a measurement. By programming, we invented machines that move meaningless letters or 0-1 pixels (which is the same) according to written instructions: the alphabet or the 0 and 1's, by replacement of one by the other, move on our screens. This is such a fascinating invention that mystics believe that it is already in nature: nature formally elaborates information, like our computers, it moves signs, 0 and 1 in the squares of a (huge) Cellular Automaton (Wolfram 2013). Instead, if computation refers to the robust science of term-rewriting and, thus, of computable functions over integer numbers, as it should since the beautiful use of these notions summarized in (Rogers, 1967) and (Barendregt, 1984), nature *does not compute*. We, humans, associate numbers to natural processes by measurement and embed them in more or less successful theories, where mathematics allows also to approximately compute – but first to understand, by organizing phenomena in very rich structures (see sect 6). Besides, computations are on integers or on computable real numbers: are the fundamental constants of physics, c, h, G, α … integer or computable reals? A question I keep asking to computationalists who project Turing Machines onto the world, against Turing, see (Longo, 2018t).

In summary, uninterpreted information, as strings of moving signs, does not stand alone. Sufficiently secure programming as well as an understanding of determination (and randomness, see the footnote) need an interpretation within a relevant, meaningful mathematical, physical or biological theory. Dehumanizing science by subtracting its notions to human interpretation and theorizing does not work. It all depends then on whether the notion of information an author refers to crosses or not the threshold of interpretation.

4 – Which information is handled by a magic demon?

Shortly after the birth of Computability Theory as the mathematical foundations of "*elaboration* of information", (Shannon 1948) set the basis of the theory of "*communication*" (of information). Once more, as Shannon stresses in several places, the transmission of strings of meaningless signs is at the core of it. The ''informational content" is only given by the inverse of the probabilities of the appearance of a sign in a pre-given list of possible signs (receiving a rare string or sign is more informative than a frequent one), see (Lesne 2014) for a recent survey. The formula used to express this property happens to be the same, by the use of logarithms of probabilities, as Boltzmann expression of entropy, with an opposite sign and modulo a coefficient (Boltzmann's *k*, a dimensional constant, see (Castiglione et al. 2008)). Now, in physics, the same mathematical formula may have very different interpretations, a fortiori when modifying coefficients and constants. For example, Schrödinger's diffusion equation in Quantum Mechanics is a "wave" equation, with a complex coefficient *i* and a constant, Planck's *h.* These coefficient and constant are crucial and yield a quantum-state function, representing the dynamics of a amplitude of probability or the probability of obtaining a value at measurement. That is, if, in Schrödinger's equation, h is replaced by a different dimensional constant or the imaginary number i by a dimensional number or by a real number, the mathematics refers to a different physical phenomenon and one obtains different diffusion equations, such as the equation for the diffusion of heath or ... of water waves. Conflating the meaning of these different writings of an equation is a physico-mathematical abuse.

In the case of entropy, one may give a conversion equation towards "information", by setting bit = $k\log 2$, where k is Boltzmann's constant (Landauer 1991). This quantity, $k\log 2$, is indeed, a fundamental physical invariant:

$k\log 2$ = *the least amount of entropy produced by any irreversible process.*

Calling "bit" this quantity, whose dimension is energy divided by temperature (that is, entropy), may be legitimate, if one knows what it refers to, as most physicists do (see the very insightful experiments in (Lutz and Ciliberto 2015)), otherwise it is a word-play. Indeed, on the grounds of a formal analogy and of this "conversion" (or dimensional "forcing"), an abuse immediately started: information is *the same as* entropy, with a negative sign. Since entropy is produced everywhere energy is transformed, information is everywhere. This understanding has even been attributed to (Schrödinger 1944), chap. 6. Yet, Schrödinger refers to negative entropy as the extraction of *order* from the environment by the organism, while increasing or maintaining its own order, a different concept. Moreover, he opposes the approach in chap. 6 to the previous one by "forgetting at the moment all that is known about chromosomes" as *code-script* (Schrödinger never mentions "information" in his book). In (Bailly and Longo 2009) we developed that approach by Schrödinger in biology by the notion of "anti-entropy" as phenotypic complexity in organisms, which differs from information, at least for its dimensionality (and more: it is a geometric notion, since metric properties, dimensions and coding do matter – in contrast to digital information). We then applied it to a mathematical analysis of an idea in (Gould 1996) as for the increase of organisms' complexity along evolution, as a random, not oriented, but asymmetric diffusion. The situation then is far from obvious and conceptual short-cuts may misguide knowledge: both information and organization may oppose to entropy production, in the broad sense of "disorder", yet they yield different notions, also dimensionally, and, more deeply, as tools for understanding, life in particular (see (Longo 2018c)) .

Brillouin is more consistently credited for conflating information and negative entropy in physics, by an argument worth recalling (Brillouin, 1956). Maxwell's demon was an inventive game, playfully opposing Boltzmann's dramatic vision of the end of the Universe by the entropic final state of equilibrium: a sufficiently smart and fast demon could decrease entropy, by separating mixed gas particles according to a measurable property of some of them, see (Leff and Rex 1990). Brillouin's relevant remark is that such a demon needs to transform energy in order to detect and let pass only certain particles through the separating door. As any energy transformation process, this would increase entropy. "We have, nevertheless, discovered a very important physical law in Eq. (13.10): every physical measurement requires a corresponding entropy increase, and there is a lower limit, below which the measurement becomes impossible" (Brillouin 1956). "We cannot get anything for nothing, not even an observation", Brillouin continues.

In Quantum Mechanics, measurement produces a new object, irreversibly, by the interaction of the classical measurement instrument and quantum "reality" (this corresponds to the irreversible projection of the state vector in Schrödinger's equation). In Classical Physics, measurement was considered to be "for free". Instead, Brillouin shows that measurement has a cost, it transforms energy, hence producing entropy - a remarkable observation. Thus, measurement, the only way we have to access the world, is irreversible, also classically. In spite of the interpretative abuses (the dimensions of constants and the amusing game-play), "We have, *nevertheless*, discovered ..." says Brillouin. Many focused more on the abuses than on the discovery of an "important physical law".

The key point is that entropy production is associated to *all* irreversible processes and *k*log2 quantifies the least amount of entropy production and this can be experimentally checked (just beautiful). Now, measuring, communicating and elaborating information as well require energy and thus produce entropy, at least in that amount, e.g. even when erasing a bit of information - Landauer's Principle (Landauer 1991), see above. By measurement we do produce information, a number, to be transmitted to colleagues, but that number (actually, an interval) is not "already there", as it is the result of the choice of an observable, the construction of a measurement tool etc.. Thus, information *is* physical in the sense that it requires quantifiable physical transformations, but physics is not information, nor information is intrinsic to inert matter. We invented this fantastic *new* observable, information, independent from its material realization, a beautiful invariance property, whose irreversible treatment does require some energy – and this is measurable.

