CAUSES AND SYMMETRIES IN NATURAL SCIENCES.

THE CONTINUUM AND THE DISCRETE IN MATHEMATICAL MODELLING[1].

Francis Bailly
Physique, CNRS, Meudon
bailly@cnrs-bellevue.fr

Giuseppe Longo
LIENS, CNRS – ENS et CREA, Paris
http://www.di.ens.fr/users/longo

Introduction

How do we make sense of physical phenomena? The answer is far from being univocal, particularly because the whole history of Physics has set, at the center of the intelligibility of phenomena, changing notions of *cause*, from Aristotle's rich classification, to which we will return, to Galileo's (too strong?) simplification and their modern understanding in terms of "structural relationships" or the replacement of these notions by structural relationships. It is then an issue of the stability of the structures in question, of their invariants and symmetries, ([Weyl, 1927 and 1952], [van Fraassen, 1994]). To the point of the attempt to completely dispel the notion of cause, following a great and still open debate, in favor, for instance, of *probability correlations* (in Quantum Physics, see, for example, [Anandan, 2002]).

The situation is even more complex in Biology, where the "reduction" to one or another of the current physico-mathematical theories is far from being accomplished (see [Bailly, Longo, 2006]). From our point of view, the difficulties in doing this reside as much within the specificities of the causal regimes of physical theories – which, moreover, differ amongst themselves – as in the richness specific to the dynamics of living phenomena. Our approach, as presented in [Bailly, Longo, 2006], has attempted to highlight certain aspects, such as the intertwining and coupling of levels of organization, which are strongly related to the phenomena of autopoiesis, of ago-antagonistic effects, of the hybrid causalities often mentioned in the theoretical reflections in Biology (see [Varela F., 1989; Rosen R., 1991; Stewart J., 2002; Bernard-Weil E., 2002; Bailly, Longo, 2003]).

We will now return to some aspects of the construction of scientific objectivity, as explication of a theoretical web of relationships. And we will mostly speak of causal relationships, since causal links are fundamental structures of intelligibility. Our approach will again be centered upon symmetries and invariances, because they enable causes to manifest themselves, namely by the constraints they impose. In a strong sense, they thus present themselves as conditions of possibility for the construction of mathematical or physical objectivity.

Now, if mathematics is constitutive of physical objectivity and if it makes phenomena intelligible, its own "internal structure", that of the continuum, for example, as opposed to the discrete, contributes to physical and biological determination and structures their causal links. To put it in other words, mathematical structures are, on the one hand, the result of a *historical formation of meaning*, where history should be understood as the constitutive process from our phylogenetic history to the construction of intersubjectivity and of knowledge within our human communities. But, on the other hand, mathematics is also *constitutive of the meaning of the physical world*, since we make reality intelligible via mathematics. Particularly, it organizes regularities and correlates phenomena which, otherwise, would make no sense to us. The thesis outlined in [Longo, 2007] and which we further develop here, is that the mathematics of continua and discrete mathematics, the latter characteristic of computer modelling, propose different intelligibilities both for physical and living phenomena, particularly for that which concerns causal determinations and relationships as well as their associated symmetries/asymmetries.

---

[1] To appear in **More Geometrico**, (Giorello et. al eds) Elsevier, 2007. A preliminary french version of this chapter will appear in "*Logique et Interaction: pour une Géométrie de la Cognition*", (Joinet ed.) Presses Universitaires de la Sorbonne, 2006 and as a chapter of [Bailly, Longo, 2006].

In a final section, we will attempt to address the field of Biology by questioning ourselves about the operational relevance and status of the concepts thus under consideration. But in this text, we will first propose to illustrate, in the case of Physics, the situation which we have just summarily described. This will enable us to "enframe" physical causality and to compare it to computational models and to Biology.

1. Causal structures and symmetries, in Physics.

The representation usually associated to physical causality is oriented (asymmetric): an originary cause generates a consecutive effect. Physical theory is supposed to be able to express and measure this relationship. Thus, in the classic expression **F** = m**a**, we consider the force **F** to "cause" the acceleration **a** of the body of mass m and it would seem downright incongruous, despite the presence of the equality sign, to consider that acceleration, conversely, may be at the origin of a force relating to mass. Yet, since the advent of the theory of General Relativity, this representation found itself to be questioned in favor of a much more balanced interactive representation (a "reticulated" representation, one may say): thus, the energy-momentum tensor doubtlessly "causes" the deformation of space, but, reciprocally, the curvature of a space may be considered as field source. Finally, it is the whole of the manifesting network of interactions which is to be analyzed from the angle of geometry or from that, more physical, of the distribution of energy-momentum. It is that an essential conceptual step has been made: to the expression of an isolated physical "law" (expressing the causality at hand) has been substituted a general principle of Relativity (a principle of symmetry) and the latter re-establishes an effective equivalence (interactive determinations) where there appeared to be an order (from cause to effect).

Here is an organizing role of mathematical determination, a "set of rules" and a reading which is abstract, but rich in physical meaning. Causes become interactions and these interactions themselves constitute the fabric of the universe; deform this fabric and the interactions appear to be modified, intervene upon the interactions themselves and it is the fabric which will be modified.

We will first of all distinguish between *determinations* and *causes* as such. For instance, we will see the symmetries proposed within a theoretical framework as related to the determinations which enable causes to find expression and to act; in this they are more general than the causes and are logically situated as "prior" despite having been established, historically, "afterwards" (the analysis of the force of gravitation, as cause of an acceleration, preceded Newton's equation).

Let's then specify that which we mean by "determination" in Physics, enabling us to return to the causal relationships which we will examine extensively. For us, all these notions are the *result* of a construction of knowledge: by proposing a theory, we organize reality mathematically (formally) and thus constitute (determine) a phenomenal level as well as the objectivity and the very "object" of Physics. We will therefore address first of all the "objective and formal determinations", particular to a theory.

More specifically, once given the theoretical framework, we may consider that:

D.1 The objective determinations are given by the *invariants* relative to the symmetries of the theory at hand.

D.2 The formal determinations correspond to the set of *rules and equations* relative to the system at hand.

To return to our example, when we represent the dynamic by means of Newton's equation, we have a formal determination based upon a representation of causal relationships, which we will call "efficient" (the force "causes" acceleration). However, when having recourse to Hamilton's equations we still have a formal determination, but one which refers to a different organization of principles (based on energy conservation, typically). It is still different with the optimality of the Lagrangian action, which refers to the minimality of an action associated to a trajectory. In this case, we have, for classical dynamics, three different mathematical characterizations of the events; and it is only with the advent of the notion of "gauge invariant" (that is, of "relativity principles") that these distinct formal determinations have been unified under an overreaching *objective determination*, related to the corresponding symmetries and invariants (manifested by transformation groups, such as the Galileo, Lorentz-Poincaré or Lie groups). A single objective determination then, for instance, the movement of a mobile with a certain mass, may account for (result from!) distinct formal determinations, based upon the concepts of force, of energy conservation and of geodesics, respectively. In the first case, the invariant is a property (mass), in the second, it is a state (energy), in the third it is question of the criticality of a geodesic (action, energy multiplied by time). If the final results of the mobile's dynamic may thus be the same, on the other hand the equations leading to them may take quite different forms unifying only under the even larger constraint of objective determinations (relating, in our example, to a mass in movement).

It is in fact the physical *objects* themselves which are the consequence of – given by – these determinations. More specifically, the physical objects are theoretically characterized by that which we designate, rather commonly, as properties and accessible states:

O.1 Properties (mass, charge, spin, other field sources…),

O.2 Accessible states, potential or actual (position, moments, quantum numbers, field intensity…),

being understood that their specific values essentially depend on empirical measurement. To highlight as simply as possible the difference we make between property and state, by means of their invariance characteristics, we may say that properties (which characterize an object) do not change when the states of the object change; conversely, if the properties change, it is the object itself which is modified.

These objective determinations thus constitute in a way the referential framework, at a given moment in time, to which are related *experience, observation and theory*, enabling to interpret and to correlate the ones to the others. In themselves, and as we have just indicated, they thus do not completely characterize the objects they construct, but constrain – among other things by extricating invariants – properties and behaviors. Thus, for instance, they impose the fact that there is a mass (sensitive to the gravitational field), but without nevertheless fixing the magnitude of this mass or, as we shall see, the manifestation of fields such as the electro-magnetic field. We are therefore facing properties which we may qualify as "categorical" and qualitative, but without necessarily specifying the associated quantities which quantitively characterize the object in direct relationship to the measurement. This is also the case for that which we call accessible states: their structure is qualitatively characterized, but the fact that the system quantitively attains such or such of these theoretically determined possible states depends on empirical factors.

Why distinguish here between properties and accessible states? It would enable us to understand as *cause*, in the traditional sense (which after Aristotle we will call "efficient cause"), all which affects (can modify) states; while we may consider that in the traditional approach, *the invariants of efficient causal reduction* are constituted by the set of properties. However, these very properties participate to a causality, which we shall relate to "material" causality.

So let's attempt to refine the analysis, not only by distinguishing between different types of "causes" but also by trying to affect the distinct elements of objectivity. Let's agree that, relatively to the effect of an object upon another:

C.1 The **material cause** is associated to the set of *properties*;

C.2 The **efficient cause** is correlated to the variation of one or more *states*.

We can recognize here a revitalization of Aristotle's classification, so dear to René Thom. In fact, if we want to maintain a parallel with Aristotelian categorization, let's observe that we have called formal determination that which the modern interpretation of the Philosopher would designate as "formal cause" (that is, that which corresponds to the set of theoretical constraints which define and measure the effects of other causes – laws, rules, theories,…)[2]. In our approach, it is the determinations,

[2] In the debate with I. Prigogine concerning determinism, R. Thom highlights the role of structural stability, even within the framework of highly unstable dynamics (the forms are maintained, all the while being deformed). It is the equations of the dynamic which determine their possible evolutions (as formal causes – determinations, for us). On the other hand, Prigogine

formal and objective, which produce the specification of objects, by means of the notions of properties and states (of which the structures and variations participate to the material and efficient causes, respectively). With regard to the causes, we will preserve the Aristotelian terminology, although material causes may be classified as "material structures". Indeed, a change in properties changes an object, as we mentioned earlier, but, at the same time, it induces – it causes! – a change of states. For example, a change in mass or charge, *in an equation*, modifies the values of the acceleration or of the electrical field.

1.1 Symmetries as starting point for intelligibility.

From the point of view we have just developed, may we consider that constraints of symmetry stem from causal constraints? According to our distinction and as we have just specified, symmetries emerge from the determinations (under the form of systems of equations, typically) where the causes manifest. Their greater generality thus also imposes itself through the relation to laws corresponding to the formal determinations (which, for example, take such or such expression according to the selected gauges). To put it lapidarily, using the example we will discuss below (Intermezzo): the phase's global gauge invariance *determines* the charge (a property) as a conserved quantity of the theory and its local invariance *determines* the existence of the electro-magnetic field (a state) under the form of Maxwell equations. The interactions, described by these equations, may, but only afterwards, be considered as giving us the *causes* (material or efficient) of the observed effects.

