



Formal Analysis of a **Triplex Sensor Voter** at Rockwell Collins France

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Perpignan



Agenda

- 1) Presentation Rockwell Collins
- 2) The Rockwell Collins Translator Framework
- 3) Analysis of a Triplex Sensor Voter

Rockwell Collins France

- Rockwell Collins France (RCF) is an **electronic systems manufacturer**
- 700+ employees, mainly located in Toulouse, France, subsidiary of Rockwell Collins Inc. (20.000 empl.)
- Systems and equipments for aircraft and rotary wing manufacturers (Airbus, Eurocopter, Augusta,...)
 - Communication, Navigation, Radar, Surveillance, Cockpit equipments
- We provide communication systems for European MODs (radio, networks)
 - Software define radio, Data Links (Link11, Link 16,...), Localization and SAR (Search And Rescue) equipments



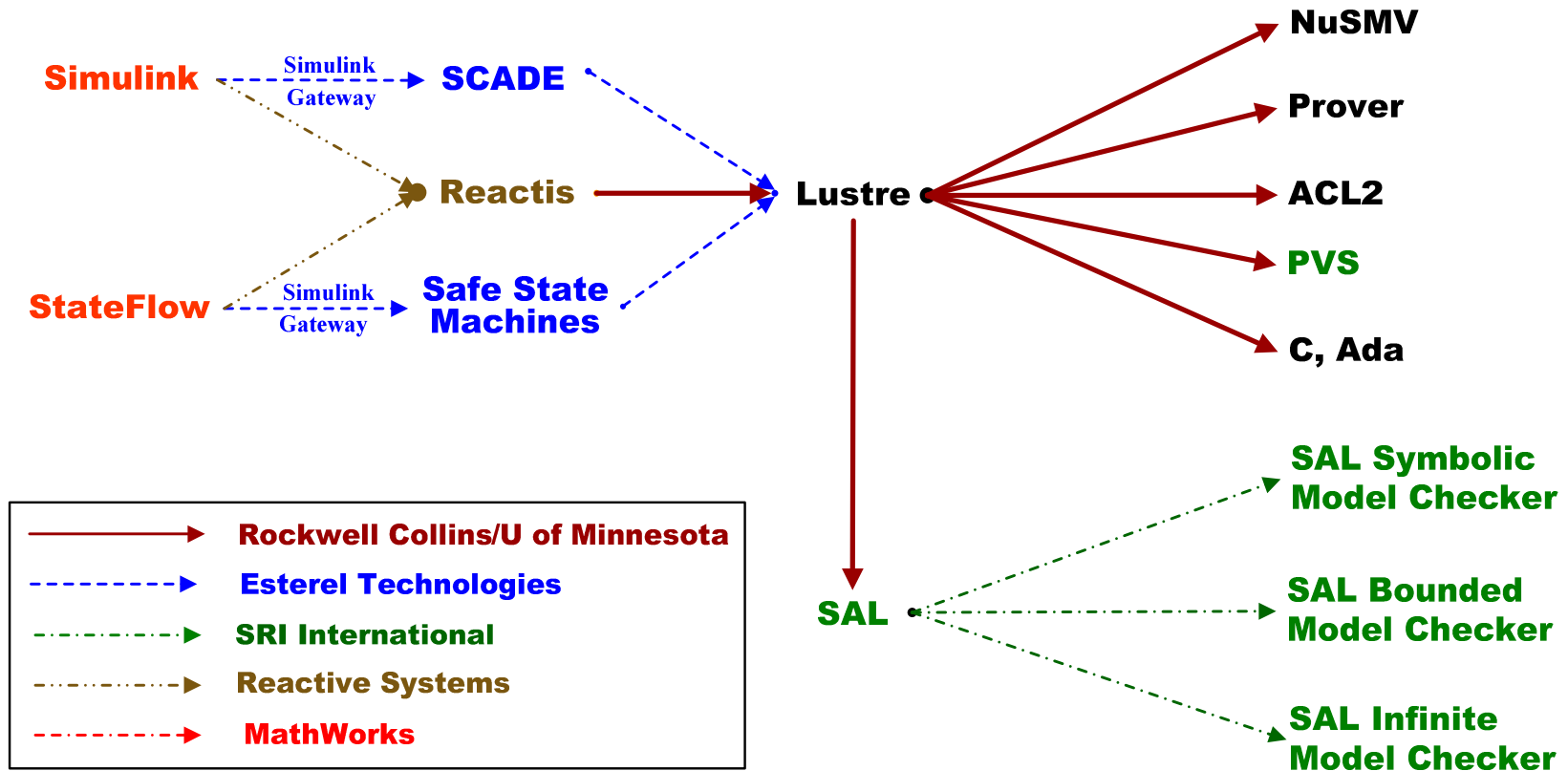
Formal Methods at Rockwell Collins

- In the US: team of ~10 research engineers (mostly PhD)
- Work on
 - model checking (MATLAB Simulink© **translator framework**)
