Equipe: Complexité et Information Morphologique
Responsable: Giuseppe Longo

Date de création: 2002

Établissements partenaires

- CNRS
- ENS, Paris

1 Composition de l'équipe au 1er janvier 2008

Liste des membres (au sens strict, pas les collaborateurs extérieurs) de l'équipe

Nom	Prénom	Fonction	Établissement	Date d'arrivée dans l'équipe
Longo	Giuseppe	DR1, CNRS	CNRS-ENS	2002
Hoyrup	Mathieu	Thésard	ENS, Lyons	2005
Rojas	Cristobal	Thésard	Polytechnique	2005

2 Thématique scientifique de l'équipe

Une Page Max In 2002, the leader of the team started a new research group, based on an "explorative interdisciplinary" project, Morphological Information and Complexity (CIM). The team has been consistently composed, since then, by this author as the only permanent member and by 2 PhD students at LIENS (two theses defended in 2006, M. Mossio, B. Saulnier; two more, M. Hoyrup and C. Rojas, in 2008). This activity has been reinforced by several collaborations, in Physics, Biology and Philosophy and from two to three joint theses directions by students working in other laboratories (A. Hazan, Laboratoire Syst. Complexes (LSC), Université d'Evry-Val d'Essonne, thesis defended in December 2007; A. Viarouge, Unicog, INSERM, since 2006; M. Devautour, CREA, Polytechnique, since 2006).

- Within the ANR project **"Singularities and Computations"** (Singul-calcul, see below), we analyzed the computational behavior of physico-mathematical dynamics, in particular in the mathematical terms of deterministic randomness. Our goal was first to establish a robust framework in order to handle probability theory as well as ergodic theory over dynamical systems from an algorithmic point of view, then to clarify the relations between classical (dynamical) and quantum randomness. How "genericity", a form of randomness for points in mathematical dynamics, or conversely, singularities, can be related to randomness or effective computations? We answered these questions in several papers, which join the geometric (morphological) analysis of dynamics to the computational approach. M. Hoyrup and C. Rojas have had a key role in this project, as well as the collaborations with mathematicians and physicists (P. Gacs, Boston Univ.; S. Galatolo, Pisa Univ.; F. Bailly, T. Paul, A. Lesne, CNRS).

- **Criticality and Singularities in Biology**. Critical states, a particular form of *singularity* in Physics, are characterized by the divergence of observables and a few "universality" properties. These states, often observed in phase transitions, may be seen as "limit" situations for physical theories, where critical values are . . . points. Our key idea has been to consider the state of a biological entity as being an "extended critical" one; this requires a non-obvious analysis of physical criticality beyond the one point-value of the control parameter. This approach is one of our "extensions" (enriching by proper observables and laws) of physical theories we work at in order to deal with the "physical singularity" of life phenomena. It is a component of the conceptual (and mathematical) transition we tackle, from Computations and Physics to Biology: in this perspective, the "living state of matter" is characterized by a long lasting far from equilibrium (and *extended*) critical state.

- **From entropy to morphological complexity**. The long-term objective of the project is the proposal and the mathematical investigation of new "observables" in the analysis of the dynamics of life. Phenotypic or morphological complexity, for example, is a major observable, both in phylogenesis and ontogenesis, but ... how to quantify it? The point is that this observable radically differs from the major observables in physical theories, energy typically, with its conservation properties and its many derived notions (momentum, action ...) and their measurement. Moreover, the complexity of organization is related to a key notion in computing, information, in a way which is far from clear. We tackled the issue from various viewpoints. First came the analysis of (negative) entropy both in Physics and Information Theory (Saulnier's thesis, 2006), then we proposed the notion of anti-entropy and handled it in original balance equations and thermodynamical inequalities (with Bailly)

- **Cognition and information**. Mossio's thesis (2006) and three more thesis by students appointed in other laboratories (one defended, two ongoing) aim at an analysis of the constitution of *invariants of action*, both in analysis of the cognitive foundations of Mathematics (Viarouge) and in the design of robots (Hazan). M. Devautour is currently working at a very ambitious mathematical project, based on an idea of the thesis co-director, D. Bennequin: Galois' theory of "ambiguity" (the relative size of the groups of automorphism over a field) could be an original and solid ground for developping this author's perspective towards a geometric (morphological) approach to information. Following Galois' theory thus, geometric information would be the opposite (in sign) of group-theoretic ambiguity: a typical way, à la Gelfand, to go from Algebra to Geometry.