In our view, the key issue is the irreversibility of physical processes, even in classical (and relativistic) frames where time is usually considered to be reversible: measurement, at least, forces, *always,* an arrow of time, or equivalently, it produces entropy. And, I insist, we have no other way to access to reality but by measurement. This equivalence, irreversibility of time and entropy production, seems to be a core, inter-theoretic physical property[6].

Unfortunately, the anthropomorphic reference to a demon encourages the use of the word information and its "projection onto the world": the demon looks and measures and decides – he actually "transforms information into negative entropy" (Brillouin 1956, p.168). Yet, as Brillouin observes, "we may replace the demon by an automatic device … an ingenious gadget", with no need to refer to information nor to an observing eye, but just to the use of some physical energy to reduce a form entropy, while producing more entropy (due to the physical work done). That is, a physical or an engineering gadget would do, by a purely mechanical activity. The reference to an impossible, playful (and inessential) demon, though, allows to revisit Maxwell's game on Boltzmann's approach and may suggest an informational agent acting on the world.

Along this line of thought, Brillouin's wording may be interpreted as a philosophy of nature (Bühlmann et al. 2015). The *human* description or "encoding" of natural phenomena requires the production of information, by measurement first. It has a cost, which can be even measured, as negentropy, a remark consistent with Brillouin's insight and the quantification above. I understand similarly the philosophical reflection by many, for example in (Ladyman and Ross, 2008): the construction of scientific knowledge requires a subject performing first the non-trivial act of measurement. Theory building is thus production of information. Consequently, it has the dimension of information and may be measured as a physical observable, whose analysis may unify science.

This understanding of information may depart from the meaningless "sign pushing", that is, from the sign transformation or communication proper to formal elaboration or transmission of information by input-output piecewise assembled machines: it may ground (and it is grounded on) human "interpretation" of natural phenomena, which begins with measurement. The latter requires a protensive gesture, starting with the choice of an observable, then a metrics, the construction of a tool for measurement etc by the knowing subject. It is an active friction on reality, along the lines of the moving amoeba above, which is also "measuring", in its own way, its Umwelt. Of course, as stressed in (von Uexküll J, 1934): "Behaviors are not mere movements or tropisms, but they consist of perception (Merken) and operation (Wirken), they are not mechanically regulated, but meaningfully organized". Multicellular embryogenesis and a long evolutionary history add on top of the movements and tropisms of a unicellular eukaryote, but all these biological processes

6 In (Longo and Montévil 2014), we added to irreversibility of time and to entropy production the coexistence of a "symmetry breaking" and of a random event: these four phenomena seem to be correlated in all existing physical theories. In our work, this correlation extends to proper biological dynamics, such as embryogenesis and evolution, where increasing organization as well (anti-entropy production) produces entropy.

organize life very differently from mechanical assemblage and regulation and set the protensive grounds or condition of possibility for meaning.

As for Brillouin, he seems to have contradicting views, sometimes crossing, sometimes not, the threshold of information as pure sign pushing or as resulting also from interpretation. On one side, one may find, in his writings, an interpreting role of human observation and experience that may be described as production of meaningful information; on the other, in many examples, such as the analysis of a game of cards in the first chapter of his book, information is presented as the formal analysis of a collection of signs and its mechanical transmission[7]. As a matter of fact, Brillouin aims to lay the foundation of a scientific discipline of information, by "eliminating the human element" (Introduction, p. viii). Can we actually construct an invariant notion of information that encompasses theory building, and is, on one side, based on the activity of a knowing human subject, while, on the other, is independent of the *historical formation of sense*?

Today we need a rigorous clarification of this issue, in view of the role that mechanically elaborated and transmitted information is having on our lives. We may improve by this both human knowledge and machines, while working towards the invention of the next machine: in spite of the impact on our society and on science, by its speed and memory size, this digital, exact, iterating machine is rather boring, per se. Typically, the mathematics of the new AI of multi-layers neural nets in continua finds a tool but also a bottleneck when implemented in discrete state machines.

5 – The biology of molecules, well before the threshold of biological meaning

"A theorem by Einstein or a random assembly of letters contain the same amount of information as long as the number of letters is the same'' observes A. Lwoff in "Biological order" (1969). His co-winners of the 1965 Nobel Prize in Biology, F. Jacob and J. Monod, in several writings, set the theoretical basis of molecular biology along the same lines. "Since about twenty years, geneticists had the surprise that heredity is determined by a message written in the chromosomes, not by ideograms, but by a chemical alphabet" (Jacob 1974). "The program represents a model borrowed from electronic computers. It assimilates the genetic material of an egg to the magnetic tape of a computer" (Jacob 1970). One should ask first where is the operating system, the compiler or the interpreter …. In Computer Science, compilers translate one language into another (a lower level one, usually) and allow the operating system to handle the mechanical computation at the level of machine language, as a rule-based transformation of 0s into 1s and vice versa. Interpreters do the same job, but are written in the same language of the source language: lambda-calculus is the mathematical paradigm for this (Barendregt 1984). In this context, thus, "interpreters" have nothing to do with the notion of interpretation that we are using here as "human proposal for the construction of meaning". As a matter of fact, within Computer Science, a clear distinction is made. On one side, "denotational semantics" is the mathematical interpretation of formal computation over meaningful, possibly "geometric", structures (Scott 1982), (Goubault 2000), (Longo 2004) – we recalled above an early example of this sort of interpretation: Argand-Gauss invention of the geometric meaning of the formal imaginary *i* over the Cartesian plane. On the other side, "operational semantics" remains a form of sign pushing, that is the rule-based job done by compilers, interpreters and operating systems within computers: meaning is reduced to the formal operations carried on by these different programs in order to have the machine work. This second notion of "semantics" is a legitimate abuse, as long as one knows what it refers to: "sign pushing" as the internal, purely formal-mechanical, handling of programs by programs (compilers, interpreters, operating systems).

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7 The information content is the logarithm of the probability, as a frequency, of the chosen card. This example opens the way to conflating Boltzmann's logarithmic formula for entropy to Brillouin's quantification of information, modulo a negative sign and a differing constant, as pointed out above, also by its dimensionality - a further major difference.