 - Theorem proving (especially **ACL2**)
- For 1,5 years, 1 research engineer in France
- Starting in october 2010, a PhD in France (CIFRE with ONERA)

The Rockwell Collins Translator Framework

- Purpose : Verification of **SCADE™** and **MATLAB Simulink©** models
- **Long term effort** in the domain of formal methods
- Used on **several projects** (see articles by Steven Miller and Michael Whalen, e.g. *Software model checking takes off*, CACM 53(2), 2010)
- Based on an extension of **Lustre** as intermediate language
- Can output **optimized descriptions** in input languages of several **different analyzers**

The Rockwell Collins Translator Framework (2)



The Triplex Sensor Voter

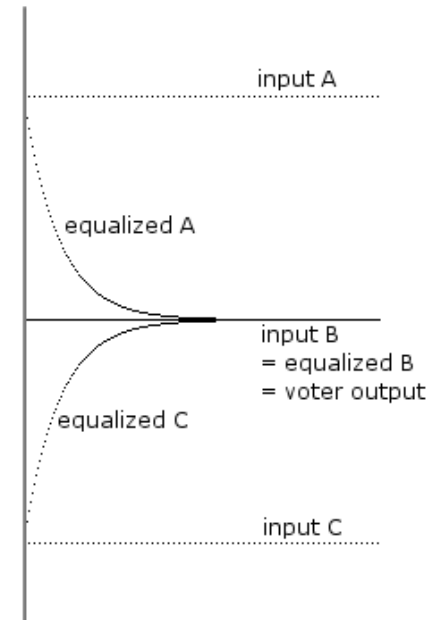
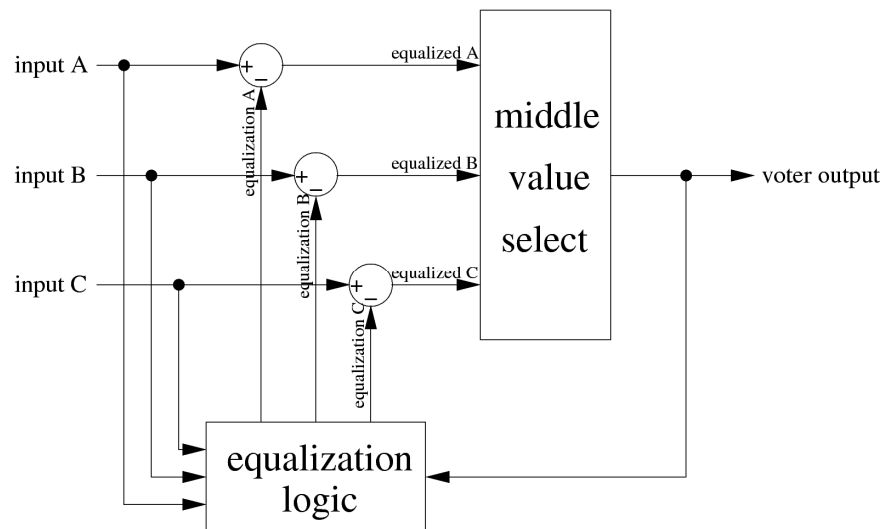
- Compute an output from input of **three redundant sensors**
- Able to detect and eliminate one **faulty sensor**
- User **reset** possible
- Implemented in **Simulink**
- Several blocks:
 - Value computation (arithmetic)
 - Fault detection (mainly boolean)
 - Reset (purely boolean)

