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Gunnar BRINKMANN
Michel DEZA

LIENS - 98 - 13

Département de Mathématiques et Informatique

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Michel DEZA

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Laboratoire d'Informatique de l'Ecole Normale Supérieure
45 rue d'Ulm 75230 PARIS Cedex 05

Tel : (33)(1) 01 44 32 30 00
Adresse électronique : deza@dmi.ens.fr

* Fakultät für Mathematik, Universität Bielefeld
D-33501 Bielefeld

Adresse électronique : gunnar@mathematik.uni-bielefeld.de

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Gunnar Brinkmann
Fakultät für Mathematik
Universität Bielefeld
D 33501 Bielefeld *

Michel Deza
CNRS and LIENS-DMI
Ecole Normale Supérieure
75005 Paris, France †

0.1 Abstract

We introduce a new notion that connects the combinatorial concept of regularity with the geometrical notion of face-transitivity. This new notion implies finiteness results in case of bounded maximal face size. We give lists of structures for some classes and investigate polyhedra with constant vertex degree and faces of only two sizes.

1 Introduction

A planar (finite or infinite) graph is called *face-transitive*, if the automorphism group acts transitively on the set of faces. For finite polyhedra (see [Ma71]) as well as for infinite graphs in the plane with finite faces and finite vertex degree (that is tilings, see [Ba90][De90]) it is well known that the graph can be realized with its full combinatorial automorphism group as its group of geometrical symmetries. Restricting the attention to polyhedra with constant vertex degree, up to combinatorial equivalence only the 5 *Platonic solids* have an automorphism group acting transitively on their faces. In the remaining text we will restrict our attention to polyhedra with constant vertex degree.

A natural generalisation of this concept – let us call it *weakly face-transitive* – is to require that only faces of the same size are equivalent under the automorphism group. If we define the 0-th corona of a face to be the face itself and the n-th corona to be the set of all those faces that are contained in the (n-1)-th corona or share an edge with it, we can further relax this concept and only require some coronas of fixed size to be isomorphic by an isomorphism mapping the central faces onto each other. A polyhedron with all n-coronas of faces of the same size isomorphic is called *weakly n-transitive*. Obviously, all polyhedra are weakly 0-transitive and if a polyhedron is weakly (n+1)-transitive, it is also weakly n-transitive. So the first interesting case to study is the case of weakly 1-transitive polyhedra. Still relaxing this condition by not requiring the first coronas to be isomorphic, but just to be isomorphic as multisets (that is: every face of a given size i must have the same number of neighbours of size i' for every i'), still gives a very restrictive condition

*e-mail: gunnar@mathematik.uni-bielefeld.de
†e-mail: deza@dmi.ens.fr

and as we will see, it already implies finiteness in case the maximal size of a face is bounded. We call this condition (strong) *face-regularity.* So the class of all face-regular polyhedra contains all weakly n-face-transitive polyhedra for any $n \geq 1$, and therefore also the weakly face-transitive or even face-transitive ones.

The same concept can be reached by strengthening the notion of a monochromatic regular dual: Let p_i denote the number of i-gons in a given polyhedron. We use the notation $p = (p_3, p_4, ..., p_i, ...p_b)$ for the face-vector (or p-vector) of a polyhedron; b is the maximal number for which a face with size b exists.

A less restrictive definition of face-regularity, but only for bifaced polyhedra, was considered in [DGr97c]. Namely, if only p_a and p_b are non-zeros and $a < b$, then the number f of i-faces, edge-adjacent to any given i-face, was required to be independent of the choice of the i-face, for i either a or b. For a k-valent polyhedron we write aR_f or bR_f, if this partial (or *weak*) face-regularity holds for a-gonal or, respectively, b-gonal faces. All such simple polyhedra with $b \leq 6$, as well as all 4-valent ones with $b = 4$, except the cases $4R_0$ for $(k; a, b) = (3; 4, 6)$ and aR_0, aR_1 for $(k; a, b) \in \{(4; 3, 4), (3; 5, 6)\}$ were found in [DGr97c]. For example, all 12 (resp. 6,4,10,26) polyhedra bR_f for all five possible cases - $k = 4$; $k = 3, b < 6$ and $k = 3, b = 6, a \in \{3, 4, 5\}$ - are listed there. (The graphs of all 26 $6R_f$ *fullerenes* (i.e. $(k, a, b) = (3, 5, 6)$) are given in list 7 below.) In these cases 8 (resp. 6,4,9,12) polyhedra are also aR_f, i.e. face-regular in the sense of the present paper.