In biology, lactose operon (Jacob and Monod, 1961) provided an early experimentally insightful example of operational control of genetic information by the genes themselves. This suggested a "microscopic cybernetic" in (Monod 1970). However, Wiener's cybernetic, as non-linear control-theory in continua (Rugh 1981), (Sontag 1990), is well beyond the programming-alphabetic approach proposed by the founding fathers of molecular biology, an exact Cartesian Mechanism, says Monod. The feed-back or circular diagrams in (Monod, 1970) are perfectly handled in programming, in particular by recursion or by impredicativity, the strongest features of lambda-calculus (Barendregt 1984), (Girard et al. 1989), (Asperti and Longo 1991). Indeed, lambda-calculus is a most advanced reference for DNA as a computer program (see below for more).

Since then, the lactose operon mechanism was proposed as a fully general paradigm and the Central Dogma of Molecular Biology was reinforced in its strongest sense: DNA contains the complete information for proteins' formation and, thus, for ontogenesis. Waddington's or McClintock's work on the epigenetic control of gene expression was forgotten and no longer quoted for many years (Fox-Keller 2000). Thus, once the human genome was decoded, it was going to be possible, according to Gilbert (1992), to encode it on a CD-Rom and say: "Here is a human being, this is me". The decoding was completed by 2001[8].

Following the early theorizing in Jacob's "Linguistic model in Biology" (1974), quoted above, (Danchin, 2003 and 2009) stress the relevance and clarify the terminology of this linguistic turn in biology: the cell is a computer that may generate another computer – a feature easily implemented in lambda-calculus, as soundly observed also in (Danchin 2003; 2009). Then, evolutionary novelty may be due to the genetic implementation of Gödel's formal method, which diagonalizes over strings of letters and numbers (see the 2018 version of (Longo 1910) for a critique). The organism is an avatar (Gouyon et al. 2002) of the selfish gene computing evolution (Dawkins 1976), (Chaitin 2012). The discovery of many fundamental mechanisms in Molecular Biology, that must be acknowledged, was mostly embedded in this cascade of computational follies or vague metaphors[9].

The consequences have been immense. As discussed in (Longo 2018c), the informational approach on discrete data types (the alphabet) diverts the causal analysis in biology, typically in the etiology of cancer, a major issue today. Still now, most cancer research in biology is guided by the Central Dogma, interpreted as "any phenotype has its antecedent in the genotype": the cancer phenotype must then be studied at the genetic level, in spite of evidence to the contrary (Sonnenschein and Soto 2011, 2013), (Versteg 2014), (Adjiri 2017). An acknowledgement of the difficulties of this approach to an increasingly spreading and deadly disease have been made also by some of the founding fathers of cancer biology, (Weinberg, 2014), (Gatenby, 2017). Yet, the focus on the molecular level, where the alphabetic program of life may be found, still dominates. The hypothesis of the completeness of genotypic information justifies the focus on (*promises* of) genetic therapies for cancer, neglecting the analysis of carcinogenes, thus prevention – a surprising and overwhelming impression to the mathematician stepping into the cancer literature (Longo 2018c). The search for the "magic bullet" that would reprogram the onco-gene or the "proto-onco-gene" or compensate for the lack of "onco-suppressor-gene", gave and gives an incredibly minor role to the analysis of ecosystemic *causes* of cancer. Instead, as Weinberg (2014) acknowledges, "most carcinogenes are not mutagenic". What is then the causal chain of cancer formation and progression? A paradigm shift (Sonnenschein and Soto 1999; Baker 2014, 2015) has found major

8 See "We Have Learned Nothing from the Genome" (Venter 2010), written by the leader of the team that first decoded a human DNA, and (Longo 2018c) for some references to the amazing promises made in early 2000s as for cancer's genetic diagnosis, prognosis and therapy, see also (Weinberg, 2014), (Gatenby 2017).

9 In a rare attempt to clarify the different role of Turing-Kolmogorof vs Shannon-Brillouin approaches in biology, (Maynard-Smith, 1999) confuses, in the explanatory examples, the dual correlation that entropy and complexity have in the two theories, see (Longo et al. 2012), (Perret and Longo 2016) for details.

obstacles, in spite of the evidence against the focus on genetic drivers of cancer (Kato et al., 2016), (Brock and Huang, 2017). The entrenched molecular-alphabetic-informational vision of the organism forbids other approaches, including the analyses of the tissue-organism-ecosystem interactions and their role in the control of cell proliferation. In our joint work in organismal biology, following (Sonnenschein and Soto 1999) and … Darwin, we consider cell proliferation with variation and motility as the "default state" of all cells, including within an organism, see (Soto et al. 2016). In particular, thus, if this Darwinian approach is correct, carcinogenes would interfere, at various levels of organization, not just molecular, with the control of cell proliferation – mutations follow, an evolutionary-developmental reaction of cells that are less constrained and/or under stress. This reaction may be present both in pathological developments and in "healthy" aging tissues (see the references above and in (Longo, 2018c)).

More generally, the need to explain how information is elaborated and transmitted in an organism forced the genocentric approach and the idea that, in a cell, macro-molecular interactions are exact, like in a "boolean algebra" … thus "evolution is due to noise" (Monod, 1970); "biological specificity … is entirely … in complementary combining regions on the interacting molecules" (Pauling, 1987). Their enthalpic random oscillations in a quasi turbulent and complex energetic environment were disregarded (Onuchic et al. 1997). Randomness, a key component of diversity production, thus of adaptivity of life, was and still is identified with "noise", (Bravi and Longo 2015), a term related to information. The production of biological diversity is then considered a pathology opposing the rule based norm (Ramellini 2002). Along these views on the exact combinatorics of macro-molecules, that would follow the formal rule of a program or Chomsky's grammatical rules (Searls 1992), the "key-lock" paradigm of the perfect fit for too long dominated the analysis of "transmission of information", for example through cellular receptors. Thus, little attention has been paid to low probability affinities, that do matter in time, a major issue for understanding endocrine disruptors and their relevance in carcinogenesis (Soto and Sonnenchein 2010). In general, the constructive role of stochasticity in gene expression and macromolecular interactions, observed since (Kupiec 1983), only recently found its way into the literature (Elowitz, 2002; Paldi, 2003; Fromion, 2013; Marinov, 2014). Of course, it is hard to conceive elaboration and transmission of information in a largely stochastic macro-molecular soup, such as the proteome of a eukaryote cell[10]. Against the informational monomania, the view of biological *constraints acting on* and *canalizing* these physical dynamics and, more generally, organizing biological functions in an organism is slowly maturing (Deacon et al 2014), (Montévil and Mossio 2015), (Soto et al. 2016) and many others.