Industrial Context of the Analysis

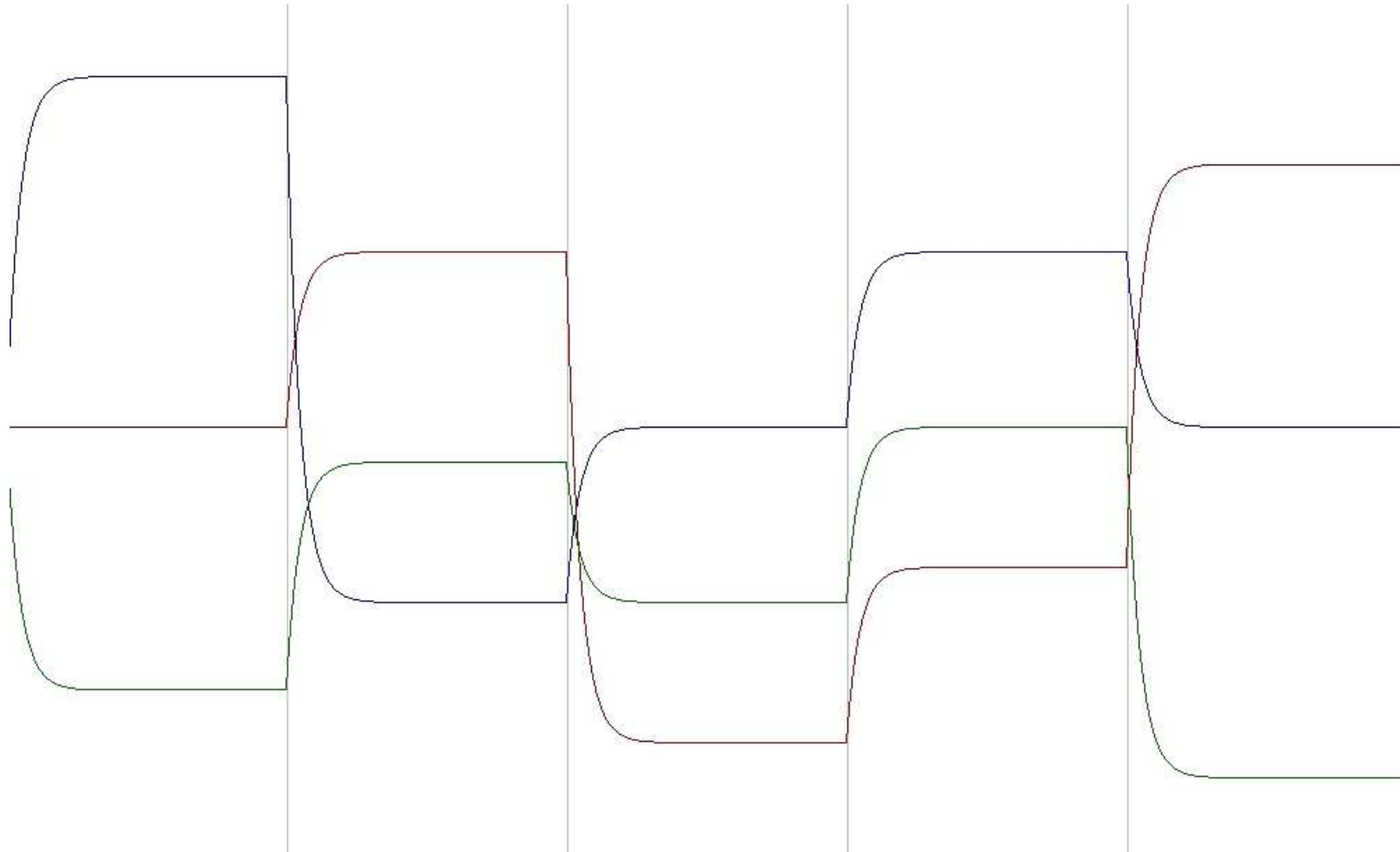
- **Legacy** model (~20 years old)
- Reverse engineering – **why** and **how** does it work ?
- Finding right **parameters** is **very time consuming**
- Has been **qualified**, high confidence
- **Modifications** are made now
 - Better usage of Simulink
 - 4th input ?
- **New application** areas

Normal Operation Mode of the Voter (no fault)