In fact, and since Galileo, that which we usually characterize as "causes" seems to correspond mainly to efficient causes, whereas, as we have just seen, the "determinations" seem to rather present themselves as a source common to causes which would derive from them (including material and formal). This results in that we may consider, "transcendentally speaking", the determinations, the symmetries, namely, to present themselves as conditions of possibility for the causes to manifest.

Now it appears to us that the natural sciences, with the exception of the biosciences, may be part of the conceptual framework we have just drawn out, including that which corresponds to extremalisation rules (the geodesics of the Lagrangian), which appear, but wrongly so in our opinion, to confer a tinge of *finality* to the processes which they model. It is only with living phenomena that the taking into account of a sort of "final causality" (to put it once more in Aristotelian terms, see [Rosen R., 1991; Stewart J., 2002]), that we have characterized elsewhere as a "contingent finality" (see also 3.1 below) and as locus of "meaning" for any living phenomenon, really becomes relevant. It is this which we will attempt to examine later, in paragraph 3.

1.2 Time and causality in Physics.

We have thus attempted to specify, very generally, the notions of objective determination, of object and physical cause, from the notion of symmetry and, more specifically, from the notion of invariance with regard to the given symmetries.

Let's also observe that, since about a century, in Physics, the laws of conservation, as formal determinations, are understood in terms of spatio-temporal symmetries; for instance, the conservation of the angular momentum is correlative to the symmetry of rotation (it is Noether's theory which is at the origin of this great theoretical and conceptual turning-point, see the Intermezzo below).

But at this stage and before continuing, it would seem appropriate to introduce a distinction in the view of clearing some possible confusion with regard to the representation of causality and to the reasoning that one may entertain about it. We propose, so as to distinguish[3], namely in the case of efficient causality, between *objective causality* and *epistemic causality*.

---

highlights the play between locally stable structures and global systems where small, amplified fluctuations induce the choice of one of these evolutions. While preserving his new view on Aristotle's finesse, but in a different way than R. Thom, we do not attribute to these different notions of causality an ontological hierarchy of the platonic type, where the formal determinations (causes) *ontologically* precede the other causes.

[3] As we have already done on the occasion of the approach and the deepening of the concept of "complexity" [Bailly, Longo, 2003].

Objective causality is associated, in our opinion, to a rather essential constraint, which is constitutive of physical phenomena, and which is the irreversible characteristic of the unfolding of time (that which we call the "arrow of time"). But even in the case where temporality does not explicitly appear, it continues to underlie any change, any process as such – including that of measurement – and constitutes in this respect a foundation to any conceptualization, observation or experience, from the moment that such a process is considered. That is to say, this time from the angle of causal analysis, that *time is constitutive of physical objectivity*.

In contrast, epistemic causality is to be considered as independent of an arrow of time. For instance, the analysis of a phase transition in function of the value of a parameter (such as temperature) does not refer to any specific temporality. It is in a way the atemporal and abstract variation of the parameter that "causes" the transition, be it in one direction (for instance, from liquid to solid) or the other (from solid to liquid). At this level, the invoked structure of causality (effect of the change of temperature on the state of the system) remains independent of the time factor, even though, at another level, it is indeed over time that the effective variation of this parameter occurs – in one direction or the other. This is also the case in the very simple example which is the law of perfect gases (pV = RT, where p represents pressure, V the volume, T the temperature and R Joule's constant). This law is independent of time and one may conceive of various "causes" at the origin of a variation in volume, for instance, leading, under constant temperature, to a variation in the pressure associated to the occurrence of a chemical reaction. The concomitant (and symmetrical, given the equational relationship) variations in volume and pressure may be considered as the causes – of the epistemic type – of one another (in fact, these variations may be said to be "correlated"), in contrast to an objective causality – temporalized, this time – which would find its source in the temporal unfolding of this chemical reaction at the origin of the considered variation in volume (our distinction may possibly help to understand the discussion in [Viennot, 2003; appendix]).

May such a distinction between the objective and the epistemic, which seems to clearly correspond to a reality in the case of an efficient causality (associated, let's remember, to modifications in the states of a system, as we have just illustrated) still be applicable to material causality?

It does appear that for material causality, one may find examples demonstrating that such is the case, inasmuch as the properties in question have different expressions depending whether they are related to their own specific system or to an external reference. This is the case in Relativity, for example, where the mass (or life-span) of particles depend on their speed with regard to the laboratory reference: in the *internal* system, the rest mass remains a characteristic property of the particle's very identity (m_0), whereas within a referential animated by a speed v with regard to the system as such, the mass takes on an epistemic character, of which the measurement is $m = m_0/(1 - v^2/c^2)^{1/2}$, where c represents the speed of light (for light itself, this also that which enables to consider that the photon's mass is null, while its energy is non-null and while Einstein's relation establishes a direct relationship between mass and energy). Likewise, the "efficient mass" which we calculate following the process of renormalization (which, in order to eliminate infinites from the calculi of perturbation, integrates with the mass some classes of interaction) takes an epistemic character with regard to the mass itself which preserves its own objective character. In this sense, we may consider that the properties which are located at the source of material causality retain an objective character in their internal system all the while acquiring an epistemic character if we relate them to different referentials.

Because we take the arrow of time into consideration while characterizing efficient objective causality, we distinguish ourselves from certain trends in relativistic and Quantum Physics that exclude such an arrow, in order to preserve any relationship by symmetry. In these approaches, the causal relationships are replaced by other concepts, for instance, in Quantum Mechanics, by probability correlations (see [Anandan, 2002], among others). The reason for this differentiation, beyond the elements of analysis we have just exposed, appears to be crucial in an epistemological respect: we will indeed often refer to dynamic systems (thermodynamic and of the critical type) and will also address certain aspects of Biology. Now, there is no analysis of these systems, even less of living phenomena, which can be performed without taking into account the existence of an arrow of time. Particularly, there would be no phylogenesis, no ontogenesis, no death… in short; there would be no life without time, oriented and irreversible. The processes of life impose an arrow of time, be it only for the thermodynamic effects to which they participate; but it even appears unavoidable to go further, because these processes require a new way of looking at complex forms of temporality, of clocks of life with causal retroactions due to intentional aims, to expectancies and previsions, characteristic of perception and action.

To conclude, our mathematical point of view is that objective determinations are given by symmetries and an efficient cause *breaks* some of the latter, be it, from an objective viewpoint, only that symmetry which is associated to the arrow of time. Reciprocally, irreversible phenomena (bifurcations, phase changes…), which are therefore oriented in time, may be read as symmetry breakings correlated to (new) causal relationships. Symmetries and their breakings therefore remain the starting point for any theoretical intelligibility.

More specifically, we will attempt to understand some causal relationships as symmetry breakings, in a very general and abstract sense. This will, among other things, enable to lay the basis of a coherent foundational framework for the analysis of the different causal regimes proposed by continuous mathematics in comparison to those of discrete arithmetics. This will therefore consist of a mathematical view upon the constitutive role of mathematics in the construction of scientific objectivity; through this approach, we aim to grasp the importance of our digital machines in this construction, since these machines are the practical realization of the arithmetization of knowledge.

The final reflection regarding Biology will bring us back to natural phenomena, in all their causal specificity. Of course, computerized modelling, in Biology as in Physics, remains a fundamental issue. It is precisely for this reason that it must be based upon a fine analysis of the different structures of relationships, particularly causal relationships, proposed within the various theoretical frameworks (physical, biological, of discrete mathematics).

1.3 Symmetry breakings and fabrics of interaction.

It is thus by the means of mathematics that we organize causal links; mathematics makes intelligible and unifies, particularly via symmetries, certain phenomenal regularities, at least those of classical Physics, both dynamical and relativistic systems. But mathematics also makes explicit the symmetries in relation to which probability correlations are quantum invariants.

In the dynamic and relativistic cases, the geodesic principles governing the evolution of systems apply to abstract spaces, "manifolds" endowed with a metric where *symmetry transformations*[4] leave equations of movement invariant[5] (typically, the trajectories defined by Euler-Lagrange equations). It is in this sense that these theories base themselves on invariants with regard to spatio-temporal symmetries: if we understand the "laws" of a theory to be "the expression of a geodesic principle within a suitable space", it is these abstract geodesics which are not modified by transformations of symmetries.

Let's return to the most classical of physical laws: the equation F = ma is symmetrical, as an equation. As we observed above, it is its *asymmetrical* reading which we associate to a causal relationship: the force F *causes* the acceleration a (the equation is read, so to speak, form left to right). We thus break, conceptually, a formal symmetry, equality, in order to better understand, following Newton, a trajectory (and its cause). More specifically, the equation *formally determines* a trajectory of which F appears as the *efficient* cause (it modifies a state, all the while leaving invariant the Newtonian mass, a property). In this sense it becomes legitimate to consider that the equation contributes to the constitution of an objectivity (the trajectory of the mobile), whereas its oriented reading (and the efficient causality it thus expresses) constitutes an interpretation and refers to an epistemic regime of causality.

We therefore propose to consider that each time a physical phenomenon is presented by an (a system of) equation(s), *a breaking in the formal symmetry* (that of equality, by an oriented reading) *makes explicit an epistemic regime of causality*. Particularly, the symmetry breaking in question may be

[4] Objective determinations, in our language.
[5] Formal determinations, for us.

correlated to an efficient cause which intervenes within the formal framework determined by the equation.

Of course, this breaking is not necessarily unique (that by which, namely, it manifests its epistemic character). For example, as we have just evoked in the preceding paragraph, one can read causally and from an epistemic standpoint pV = RT from left to right and *vice versa*. By reference to relativistic systems, we have already read the equation F = ma (or more exactly, its relativistic equivalent), inversely, while highlighting the fact that, reciprocally, the curvature of a space may be considered as a field source. This interpretive reversal, which reorganizes phenomena radically, is legitimate; indeed, in our spatial manifolds, *the transformations* (of gauges), which enable to pass from one referential to another, are supposed to leave invariant the equations of movement, and by doing so they preserve the *symmetries*, but without necessarily preserving the asymmetrical readings of the formal determinations (among which the epistemic causal reading we have just discussed).

We thus propose to consider formal interactions, organized by the asymmetrical structures of equations, but also (efficient) causes, which can be associated to possible asymmetries in the reading of these very equations. Let's observe, once more, that certain coefficients, such as the mass m in F = ma, are correlated to that which we have categorized as material causes (whereas acceleration is correlated to states". And, when an "external" cause (efficient or material) is added to a given determination (equations of evolution), the geodesics of the relevant space are deformed and symmetries associated to this space may be broken, following the variation of states and properties.