Finally, observe that Genetically Modified Organisms (GMOs) are the direct children of the Central Dogma, in its strict interpretation: by modifying the DNA encoded information, a "blue-print" of the organism, one may completely pilot the plant in the ecosystem. This is false. For example, the microbial communities in the roots and soil are heavily affected by the use of GMOs, while they contribute to plants phenotypes in an essential way (Kowalchuck et al 2003). However, the medium-long term disaster may partly be balanced out, in the short term, by enough chemical fertilizers – see the recent Bayer-Monsanto fusion.

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10 Even more radically "… proteins never do fold into a particular shape, but rather remain unstructured or "disordered" ... In mammals, about 75% of signaling proteins and half of *all* proteins are thought to contain long, disordered regions, while about 25% of all proteins are predicted to be "fully disordered'' … Many of these intrinsically unstructured proteins are involved in regulatory processes, and are often at the center of large protein interaction networks" (Gsponer and Madan Babu 2009). See also the increasingly acknowledged important role of long non-coding RNAs (Hadjiargyrou and Delihas 2013). Dogmas on the exact mechanisms inspired by the need to transmit and elaborate alpha-numeric information are collapsing one after the other (Mouilleron et al. 2016).

In conclusion, under the slogan "biology is information", specified under the form of alpha-numeric information, thus genetic information, we witnessed a major distortion of knowledge construction. Yet, we should not be distracted by this. DNA is an amazingly important physico-chemical trace of evolution, used by the cell according to the context (tissue, organism, ecosystem). It is an *internal* constraint to development (Montévil and Mossio, 2015). Its biological *meaning* lies in its use as a constraining template for the production of molecules, in ways largely incompatible with Turing's and Shannon's theories of information. In particular, it strictly depends on dimensionality and its specific physico-chemical matter, on torsions and pressures on chromatin (Cortini et al 2016) etc, far away from the uni-dimensional encoding and hardware independent theories of elaboration and transmission of information.

Besides the increasingly acknowledged role of stochastic gene expression mentioned above, the very notion of "gene" is being deeply revised, as it has been several times in the XX century (Fox-Keller 2000): alternative splicing, overlapping genes[11], the "Physics of Epigenetics" (Cortini et al. 2016) … totally modify our understanding of DNA and bio-molecular dynamics in their physical and geometric context. The reader who absolutely cares of the informational terminology at least in mathematical modeling, should at least look at the novel approaches in Geometry of Information, where the group-theoretic notion of "reduction of ambiguities" provides a beautiful interpretation of some dynamics in terms of continuous symmetries and their breaking, (Barbaresco and Mohammad-Djafari 2015). So far, this geometric approach to information is ignored by the tenants of the rule-based world, where agents must follow the formal, alpha-numeric norm, possibly independently of meaning, from biology to economy, as we shall argue.

6 - From geodetics to formal rules and back again

Galileo's inertia is a principle of intelligibility. It is a limit principle, since no physical body moves like a point-mass on a straight line. Yet, it allows to understand all classical movements and to focus on what modifies them: gravitation and frictions, both closely analyzed by Galileo. Einstein unified inertia and gravitation, as the latter may be understood as inertial movement in a curved Riemannian space. Conservation principles, such as inertia (conservation of momentum) and energy conservation, are the main unifying assumptions encompassing even incompatible physical theories (Quantum Mechanics, Relativity Theory, Hydrodynamics …). These principles may be described as *continuous* symmetries in equations, (Kosman-Schwarback 2010). Thus one may say: I do not know the result of this coin flipping nor where that river will go exactly, but they will all move along geodetics (optimal paths), *for symmetry reasons*[12]. Falling stones, coins, rivers ... will never "go wrong": their unpredictability, if any, is a matter of non-linearity of our mathematical approach

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11 Overlapping genes are parts of a given DNA or RNA region that can be translated in at least two different reading frames, yielding very different proteins. In fact, a shift in the reading frame of a nucleotide sequence, by one-two bases, totally changes the resulting protein. Discovered in the late '70s in a small viral genome (Barrell et al. 1976), overlapping genes are coming to the limelight only recently since many studies have shown they are not restricted to viruses (Chirico et al., 2010), but they are present as well in cellular organisms, man included (reviewed in Pavesi et. al., 2018). M. Granero and A. Porati, among the pioneers in this field, nicely described the phenomenon in Italian: by a one letter shift CARABINE MICIDIALI becomes ARABI NEMICI DI ALI , and GAS INODORO becomes ASINO D'ORO (A. Vianelli, personal communication). This sort of shifts in ''reading'' is not recommended in linguistic analysis nor in programming, where it is avoided by the use of parenthesis, typically - like in lambda-calculus, a consistently evoked reference for the genetic program, see (Danchin, 2003; 2009). Actually, even in genomes (at least bacterial ones) there might be more STOP off-frame codons than expected to avoid unneeded reading frame shifts (Seligmann and Pollock, 2004; Abraham and Hurst, 2018). What is a gene, then?

to their dynamics jointly to physical measurement that is always an interval - and a minor fluctuation below the best possible measurement may give a very different geodetics. The point is that, in general, we do not obtain a number from physical measurement, but an interval; this allows to *prove* and even evaluate, in some cases (by Lyapounov exponents, (Devaney 1989)), the unpredictability of non-linear dynamics and to *understand* quantum non commutativity (up to Planck's h). The discrete replaces measurement and the enumeration of acts of measurement, proper to theories over continuous manifolds, with solely enumeration: in the discrete, one can only count - a beautiful remark in (Riemann 1854). And continuous deformations of Riemannian manifolds and their relation to the metrics (thus measurement) found Relativity Theory. Riemann's distinction, measuring and counting vs just counting, allows to pose explicitly the challenges in computer modeling.

Note first that randomness is unpredictability relative to the intended theory (cf. classical vs quantum randomness), and its analysis increases intelligibility, (Calude and Longo 2016). That is, the understanding of determination in terms of symmetry principles and, by a fine analysis, of the nature of randomness (from Poincaré's non-linear dynamics to indeterminism and non-commutativity in Quantum Mechanics) may not imply predictability. On the contrary, predicting, given a theory, is a further issue: it is computing a number, actually an interval, that is a future possible evolution of a "trajectory" in the broadest sense. We understand very well the dynamics of dice and rivers in complicated landscapes, with little or no predicting power – and a rather limited one even for Planets (Laskar 1994). However, we are theoretically sure that, much like a falling stone, they never go wrong[13].