3 Principaux résultats obtenus sur la période 2004-2007

Deux pages Max. 5 entrées maxi.

- **Dynamics and Computations**. In a note of 2001, the leader of the team observed an epistemological connection and, later, conjectured a mathematical one between Poincaré's Three Body Problem, in Mathematical Physics, and Gödel's theorem, in Logic. Note that Poincaré showed the existence of finite time unpredictability for basic dynamics (three celestial bodies in their gravitational field) and, today, astronomers can compute this time for the Solar System on the grounds of the best conceivable approximation or measure interval of the barycenter' coordinates of the planets. Intuitively, one may express this modern quantification of dynamical unpredictability as effective undecidability, in Gödel's sense: "the (formal) assertion on the future is *undecidable* w.r. to the given formal frame of the equations". Yet, the mathematical connection is not obvious, in particular because, following (the negative answer to) Hilbert's program, undecidability is a matter of a purely mathematical assertion. This author's idea was to compare the consequence of Poincaré's analysis, the existence of *deterministic randomness* in dynamical systems, with *algorithmic unpredictability*, a notion derived from Gödel's undecidability. Both notions may be expressed in purely mathematical terms, as asymptotic phenomena; in particular, as for deterministic randomness this may be also seen under the form of Birkhoff ergodicity.
  A major result has then been obtained by the team in the last two years: a mathematical relation between a very interesting notion of algorithmic randomness, due to Martin-Löf, Kolmogorov computational complexity and dynamical randomness. In particular, Hoyrup's and Rojas' theses proved that:
  - algorithmically random points are dynamically well behaved, in the ergodic sense,
  - the computational unpredictability of their orbits is equivalent to the chaoticity of the dynamical system.
  This result spans two theses, as it first required the reconstruction of key notions for physico-mathematical systems, in Recursion Theoretic terms. Typically, it was needed to develop an effective Measure Theory, in the sense of Lebesgue, effective probability measures and a suitable effective Ergodic Theory; notions and theorems for algorithmic entropy were also given, extending Shannon's approach to entropy on computable metric spaces. Strong relations between randomness, ergodic theorems, orbit complexities and entropies were established in these general, but effective, dynamical frames.

As for actual computing, that *pseudo*-random generators, in sequential machines, were characterized as generating sequences that pass *some* statistical tests, in contrast to proper random sequences that pass *all* of them. A close analysis was finally developed in order to distinguish dynamic flows vs. computations, classical vs. quantum randomness. (For this work one should consult Hoyrup's and Rojas' theses, their several submitted papers - downloadable - and [10], [5], [2] and [7]).

- **Extended Criticality**. In our exploratory systemic approach to Biology, we proposed to consider living systems as "coherent critical structures". These are extended singularities in space and time, in contrast to the pointwise singularities analyzed by physical theories of critical states. Our work may be seen as an "independent-track" contribution to the large amount of existing work on "self-organized criticality". More precisely, our main physical paradigm is provided by the analysis of "phase transitions", as this peculiar form of critical state presents interesting aspect of emergence, which oppose entropy growth: the formation of extended correlation lengths and coherence structures, the divergence of some observables with respect to the control parameter(s), ... Our claim is that in the case of living systems, these coherent critical structures are "extended" and organized in such a way that they persist over intervals of the parameters' space.
  To summarize certain aspects of our work, we could characterize the extended critical situation for a living organism by means of the following (non-exhaustive) traits:
  - a spatial volume enclosed within a semi-permeable boundary
  - correlation lengths of the order of magnitude of the greatest length of this volume
  - a confinement within a non-null volume of a many dimension parameters' space (temperature, pressure ...).
  Different levels of organization alternate (by integration/regulation) biological units (organisms) and organs, which are likely to be distinguished by the existence of fractal geometries (membranal or arborescent); the fractal geometries can be considered as the trace (or model) of effective passage to the infinite limit of an intensive magnitude of the system, in order to optimize exchanges of energy or of matter. The lengths of correlation manifest both within and between these levels. A preliminary insight into this approach has been hinted in [1], further work is presented in [4].