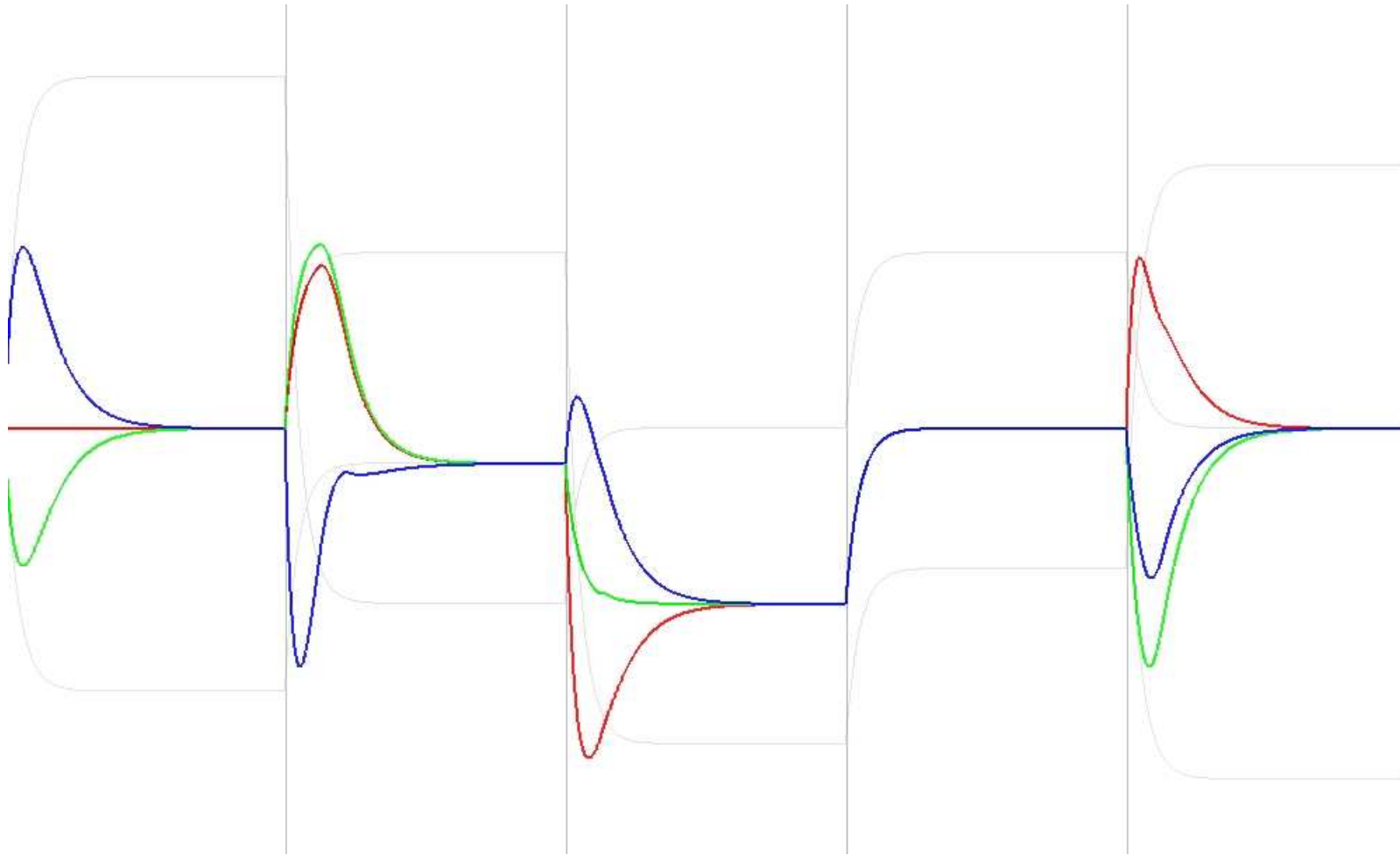
- From each of the three inputs, subtract an equalization value
- Output is middle value of equalized values
- Equalization based on integration – 3 memories of rational type