A computational approach to theorizing in physics conflates two notions: understanding and predicting. This is particularly absurd in biology: Darwin's evolution is a bright light for knowledge, but it has little to do with predicting. If theorizing boils down to creating Turing or Shannon information, over discrete databases[14], a deterministic theory would always compute an exact value, thus predict, even in the most chaotic non-linear case (see below). In non-linear dynamics, instead, given a continuous solution of a system of equations, any discrete time and space implementation "soon" diverges from the continuous description proposed by the theory – usually written on the grounds of a conservation principle (write the Hamiltonian, says the physicist, as continuous symmetries). At most, under some reasonable mathematical assumptions, the discrete time and space implementation of equations given in continua is approximated by a continuous trajectory (Pilyugin 1999), *not the opposite*. That is, one can find a continuous trajectory that approximates the discretized one, *but not a discrete one approximating the continuous trajectory*, except, sometimes, for a "short" time-space trajectory. The point is that measurements do not produce discrete series of integer (or rational) numbers (see the footnote), but series of interval and what may happen a fluctuation below those intervals of approximation yields (classical)

12 As observed (and promoted) by H. Weyl (1949), physics moved "from causal lawfulness to the structural organization of time and space (structural lawfulness), nay, from causal lawfulness to intelligibility by mathematical (geometric) structures", see also (Bailly, Longo, 2011).

13 This is in blatant contrast with biological organisms, which "go wrong" very often or most of the time – but, in absence of a pre-given phase space for biological dynamics, it is hard to pre-define "wrong/right": very rarely, but crucially, hopeful monsters may be "right" in evolution, and contribute to speciation, as we may observe *a posteriori* (Longo, 2017). The point is that ecosystemic compatibility and viability is not optimality, in particular because the ecosystem is not pre-defined, but co-constructed with/by the organism.

14 "… due to the inherent limitations of scientific instruments, all an observer can know of a process in nature is a discrete time, discrete-space series of measurements. Fortunately, this is precisely the kind of thing — strings of discrete symbols, a "formal" language—that computation theory analyzes for structure." (Crutchfield 1994).

unpredictability, in non-linear or similar systems that would amplify that fluctuation - this makes no sense in discrete dynamics. Missing this point is passing by “the fundamental aporia of mathematics” (Thom), discrete vs continuum, already singled out by Riemann

Thus, one understands, frames and discusses falling stones, planets’ trajectories, river’s paths, hurricanes, Schrödinger’s diffusion etc by writing equations that are believed pertinent for good reasons (general conservation principles). If the system is sufficiently “complex” (non-linearity describes “interactions”, usually, the least required to express “complexity”), then predictability is possible at most for short time scales. Computers enhanced immensely our predictive power by increasing approximations and computing speed. Yet, the theoretical challenge should be clear: digital computers do not follow continuous, mathematical, nor actual dynamics. Thus, the transfer of intelligibility principles, given as continuous symmetries, into discrete sign pushing presents major epistemological and modeling problems (Lesne 2007), (Longo 2018c). And here, we discuss exactly the move from principles of intelligibility to computational rules, as formal norms to follow. Of course, computers’ implementation is essential to modern science, but exactly because of this, a close analysis is needed, in order to improve the use of these fantastic computational tools, beyond myths.

6.1 – Computations as norms

By the previous remarks, if theorizing is identified with active production of information, a major shift is implicitly produced. On the one side, a clear and important role is (at last) given to the knowing subject: he/she has to fix observables and parameters, measure etc. in order to produce information. In Weyl’s terms, scientific objectivity is the result of the passage from “subjective-absolute” judgments on space and time to the “objective-relative” construction of invariants[15]. That is, relativizing knowledge is the production of invariants w.r.to changing reference frames and knowing subjects, with his/her measurement tools. This relativization yields objectivity. In this sense, we do actively produce “information” in the broadest sense and radically depart from “science as the search for absolute or intrinsic truth” proper to naive philosophies of knowledge, which place the subject in an absolute position.

On the other side, if theorizing is production of information and this refers to Turing's or Shannon’s elaboration or transmission of information, we move from intelligibility to *formal normativity.* Typically, we understand all trajectories, including the trajectory of an amplitude of probabilities in Schrödinger’s equation, as geodetics in suitable phase spaces. These locally maximize a gradient and the integral of these gradients yields the optimal path. However, this is just a principle of intelligibility, as mentioned above: in no way can we, the observers, know exactly and even less force where the object will actually go. The actual path depends also on local fluctuations possibly below measurement. This is very different from a computational rule, which describes *and* prescribes: go from point (5,3,7) to (8,1,3) in space, say. This is exactly what a program does on your screen, even when simulating the wildest non-linear dynamics. Nothing moves on a digital computer’s screen. The formal rule based implementation, a term re-writing system, prescribes exactly how signs/pixels must be written and then re-written, that is switched on-off, in order to implement movement[16].

By contrast, we understand how falling objects go by the gradient principle, and this according to the context (local frictions, fluctuations …), often with little predictability. Instead, the computer implemented image of that very object will follow exactly the pixel by pixel rule: frictions, fluctuations … are either excluded or explicitly formalized in the rules. The writing of the equations

15 “Subjective-absolute and objective-relative seems to me to contain one of the most fundamental epistemological insights that can be extracted form natural sciences” (Weyl, 1949)

16 Note that also declarative or functional languages are “imperative”: they apply reduction rules to get to “normal forms” or implement the equality (and this on the grounds of fundamental theorems, such as Normalization (Girard et al 1989) and Church-Rosser (Barendregt, 1984)).

does not norm, but describes. The writing of a program describes *and* norms. Moreover, equations, as sequences of signs, do not move; instead, programming is a writing and re-writing sequences of signs, including the programs themselves, the extraordinary idea underlying Turing's invention - and fully expressed in lambda-calculus by programs acting on themselves and on their types, see (Barendregt, 1984), (Girard et al, 1988), (Asperti and Longo, 1991). Compilers, operating systems, interpreters … are all written in sequences of 0s and 1s, like all programs, in one dimension, a line of discrete entities: they are implementations of Turing's Universal Machine, the beautiful invention of a major mathematical invariant - the class of computable functions. Mystics transformed it into an absolute.

Again, the focus on the production of information while doing science, first by measuring, is one thing – it may help to clarify and unify in different sciences the role of the knowing subject (Rovelli 2004), (Zeilinger 2004). Claiming instead that theorizing is sign pushing on discrete data types (Crutchfield 1994) is another: the structures of intelligibility are then transformed into normative frames. Of course, one reaches the bottom line when claiming that a body falls because it is programmed to fall: "We can certainly imagine a universe that operates like some behaviour of a Turing machine" (Wolfram, 2013)[17].