Now, this mathematical intelligibility, conceptual fabric of symmetries and asymmetries which correlates the regularities of the world, is constitutive of physical phenomena as well as of scientific objectivity. As we shall see, they profoundly change if the world's proposed reading grid is rooted in continuous or in discrete mathematics. And they must subsequently be enriched, if one hopes to better conceptualize certain phenomena pertaining to life.

Intermezzo: Remarks and technical commentaries.

Inter.1: More on symmetries and symmetry breakings in contemporary Physics.

Let's consider the previously analyzed three great types of physical theories which are the relativistic, the quantum and the critical types (dynamic and thermodynamic systems).

Relativistic theories are essentially tributary of *external* symmetries (sets operating over space-time). Classical mechanics already presents these relativistic traits, with its constraints of invariance under the Galileo group (within the Euclidean space), but it is especially with classical electro-magnetism and Special Relativity that symmetries begin to play a determining role under the Lorentz-Poincaré group (group of the rotations and translations within a Minkowski space). Regarding General Relativity and cosmology, it is the group of the set of diffeomorphisms of space-time which plays the determining role. The corresponding symmetry breakings principally manifest themselves through phenomena of dissipation, or of the arrow of time.

Q*uantum* type theories for their part mobilize essentially *internal* symmetries operating on the fibers of the corresponding fibrates: it is the gauge sets which generate the gauge invariances and which present themselves as Lie groups (continuous groups). In quantum field theory, the most important symmetry breakings (Goldstone, Higgs fields) are considered as sources of the masses of quantons.

C*ritical* type theories constitute theories par excellence of symmetry changes (namely by breakings): it is phase transitions, spontaneous symmetry breakings (or, conversely, of the apparition of new symmetries), of which the effects are processed this time by means of the renormalization semi-group process in order to characterize the critical exponents and to established rules of universality which constitute, in a certain way, over the classes of equivalency which they bring forth, the basis of new relativities and symmetries (very different symmetries may present critical exponents, and thus behaviors, which are identical, depending only on parameters as general as the magnitudes of the prolongation spaces or that of the order parameters).

We will also mention the fact that the unification processes, in the relativistic theories as well as in the quantum theories, or even between themselves, involve the expansion of the concerned symmetry groups (often at the same time as the spaces within which they operate).

Inter.2: From Noether's theorem and physical laws of conservation.

One of the principal foundations of the role of symmetries for Physics can be found in Noether's theorem, according to which any transformation in symmetry, operating upon a Lagrangian and conserving the equations of movements, associates conserved quantities. By a more precise analysis, one may observe that this theorem narrowly couples such laws of conservation – physical invariants, that is, objective determinations – to indeterminations of reference systems (space-time, fibers) by the fact of the principles of relativity and of the symmetries supposed to operate there (for instance, the impossibility to define a temporal or positional origin, an origin of phases, etc.).

One of the simplest and most spectacular cases we may evoke in this regard is that of quantum electrodynamics, for which the gauge group is the phase group U(1). In this case, it is required that the form of the density of the Lagrangian remain invariant under the multiplication of the state vector by a phase term (exp(iL)). The global gauge invariance (L *independent* of the position), conduces, by the application of Noether's theorem, to the conservation of a quantity which we identify to the charge and which corresponds, according to the classification which we propose, to a "property", that is, to a material characteristic. Moreover, the local gauge invariance (L *dependent* of the position) requires, in order to re-establish the broken Lagrangian covariance, to introduce a gauge potential, where there results a gauge field which is no other than the electro-magnetic field itself, expressed by the Maxwell equations (the gauge potential corresponding to its vector potential), which corresponds itself to the source of an efficient causality. Thus, it is indeed the indetermination of any phase origin (an aspect of the referential universe) which very strongly determines the conservation of the charge (an aspect of the determination of the physical object) and, most of all, for local invariance, determines the electro-magnetic field itself, nevertheless generally interpreted as "cause" of electro-magnetic phenomena. Here, one may also notice (but we will not proceed further in the analysis, at this stage) that it is the global gauge invariance which finds itself to be coupled with a property (the charge), whereas the local invariance is coupled with a phenomenon intervening upon the states (with an effect of efficient causality): the field. So there are two forms of invariance, with regard to spatio-temporal symmetries, which we understand as objective determinations of a property and of a state, respectively.

In fact, in theoretical work as in the quest for unification, it is indeed these properties of symmetry – these forms of indetermination of the referential universes – which play an essential heuristic role in the determination of physical phenomenality. As if we were passing from the prevalence of the representation by *efficient causality* to that of a representation by *formal determination* with its symmetries and equational invariants. It is appropriate to recall here the remark, already quoted, by C. Chevalley in his preface to B. van Fraassen's book, where it is question of "*substituting to the concept of law, that of symmetry"*. This is emphasized by van Fraassen himself, when he writes on symmetries "… *I consider this concept to be the principle means of access to the world we construct through theories"*.

We will also note, besides of the continuous symmetries that we have mainly evoked, the important role played by discrete symmetries, as in the CPT theorem, according to which the result of the three transformations T (reversal of time), C (charge conjugation: passage from matter to anti-matter), P (parity, mirror symmetry in space) is conserved in all the interactions, whereas we know that P is broken by chirality in the weak interaction, and that CP is broken in certain cases of disintegration (which led Sakharov to see in this a reason for the weak prevalence of matter over anti-matter and therefore the existence of our universe). We may understand here the particularity of the approaches, in Quantum Physics, which do without the arrow of time ([Anandan, 2002], for instance). The breakings of the CP symmetry are not taken into consideration, enabling to have no asymmetry in the T transformations, and therefore to not have an oriented time factor.

With regard to spontaneous symmetry breakings, we have also evoked phase transitions and, from the quantum viewpoint, the Goldstone fields (for the global level) and the Higgs fields (for the local level), supposed to confer to particles their masses. But it is necessary to emphasize that from a cosmological standpoint, the decoupling of the fundamental interactions between themselves (gravitational, weak, strong and electromagnetic), also constitute such breakings: they then correspond to differentiations which have enabled our material universe to evolve into its current form. Without mentioning the fact that the big-bang itself may be considered as a very first symmetry breaking (due to quantum fluctuations) of a highly energetic void.

But it is doubtlessly with regard to living phenomena that these symmetry breakings play an eminently sensitive role. Thus, Pasteur, lets recall, who had lengthily worked on the chirality of tartrates, did not hesitate to assert: "*Life as it presents itself to us is a function of the asymmetry of the universe and a consequence of this fact"*. More recently, dynamic models involving sequences of bifurcations have also been proposed to represent processes of organization of which living phenomena could be the locus ([Nicolis, 1986; Nicolis, Prigogine, 1989]).

2. From the continuum to the discrete.

Differential and integral equations, as limits, but also variations and continuous deformations, are present everywhere, in the physico-mathematical analyses that we have evoked. From Leibniz and Newton to Riemann, the phenomenal continuum, with its infinity and its limits in action, is at the center of mathematical construction, from infinitesimal calculus to differential geometry: it constitutes the space of meaning for the equations (formal determinations) of which we have spoken, the structure underlying any spatial manifold (Riemannian). However, a discretization, or a representation that is finite, approximated, but "effective", should be possible. It is the dream, implicit in Laplace's conjecture, which will find its continuation in the foundational philosophy of arithmetizing formalisms. If, as Laplace had hoped for the solar system, to a small perturbation always responds a consequence of the same order of magnitude (except in critical situations, cases which are "isolated" – topologically – such as a mountain peak, of which Laplace was well aware), then today it would be possible to organize the world by means of well delimited little cubes (corresponding to the approximation of digital rounding; to the pixels on our machines' screen) and to proceed to the arithmetic calculi upon these discrete values (the encoding of pixels by integers, sequences of 0s and 1s), which would then provide a "complete" theory (any statement concerning the future and the past would be decidable, *modulo* the concerned approximation). Indeed, arithmetical rounding, which associates a single number to all the values contained within a "little cube", does not perturb the simulation of a linear or Laplacian system, because the approximation which is inherent to it is preserved (modulo a linear growth) over the course of the calculi, just as over the course of physical evolution. Let's explain ourselves, because a whole philosophy of mathematical foundations and, in fact, of nature, stems from this approach, with its own view on causality and determination.

2.1 Computer science and the philosophy of arithmetics.

Digital computers are in the course of changing our world, by means of the very powerful tools for knowledge they provide us with and by the image of the world they reflect. They participate to the construction of all scientific knowledge via simulation and the elaboration of data. But they are not neutral: their theory, as formal machines, dates back to the 30s, when effective computability, a theory of functions upon integers of integral value, imposed itself as the paradigm for logico-formal deduction. Induction and recursion, arithmetic principles, are at its center. Our arithmetic machines and their techniques for the digital encoding of language (Gödelization) thus derive from a strong vision of mathematics, of knowledge in fact, rooted upon arithmetics; the latter has been proposed as locus of certitude, and of the absolute (the integer number, "an absolute concept", for Frege), as locus of the possible encoding of any form of knowledge ("of all which is thinkable", Frege), of geometry in particular (Hilbert, 1899), as an organizing theory of time and space. And certitude would be attained without preoccupation for the revolution caused by the geometrization of Physics within non-Euclidean continua, with their variable curvatures, those of Riemann geometry (a "delirium", with regard to intuitive meaning – Frege *dixit*, 1884); without this incertitude of determinism deprived of predictability, characteristic of the geometry of dynamic systems since Poincaré. So a Philosophy of arithmetics imposed itself upon foundational reflection, all the while departing from the new Physics, which will mark the XXth century. And it proposes that we read the world *modulo* an arithmetic encoding, the same one enabling us to construct, from the world, the basis for modern digital data.

For this reason, the analysis of the constitution of intelligibility and of meaning, as intrication of mathematics with the world, is not traditionally part of foundational analysis in mathematics. The mathematical logic of Frege and Hilbert, with the profoundness of its achievements and the force of its philosophy, has lead us to believe that any foundational analysis could be reconducted to the analysis of an adequate logico-formal system, a logical system (Frege), or a finite collection of sequences of meaningless signs (Hilbertian school), of which the meta-mathematical investigation would then become an arithmetic game (following the digital encoding of any finitary formal system), perfectly removed from the world. And, since Hilbert, as we have seen, the *formal coherence* of these calculi of signs claims to provide the sole justification of these systems, even those of the geometry of physical space and of theories of continua, as they be reduced to Arithmetic.

This has definitely separated mathematical foundations from the foundations of other sciences, including Physics, despite the roles of construction and reciprocal specification between these two disciplines, having a common constitution of meaning. With regard to Biology, the foundational interaction has been lesser, for the time being, following the lesser mathematization of this discipline. However, the ideology of the construction of the computational model as main explicative objective has already marked the interface between mathematics and Biology, all the while forgetting the strong commitment to structuring the world, implicit in its computational arithmetization; and few discussions have attempted to correlate the foundations of the arithmetizing theories to those of the theories of life (see [Longo, Tendero, 2005]). This epistemological separation makes difficult the interdisciplinarity and applications from one discipline to another, because foundational dialog is a condition of possibility for a thought-out interdisciplinarity, a starting point for a parallel constitution of concepts and practices and for a common formation of meaning.