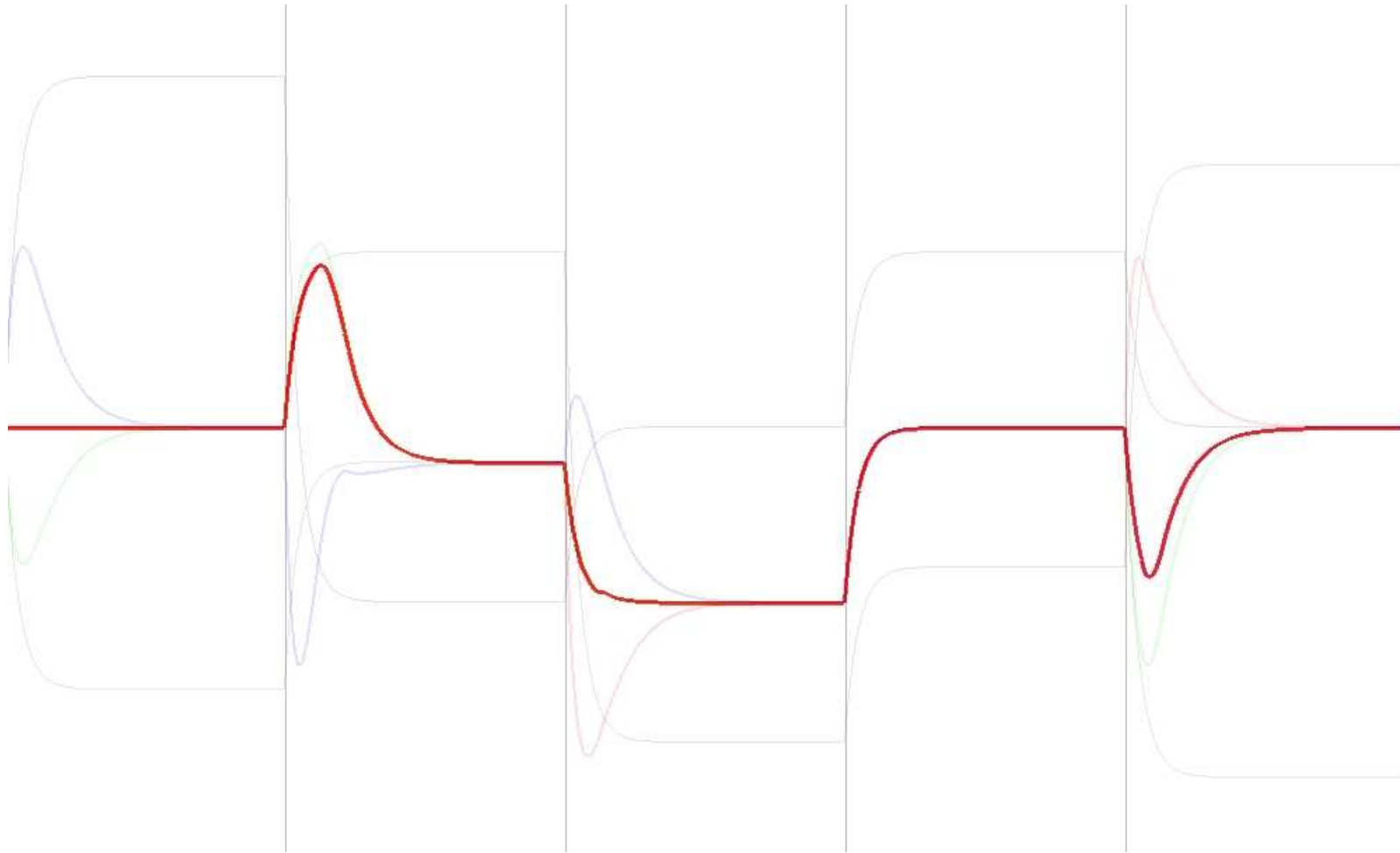
Simulation: Input Values



Simulation: Equalized Values



Simulation: Output Value



Equations of the Normal Operation Mode

$$\text{Equalization}A_0 = 0.0$$

$$\text{Equalization}B_0 = 0.0$$

$$\text{Equalization}C_0 = 0.0$$

$$\text{Centering}_t = \text{middleValue}(\text{Equalization}A_t, \text{Equalization}B_t, \text{Equalization}C_t)$$

$$\text{Equalized}A_t = \text{Input}A_t - \text{Equalization}A_t$$

$$\text{Equalized}B_t = \text{Input}B_t - \text{Equalization}B_t$$

$$\text{Equalized}C_t = \text{Input}C_t - \text{Equalization}C_t$$

$$\text{VoterOutput}_t = \text{middleValue}(\text{Equalized}A_t, \text{Equalized}B_t, \text{Equalized}C_t)$$