The face-regularity which we consider, is a purely combinatorial property of the skeleton of a polyhedron. It is different from the affine notion of *regular-faced* (i.e. all faces being regular polygons) polyhedra.

We use the abbreviation *frp* for *face-regular polyhedron.* An frp in one of the lists below is described by i_j, where j is the number of the List and i is its number in List j. We also use the notation i for i_1.

We call two frp *fr-isomers*, if they have the same parameters as frp, i.e. v, the p-vector and the numbers $f(a, b)$, i.e. the number of b-faces, edge-adjacent to each a-face for any a, b, coincide.

All fr-isomers in List 1 are bifaced. They are: $11, 12 (v = 16)$; $20, 21 (v = 32)$; $32, 33 (v = 80)$ and 3-faced $49, 50 (v = 20)$

All fr-isomers in List 2 are:

for $v = 20$: $10_2, 11_2$;

for $v = 24$: $61_2, 62_2$;

for $v = 26$: 16_2—19_2;

for $v = 28$: $66_2, 67_2$; $69_2, 70_2$; 72_2—74_2; $75_2, 77_2$; $76_2, 78_2$;

for $v = 32$: 28_2—31_2; 32_2—34_2; $87_2, 88_2$; $89_2, 90_2$;

for $v = 36$: $2_2, 3_2$; $95_2, 96_2$; $102_2, 103_2$;

for $v = 40$: $42_2, 43_2$.

for $v = 44$: $5_2, 6_2$; 49_2—51_2; $119_2, 120_2$; 137_2—139_2

All fr-isomers in Lists 4 and 5 are $6_4, 7_4$ with $v = 14$.

Considering the polyhedra of Lists 1,2 and 3 with respect to collapsing of all triangular faces to points, (i.e. the inverse to vertex-truncation), we see that in List 1, any such collapsing gives a member of List 1. But in List 2 there are polyhedra, such that this collapsing does not give an frp. The smallest one is 116_2.

Examples of sequences of frp, such that each of them comes from the previous one by 1-edge truncation are: $1, 4, 2, 6, 7, 8, 9$; $1, 4, 35, 36, 59, 39, 11$; and $1, 4, 2, 6, 14, 4_2$.

Some infinite families of 3-valent frp:

Bifaced : Prism_n and Barrel_n (i.e. two n-gons separated by two layers of 5-gons);

3-faced : Prism_n, Barrel_n, truncated on all 2n vertices of both n-gons;

Prism_{2n}, edge-truncated on n disjoint edges of only one n-gon;

Prism_{3n}, edge-truncated on n edges, separated by at least 2 edges, of only one n-gon;

4-faced : Prism_n, (vertex-) truncated on all vertices of only one n-gon;

5-faced : Barrel_n, truncated on all vertices of only one n-gon.

In fact, many of the frp in the lists are some partial truncations of Prism_n and Barrel_n. For example, there are exactly 10 frp, which are partial truncations of the Cube: There are 1 (resp. 3,1,3,1,1) possibilities for truncations on 1 (resp 2,3,4,6,8) vertices.

Remarks:

(i) Among the chiral polyhedra in the lists are, for example, Nrs 41, 61, 62, 63, 100, 104 in List 2; Nr.9 in List 3; and, especially, Nrs 13,22,34 in List 1 and 9 in List 4 with symmetry T, O, I and O, respectively.

(ii) None of the polyhedra in any of our Lists has a trivial symmetry group.

The Finiteness of Classes with Bounded Face Size

Theorem 1 *For every $n \in \mathbb{N}$ there is only a finite number of face-regular polyhedra with constant vertex degree and face sizes not exceeding n.*

Proof.

We will assume that the polyhedra in question all contain an n-gon. The total number can be obtained by summing up over all $m \leq n$.

Remind that for $i, j \in \mathbb{N}$ the number $f(i,j)$ denotes the number of neighbouring j-gons of an i-gon. So $f(i,j)p_i = f(j,i)p_j$ is the number of edges between i-gons and j-gons and we can express p_j as $p_j = \frac{f(i,j)}{f(j,i)}p_i$ in case i-gonal and j-gonal faces share at least one edge.