2.2 Laplace, digital rounding and iteration.

So let's return to the "bifurcation" having taken place in history: on one side, the arithmetization of the foundations of mathematics (from Frege and Hilbert, although in different frameworks), and on the other, the geometrization of Physics (Riemann and Poincaré, in particular). The two branches have been quite productive: one the one hand we have the theory of effective computability and, therefore, our extraordinary arithmetic machines and, on the other hand, two fundamental aspects of modern Physics. The first branch of the bifurcation, however, in its foundational autonomy, has continued to base itself upon Newtonian absolutes (Frege) and upon the Laplacian determination (Hilbert), the one which involves the predictability (and which has its counterpart, in meta-mathematics, in the "*non ignorabimus*", Hilbert's decidability: once a mathematical statement is well formalized, one must be able to demonstrate either it or its negation – to falsify it).

It is actually quite clear that Laplace's hypothesis explicitly and first of all aims for predictability ("any deterministic system is predictable"; that is, in a formally determined system, any statement – concerning the future/past – is decidable). However, it bases itself precisely upon this "conservational" interpretation of perturbation evoked earlier: Laplace is very well aware that physical measurement always constitutes an interval (it is necessarily approximated), but he believes that the solutions for the world's systems of equations, approximated if necessary by means of series (of Fourier), will be "stable" with regard to small perturbations, particularly those of which the amplitude remains below possible measurement. The perturbation which by "almost insensible variations" could even induce quite important secular changes – in his words, should not impede the stability of the solar system. It is this which guarantees predictability: in a system which is deterministic (therefore, in principle, formally determined by means of equations), predictability is ensured by the resolvability of the system and/or the *preservation of the approximations* under certain conditions (given the values of the initial conditions, with a given approximation, it will be possible to describe the system's evolution by an approximation of the same order of magnitude). There is the conceptual (and historical) continuity, which we have already addressed, between Laplace's conjecture and the myth of the arithmetization of the world: approximation and rounding (discretization) do not modify the evolutions under consideration, physical and simulated.

Yet it is nowhere like this. Even within a system which is (relatively simple and) explicitly determined by equations (the symmetric structure of formal determinations: the nine equations of three bodies within their gravitational fields, for example), unpredictability arises, Poincaré has explained. What happens? Even within such a simple system, almost everywhere small perturbations may give rise to huge consequences; in fact, "small dividers" (which tend towards 0) within the coefficients of approximating series (of Lindstedt-Fourier), amplify the slightest of variations in the initial values. Ninety years later, this phenomenon will be defined as "sensitivity to the initial conditions" (or at the border, or "at the limits"). Particularly, even perturbations of which the amplitude is below the threshold of possible physical measurement may, after a certain amount of time, produce measurable changes.

So, in our interpretation, a perturbation, a "small force" which perturbs a trajectory even below that which is measurable, *breaks an aspect of the symmetry* described by the equation of the system's evolution; it is the cause (efficient or material) of a variation in the initial conditions, which may produce observable consequences, even very important ones. Sometimes, it may even be question of a fluctuation, such as a local or momentary breaking of the symmetry internal to the system, without the influence of "external" causes: in the Intermezzo, we have evoked the big-bang in cosmology, as a very first symmetry breaking (due to quantum fluctuations) of a highly energetic void. Once again, it is a broken symmetry that is the *material or efficient cause* of a specific observable evolution, that of our universe[6].

Now, the intelligibility of these phenomena, present at the center of modern Physics, is conceptually lost if we organize the world by means of the exact values that arithmetical discretization imposes. Or, rather, and here lies our thesis, we obtain a different intelligibility. Particularly, the perturbation or fluctuation, which have their origin in efficient or material causes, and which manifest *below the proposed discrete approximation*, elude arithmetic intelligibility, or are neglected in favor of a forced stability of phenomena. And arithmetic calculus shows us the passing from one state to another by little iterated jumps of trajectories that are imperturbable, because perfectly iterable. Better, it shows us trajectories which are affected by their own intrinsic perturbation, at each increment of calculus, and always identically iterated and iterable: *the rounding-off*. A new cause, our computational invention, which, projected into the world, becomes a relevant cause (efficient or material) with regard to the properties and states of a system. Because digital rounding modifies the simulated geodesics and may even change, in certain cases and in its own way, the conservation phenomena (of energy, of moment…) by breaking the associated symmetries. We will return to this point.

In what sense then, do we obtain, when we superimpose upon the world an arithmetic grid, a "forced stability", as well as evolutions and perturbations which are very specific and "iterable" at will? We will understand this thanks to the digital computer, because, when this arithmetic machine is used as *model of the world*, it organizes the world according to its own causal regime, its own symmetries and symmetry breakings. In fact, the digital simulation of a physical process if constitutive of a new objectivity, to be analyzed closely because very important, due to the mathematical tools which are at its center: the arithmetic calculi and discrete topology of its digital databases and of its working memory space, exact and absolute.

It is clear that our analysis does not aim to oppose what would be an "ontology" of continua to what would be an ontology of discrete mathematics (we are not defending the idea according to which the world itself would be continuous as such!). We are rather attempting to highlight the difference of the views proposed by discrete mathematics in relation to those proposed by continuous mathematics, in our efforts to make the world intelligible. It is the constructed objectivity of mathematics which changes and not, we repeat, any ontology.

Moreover, the relative incompleteness of computational simulation, which we emphasize here, goes hand in hand with the mathematical incompleteness of arithmetic formalisms which, it too, is relative (to the practice of mathematical proof, in this case). But incompleteness does in no way mean

---

[6] According to the Curie principle, "the symmetries of the causes can be found in the symmetries of the effects". In the approach followed here and as we have observed, one would say that in these cases, at the level of the observable, this is not the case: to an apparently symmetrical initial situation may follow an observable evolution which does not reproduce the same symmetries, following a breaking of symmetry, initial or at the edges, of which the amplitude, initially, is below the threshold of possible measurement (therefore non-observable). In these cases, therefore, certain symmetries are not conserved when passing from (observable) causes to (observable) consequences.

"uselessness": to the contrary, we emphasize the need for a fine conceptual analysis of algorithmic methods, precisely for the essential and strong role they play today in any scientific construction.

2.3 Iteration and prediction.

Computers iterate; that is their strength. From primitive recursion, at the center of the mathematics of computability, to the software application, the program, the sub-program, re-run a thousand times, a billion times, once every nanosecond, all reiterate with absolute exactitude. For this reason, there is no randomness as such in a digital world: (pseudo-)randomness generators are small programs, perfectly iterable, which generate periodic sequences endowed with very long periods (they are functions iterated upon finite domains).

In the algorithmic theory of information, we call random any sequence of integers for which *we do not know* a generating program shorter than the sequence itself. That is, that we do *not see* any sufficient regularities within the sequence to be able to deduce a rule by which to generate it. This definition identifies with randomness the informational characteristics of a series of throws of dice or of roulette, in fact, their incompressibility. This identification, applied to algorithms, (to pseudo-)random generators for instance, leads to a confusion between an epistemic notion of randomness, specific to Physics, and "randomness by incompetence", (the programmer has not told us how the program is designed, usually a one-line program). And iteration reveals the trick: if we re-launch a programmed (pseudo-)randomness generator, using the same initial values, we will obtain the same sequence, exactly. On the other hand, dynamical systems give us the good (epistemic) notion of randomness: a process is random if, when we iterate it with "the same" initial conditions, it does not follow, generally, the same evolution (dice, roulette… planetary systems having at least three bodies, if we wait long enough). All the difference lies in the topological signification of this notion of "same" (*same* discrete values and *same* initial conditions): a digital database is discrete and *exact*; whereas physical measurement is necessarily an interval.

In short, in the mathematical universe of effective computability, there is no randomness as such, at best there is incomprehensible information (which may provide a good "imitation", see below, of randomness). And one may say synthetically with regard to our approach: the time of calculation processes is subject to a "symmetry" in terms of iterability (identical repeatability), which does not have an absolutely rigorous meaning in the physical world and even less so in that of living phenomena. This iterability is essential to computer science: it is at the center of software portability, therefore, of the very idea one may have concerning software; that one may transfer it onto any adequate machine and run it and re-run it identically as often as one wants. And it works, in fact.

Of course, computers are in the world. If we come out of the discrete arithmetics internal to the machine, we may plug them upon physical randomness (epistemic – dynamic systems – or intrinsic – Quantum Physics, see Appendix). We may, for instance, use temporal shifts within a network (a distributed and concurrent system, see [Aceto and al., 2003]) upon which humans also intervene, randomly; or using little boxes, sold in Geneva, which produce 0s and 1s following quantum "spin-ups/spin-downs". But normally, if you run the simulation of the most complex of chaotic systems, a Lorentz attractor, a quadruple pendulum… and iterate with *the same initial digital data*, you will obtain the same phase portrait, the same trajectory. The same initial data, there is the problem. As we have emphasized earlier, this physical notion is conceived *modulo* possible measurement, which is always approximated, and the dynamic may be such that a variation, including below the threshold of measurement – the material or efficient cause -, (almost) always generates a different evolution. On the other hand, in a discrete state machine, "the same initial data" signifies "exactly the same integers". This is what leads Turing to say that his logico-arithmetic machine is a Laplacian machine (see [Turing, 1950], [Longo, 2007]). Like Laplace's God, the digital computer, its operating system, has a complete mastery over the rules (implemented in its programs) and a perfect knowledge of (access to) its discrete universe, point by point. As for Laplace's God, "prediction is possible" [Turing, 1950].

And so thus is the philosophy of nature implicit to any approach which confounds digital simulation with mathematical modelling, or which superimposes and identifies algorithms to the world. Discrete simulation is rather an *imitation*, if we recall the distinction, implicit in Turing, between model and imitation (see [Longo, 2007]). Very briefly: a *physico-mathematical* model tries to propose, by means of mathematics, the *constitutive* formal determinations of the considered phenomenon; a functional imitation only produces a similar behavior, based, generally, upon a different causal structure. In the case of continuous *vs.* digital modelling, the comparison between different causal regimes is at the center of this distinction.

Turing has the huge merit of having invented the machines and of having, in 1950, when he abandoned the myth of the great digital brain, highlighted the difference between *computational imitation*, the game which should demonstrate his machine to be indistinguishable from a woman (modulo the sole intermediary of a written interface, [Turing, 1950]), and *modelling*. In fact, in his 1952 article on morphogenesis, he presents a *model* of chemical actions-reactions which generates forms and which bases itself upon a system of differential equations: tiny variations generate the variety of forms in certain natural phenomena (Turing calls this sensitivity "exponential drift", a quite relevant and original name! The notion of "sensitivity to the initial conditions" dates back to the 70s). This mathematical model, which he explicitly says could be false, nevertheless attempts to make the world intelligible; the 1950 imitation tends for its part to trick the observer (and will thus be considered at the origin of Classical Artificial Intelligence).