$$\begin{aligned} \text{Equalization}A_{t+1} = & \text{Equalization}A_t + \\ & 0.05 * (\text{sat}_{0.5}(\text{Equalized}A_t - \text{VoterOutput}_t) - \text{sat}_{0.25}(\text{Centering}_t)) \end{aligned}$$

$$\begin{aligned} \text{Equalization}B_{t+1} = & \text{Equalization}B_t + \\ & 0.05 * (\text{sat}_{0.5}(\text{Equalized}B_t - \text{VoterOutput}_t) - \text{sat}_{0.25}(\text{Centering}_t)) \end{aligned}$$

$$\begin{aligned} \text{Equalization}C_{t+1} = & \text{Equalization}C_t + \\ & 0.05 * (\text{sat}_{0.5}(\text{Equalized}C_t - \text{VoterOutput}_t) - \text{sat}_{0.25}(\text{Centering}_t)) \end{aligned}$$

Sensors and their Faults

- Non-faulty sensors furnish a value within an interval around true value determined by constant **MaxSensorError**

$$\text{abs}(\text{SensorValue} - \text{TrueValue}) \leq \text{MaxSensorError}$$

- Fault detection is based on **equalization** values

Objectives of the Analysis

- Analyse output to show that **transient peaks** cannot occur
- Find good **parameters** for fault detection and prove that a non-faulty sensor is never eliminated
- **Correct behaviour** : output tends to middle input value
- No **overflows**
- In general, what can we do with our **translator framework** ?

Approach of the Analysis

- Check on **model level**
- Handle **real** values in model as **rational** values
- Proof by induction -> **invariants** necessary

Example Properties

- What is the **maximal output error** for a given maximal sensor error ?

$$\text{abs}(\text{VoterOutput} - \text{TrueValue}) \leq ?$$

- What is the **maximal difference** of two equalization values for a given maximal sensor error ?

$$\text{abs}(\text{EqualizationA} - \text{EqualizationB}) \leq ?$$

Inductive Invariant

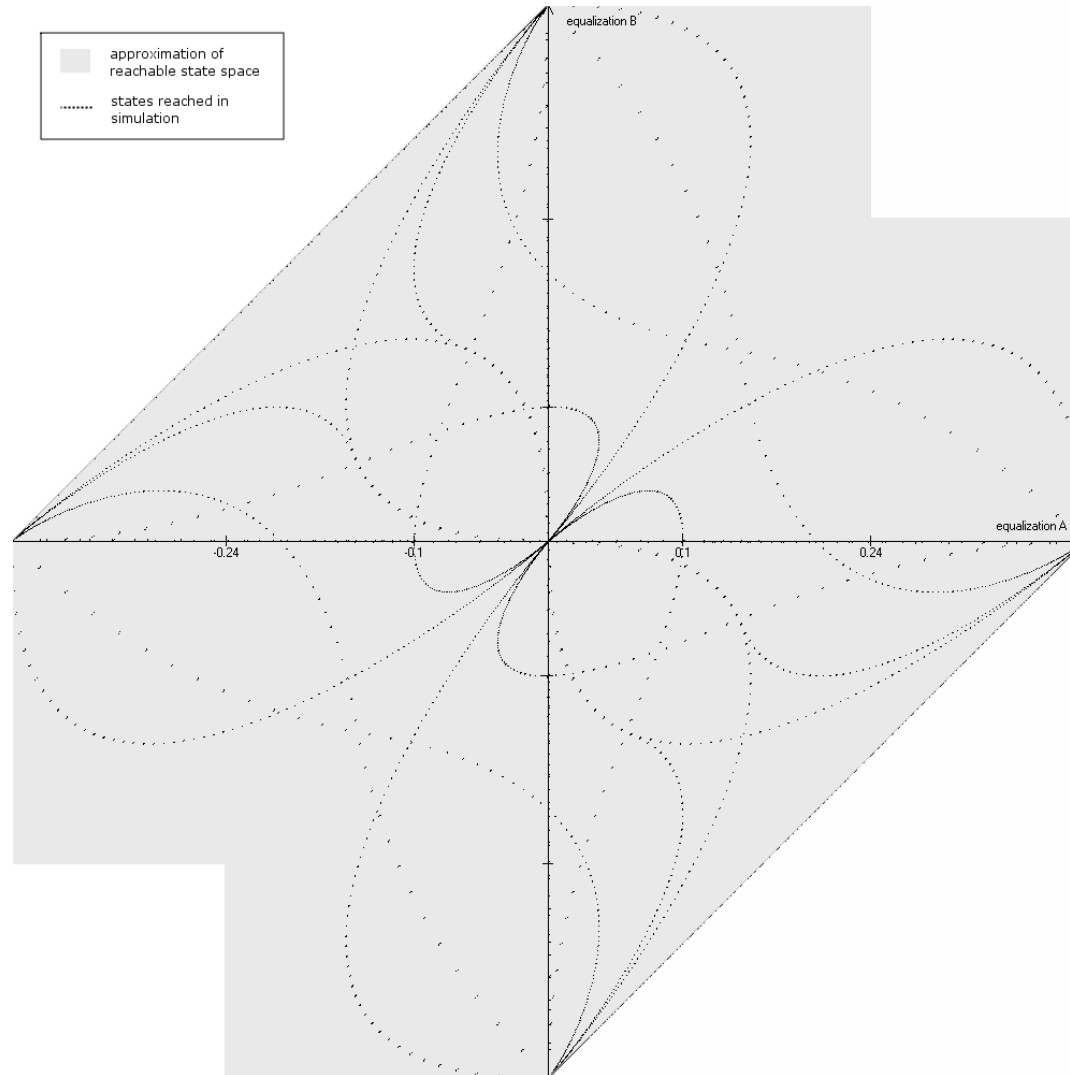
- $\text{Abs}(\text{EqualizationA} - \text{EqualizationB}) \leq 0.4$
- $\text{Abs}(\text{EqualizationA} - \text{EqualizationC}) \leq 0.4$
- $\text{Abs}(\text{EqualizationB} - \text{EqualizationC}) \leq 0.4$