Look at the *f-graph* G with vertex set $V = \{i|p_i > 0\}$ and edge set $E = \{\{i,j\}|f(i,j) > 0\}$. This graph is connected since the dual of the underlying polyhedron is connected. We can express every other value p_i by a formula of the kind $\frac{f(i_1,i)}{f(i,i_1)}\frac{f(i_2,i_1)}{f(i_1,i_2)}\cdots\frac{f(i_b,i_k)}{f(i_k,i_b)}p_n =: g(i)p_n$ if $i, i_1, \ldots, i_k, n$ is a (e.g. shortest) path from i to n in G.

Since for fixed n all the $f(i,j)$ as well as the length of the path are bounded and since the number of graphs on n vertices is also finite, we have only a finite number of possible sets of equations $p_i = g(i)p_n$ $(3 \leq i \leq n)$.

As a well known consequence of Euler's formula we get $\sum_{i=3}^{n}(6-i)p_i = 12$ in the 3-valent case, $\sum_{i=3}^{n}(4-i)p_i = 8$ for 4-valent polyhedra and $\sum_{i=3}^{n}(10-3i)p_i = 20$ for 5-valent polyhedra.

Substituting p_i by $g(i)p_n$ in this formula, every set of equations gives exactly one solution for p_n and therefore also for each p_i. So for every set of equations there is a well determined number of faces and therefore there is a maximum number of faces that is possible.

□

Corollary 1 *If in the cubic case the number of non-hexagons is bounded or in the quartic case the number of non-squares is bounded, then there is only a finite number of face-regular polyhedra.*

Proof.

The fact that the number of faces smaller than 6 (resp. 4) is bounded gives an upper bound on the maximum face size, implying the result by the previous theorem.

□

Statistics

In this section we will give some statistics about the number of face-regular polyhedra compared to the number of all polyhedra for some classes.

vertices	polyhedra	face-regular polyhedra
4	1	1
6	1	1
8	2	2
10	5	4
12	14	7
14	50	5
16	233	15
18	1 249	9
20	7 595	33
22	49 566	11
24	339 722	58
26	2 406 841	29
28	17 490 241	99
30	129 664 753	44
32	977 526 957	194
34	7 475 907 149	25
36	57 896 349 553	318

Table 1: Cubic polyhedra

vertices	polyhedra	face-regular polyhedra
8	1	1
10	1	1
12	2	2
14	5	3
16	12	3
18	34	1
20	130	10
22	525	2
24	2 472	8
26	12 400	5
28	65 619	10
30	357 504	7
32	1 992 985	30
34	11 284 042	1
36	64 719 885	22
38	375 126 827	16
40	2 194 439 398	18

Table 2: Cubic polyhedra without triangles

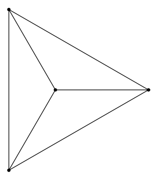
vertices	polyhedra	face-regular polyhedra
4	1	1
6	1	1
8	2	2
10	5	4
12	10	7
14	15	3
16	30	7
18	44	2
20	77	10
24	184	6
26	267	2
28	420	3
30	595	1
32	883	5
38	2 445	1
44	6 319	1
52	19 345	1
56	32 219	2
60	52 293	1
68	128 343	1
80	425 998	2
140	???	1

Table 3: Cubic polyhedra without faces larger than a hexagon. For all vertex numbers not mentioned, no face-regular polyhedra exist.

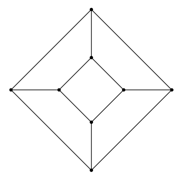
2 List 1: all 64 face-regular simple polyhedra with $b \le 6$.

Among the 64 polyhedra of the List, the first three are regular, then there are 31 bifaced ones: six with $b \le 5$, four 3_n (for n = 12, 16, 16, 26), nine 4_n (for n = 12, 14, 20, 20, 24, 26, 32, 32,36) and 12 fullerenes 5_n (which are $F_{24}(D_{6d})$, $F_{28}(T_d)$, $F_{32}(D_{3h})$, $F_{38}(C_{3v})$, $F_{44}(T)$, $F_{52}(T)$, $F_{56}(T_d)$, $F_{60}(I_h)$, $F_{68}(T_d)$, $F_{80}(I_h)$, $F_{80}(D_{5h})$, $F_{140}(I)$). Nrs. 35–57 have three types of faces and last seven polyhedra, Nrs. 58–64, have four types of faces.