- **Anti-entropy,** *or from entropy to morphological complexity as "negative entropy"*. The notion of entropy appeared several times in all research themes above. As a matter of fact it provides, in very different contexts, a unified insight into randomness, lowering of information, relational dispersion, energy degradation or disorganization .... Its opposite, in sign, may provide a mathematical tool for the analysis of "organization" and the formation of morphological complexity, both in Evolution and in ontogenesis. Thus, by ongoing work with F. Bailly, first presented in [6] (a much longer and technical paper has been just accepted for publication, [3]), we proposed a systemic perspective for some aspects of both phylogenesis and ontogenesis by expressing biological organization in terms of "anti-entropy", a notion close but conceptually different from the common use of "negative entropy" (for a critique of "informational" approaches in Biology, see [8]).
  To this purpose, we introduced two principles, in addition to the thermodynamic ones, which are mathematically compatible with the classical principles but which have no meaning with regard to inert matter. A traditional balance equation for metabolism has then been extended to the new notion specified by these principles. We examined far from equilibrium systems and we focused in particular on the production of global entropy associated to the irreversible character of the processes. A close analysis of anti-entropy has been performed from the perspective of a diffusion equation of biomass over "complexity" and, as a complementary approach and as a tool for specifying a source term, in connection to Schrödinger's method regarding his equation in Quantum Mechanics. We borrow only the operatorial approach from this equation and do so using a classical framework, since we use real coefficients instead of complex ones, thus outside of the mathematical framework of quantum theories. The first application of our proposal is a mathematical model of the biomass distribution over complexity along Evolution. Thus, the asymmetric diffusion equation mathematically justifies the paleontological evidence presented by Gould.
  On the grounds of this approach, we analyzed metabolism and scaling laws. By this, we could compare various relevant coefficients appearing in these laws, which are shown to fit empirical data. This analysis was first

developed in B. Saulnier's thesis on (Shannon's) entropy in Biology and Physics (defended in October 2006), further developed in [3], where a quantitative evaluation of phenotypic complexity is proposed, also in relation to specific empirical data (caenorhabditis elegans). The results in [3] are the other main achievement in the team's activity.
Finally, a proper analysis of biological time is instrumental to our approach. We formalized this by representing time over a two dimensional manifold, that is by a temporal bi-dimensionality, one thermodynamical, associated to the physico-chemical processes in an organism, and the other, compactified, associated to the endogenous physiological rhythms of organisms, manifested by dimensionless numerical quantities (paper in press).

- **Cognition.** M. Mossio defended a thesis (LIENS & LPPA, joint supervision with A. Berthoz, Collège de France, October 2006) on the relation between biological autonomy and the constitution of cognitive invariants, [9]. These are analyzed as the result of sensorimotor activities. A. Hazan's thesis (December 07, Laboratoire Syst. Complexes (LSC), Université d'Evry, supervisor H. Maaref; co-director: this author) focused on statistical measures concerning a robot's activity within an ergodic sensorimotor network. The part pertinent to this team's work concerned the analysis of stabilization of morphogenetic and cognitive invariants by action in robots. The ongoing co-directed work by A. Viarouge and M. Davautour analyze "gestalts" and relation to space (geometry) in the formation of mathematical invariants and the proposal of a notion of morphological information (see the last theme above).

4 Objectifs scientifiques 2009-2011

Une Page Maxi. 5 entrées maxi

Rationale of the scientific objectives. In the last 4 years we focused on "principles", as the CIM exploratory project was meant to deal with *theoretical* issues bridging Computations, Physics and Biology. So we compared dynamical vs. algorithmic randomness; we modified (extended) criticality; we added anti-entropy to fundamental thermodynamic (in-)equalities and balance equations; we modeled rhythms and biological time by a doubling its manyfold's dimension. All papers are motivated by and contain references to empirical evidence, derived from the literature and corroborating the various approaches, yet a relevant part of the forthcoming work will be based on further collaborations with biologists, in particular, with an empirical research experience (A. Berthoz, N Peyreras, J. Ricard - France; C. Sonnenschein, A. Soto - USA; M. Buiatti - Italy). More specifically:

1. **Randomness and determination.** In [2], [7] and two theses (Hoyrup, Rojas) we presented the mathematical relations and differences between various approaches to randomness: classical (dynamical), computational, quantum. We claim that biology needs a proper notion, based on the indetermination or unpredictability of the very "phase space", that is of the pertinent observables (which will be the species or the phenotypes in the "next" ecosystem?). This is an unexplored indetermination in Physics (by Fock's spaces, quantum physicists list even the possible but unpredictable creations of new particles). Is this due to the intrinsic entanglement of classical and quantum randomness in life phenomena (morphogenetic effects vs. mutations)? Does one need to add to this the randomness in networks, from cellular networks to computers' networks? (collaborations: T. Paul, Math-Physics ENS; C. Palamidessi (INRIA) and C. Calude, Comp. Sci., New Zeeland; C. Sonnenschein, Biology, Tufts U., Boston).