2.4 Rules and the algorithm.

Computerized simulation transforms all physical evolutions into an elaboration of digital information. Particularly, the simulation of a geodesic in a discrete universe should make a digital computation correspond to a trajectory, and make the conservation of information correspond to laws of conservation (energy, movement…). Any state or property, in short, any physical quantity, as determination of objects (in the sense of D.1), are in fact encoded by digital information; the quantity of movement is encoded using 0s/1s, just as is the intensity of a field or mass, and their evolution is a calculus approximated by these 0s/1s. Is this encoding "conservational" (does is preserve that which is important)? If a physical trajectory is a geodesic, which geodesic do we associate to the calculus in its digital universe, which symmetry breaking do we associate to the rounding?

Let's begin by recalling the generality of the geodesic principles in Physics at the center of this science since Copernicus, Kepler, and Galileo. As we observed already, any fundamental law of Physics is the expression of a geodesic principle applied within the appropriate space. The work of the physicist, who organizes and, by that, makes the phenomena intelligible, consists in a good measure to the search for this space (conceptual, or mathematical) and for its relevant metric.

This approach leads us to understand the slide of meaning around the concept of law, which intends to justify the identification of mathematical modelling with computational imitation. The notion of "law" has a social origin: law is normative to human behavior. The transferal of the concept as it is to Physics corresponds to an ordinary metaphysics: an *a priori* (divine, if possible) which would dictate the world's laws of evolution. Matter would then conform itself to this pre-existing and normative ontology (as mathematical laws of a platonic universe, for instance). On the other hand, the comprehension of the notion of law as *explicitation of the regularities and criticalities of a landscape*, with its mountain passes, valleys and peaks, its geodesics, inverses this and highlights the transcendental constitution at the center of any construction of knowledge. Mathematics, the tool of formal determination, then elaborates itself upon the phenomenal veil at the interface between us and reality, the reality which, of course, causes friction and canalizes the cognitive act, but which is also organized by this same act. Laws are not "already there", but are a co-constituted in the intrication between ourselves and the world: the discernability of geodesics, as formal determinations within the framework of a network of interactions, is its main result. And their mathematical processing coincides with the beginning of modern science. The normativity of physical law then becomes only cognitive (in order to construct knowledge), and not an ontology. The different forms of formal determination (laws) propose us different causal regimes, ulterior tools for intelligibility.

The identification between algorithm and law causes us to make a backwards step: the algorithm is normative for the machine, for its calculi, exactly as God's law governs any trajectory. The machine would not know where to go; it would be static, without its primary motor, the program. Once again, the myth of the computer-Universe (the genome, evolution, the brain … all governed by algorithms) consists in a metaphysics and a notion of determination which precedes the science of the XXth century and for good historical reasons: the re-centering of the foundations of mathematics upon a philosophy of the arithmetic absolute, at the fringes of the time's great scientific turning points, as we have mentioned a few times.

This way of understanding law should highlight the very first difficulty for the computational simulation of a physical trajectory by means of a calculus. Physical law and algorithm therefore do not coincide: they do not have the same epistemological status. Law is also not an algorithm for another reason we have already evoked: the formal determination, as mathematical explanation of laws, does not imply the predictability of physical evolution. On the other hand, any algorithm, implemented within a discrete state machine, generates a predictable calculus, at least thanks to the "symmetry" by repetition within time which we have mentioned earlier (identical iteration always being possible for a sequential computer).

However, we absolutely need digital simulation, a tool which is indispensable today to any sort of construction of scientific knowledge: by highlighting the differences, outside of any computational myth (the world would be like a computer), we aim to better identify that which is doable, and afterwards to do better, in terms of simulation-imitation. Let's attempt then to understand the evolution of a calculus in physical terms.

In the case of an isolated computer – a sequential machine – we remain within a Newtonian framework: the absoluteness of the clock and of the access to a database are its essential characteristics. The situation is more complex with modern networks: the distribution of machines in physical space and the ensuing relational time changes the situation. Certain aspects of the absoluteness of Turing machines are thrown into question (we discuss this in detail in [Longo, 2007] and, more technically, in [Aceto and al., 2003]). However, the *exactitude* of the discrete database subsists, as well as the issue of rounding, of course.

In both cases, of sequentiality and concurrence, we may nevertheless understand calculus as a geodesic within *the space (pre)determined by the program* (more specifically: by the programming environment, or all aspects of software – operating system, compilers and interpreters, programs…). Shortly, while accepting the divine *a priori* of the programmer who establishes, beforehand, the rules of the game, the notion of "following a geodesic" would be defined by *following the rule* correctly. The hardware or software bug would then be the fluctuation or perturbation that causes the evolution to derail. However, this type of bug is not integrated to the theory; it is not inherent to it, contrarily to the theory of dynamic systems which integrates the notion of sensitivity to the conditions at the edges as well as measurement by interval. Moreover, hardware bugs and logical errors are very rare (therefore statistically very different to the variation due to approximation within a dynamic); they are to be avoided and, in principle, are avoidable (or they may belong to another phenomenal level, which is far from being integrated within the mathematics of effective calculus: Quantum Physics).

We are left with the issue of rounding, which is inherent to calculus. The use of rounding, today, can be very dynamic and mobile: one can aim to a desired approximation and the end of a calculus and increase beforehand the available decimals, up to hundreds, to end within the targeted interval, if possible. The modern approach to analysis by intervals provides a powerful theoretical framework for these processes ([Edalat, 1997]). Of course, the speed of the calculi is inversely proportional to the improvement of the approximation. An excess of the latter may prevent from following any dynamic long enough.

This being said, this bound, the rounding, constitutive of the arithmetization of the world (a quite necessary arithmetization if one wants digital machines to perform calculi and therefore to participate to science today), modifies the causal regime and the symmetries correlated to it, as we hinted and keep demonstrating.

Firstly, let's evacuate a possible confusion: the interval inherent to classical physical measurement and quantum incertitude have nothing to do with digital approximation. First, measurement as interval is a physical principle, a classical one; it is not a "practical" issue: thermal fluctuation, at least, is always present above the absolute zero, by principle. And, as we have already and often observed, the fluctuation or perturbation below the observable amplitude participate to the evolution of a dynamic system, which is somewhat instable, because it can break the symmetries of the evolution, and thus be one of the causes of a specific trajectory. In computer science, a bug which manifests below the rounding is without effect. Afterwards, the analogy sometimes made – naively –

between digital discretization and that of elements of "length" (time and space) induced by the Plank constant, h, is not relevant. The non-separability, the non-locality, the essential indetermination of which we speak in Quantum Physics are almost the opposite of the certitude of the little boxes, well *localized* and stable, well *separated* by predicates (the memory addresses), within which is distributed the digital universe.

So we now face the principal issue: rounding entails a loss of information, at each step of the calculus. It can be associated to the irreversible growth of a form of entropy, defined as neg-iformation. So if one encodes all determinations, formal and objective, of a physical object and process, all properties and states, in the form of digital information, the elaboration of the latter, the digital calculus, will follow a geodesic which is, normally, perturbed at each step by a loss of information. This perturbation does not correspond to any phenomenon intrinsic to the process we intend to simulate: the loss of information is not, generally, the encoding of the change in objective determination. It is a new type of symmetry breaking. Will this influence the proximity of the virtual reality to the physical phenomenon? Will it influence the quality of the imitation?

As we have already observed, arithmetic approximation does not affect the simulation of a linear or Laplacian process: just as the approximation of the measurement, the rounding, does not remove the computational geodesic (the following of the rule) from the physical geodesic. The initial loss of information is preserved, it remains of the same order of magnitude or it increases in a controlled way (technically: the extremes of the approximation intervals are preserved). This is not the case for non-linear cases. Let's consider, for illustrative purposes, one of the simplest dynamics, one that is well-known and one-dimensional: the discrete logistic equation,

$$x_{n+1} = k\, x_n\, (1- x_n).$$

For $2 \leq k \leq 4$, this equation formally defines a sequence $\{x_i\}$ of real numbers, between 0 and 1 (a "time discrete" trajectory within a continuous space). Particularly, for k = 4, it generates chaotic trajectories (sensitive to the initial conditions, dense in [0,1], with an infinity of periodic points…). Can we approximate any sequence of real numbers thus generated by a digital computer? Out of the question, at least in what concerns an initial value of x_o taken from a set of measure 1 (that is, for almost any real value in [0,1]). Even if we choose a x_o that can be represented exactly by a computer, at the first rounding in the course of the calculus, the digital sequence and the continuous sequence will begin to diverge. By improving the approximation/rounding of 10^{-14} to 10^{-15}, for instance, after approximately 40 iterations, the distance between the 2 sequences will start to oscillate between 0 and 1 (the greatest possible distance). Likewise if, with a rounding of 10^{-15}, we begin using values that differ from 10^{-14} (of course, if we want to, nothing would prevent us from restarting the digital machine upon the exact same values and from calculating, with the same rounding, exactly the same discrete trajectory…). The technical problem can be summed up by the observation that the dynamic is a "shuffling" one: the boundaries of the interval are not preserved.

We therefore may not, in general, approximate, with the machine, a continuous trajectory; however, we may do the opposite. In fact, all that can be proved, in dynamic (metric) contexts we will not specify here, is the following "pursuit" lemma (see the *Shadowing Lemma* [Pilyugin, 1999]: notice the order of the logical quantification):

> For any x_0 and δ there is a ε such as that for any trajectory f, ε-approximated (or with a rounding-off not greater than ε, at each step), there exists a continuous one, g, such as g approaches f by a difference of δ, at each step.

Even when considering the lucky case where we have $\delta = \varepsilon$ (this is possible in certain cases), it comes to say that, globally, your digital sequences are not so "wild": they can be approximated by a continuous sequence, or … there are so many continuous trajectories that, if we take a discrete one, one can easily find a continuous one which is close to it. Thus, the image of an attractor on the screen provides qualitatively correct information: the digital trajectories are approximated by trajectories of continuous dynamics (determined by equations). But the reverse is not true: that is, it is not true in general, for a trajectory given by analytical means, that the computer may always approximate it. Different versions of the pursuit lemma apply to sufficiently regular chaotic systems. However, many dynamic systems do not even satisfy weak forms of this lemma (see [Sauer, 2002]). This signifies the existence of initial values and intervals such that, within these intervals, any rounding and any other initial value cause any continuous sequence to diverge from the given discrete sequence.