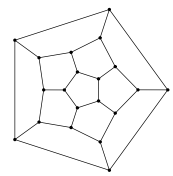
Among the 64 polyhedra of the List 1, three are regular ones (Tetrahedron, Cube and Dodecahedron), five are semi-regular (3-, 5-, 6-gonal prisms, truncated octahehedron and truncated Icosahedron) and no one is regular-faced from the list of 92 in [Joh66]. But there are three, which are dual to regular-faced snub disphenoid, 3-augmented 3-gonal prism and gyroelongated square dipyramid (last three have number 84, 51 and 17, respectively, in the list of [Joh66]). Together with three regular ones and 3-, 5-gonal prisms, it gives the duals of all eight convex deltahedra.



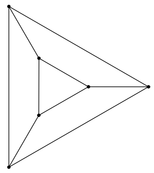
Nr.1 $v = 4$
$p_3 = 4 : 3$
Groupsize: 24
Group: T_d



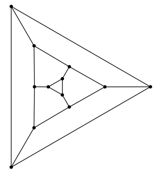
Nr.2 $v = 8$
$p_4 = 6 : 0{,}4$
Groupsize: 48
Group: O_h



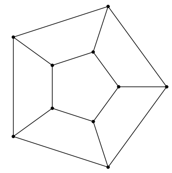
Nr.3 $v = 20$
$p_5 = 12 : 0{,}0{,}5$
Groupsize: 120
Group: I_h



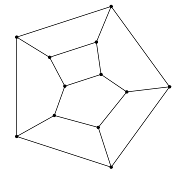
Nr.4 $v = 6$
$p_4 = 3 : 2,2$
$p_3 = 2 : 0,3$
Groupsize: 12
Group: D_{3h}



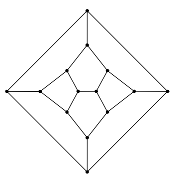
Nr.5 $v = 12$
$p_5 = 6 : 1,0,4$
$p_3 = 2 : 0,0,3$
Groupsize: 12
Group: D_{3d}



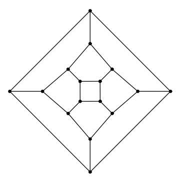
Nr.6 $v = 10$
$p_5 = 2 : 0,5,0$
$p_4 = 5 : 0,2,2$
Groupsize: 20
Group: D_{5h}



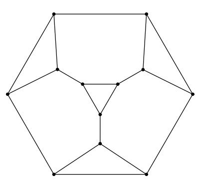
Nr.7 $v = 12$
$p_5 = 4 : 0,3,2$
$p_4 = 4 : 0,1,3$
Groupsize: 8
Group: D_{2d}



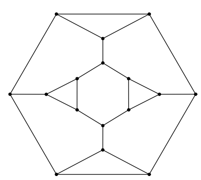
Nr.8 $v = 14$
$p_5 = 6 : 0,2,3$
$p_4 = 3 : 0,0,4$
Groupsize: 12
Group: D_{3h}



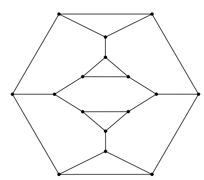
Nr.9 $v = 16$
$p_5 = 8 : 0,1,4$
$p_4 = 2 : 0,0,4$
Groupsize: 16
Group: D_{4d}



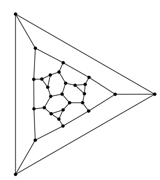
Nr.10 $v = 12$
$p_6 = 4 : 3,0,0,3$
$p_3 = 4 : 0,0,0,3$
Groupsize: 24
Group: T_d



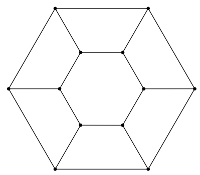
Nr.11 $v = 16$
$p_6 = 6 : 2,0,0,4$
$p_3 = 4 : 0,0,0,3$
Groupsize: 24
Group: T_d