2. **Extended criticality and cognition.** The least level of cognition seems to be a discriminating activity (a unicellular can "discriminate"): can this be described in terms of the internal unstability (an attractor moving from one point to another) in a dense subset of the hypercube of extended critical zones?
In a tissue, the cellular dialog seems to generate a zone of extended criticality: can this be checked empirically by providing the exact values of the critical extension and its relation to tissue degeneration as unstability? What about the (simpler) bacterial films ? (A PhD thesis grant has been just assigned to CIM on this topic by ED "Frontières du vivant", Oct. 2008; M. Montevil, Mathematics, ENS, Cachan, will have it: he is currently

working at a second Master Thesis in Biology, in the team of F. Taddei, INSERM)

3. **Anti-entropy.** We plan to compare our theoretical approach to anti-entropy, analyzed in balance equations which include entropy production, and empirical data. Are there correlations between the individual mass and the speed of entropy production in the evolution of the metabolism? Our most recent paper gives a precise mathematical frame to check theory against data, but a further collaboration with biologists is needed for this (A. Soto, Biology, Tufts U., Boston; M. Buiatti, Florence; F. Taddei, INSERM, by Montevil's work).

4. **Ambiguity as Negative Information.** In the last research theme above we mentioned the idea of understanding morphological information by Galois' theory of ambiguity (with an opposite sign): can this give a precise quantification of our idea of morphological information? Can Shannon's principles (entropy, say) be suitably transferred to the new frame? (co-directed thesis by M. Devotour, with D. Bennequin)

5. **Philosophy.** Two very active interdisciplinary groups, CHPS (Collectif Histoire et Philo Sciences) at ENS, and LIGC (Logique et Interaction comme Géométrie de la Cognition, international, co-founder and co-director with J-Y Girard) will be an occasion for further collaborations with philosophers (see downloaded papers, not quoted here). The current 20% time-commitement of the head of the team at CREA provided a chance for many meetings and discussions, which are leading to a better understanding of the strong philosophical comitment of the CIM project. In the proposed mathematical interactions between Computations, Physics and Biology there is a more or less implicit proposal for a "Philosophy of Nature", to put it in an old-fashioned and daring way, to be made philosophically explicit. The collaborations (ongoing or to be based on expected PhD co-theses by G. Frezza, A. Danielli) with J. Petitot, J. Lassegue, M. Bitbol at CREA, C. Debru at ENS, R. Fabbrichesi, M. Abrusci, E. Gagliasso in Italy, R. Tiestzen in USA will be part of this project (see the individual report for more).

5 Enseignement et Encadrement

Enseignements pour la période 2004-2007

Cours de niveau au moins égal à M1

Enseignant	Niveau	Titre court	Établissement	Vol. Horaire	N. années
Longo	M1	Cognition: modèles discrets/continus	EHESS-ENS	12h	3
Longo (avec T. Paul)	Cours libre	Incomplétude en logique et en physique	ENS	12h	1

Thèses soutenues dans la période 2004-2007

Nom	Prénom	Date de Soutenance	Établissement	École Doctorale	Encadrant	Situation Actuelle
Saulnier	Boris	15 septembre 2006	Paris VII	Informatique	G. Longo	Londres, M. Lynch
Mossio	Matteo	16 octobre 2006	EHESS-ENS	Cognition	G. Longo	Post-Doc, Paris I
Hoyrup	Mathieu	17 Juin 2008	Paris VII	Informatique	G. Longo	Post-Doc, INRIA-X
Rojas	Mathieu	18 Juin 2008	Polytechnique	Maths-Info	G. Longo	Post-Doc, Fields Inst., Toronto

HdR soutenues dans la période 2004-2007

Post-doctorants accueillis dans la période 2004-2007

6 Collaborations internationales avec publications conjointes

Hoyrup and Rojas have been developing extensive collaborations with P. Gacs (Computer Science, Boston Univ.) and S. Galatolo (Mathematics, Univ. of Pisa). The ongoing work lead to the following papers:

- Peter Gacs, Mathieu Hoyrup and Cristobal Rojas. Randomness–A dynamical point of view. *Submitted to Ergodic Theory and Dynamical Systems*.
- Stefano Galatolo, Mathieu Hoyrup and Cristobal Rojas. Effective symbolic dynamics, random points, statistical behavior, complexity and entropy. *Submitted to Information and Computation*.