What happens, in the terms of our approach, which is geometric in nature? To understand this in detail, it would be necessary to refer to the technical analysis which the authors are also developing. In this work of reflection, which nevertheless guides the mathematical and computational analysis, let's try to see it in a very informal manner. The first difficulty resides in the necessity to place oneself within the appropriate space, in order to better understand. In short, it is necessary to analyze the evolution of a system, such as for instance the discrete logistic function, within a space where the notion of neighborhood, between real numbers, corresponds to digital approximation. A space which provides for such a metric is called a "Cantor space". In this space, which we will not define here, two real points are close if and only if they have close binary or decimal representations (for instance, 0.199999…. to infinity and 0.2 are very far from one another in the Cantor space, whereas they are identical with regard to the usual real line, this posing numerous problems from the computational standpoint, when we try to operate upon their approximations).

We then see that at each of the digital calculus' iterations, the rounding induces a loss of information corresponding to the deterioration of the approximation around the point of the trajectory. If we measure the phenomenon in terms of isotropy of space (the points within this widening neighborhood are "indistinguishable", in a manner of speaking), this "grey" zone, of isotropy, grows, thus augmenting the symmetries of space. A notion of entropy as negative information also enables to grasp this change in symmetries, as loss of information. Now, all that we have in the machine is encoded information. Independently of what it encodes, the physical object's formally determined properties or states, all is in the form of digital information. So the objective determination, which is given by the preservation of the theoretical symmetries, radically changes: we are facing a change in symmetry that does not model a component of the evolution of the natural phenomenon, because it depends only upon the discrete structure of the simulation universe and upon the imitation of the formal physical determination by algorithms (or, when we make a philosophy of it, of the epistemological identification of *law* with *algorithm*).

So there is, in terms of symmetry, the explanation of the causal regime of which we were speaking. The discretization, in fact the organization of the world by means of discrete mathematics proposes a causal regime (in this case, an evolution of symmetries) which is different than that which is proposed by continuous mathematics. It is not an issue of finitary translations of a same physical world, but of scientific construction, because this world is itself co-constituted by our formal and objective determinations. When they change, its organization and its intelligibility also change. Once more, this does not imply that the world is continuous "in itself": we are only observing that, since Newton, Leibniz, Riemann, Poincaré … we have organized and made intelligible some physical phenomena by means of historical notions of continuity and of limit. If we want to do without them, causal organization and intelligibility will be altered.

Another issue would also merit to be detailed, but we will leave it as an ulterior venue for work. Singularities in modern Physics play an essential role. We know for instance of shock situations, in non-linear systems, where the digital calculus does not come even remotely close to the critical situation. We have the continuous description; the mathematics are clear, explicative, organizing for the physical phenomenon, we understand qualitatively, but the numerical calculi chaotically revolve around the singularity, without coming close to it. In fact, the current notions of limit and of singular point, which are absolutely necessary to the analysis of the phase changes, the shocks, in order to even speak of renormalization processes in Physics, are not always coherently approximable. The loss of symmetries and the change of correlated causal regime constitute our way of understanding this problem, which is specific to the digitalization of phenomena, without referring to Laplacian myths and computational metaphysics. Computer science, a science which is now mature, deserves, from an epistemological and mathematical standpoint, more attention and an internal view which is able to assume the force and the limits of its own methods.

3. Causalities in Biology.

While focusing now on Biology, we will not address the discussions concerning the biological levels of organization, the intertwined hierarchies, the crossed causalities, the ago-antagonistic effects, the variabilities within phenomena, the autopoetic processes, which we find in Biology. Of course, all these properties will remain part of the backdrop of the approach which we propose here, but the approach will be more conceptual or, better, "schematic" (in the geometric sense of simple shemata or diagrams) than thoroughly theoretical or descriptive: it will seek rather to contemplate a framework of representation enabling to extricate heuristic categories of thought than to account for the effective phenomenality of life. Indeed, the "relationships" which we highlight, by means of very abstract little patterns, do not necessarily correspond to "material relationships" or to physical configurations; they are only organizing structures of thought, which should aid the comprehension of phenomena, by proposing a conceptual framework. Moreover, is $F = ma$ – at a much more elaborate and mathematized level indeed – not a correlation which organizes a phenomenon by making it intelligible? Let's also recall that this equation as been preceded by the general concept of inertia, or even, way before Galileo, by cosmological speculations and concepts as eminently philosophical as profound (see, for instance, the remarks of Giordano Bruno in "*L'infinito universo e mondi*", 1584). The physical intelligibility specific to this equation can be the object of highly differing conceptual "readings": it may no longer be primitive, but derived (from the Hamiltonian, from the Lagrangian, as we have mentioned in the first part), where it may be correlated to distinct symmetry breakings, as we have also seen.

So, as we insist on emphasizing from the onset, our approach remains very speculative here: it is for us the beginning of an attempt at a conceptual categorization and schematization which seeks to open new venues without being sure of their outcomes and which will require, in order to be continued, more discussion with biologists and the sanction of a certain fecundity in the quest for a greater understanding of living phenomena. Of course, we will remain within the framework which we have determined for ourselves, that is in the geometric terms of symmetries as an analysis of physical causality; yet, we will *also* take into account here aspects which are specific to Biology, related to forms of telenomies or of anticipation and which we have already summed up with the concept of "contingent finality" (see the next section).

3.1. Basic representation.

Let's consider the dynamic functioning of a centrifugal governor (or Watt governor: two weights are lifted in rotation by pressure, making a valve open so as to lower the pressure…). This functioning is completely determined by the data, obtained afterwards, concerning the initial conditions and the physical laws. In this sense, its "behavior", though being well regulated and leading to a dynamic equilibrium, is determined in a univocal and oriented way (geodesics within a well – and pre-established phase space).

In the case of living phenomena, the behavior (and functioning) of an organism does not appear to be determined in the same way. What appears to be determined like this in a more or less rigid way (within a given domain, that is compatible with the organism's survival), is what we could call the aim of the functionings and behaviors, the functions to be fulfilled in order to ensure homeostasis (-rhresy); but what is not determined in such a manner, are on the one hand the possible ways to achieve this and on the other hand the adaptations and modulations which would ensure the achievement of the functions and behaviors. Specifically, to put it in the language of Physics, there are not only phase changes but also, changes in phase spaces, that is, observables and relevant variables.

We know that the mathematical and equational formalization of this situation (as took place for Physics and as we hope, in time, could be the case for life sciences inasmuch as the adequate mathematics would be elaborated) is confronted with profound difficulties, sometimes even difficulties of principle, which the numerous and successive attempts at modelling have encountered. Also, before any reiterated attempt in this regard, it appears to be necessary to try to illustrate and to represent – in this case by means of schematics, the first abstract conceptual stage – that which appears to characterize these modes of functioning and that which we could call the "finalities" which interpret them, in the sense in which Monod could speak of a *telenomy* of life. These finalities, of course, are neither necessary nor absolute; they rather participate to our view upon living phenomena and, most of all, they are contingent, as they are specific to living matter and relative to its contexts. In short, they could not be present (no life, no specific specie or individual); they are relevant to various levels of organization and to their correlations, or to their intertwining and looping, particularly in the form of integration and regulation (see [Bailly, Longo, 2003]).

To make intelligible the notion of contingent finality, we will try to organize into networks the interactions between the "material structures" and "functions" of living matter. It is at this level in fact that telenomy manifests: for instance, when an organic structure appears to be finalized in relation to a certain function.

We therefore propose a conceptual framework, to organize knowledge by means of our recourse - temporarily at least – to a description of this sort (for a better, active, understanding, the reader may easily draw the simple intended diagrams):

1) We have a target set, constituted of several domains – the target domains, which may or may not overlap – corresponding to the functions to be ensured for the maintenance and the perdurance of the organism and its species.
2) We have a source set constituted of all the organic possibilities likely to be mobilized to this end (biochemical reactions, transport agents, etc), also represented by this set's domains (source domains).
3) We have a set of arrows, originating in the source domains to reach to the target domains (these arrows correspond to the orientations and functioning modes aiming to ensure the functions) and which presents the following particularities:
   - **i)** Any target domain is reached by at least one arrow; usually, several source domains are at the origin of the arrows reaching a same target domain. An example of this situation is the conjunction of "oxygen metabolisms" and of "glucose metabolisms" to ensure the maintenance of muscular tissue such as the heart; in this case both source domains are respectively defined by the chemical reactions related to the specific energetic sources (availability of glucose) and by the cellular assimilation processes of the intake of oxygen obtained through breathing (availability of oxygen), the arrows corresponding for their part to the various transport and transformation systems which enable the effective transferals from sources to targets.
   - **ii)** The arrows pointing to a same target domain are endowed with different widths depending on the prevalence of the usual modes of functioning (in the preceding example, we would have, in the normal case, an arrow width for the "oxygen metabolism" that is much greater than that of the "glucose metabolism". In the case where a dominant mode of functioning would fail (pathology), the corresponding arrow could narrow down to the benefit of another whose initial width was smaller (and this, without necessarily reaching the width of the first one: functional weakening all the while attempting to preserve the function): this mechanism would correspond to a property of *plasticity*.
   - **iii)** The arrows stemming from a same source domain and extending towards several target domains exist, but may be relatively rare in adult homeostatic (-rhersic) functioning. They refer mainly to potentialities, preceding ulterior actualizations or differentiations (cf. stem cells, for instance), or to other possibilities of plasticity (cerebral, for instance). On the other hand, in the case of the representation of a *genesis* (embryogenesis, namely), these arrows are dominant and play an essential role in the representation of the organic differentiations from totipotent eggs or from pluripotent stem cells. There is therefore a dynamic for the "topology" and the width of the arrows over the course of development to reach the adult situation.

It could be interesting and enlightening to note here that the joining of the characteristics i) and iii), for the arrows, corresponds rather well to the concept of *degeneracy* such as it has been introduced by [Edelman, Tononi, 2000] with regard to cerebral functioning (that non-isomorphic structures may participate to a same functionality and that a given structure may participate to several of these functionalities), concept which returns to and generalizes that of *redundancy*, but while differing somewhat from it (computer redundancy, for example, is achieved by iteration of identical components).

In this perspective, we could qualify the described situation by the characteristics in (i) of "*systemic" degeneracy* (a same system participating to distinct functions) and the characteristics in (iii) of *"formal" degeneracy* (non-isomorphic systems participating to a single and same function).

Let's also point out right away that the concepts of "source domain" and "target domain" do not necessarily refer to "absolute" categorizations, but are relative to a given functionality (or to a set of functionalities): a target domain for a functionality can very well operate as a source domain for another[7] on the same level of organization or between levels, thus the many possible intertwinings.

Notice on the other hand that, in this approach, the environmental, feed-back or adaptation effects can be represented by variations in the widths of the arrows ("metric" aspect), or that the fundamental changes would rather correspond to changes in the structuring of the set of arrows ("topological" aspect). Moreover, pathology is likely to occur (in order of "seriousness"):

- either with a variation in the width of the arrows,
- either with the disappearance of certain arrows (without nevertheless a target domain being no longer concerned at all)
- or with the disappearance of the source domains (in this case, grafts and prostheses can play an "artificial" regulatory role).