One more issue. It should be clear that in theoretical universes of discrete signs there is no classical randomness. Discrete data types, where one can only count, are given exactly, and access to data is exact. Classical randomness requires the interplay of *non-linearity*, or some form of mathematical chaos, and *approximated measurement* as an interval (Devaney 1989). Of course, Quantum Physics adds the intrinsic indetermination of measurement, up to Planck's h, and of the discrete spin-up or down of a quanton, say. But, Turing Machines and von Neuman's Cellular Automata (CA) or similar devices are treated classically[18]. Now, in classical discrete state machines there is no unpredictability: they just "follow the written rule". Typically, if one iterates (restarts) a programmable discrete state machine, including CAs, on the same initial data, one obtains *exactly the same trajectory*. Instead, a necessary property of random processes, in physics, is that, when iterated in the "same" initial/border conditions (the same intervals of measurement), in general, they do not follow the same trajectory. In digital machines there is no randomness, at most noise, and this is successfully eliminated. Even in networks of computers, given in space-time continua where noise is massively present, it is largely reduced, it is "do not care" (Longo et al. 2010) – a remarkable feat of network and concurrent computing (Aceto and al. 2003). Finally, "Indeterministic" Turing Machines are just "one input – many outputs" machines: once the latter is encoded in one number, they become faster, but perfectly deterministic machines (Longo et al. 2010).

As a consequence, there is no "emergence" in deterministic discrete state machines, including CAs. Emergence should, at least, imply unpredictability, thus randomness (Calude and Longo 2016): instead, restart your CA, on the same, exact, discrete, initial data, and you get exactly the same complicated "emergent" shape – noise and statistic must be introduced on purpose, from outside. Moreover, the definition of randomness as irreducibility in (Crutchfield 1994) and

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17 Unfortunately, this attitude is shaping minds. A student in mathematics, in my top institution in higher education (ENS, Paris), asked me not long ago: "how can the Universe compute at each instant where it will go in the next instant?". I answered: "there are plenty of very small Turing Machines, hidden everywhere, that do the computations!". And I wrote a letter to Alan Turing (Longo, 2018t) - he explicitly and radically departed from these views (the brain ... the Universe, are surely not discrete state machines, i.e. Turing Machines, he observes - see references in my letter).

18 Quantum Computing formally introduces a major feature of Quantum Mechanics, entanglement (Zorzi 2016). By this, it forces a connection between hardware and software, a major computing and engineering challenge, beyond Turing's fruitful split, hardware vs software, perhaps along the way of a major revolution in computing.

(Wolfram, 2002) is ill-defined in two dimensional CAs, as a mathematical invariant: it depends on coarse graining (Israeli and Goldenfeld 2004) and it is subject to "speed up" - in more than one dimension there is no lower bound in program size, by using concurrent processes, see (Collet et al. 2013). Finally, even when incompressibility is well defined, as it is on one dimensional, finite strings of numbers a la Kolmogorof, it does not corresponds to physical randomness[19].

In short, von Neuman universes of exact formal rules handling information on discrete data types are the grounds for von Neuman's nightmare games in life, economy and wars, a formally normed Informational Universe that still dominates the collective imaginary: just follow the meaningless rule on discrete data and iterate exactly – any classical dynamics on discrete space and time is perfectly predictable, it is normed once and for all, it is computable, it is thus doomed to iterate identically on the same data, for ever. Fortunately, these nightmares are slowly fading away even inside one of the areas where the de-humanization of knowledge construction was first brought to the limelight, AI.

6.2 – Back to geodetics in Artificial Intelligence and to sense construction

Hilbert's attitude to mathematics often wavered between a strict formalist credo, as recalled by H. Weyl (see the footnote above) and a practice of inventive and meaningful constructions, in actual work. In a sense, he wanted to get rid of the foundational issue for good ("a definitive solution"): once consistency and completeness of major formal theories were proved, one could work freely in "Cantor's paradise", rich in infinities, intuition and sense (Hilbert, 1926). In formalist mathematical circles, formal deduction is rarely extended to an understanding of human general cognition and the threshold of interpretation and meaning is often crossed. Yet, the myth of intelligence as sign pushing has been at the core of Classical Artificial Intelligence, of which (Turing, 1950) is considered the founding paper. Note that Turing explicitly says that "The nervous system is certainly not a discrete-state machine" (p. 451). He thus invents an "imitation game" whose aim is to simulate human intelligence independently of how the brain could actually be organized, but in such a way that an interrogator would not be able to make a difference between his machine and a woman – in 30% of the cases, says Turing, for a game no longer than five minutes and ... by year 2000 (an enormous literature is available on this paper, I dare to refer to my personal letter to Turing also for references, (Longo, 2018t)).

Soon after 1950, Hilbert's deductive formalisms and Turing's cautious imitation of cognitive capacities were transformed in a vision of intelligence as formal operations on signs, since "a physical symbol system has the necessary and sufficient means of general intelligent action." (Newel and Simon, 1976). The biological structure of the brain had no interest in this perspective

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19 Many projected into nature Kolomogorof's effective notion of "incompressibility" for finite strings, as the dual component of predictive determination, i.e. as randomness. A finite sequence or string of letters/numbers *w*, say, is *incompressible* if no program to generate it is shorter than *w* (Calude 2002). Today, this is a crucial notion since our machines need to compress strings. However, it does not describe physical randomness, as thought by many, yet another abuse of a (remarkable) computational invention. Only asymptotically, infinite random sequences, as defined by Martin-Löf, relate to classical and quantum randomness (Calude, Longo, 2016). This is to be expected: limit constructions already unified, by Boltzmann work, random particle trajectories and thermodynamic entropy (Chibbaro et al. 2015). In the finite, all sufficiently long strings are compressible, by Van der Waarden theorem (Calude, Longo 2017). Thus, in terms of incompressibility, there would be no long random sequences …. And a string to be generated, in the *future* (next year lottery drawings), is blatantly not random if one can compute, *now,* just one element of it, independently of its ''a posteriori" compressibility or not. Once more, there is no way to analyze such a meaningful notion for physics, randomness for finitely many events, in abstract computational terms: one has first to specify the intended theory, as an interpretation and organization of (a fragment of) nature, and then define randomness relatively to it (Calude and Longo 2016).

that dominated AI for decades. In contrast to this approach, a few tried to “model”, not just imitate, brain activities as abstract “nets of neurons”, (McCulloch and Pitts 1943), (Rosenblatt 1958). Mathematical neural nets function by continuous deformations and reinforcement of connections, following the ideas in (Hebb 1949). This *connectionist* modeling was rather marginalized for decades, or its nets were considered only for their computational power: as for input-output functions on integer numbers, they compute no more than a Turing Machine[20].