7 Logiciels et Brevets

8 Principaux Contrats

Sans partenaire industriel

- **Projet ANR** "Singul-Calcul" (**"Singularities and Computations"**, in collaboration with A. Lesne, Physique, CNRS-IHES), 2005-08 (see the first research theme)

9 Éléments de Visibilité

Prix, Distinctions, Organisations de conférences majeures, Comités éditoriaux, . . . 10 Maxi; Indiquer aussi les actions de vulgarisation.

Journal Boards :

- **Editor - in - Chief**: Mathematical Structures in Computer Science, Cambridge Univ. Press
  (far away the main editorial activity)
- Information and Computation, Academic Press
- Theoretical Informatics and Applications, EDP Sciences
- La Nuova Critica (Philosophy of Sciences), Roma

Volumes edited :

- "Géométrie et Cognition", G. Longo (Editeur), num. spécial, Revue de Synthèse, Ed. rue d'Ulm, 124, 2004.
- "Images and Reasoning", P. Grialou, G. Longo, M. Okada (Eds), Keio University Press, Tokio, 2005.

Major Conference Organisation (16 more were (co-)organized by GL, 2004-08, see web page):

- "Computability in Europe 2006 *and* 2007: New Computational Paradigms", Swansea, GB, June 30 - July 5, 2006 *and* "Computation and Logic in the Real World", Siena, Italy, June 18-23, 2007.
- 4th (2007) *and* 5th (2008) annual Conference on "Theory and Applications of Models of Computation (TAMC07)", Shanghai, China, May 22-25, 2007 *and* (TAMC08) Shanghai, China, April 25-29, 2008.

20 Invited Lectures at national and international conferences by GL, 2004-08

- (see http://www.di.ens.fr/users/longo/exposes-tous.html)

2 Lectures for the general public, France Culture, Radiofrance, Paris:

- présentation du livre [1], le 14/6/2007 et "Modèles: mathématique vs. informatique", le 13/9/2007.

10 Publications Majeures (10 Maxi.; see http://www.di.ens.fr/users/longo/ for more)

References

[1] Francis Bailly and Giuseppe Longo. *Mathématiques et sciences de la nature. La singularité physique du vivant*. Hermann, Paris, 2006. (pp. 1-280; ongoing translation in English) (After submitting this book to the publisher, GL was invited to co-direct a new Hermann book series: Visions des sciences).

[2] Francis Bailly and Giuseppe Longo. Randomness and determination in the interplay between the continuum and the discrete. In T. Paul, editor, *3-bodies, classical-quantum, discrete-continuum*, pages 289–307. Special issue, Mathematical Structures for Computer Science, vol. 17, n.2, Cambridge U. Press, 2007.

[3] Francis Bailly and Giuseppe Longo. Biological Organization and Anti-entropy. *to appear in Journal of Biological Systems*, 2008.

[4] Francis Bailly and Giuseppe Longo. Extended Critical Situations. *Journal of Biological Systems*, 16(2):309–336, 2008.

[5] Mathieu Hoyrup, Arda Kolcak, and Giuseppe Longo. Computability and the morphological complexity of some dynamics on continuous domains. *Theoretical Computer Science*, 398(1-3):170–182, 2008.

[6] Giuseppe Longo. From exact sciences to life phenomena: following Schrödinger on Programs, Life and Causality. In *Information and Computation, Longo's conference special issue (to appear)*. Concluding lecture at "From Type Theory to Morphological Complexity: A Colloquium in Honor of Giuseppe Longo". Version préliminaire, en français, dans "Explication, modélisation et simulation en Biologie" (Barberousse et al. dir.) n. spécial de "Matière première", n. 3/2008, éd. Syllepse, Paris.

[7] Giuseppe Longo and Thierry Paul. The mathematics of computing between Logic and Physics. In B. Cooper et al., editor, *Computability in Context: Computation and Logic in the Real World*. Imperial College Press/World Scientific, 2008.

[8] Giuseppe Longo and P.-E. Tendero. The differential method and the causal incompleteness of Programming Theory in Molecular Biology. *Foundations of Science*, 12:337–366, 2007. Version française dans "Evolution des concepts fondateurs de la biologie du XXIe siècle" (P. A. Miquel ed.), DeBoeck, Paris, 2007.

[9] Matteo Mossio, Catherine Vidal, and Alain Berthoz. Traveled distances: new insights into the role of optic flow. *Vision Research*, 48(2):289–303, 2008.

[10] Cristóbal Rojas. Computability and Information in models of randomness and chaos. *Mathematical Structures in Computer Science*, 18:291–307, 2008.