- $\text{Abs}(\text{EqualizationA} + \text{EqualizationB} + \text{EqualizationC}) \leq 0.66$

- $\text{Abs}(\text{EqualizationA}) \leq 0.4$
- $\text{Abs}(\text{EqualizationB}) \leq 0.4$
- $\text{Abs}(\text{EqualizationC}) \leq 0.4$

- $\text{Abs}(\text{middle}(\text{EqualizationA}, \text{EqualizationB}, \text{EqualizationC})) \leq 0.24$

Simulation and Proof



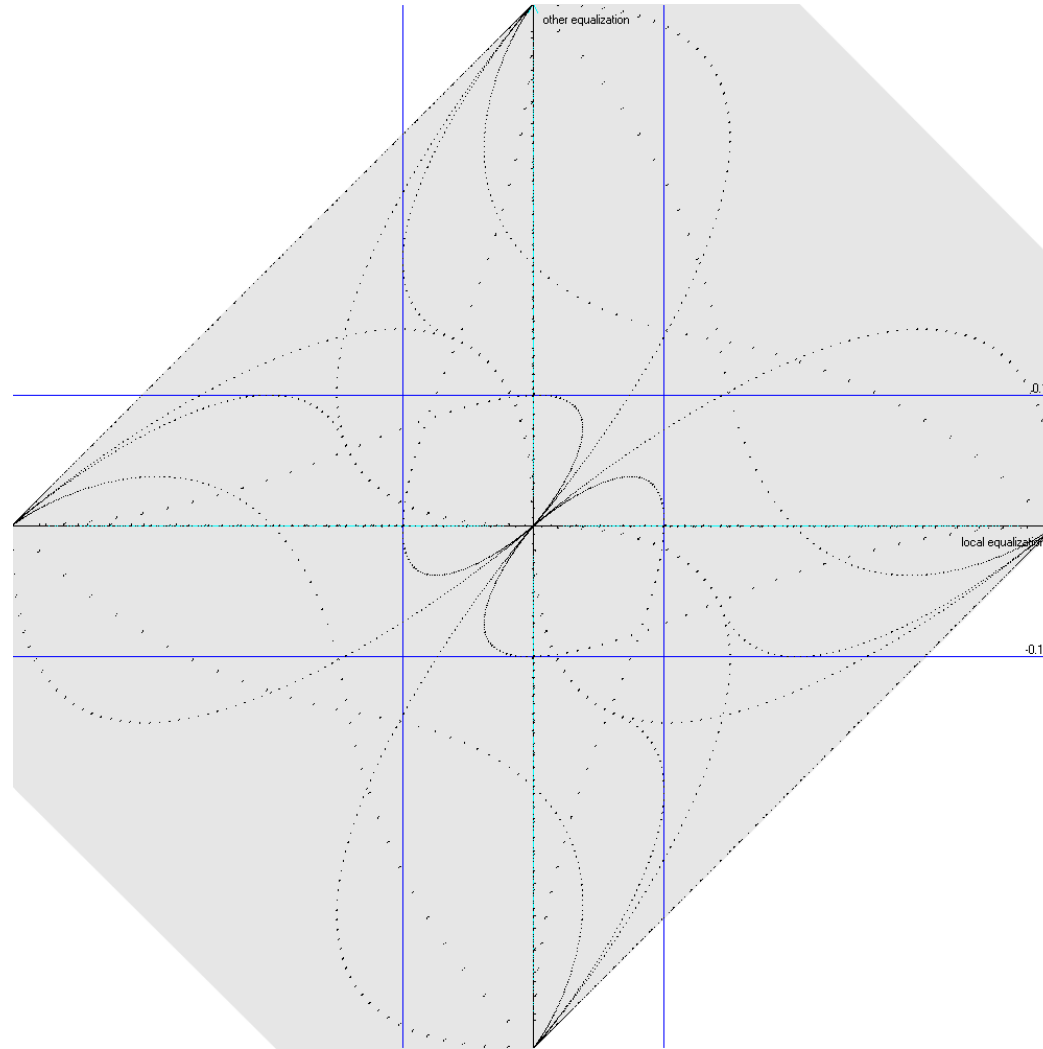
Inductive Octagonal Invariant

- $\text{Abs}(\text{EqualizationA}) \leq 0.4$
- $\text{Abs}(\text{EqualizationB}) \leq 0.4$
- $\text{Abs}(\text{EqualizationC}) \leq 0.4$

- $\text{Abs}(\text{EqualizationA} - \text{EqualizationB}) \leq 0.4$
- $\text{Abs}(\text{EqualizationA} - \text{EqualizationC}) \leq 0.4$
- $\text{Abs}(\text{EqualizationB} - \text{EqualizationC}) \leq 0.4$

- $\text{Abs}(\text{EqualizationA} + \text{EqualizationB}) \leq 0.6$
- $\text{Abs}(\text{EqualizationA} + \text{EqualizationC}) \leq 0.6$
- $\text{Abs}(\text{EqualizationB} + \text{EqualizationC}) \leq 0.6$

Inductive Octagonal Invariant



Simple Automatic Generation of Inductive Invariants

Choose a set of expressions $\text{expr}_1, \dots, \text{expr}_n$ over state variables

$v_1, \dots, v_n = 0.0$

Repeat

Check if $(\text{abs}(\text{expr}_1) \leq v_1 \text{ and } \dots \text{ and } \text{abs}(\text{expr}_n) \leq v_n)$
is inductive invariant

For all i

If step counter example exists with $\text{abs}(\text{expr}_i) > v_i$
 $v_i += 0.01$

Until no counter example exists

Analysis with Astrée

- Implementation of the (reduced) voter in C
- Astrée casts **false alarms** on **overflow** of equalization values
- Communicated to AbsInt
- Confirm the inductive invariant on code level ?

Lessons Learnt from Analysis

- Inductive proof was used, finding invariants was very **time consuming**
- All terms in invariant are **linear** (sums and differences)
- For max output error : still **gap** between value found by simulation and value proved
- **BMC not helpful** : too many steps necessary

Ongoing Work

- Extension to **fault case**
- **Speed up analysis** by adding lemmas
- Try to find **closer approximation** of state space
- Experiment **different proof engines** (e.g. new version of Kind)

Future Directions

- Can invariants be found **automatically** ? By abstract interpretation ?
- **Other forms** of invariants (non-linear, combined with boolean conditions, etc.) ?
- Potential case study for **combining** model checking and abstract interpretation (CMACS, PhD RCF/ONERA)
- Relevance for implementation with **floating point numbers** ?

Thank you !