We may consider that the disappearance of the target domains corresponds at best to a mutation, and in the worst of cases, to death.

To give a few "systematic" examples of the functioning thus represented, we can propose the following triplets (by starting by the source domains, then arrows – in fact corresponding to functions – and finishing by target domains):

- Vascular system / circulation (transport) / local essentials (nutrients, oxygen, etc.);
- Respiratory system / breathing / oxygenation;
- Nervous system / information, command / adaptation, initiative;
- Genes / expression / proteins, regulation;
- Mitochondria / biochemical reactions / energy produced;
- Digestive system / digestion and transport / metabolism;
- Immune system / reconnaissance / tissue identity, struggle against aggressions.

3.2. On contingent finality.

Based upon these considerations, we can propose to call *contingent finality* the abstract structure formed,

1) by the triplet {source domain, arrows, target domains}
2) endowed with the "measurement" constituted by the set of real numbers E, of the widths of the n arrows: $E = \{e_1, e_2,\ldots, e_n\}$
3) ensuring a *structural stability* for these characterizations. We mean, by such structural stability, the conservation of the target domains in the sense that there will always be at least one arrow for which the width is non null and which points to these targets, regardless of the source domains.

Let's go back to the preceding example and let's attempt to compare a normal state to a pathological state. In the normal state, the "oxygen metabolism" arrow has a width of e_{O1} and the "glucose metabolism" arrow has a width of e_{G1}, with $e_{O1} >> e_{G1}$ and $e_{O1}+e_{G1} = e_1$. The establishment of the pathological state is translated by a narrowing of the "oxygen" arrow and the widening of the "glucose" arrow; finally, we have $e_{O2} < e_{O1}$, $e_{G2} > e_{G1}$, $e_{O2} + e_{G2} = e_2 < e_1$.

The fact that the arrows do not cancel each other out and that the target domain remains translates a partial *plasticity*, whereas the decrease of the total width, on the one hand, and the internal rebalancing of the widths, on the other, demonstrates the pathological character. The influence of these two factors

[7] For instance, the putting into effect of ionic equilibrium processes may constitute a source domain for the functioning of the target domain represented by a cell, itself constituting a source domain for the good functioning of the tissues to which it participates, good functioning representing one of its target domains. It would go likewise for cerebral functioning, for example, as target domain for an oxygenation and as source domain for a control or behavior.

(total width and respective widths) could indicate that total plasticity (in the sense where we would finally have $e_2 = e_1$) does not however restore a completely "normal" situation.

From a much more general point of view, we will notice that – as we have already highlighted through the approach by levels of organization, previously considered – the same structure of "contingent finalization" thus defined, replicates itself at various levels of organization of biolons (cell, organism, species), even if the characterizations (triplets and measurements) may differ in their specific content, according to the level. This structural likeness is doubtlessly the result of a certain form of equivalence of the objective complexities associated to these levels, as we have already noted, [Bailly, Longo, 2003][8].

3.3. "Causal" dynamics: development, maturity, aging, death.

Let's note here that if we accept the schema we have just exposed, it proves likely to represent, thanks to the topological and "metric" plasticity it is able to demonstrate, the great dynamic processes of which life can be the locus: the beginning of development is characterized by the prevalence of arrows which stem from a source domain to point towards several target domains which they even contribute to constitute (differentiation of tissues and of anatomical and physical systems). As the process unfolds and at the same time as the number and structure of the target domains stabilizes, these arrows narrow down (some may even disappear) at the same time as the arrows originating in several source domains ending in a same target domain (functional aims) start to prevail. The set stabilizes once again following development, the period of maturity.

Once the stability of the maturation achieved, the topology maintains itself "on the whole" and aging manifests itself mainly in a "metric" fashion (by the variation of the *measurement* of the narrowing of arrows). It is even possible that in borderline cases one may witness disappearances of arrows by cancellation of their widths, amounting, beyond metrics, to tapping onto the topological structure of the schema. And finally, to represent the death of an organism, we may agree, as suggested above, that it manifests itself by the disappearance of one or more target domains (corresponding to vital functions), in that there are no more arrows pointing towards them.

We will note that if most of an individual's target domains are oriented towards the individual's perdurance, at least one of them, corresponding to the reproductive function – is likely to produce a new source domain (child cell, fertilized egg) as origin of the reiteration of the process for a new individual. It is the set formed by the abstract joining of this particular target domain to the newly produced source domains which may constitute – at a different level – the source domain at the origin of the genesis of individuals of the level thus considered (organism for cells, species for individuals).

From the standpoint of an attempt at a more precise "phenomenal" identification of the characteristics we have just introduced abstractly, we may consider that in the initial "transitory regime" (time of genesis) the source domains are principally constituted by biolons (embryonic cells, individual organisms, species, as defined in [Bailly and al., 1993] and further analyzed in [Bailly, Longo, 2006]), the target domains being mainly constituted by orgons (cell's organites, organs and tissues to be set). The arrows corresponding for their part to the phenomena of differentiation, of migration and of structuration, whereas, conversely, in the "stationary regime" (adult organism), the source domains are mainly constituted by the constitutive orgons, whilst the target domains would be constituted by the variety of vital functions ensuring the organism's maintenance and autonomy, the arrows corresponding this time to the biochemical and physical processes enabling to ensure these functions (integration and

---

[8] Let's recall that, according to our analysis in [Bailly, Longo, 2006] (where we had distinguished between objective complexity and epistemic complexity in Biology, also providing examples), the elements of living matter which are biolons present an objective complexity which may be considered as being infinite, w; r. to any physical measure (crossing of the essential level of organization which enables to pass from inertia to life). From this point of view, and still with regard to physical complexity, the objective complexity of biological objects is comparable regardless of what these biolon-type objects are (the living cell presents an objective complexity almost identical to that of an organism such as a mammal). What is modified along life's scale of complexity is epistemic complexity, related to the enriching struture of phenotypes, to the increasing, along evolution, levels of organization, their intertwining, the proliferations of structures and functions, the conditions of description, etc.

regulation). Such an approach would enable to propose a sort of "temporalized" schema of biological functioning.

Could we refine the analysis by taking more precisely into account the nature of the fluxes which link source and target domains to one another? Namely by putting the distinction between energy and information to work? In a first approach, it appears legitimate to consider that the fluxes in the source/target direction mainly have an energetic character (transport of matter or energy), responding to fluxes which are mainly of information (gradients, divergences from the dynamic equilibrium) going in the target/source direction. The arrows are then supposed to integrate and represent both types of fluxes, their width alterable in the event of a failing of either the correlated "informative" or of the "energetic" character (we could, in a first approximation, take as parameter the product of these two types of fluxes, for instance[9]). Let's try to take an example at one of the most elementary levels, that of the cell: in this case, a particular source domain can be associated to the functioning of ionic channels enabling the ions to cross the cellular membrane and a corresponding source domain would be the stationarity (dynamic equilibrium) of the cell's the internal ionic state (homeostasis – homeorhesis) which enables it to function in the best conditions. The arrows would then correspond to the taking into account of both "fluxes": on the one hand, the "information" flux which would be generated by a difference in the internal ionic concentration with regard to the stationary state (gradient, difference in osmotic pressure, electric field, …) and which would lead for instance to the opening of certain channels, and on the other hand the concomitant flux of matter (these same ions) coming from the outside in view of re-establishing homeostasis and entering through these channels.

Notice that, from a standpoint analogical to this stage, the taking into account of these two aspects (matter/energy and information) highly resembles the thermodynamic situation where the definition of free energy (of which the variations govern the system's evolution) entails the intervention on the one hand of an enthalpy (or internal energy) and on the other hand of an entropy, these two magnitudes being associated by means of temperature.

3.4. Invariants of causal reduction in Biology.

As in the case of Physics, we may question ourselves regarding the invariants of causal reduction (if they exist) specific to life and their relationships with what could constitute, in the field of Biology, the determinations associated to the symmetries which we have encountered for Physics.

It very well appears that these biological invariants exist indeed and that they are constituted by sets of pure, numerical invariants (and not dimensional invariants as in Physics). It also appears that the determinations which enframe, modulate and actualize them are now rules of "scaling" in function, let's say, of the size or mass of the organisms (sorts of dilatation or scale symmetries), see also [Schmidt-Nielsen, 1984].

Thus, for instance, the average life-spans of the set of organisms do appear to scale as power 1/4 of their masses and their metabolism as power 3/4 of these masses ([Peters, 1983]). Likewise, on a level that is somewhat different but which is in relationship with these properties, we would recall here that mammals are characterized by an invariant average number of heartbeats or breathings (of the order of 10^9 heartbeats or of 2.5×10^8 breathings over the course of an average life), these numbers conducting to frequencies (or periods) – dimensional magnitudes, this time – which are submitted to these rules of scaling in function of the average mass of the individuals of the considered species (for instance with a power -1/4 of this mass for the frequencies).

But such characteristics of invariance do not manifest solely at the high level of the biological functions of evolved organisms; they can also be found at much more elementary levels such as those of

[9] If E is the matter-energy flux extending from the source domain to the target domain in order to "respond" to the information flux ("request") going from target to source domain "within" a given arrow, we could take as one of the parameters of functioning – which participates to the width of this arrow – the product $E \times F$. Thus, the failing of a flux in one or the other direction would translate as a diminution of this product, corresponding to a decrease of the width of the arrow and thus expressing an alteration of the functional process summarized by this arrow.

cellular metabolic networks[10] [Ricard, 2003; Jeong and al., 2000], of which the diameter[11] remains invariant along the phylogenetic tree and of which the connectivity distribution, over at least 43 organisms belonging to the three categories of life forms[12], presents the same characteristic power (2.2 approximately). As emphasized by J. Ricard, such an invariance of the network's diameter implies that the degree of connection of nodes increases with the number of these nodes, that is, with the number of stages likely to connect them. Here again, one will notice that it is a case of numeric and not dimensional invariants.

3.5. A few comments and comparisons with Physics.

We see that viewed from this angle, the causality which seems to manifest in living phenomena presents similar traits as well as different traits with regard to those which we have noted in the case of Physics which only addresses inertia. Material and efficient causality are manifestly present there – this having doubtlessly favored the idea of a possible physicalistic reduction – although in a much less rigorous and structured way than for Physics (namely in connection with the plasticity and adaptability capacities). The formal determinations are relatively weakly represented there, despite the advances made in terms of various local modellings (we have mentioned the metabolic networks, but at a different level, we could evoke population dynamics, for instance, or the transport properties close to the fluid dynamics). Likewise, the essential objective determinations do not really appear to have been extricated, despite the observation of variegated properties of symmetry – or of symmetry breakings – (over the course of development or in certain anatomies, for instance) and the identification of certain digital invariants. On the other hand, the dimension of "final causality" (to use old categorizations) or of "contingent finality" appears to play here a role which is rather important and unknown to Physics. As if the fact of finding itself in an extended critical state (therefore potentially highly unstable, although necessary to an elaborate organization) could be compensated for the structural (momentary) stabilization of living matter only with the introduction of these factors of telenomy/anticipation which appear to characterize it.