In the late '80s, it was observed that an animal brain is actually in three dimensions. Multi-layer neural networks were then invented for recognizing patterns by filtering and back propagation algorithms over the layers (Boser et al 1991). Very refined gradients methods entered this new, radically different AI, including difficult “wavelets” techniques from mathematical physics (Mallat and Hwang, 1991), (Mallat 2016). By these methods, in continuous structures, invariants of vision may be constructed: roughly, after inspecting thousands, millions of images of the same object under different perspectives, a sort of “optimal” result or fundamental invariant structure of that object is singled out – a geodetic that results from the depth of the many layers (thus the well selling name, Deep Learning). The mathematical challenge is a major one as well as the implementation on discrete state machines – the usual problems of approximation and shadowing pop out, as these methods are mostly non-linear (see above). Moreover, they are generic, in the sense that, so far, they uniformly work for the recognition of images, sounds, language ….

It is interesting to note that the more these techniques advance and work (there exist already fantastic applications), the more the geometric and topological structures and methods they use radically depart from the brain structure. The visual cortex is the best known, and its “neuro-geometry” is better understood as a complex manifold of multidimensional hyper-columns, far from multi-layered networks (Petitot 2017). Moreover, different brain functions are realized by other, much less known, yet very different structuring of neurons (Cant and Benson 2003). Finally, emotions, (Violi 2016), or pregnances, (Sarti et al 2018), that is meaning, essentially contribute to the shaping and the sensitivity of the brain: that is, its dynamic structuring depends also on emotions and meaning, as its activity takes place only in the brain’s preferred ecosystem, the skull of a material living body, in interaction with an ecosystem and on the grounds of an evolutionary and individual history that gives meaning to its frictions with the world.

7 – Input-output machines and brain activity

In a time when fantastic discrete state machines for elaborating and transmitting information are changing science and life, we need to focus with rigor on the notion of information. The limits of the arithmetising linguistic turn, which spread from the foundations of mathematics to biology and cognition, has been shown from inside several times. In Logic, by purely formal methods, Gödel, Church and Turing’s (very smart) diagonal tricks proved its incompleteness from within arithmetic itself, (Longo, 2010; 2018t) – we can do better today, (Longo 2011). Biology is, slowly, undergoing a major paradigm shift away from the myth of the completeness of the alphabetic coding of the organism in DNA – the decoding of DNA greatly helped in this, as hinted above. Classical sign pushing AI is being silently replaced by non-trivial mathematical methods in

20 A red-herring, the so-called *extended* Church Thesis, often blurred computational novelties: any physical finite structure computes at most Turing computable functions. Now, mathematical neural nets are not meant to compute number-theoretic functions. Of course, in this and other cases, if one forces a physical dynamics to take a digital input and then produce at most one digital output and formalizes the dynamics “a la Hilbert”, one can prove that that formal system computes no more than a Turing Machine. This says nothing about the proper expressivity and possible functions of a continuous dynamics of networks (see below), whose job is not to compute functions on integers (see below for more). A similar trivializing game has been played also with Quantum Computing and Concurrent Networks (Aceto et al 2003).

continua etc. where "information" is elaborated in a totally different way, before being passed over to digital computers.

As for the latter issue, note that an input-output machinery underlies both classical AI and Deep Learning. Also multi-layers neural nets are *a priori* static. They receive inputs and then output invariants of vision, audition, language etc. Moreover, so far, their continuous dynamics must be encoded in discrete state machines.

The animal brain works in a totally different way. First, it is always active, indeed, super-active. The friction with the world, by the body, canalizes, constrains, selects its permanent re-structuring and activities. Neuron's continuous and continual deformations of all sorts include electrostatic critical transitions, that are too poorly schematized as 0 or 1, as well as moving synaptic connections. In case of sensory deprivation (perfect darkness and silence, lack of skin sensations … a form of torture), one gets crazy by the increasing chaoticity of neural activities. Even the formation of the visual cortex, the best known so far, goes by, first, an explosion of connections, later selected by their activity (Edelman 1987). Action on the world, beginning with the restless uncoordinated activities of the newborn, is at the origin of the construction of meaning, as hinted in sect. 1: meaning results from an active friction with the world. Moreover, as analyzed in (Violi 2016) as for humans, the mediation of the mother contributes to the earliest form of interpretation of the environment: the baby feels or stares at the mother in order to make sense, in the interaction, of a new event. Meaning, in humans, requires and accompanies the historical constitution of the individual. Emotions and the material bodily structure contribute to it: this skin, smell, cells' membranes, chemical structure of DNA ....

And this is a crucial issue. All theories of information mentioned above, are based on an essential and radical separation of signs from their material realization: the great idea by Turing to formalize the split hardware vs software, practiced since Babbage and Morse or earlier, is at the core of computing as well as of Shannon's transmission of information. The same signs may be transmitted and elaborated by drums, smoke signs, electric impulses, wave frequency modulations … valves, diodes, chips …. In general, we use gradients of energy or matter for this purposes. The claim that in nature, in cellular chemical exchanges for examples, there is "information" any time the variation of a gradient matters more than an energy or matter flow, is an amazing anthropomorphic projection. In the perspective of the immateriality and uni-dimensionality of discrete information, the radical materiality of life, its intrinsic space dimensionality are lost in vague abstract notions of information, rarely scientifically specified. Some of the consequences are hinted in (Longo, 2018c).

It may be appropriate now to ask whether the construction of invariants of vision, for example, in Deep Learning, soundly models the brain's behavior, once observed that it does not model its architecture nor biological activity. Memory or, more generally (pre-conscious) retension, selects the relevant invariants by forgetting the details that do not matter, both at the moment of the memorized action and when memory is used to act in a new context. Retension exists for the purpose of protension (Berthoz, 1997), (Longo and Montévil, 2014), e.g. for capturing the prey by preceding its trajectory. Forgetting is crucial to animal memory: the child must remember what matters of the trajectories of a ball in order to grip it, not the color of the ball, say. We recognize a friend 20 years later because we remember what mattered to us, his/her smile, the expression, a movement of her/his eyes. These are fundamental cognitive constructions of invariants of action and intentions. Intentionality and pre-conscious protension are at the core of them, while they are based on and constitute meaning: because of the protensive nature of action, the friction on the world interferes and give sense to its deformations. With no emotion nor protensive affection, without the joy of gripping a ball thrown by the father or the need to capture a prey, the selection of what matters for an aim is hard to conceive: what is interesting for action and what may be relevant to recall for a new activity is largely related to the meaning we attach to an object of desire. It is hard to see anything of this nature, surely not a model, in the gradients' or wavelets' methods used by the excellent mathematics of multi-layered neural nets. Yet, they may provide increasingly effective imitations of key cognitive activities. As for the biological brain, note finally that its continuous material deformations are not implemented by a digital computer "in the background", in contrast to the non-linear mathematics of Deep Learning.