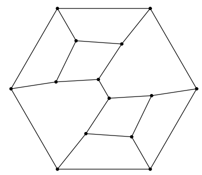
Nr.12 $v = 16$
$p_6 = 6 : 2,0,0,4$
$p_3 = 4 : 0,0,0,3$
Groupsize: 8
Group: D_{2h}



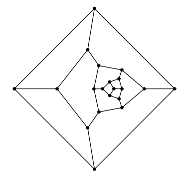
Nr.13 $v = 28$
$p_6 = 12 : 1,0,0,5$
$p_3 = 4 : 0,0,0,3$
Groupsize: 12
Group: T



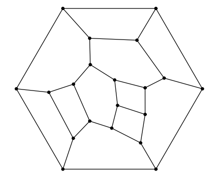
Nr.14 $v = 12$
$p_6 = 2 : 0,6,0,0$
$p_4 = 6 : 0,2,0,2$
Groupsize: 24
Group: D_{6h}



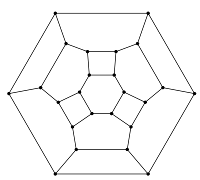
Nr.15 $v = 14$
$p_6 = 3 : 0,4,0,2$
$p_4 = 6 : 0,2,0,2$
Groupsize: 12
Group: D_{3h}



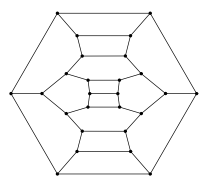
Nr.16 $v = 20$
$p_6 = 6 : 0,2,0,4$
$p_4 = 6 : 0,2,0,2$
Groupsize: 12
Group: D_{3d}



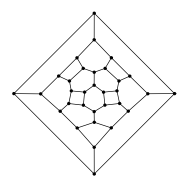
Nr.17 $v = 20$
$p_6 = 6 : 0,3,0,3$
$p_4 = 6 : 0,1,0,3$
Groupsize: 6
Group: S_6



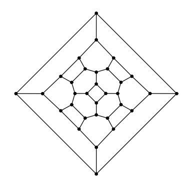
Nr.18 $v = 24$
$p_6 = 8 : 0,3,0,3$
$p_4 = 6 : 0,0,0,4$
Groupsize: 48
Group: O_h



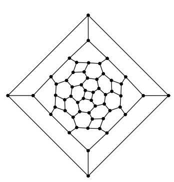
Nr.19 $v = 26$
$p_6 = 9 : 0,2,0,4$
$p_4 = 6 : 0,1,0,3$
Groupsize: 12
Group: D_{3h}



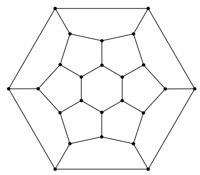
Nr.20 $v = 32$
$p_6 = 12 : 0,2,0,4$
$p_4 = 6 : 0,0,0,4$
Groupsize: 12
Group: D_{3d}



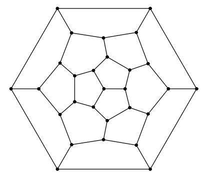
Nr.21 $v = 32$
$p_6 = 12 : 0,2,0,4$
$p_4 = 6 : 0,0,0,4$
Groupsize: 48
Group: O_h



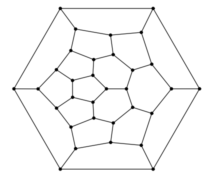
Nr.22 $v = 56$
$p_6 = 24 : 0,1,0,5$
$p_4 = 6 : 0,0,0,4$
Groupsize: 24
Group: O



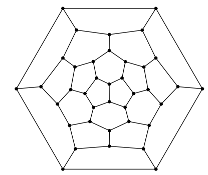
Nr.23 $v = 24$
$p_6 = 2 :$ 0,0,6,0
$p_5 = 12 :$ 0,0,4,1
Groupsize: 24
Group: D_{6d}



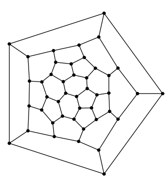
Nr.24 $v = 28$
$p_6 = 4 :$ 0,0,6,0
$p_5 = 12 :$ 0,0,3,2
Groupsize: 24
Group: T_d



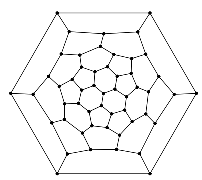
Nr.25 $v = 32$
$p_6 = 6 :$ 0,0,4,2
$p_5 = 12 :$ 0,0,3,2
Groupsize: 12
Group: D_{3h}