4. Synthesis and conclusion.

We have attempted to briefly characterize the different aspects of physical causality such as they may appear and be analyzed through contemporary theories. We have emphasized the fact that symmetries and invariances constitute determinations which are even deeper than those which manifest causal laws in that they present themselves, in a way, as the conditions of possibility for the latter and as frames of reference to which they must conform.

We have also sketched out an analysis of the causality internal to the systems of effective computability, of which the symmetries and invariances obey a specific regime, rooted in the discrete arithmetic structure of databases and algorithms. The intelligibility structure proposed by these methods differ by that which is inherent, on one side, to the geometry and mathematics of the phenomenal continuum, particularly by the difference between the (modern) notion of physical law, to which we refer, and, on the other side, to the notion of algorithm. The consequences to these two aspects can be measured in terms of different causal regimes, following differences (breakings) in symmetry. Iteration, as a particular symmetry in time, is also mentioned as one of the characteristics of digital simulation, in fact as one of the strong points of computational imitation (and a starting point for effective recursion, as a mathematical theory). It is also at the center of the particular status of predictability, even in the case of the computer implementation of highly unstable non-linear systems, because the possibility of identically iterating a process (or of accelerating a simulation) is a form of prediction. Iterability, that is,

[10] We know that a network is a graph formed by nodes which are connected according to certain rules. In the case of metabolic networks, these may be constituted by metabolites or by enzymatic reactions and the connection links representing the mutual biochemical interactions.

[11] The diameter of a metabolic network is defined by the average of the shortest paths (in terms of steps), leading from one of the network's nodes to another.

[12] That is, archaebacteria, bacteria, eukaryotes.

digital calculi, also enables to grasp the difference between the randomness of the theory of algorithmic information and the randomness of physical processes of the critical and quantum type. In the case of algorithms, randomness coincides with incompressibility. On the other hand, in the first of the physical cases (deterministic, dynamic, and thermodynamic systems), it is of an epistemic nature and it implies the non-iterablity of processes; in the second (Quantum Physics) it is intrinsic to the theory (it is part of the objective determination, [Bailly, Longo, 2006]). These two last cases are incompatible with the individual iterability of an individual process (which is typical of algorithmics); although it may have a statistical iterability, as in quantum mechanics.

We have then attempted to widen the causal problematic to encompass life by taking into account its specific character through what appears as a sort of finalization of its functioning which we have attempted to conceptually systematize. This led us to refer to specific concepts, such as that of "contingent finality", and to propose new representations (topologico-metrical) in order to attempt to account for it in a more or less operational fashion. Finally, we have evoked the possibility of extricating that which – through numerical constants and scaling properties – could be considered as invariants of causal reduction specific to life.

If the considerations relative to Physics and to calculus base themselves upon well elaborated and mathematized theories, enabling us to refer to a *corpus* we can say to be almost completely objectivized and thus lending itself particularly well to a thorough conceptual and epistemological analysis, moreover situating itself within the framework of a well established tradition, it seems that for Biology the situation is much more fragile in this regard. Also, with regard to life, the analyses we propose are of a much more speculative nature and require with greater necessity the theoretical and conceptual sanctions concerning experimental practices in this field. More so that the causal representations, in the case of life, if we seek to detail them, must take into account multiple interactions, which present themselves simultaneously all the while remaining of a highly different nature, whereas in Physics, for example, the interactions may in many cases be sufficiently decoupled from one another for us to be able to approach and study them separately; even if it implies, in a second stage, to seek the conditions and procedures for their unification. In other words and in particular, in Biology, the causal representations must take account of massive retroactions, which prevent them most often from partitioning systems into weakly coupled sub-systems in order to facilitate analysis, as is done in Physics, as well as to consider holistic telenomies, according to which the local organization is dependent upon the global structure and reorganizes itself according to the necessities of optimization or of perdurance of this structure according to criteria which is still not well known[13].

Nevertheless, it appears that one of the points common to these disciplinary fields – and this is what we have wanted to highlight and to emphasize in this text – resides in the fact that the causal analysis, all the while remaining useful and efficient – must now be relativized and henceforth leave way, for the purpose of a better comprehension of the theoretical and conceptual structures of these fields, to a more general approach relying much more upon the properties of invariance, of symmetry (and their breakings) and of conservation we stressed. These properties underly the manifestations which we tend to spontaneously (at least since the Renaissance) interpret in terms of objective causal actions. We will maybe see there the trace of a process of conceptual rehabilitation of the "geometrical" (taken in a broad sense) in relation to the "arithmetical"[14], which is not without echoing the most profound preoccupations of this volume.

---

[13] We know, for example, that the genetic constraints themselves manifest "normally" only in adequate extragenomic or environmental frameworks. Moreover, some phenomena, such as *local* "death" – apoptosis – apparently in contrast to evloutionary-survival paradigms, prove to be necessary to the *global* viability, as an integral part of "normal" life and even more of the adaptation faculties of organisms in the event of the modification of the exterior environment.

[14] In reference to the debates of the beginning of the XXth century concerning the foundations of mathematics, but all the while emphasizing the fact that the geometrization of Physics, for its part, has never ceased to develop, probably explaining why it is in this discipline that symmetries and invariances have acquired a determining explicative and operational status rather early on.

References

(preliminary or revised versions of Longo's papers are downloadable: http://www.di.ens.fr/users/longo or Google: search : Giuseppe Longo).

Aceto L., Longo G., Victor B. (eds.) **The difference between Sequential and Concurrent Computations**. *Special issue,* **Mathematical Structures in Computer Science**, Cambridge U. Press, n. 4-5, 2003.

Anandan J. "Causality, Symmetries and Quantum Mechanics" **Foudations of Physics Letters**, vol. 15, no. 5, 415 – 438, October, 2002.

Bailly F., "L'anneau des disciplines", **Revue Internationale de Systémique**, vol.5, n°3, 1991

Bailly F., "Invariances, symétries et brisures de symétries", in L. Boi (ed.) **New Interactions of Mathematics with Natural Sciences,** Wordl Scientific, 2005 (also in [Bailly, Longo, 2006]).

Bailly F., Gaill, F., Mosseri R., "Orgons and Biolons" in Theoretical Biology : Phenomenological Analysis and Quantum Analogies, **Acta Biotheoretica**, Vol.41, p.3, 1993.

Bailly F., Longo G. "Objective and Epistemic Complexity in Biology" Invited lecture, Proceedings of the **International Conference on Theoretical Neurobiology,** (N. C. Singh, ed.), NBCR, *New Delhi,* pp. 62 – 79, 2003.

Bailly F., Longo G. "Space, time and cognition. From The Standpoint of Mathematics and Natural Science". Invited paper, **Causality and Mind**, (Peruzzi ed.), Benjamins, pp. 149-199, Amsterdam, 2004. (Version française dans la **Revue de Synthèse,** Paris, 2004, et dans [Bailly, Longo, 2006]).

Bailly F., Longo G. "Incomplétude et incertitude en Mathématiques et en Physique". *Article invité*, **Il pensiero filosofico di Giulio Preti**, (Parrini, Scarantino eds.), Guerrini ed associati, Milano, pp. 305-340, 2004 (et dans [Bailly, Longo, 2006]).

Bailly F., Longo G. **Mathématiques et sciences de la nature. La singularité physique du vivant**. Hermann, Paris, 2006.

Bernard-Weil E. "Ago-antagonistic Systems", in M. Mugur-Schächter and A. Van der Merwe (eds.) **Quantum Mechanics, Mathematics, Cognition and Action***,* Kluwer, 2002

Edalat A. "Domains for Computation in Mathematics, Physics and Exact Real Arithmetic", **Bulletin for Symbolic Logic**, 3 (4), 401 – 452, 1997.

Edelman G., Tonni G., **A Universe of Consciousness. How Matter Becomes Imagination**, Basic Books, 2000.

van Frassen B. **Lois et symetries,** Vrin, Paris, 1994.

Jeong H., B. Tombor, R. Albert, Z.N. Ottvai, A-L. Barabasi, "The large scale organization of metabolic networks", **Nature**, vol. 407, p.651, 2000.

Longo G. "Laplace, Turing and the "imitation game" impossible geometry: randomness, determinism and programs in Turing's test." *In* Epstein, R., Roberts, G., & Beber, G. (Eds.). **The Turing Test Sourcebook: Philosophical and Methodological Issues in the Quest for the Thinking Computer.** Dordrecht, The Netherlands: Kluwer, 2007.

Longo G. “The Constructed Objectivity of Mathematics and the Cognitive Subject”. *In* **"Quantum Mechanics, Mathematics, Cognition and Action - Proposals in Epistemolgy."** (M. Mugur-Schachter ed.), Kluwer, pp. 433-463, 2002.

Longo G. "The reasonable effectiveness of Mathematics and its Cognitive roots", *in* **"New Interactions of Mathematics with Natural Sciences"** (L. Boi ed.), World Scientific, 2005.

Nicolis G., **Dissipative systems**, Rev. Prog. Phys., IL, p.873, 1986.

Nicolis G., Prigogine I., **A la rencontre du complexe**, Paris, Presses Univ. France, 1989.

Peters R.H., **The Ecological Implication of Body Size**, Cambridge University Press, 1984.

Pilyugin S. Yu. **Shadowing in dynamical systems**, Springer, 1999.

Ricard J., “Emergence, organisation et causalité dans les systèmes biologiques”, *in* : **Enquête sur le concept de causalité** (L. Viennot, C. Debru Eds.), PUF, 2003.

Rosen, R. **Life Itself,** New York: Columbia University Press, 1991.

Sakharov A., **Œuvres scientifiques**, Anthropos, 1984.

Sauer T. “Shadowing breakdown and large errors in dynamical simulations of physical systems”, preprint, George Mason Univ., 2003.

Schmidt-Nielsen K., **Scaling**, Cambridge University Press 1984

Stewart J. “La modélisation en biologie” *dans* **Enquête sur le concept de modèle**, Nouvel P. (ed.), Presses Univ. de France, 2002.

Turing A. “Computing Machines and Intelligence”. **Mind**, 59: 433-466, 1950.

Varela, F. **Autonomie et connaissance**, Paris: Seuil , 1989.

Viennot L. “Raisonnement commun en physique : relations fonctionnelles, chronologie et causalité” en **Enquête sur le concept de causalité** (Viennot, Debru eds.) Press Univ. de France, 2003.

Weyl H. **Philosophy of Mathematics and of Natural Sciences**, 1927 (english transl., Princeton University Press, 1949).

Weyl H. **Symmetry**, Princeton University Press, 1952.