8 - A societal conclusion

Besides a hint to the Geometry of Information and to neural nets, I only quoted the main mathematical frames for elaboration and transmission of information on *discrete* data types. By a historical simplification, I named them from Turing and Shannon. Wiener's approach (1948) belongs more to, and greatly enhanced, the theory of continuous control, based on the calculus of variations and other areas of optimization theory, (Rugh 1981), (Sontag 1990). The harmony of a noiseless world is obtained by smooth feed-back control of a dynamics, with a prefixed goal. Besides the construction of analogical machines, the social aim is explicit: a harmonious society governed independently of ideologies and passions, adjustable, by a fine tuning in continua, to the smooth governance of any political system, once its goals are given. A world where each society would move along a unique, mathematically defined geodetics, beyond the painful frictions caused by the democratic debate and opposing goals[21].

There is a similar commitment, but even more "dry" and radical, in the myth of digital information as ruling nature. Smooth control is replaced by lists of "formal instructions", of pitiless "follow the rule" orders, exactly like in von Neuman's Automata. As often in science, from classical Greece to the Italian Renaissance, a vision of nature is also (or first) a perspective on the human condition. This is for example clear when rule-based classical AI is (was) considered an imitation or even a model of the human brain: it was first an image of what human cognition is, a rule based sign pushing (see the quotations and references above).

The formalization of information as a new observable by the new fantastic sciences of elaboration and transmission of data has had a role in this. In order to be implemented in machines, information was separated from its use as meaningful symbolic human exchange, from our biological body and its historicity. It now seems to provide tools that float in between two possible abusive transfers to the governance of human societies, I discuss below. Their common effect or, perhaps, aim is to subtract the economic, thus social and political decisions, to the democratic debate, anchored on possibly different interpretations. It is now beginning to modify human justice, increasingly affect by automated tools (Garapon and Lassègue 2018), while the human and historical "interpretation of the law" is crucial.

One of these abuses is the transfer of the mathematics of physical equilibrium processes to economic equilibria, since (Walras 1874), now enriched by the informational terminology[22]. Control theory is a variant of it and some more general versions in non-linear mathematics pretend to be applicable to all sorts of social dynamics, an approach also inaugurated by (Wiener 1954). The other abuse is the new role of information sciences in finance. Non-trivial mathematics, see (Merton, 1990), (Bouleau, 2017), (Biasoni 2020), joins sophisticated algorithms in elaborating abstract market information. Largely automatized information processing even more radically excludes human interpretation and social meaning from the financial dynamics that drive economy[23].

In either case, a supposed (mathematical, numerical) objectivity leaves no alternative: when freely moving along a geodetics, the unique optimal path, with no human/economic meaning nor

21 See (Supiot 2017) for the difference between democratic government and governance, and (Longo 2018d) for the entanglement of science and democracy.

22 "Optimal pricing of goods in an economy appears to arise from agents obeying the local rules of commerce" (Fama 1991), quoted in (Crutchfield 1994), where it is stressed: "global information processing plays a key role".

political friction, economy and finance cannot go wrong. Its implementation by formal rules, in digital machines, even further dehumanizes the government of the common house (the Greek oiko-nomos) as well as human conducts. In my view, the new hegemony of this autonomous universe of senseless financial data, in the governance of the world, plays a key role in encouraging the unbounded diffusion of the informational and computational terminology, superposed to all phenomena, independently of meaning and interpretation. The cultural hegemony of the "all is computation and digital information" fashion is further reinforced by today's general role of our extraordinary information networks and their computing machines. For example, what do Ladyman et al. (2008) mean when claiming that "what cannot be computed cannot be thought" (p. 209)? Are they strictly referring to computing as what can be implemented in networks of digital computers? Or do they refer to some broader notion of computing that would vaguely include the *thinkable meaning* produced by a dancer, a painter or by the "geometric judgments" needed to prove the formally unprovable statements of number theory (Longo, 2011)? They seem to lean more towards the first interpretation and, thus, say: the construction of sense that cannot be produced/computed by the machines currently owned/developed by Google, Apple, Microsoft etc. *cannot be thought*. Perhaps, this is more a "normative" statement than a scientific analysis; that is, in view of the role of these corporations, this claim is meant to become a norm: you are not allowed to think what the GAFAM cannot compute - the digital information they handle *is* the/your world, and nothing else.

We need to oppose this trend and analyze and work at meaningful knowledge constructions, as the result of an historical formation of sense. An analysis of the historicity of biological evolution may be one of the possible links of science to the humanities, with no subordination. A modest attempt is carried on in the interplay we hinted to in (Koppl et al, 2015), by stressing the role of changing spaces of possibilities (phase space) and rare events, further analysed in biology in (Longo, 2017); this role is shared by theorizing both in evolution and in the historical humanities, economy in particular. Some mathematics is being invented on these and on semeiotic grounds in (Sarti et al, 2018) - see (Montévil et al, 2016) as for variability and diversity production in biological morphogenesis. These approaches to "heterogenesis", as called in (Sarti et al, 2018), are a tentative way to depart from an analysis and a governance of biological processes and of our communicating humanity by optimal paths in pre-given spaces of possibilities, towards pre-given goals, possibly by mechanical rules.

We may then conclude by quoting Simondon (1989, p.272): "La machine peut se dérégler et présenter alors les caractéristiques de fonctionnement analogues à la conduite folle chez un être vivant. Mais elle ne peut se *révolter*. La révolte implique en effet une profonde transformation des *conduites finalisées*, et *non un dérèglement de la conduite*[24]."

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23 Statistical analysis of "abstract" data govern finance and, thus, economy, (Bouleau 2017). These analysis are independent from the underlying assets, that is from any reference to "tables, chairs, mugs", as Hilbert would put it (footnote 3).

24 "The machine may be out of order and present the operating characteristics similar to a mad behavior of a living being. But it cannot revolt. The revolt indeed implies a profound transformation of the finalized behavior, and not a malfunction of the conduct".

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