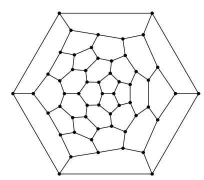
Nr.26 $v = 38$
$p_6 = 9 :$ 0,0,4,2
$p_5 = 12 :$ 0,0,2,3
Groupsize: 6
Group: C_{3v}



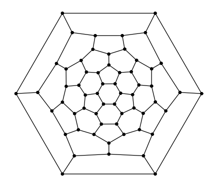
Nr.27 $v = 44$
$p_6 = 12 :$ 0,0,3,3
$p_5 = 12 :$ 0,0,2,3
Groupsize: 12
Group: T



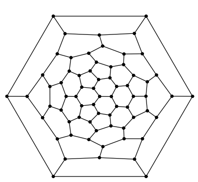
Nr.28 $v = 52$
$p_6 = 16 :$ 0,0,3,3
$p_5 = 12 :$ 0,0,1,4
Groupsize: 12
Group: T



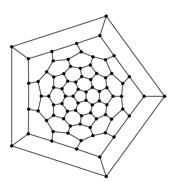
Nr.29 $v = 56$
$p_6 = 18 :$ 0,0,2,4
$p_5 = 12 :$ 0,0,2,3
Groupsize: 24
Group: T_d



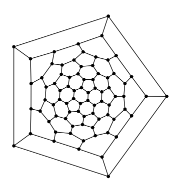
Nr.30 $v = 60$
$p_6 = 20 :$ 0,0,3,3
$p_5 = 12 :$ 0,0,0,5
Groupsize: 120
Group: I_h



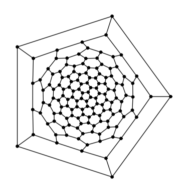
Nr.31 $v = 68$
$p_6 = 24 :$ 0,0,2,4
$p_5 = 12 :$ 0,0,1,4
Groupsize: 24
Group: T_d



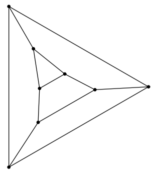
Nr.32 $v = 80$
$p_6 = 30 :$ 0,0,2,4
$p_5 = 12 :$ 0,0,0,5
Groupsize: 120
Group: I_h



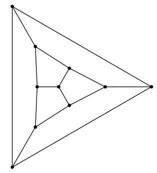
Nr.33 $v = 80$
$p_6 = 30 :$ 0,0,2,4
$p_5 = 12 :$ 0,0,0,5
Groupsize: 20
Group: D_{5h}



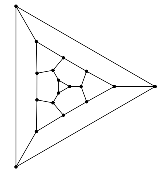
Nr.34 $v = 140$
$p_6 = 60 :$ 0,0,1,5
$p_5 = 12 :$ 0,0,0,5
Groupsize: 60
Group: I



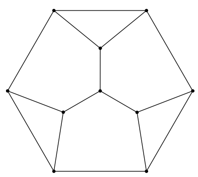
Nr.35 $v = 8$
$p_5 = 2 : 2,2,1$
$p_4 = 2 : 1,1,2$
$p_3 = 2 : 0,1,2$
Groupsize: 4
Group: C_{2v}



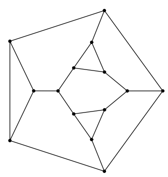
Nr.36 $v = 10$
$p_5 = 3 : 1,2,2$
$p_4 = 3 : 0,2,2$
$p_3 = 1 : 0,0,3$
Groupsize: 6
Group: C_{3v}



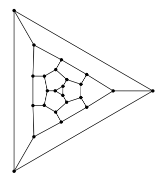
Nr.37 $v = 18$
$p_6 = 6 : 1,2,0,3$
$p_4 = 3 : 0,0,0,4$
$p_3 = 2 : 0,0,0,3$
Groupsize: 12
Group: D_{3h}



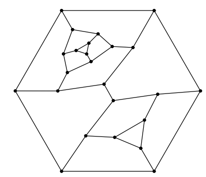
Nr.38 $v = 10$
$p_6 = 1 : 3,0,3,0$
$p_5 = 3 : 2,0,2,1$
$p_3 = 3 : 0,0,2,1$
Groupsize: 6
Group: C_{3v}



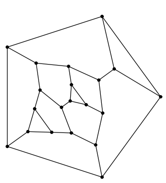
Nr.39 $v = 14$
$p_6 = 3 : 2,0,2,2$
$p_5 = 3 : 1,0,2,2$
$p_3 = 3 : 0,0,1,2$
Groupsize: 6
Group: C_{3v}



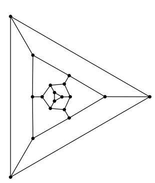
Nr.40 $v = 24$
$p_6 = 6 : 1,0,3,2$
$p_5 = 6 : 0,0,2,3$
$p_3 = 2 : 0,0,0,3$
Groupsize: 12
Group: D_{3d}



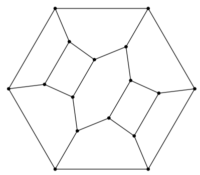
Nr.41 $v = 24$
$p_6 = 6 : 0,0,2,4$
$p_5 = 6 : 1,0,2,2$
$p_3 = 2 : 0,0,3,0$
Groupsize: 12
Group: D_{3h}



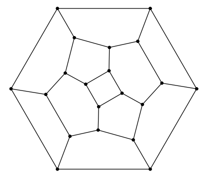
Nr.42 $v = 20$
$p_6 = 6 : 1,0,2,3$
$p_5 = 3 : 1,0,0,4$
$p_3 = 3 : 0,0,1,2$
Groupsize: 6
Group: C_{3h}



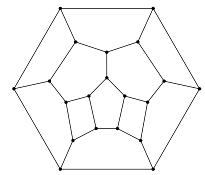
Nr.43 $v = 18$
$p_6 = 3 : 0,0,4,2$
$p_5 = 6 : 1,0,2,2$
$p_3 = 2 : 0,0,3,0$
Groupsize: 12
Group: D_{3h}



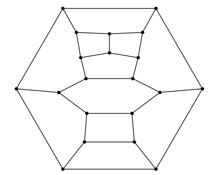
Nr.44 $v = 16$
$p_6 = 2$: 0,2,4,0
$p_5 = 4$: 0,2,1,2
$p_4 = 4$: 0,1,2,1
Groupsize: 8
Group: D_{2h}



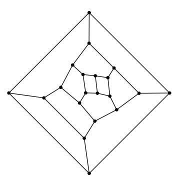
Nr.45 $v = 20$
$p_6 = 3$: 0,2,4,0
$p_5 = 6$: 0,1,2,2
$p_4 = 3$: 0,0,2,2
Groupsize: 12
Group: D_{3h}



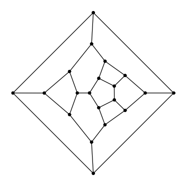
Nr.46 $v = 20$
$p_6 = 4$: 0,2,3,1
$p_5 = 4$: 0,2,0,3
$p_4 = 4$: 0,0,2,2
Groupsize: 8
Group: D_{2h}



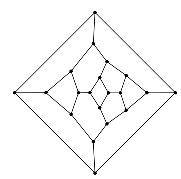
Nr.47 $v = 20$
$p_6 = 4$: 0,1,3,2
$p_5 = 4$: 0,2,0,3
$p_4 = 4$: 0,1,2,1
Groupsize: 8
Group: D_{2d}



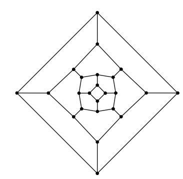
Nr.48 $v = 20$
$p_6 = 4$: 0,2,3,1
$p_5 = 4$: 0,1,1,3
$p_4 = 4$: 0,1,1,2
Groupsize: 4
Group: C_{2v}



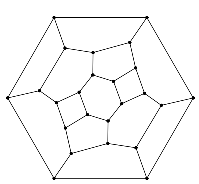
Nr.49 $v = 20$
$p_6 = 4$: 0,2,2,2
$p_5 = 4$: 0,2,1,2
$p_4 = 4$: 0,0,2,2
Groupsize: 8
Group: D_{2d}



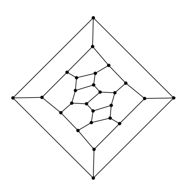
Nr.50 $v = 20$
$p_6 = 4$: 0,2,2,2
$p_5 = 4$: 0,2,1,2
$p_4 = 4$: 0,0,2,2
Groupsize: 8
Group: D_{2h}



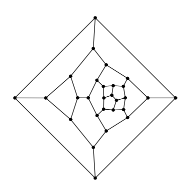
Nr.51 $v = 24$
$p_6 = 4$: 0,0,4,2
$p_5 = 8$: 0,1,2,2
$p_4 = 2$: 0,0,4,0
Groupsize: 16
Group: D_{4h}



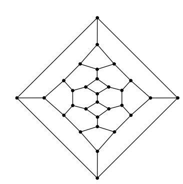
Nr.52 $v = 24$
$p_6 = 6$: 0,2,2,2
$p_5 = 4$: 0,1,1,3
$p_4 = 4$: 0,0,1,3
Groupsize: 4
Group: D_2



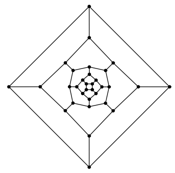
Nr.53 $v = 26$
$p_6 = 6$: 0,1,3,2
$p_5 = 6$: 0,1,1,3
$p_4 = 3$: 0,0,2,2
Groupsize: 6
Group: D_3



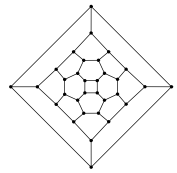
Nr.54 $v = 28$
$p_6 = 8$: 0,1,1,4
$p_5 = 4$: 0,2,1,2
$p_4 = 4$: 0,0,2,2
Groupsize: 4
Group: D_2



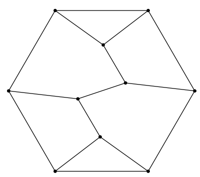
Nr.55 $v = 30$
$p_6 = 10$: 0,2,1,3
$p_5 = 2$: 0,0,0,5
$p_4 = 5$: 0,0,0,4
Groupsize: 20
Group: D_{5h}



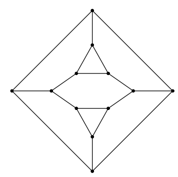
Nr.56 $v = 32$
$p_6 = 8 :$ 0,0,2,4
$p_5 = 8 :$ 0,1,2,2
$p_4 = 2 :$ 0,0,4,0
Groupsize: 16
Group: D_{4h}



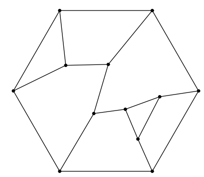
Nr.57 $v = 32$
$p_6 = 8 :$ 0,1,3,2
$p_5 = 8 :$ 0,0,2,3
$p_4 = 2 :$ 0,0,0,4
Groupsize: 16
Group: D_{4d}



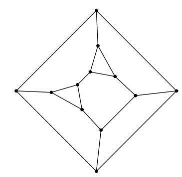
Nr.58 $v = 10$
$p_6 = 1 :$ 2,2,2,0
$p_5 = 2 :$ 1,2,1,1
$p_4 = 2 :$ 1,0,2,1
$p_3 = 2 :$ 0,1,1,1
Groupsize: 2
Group: C_2



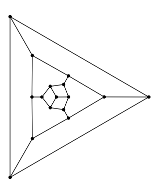
Nr.59 $v = 12$
$p_6 = 1 :$ 2,0,4,0
$p_5 = 4 :$ 1,1,2,1
$p_4 = 1 :$ 0,0,4,0
$p_3 = 2 :$ 0,0,2,1
Groupsize: 4
Group: C_{2v}



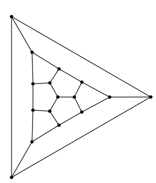
Nr.60 $v = 12$
$p_6 = 2 :$ 1,2,2,1
$p_5 = 2 :$ 1,1,1,2
$p_4 = 2 :$ 1,0,1,2
$p_3 = 2 :$ 0,1,1,1
Groupsize: 2
Group: C_2



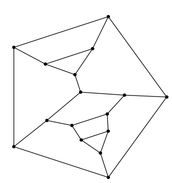
Nr.61 $v = 12$
$p_6 = 2 :$ 2,1,2,1
$p_5 = 2 :$ 1,2,0,2
$p_4 = 2 :$ 0,1,2,1
$p_3 = 2 :$ 0,0,1,2
Groupsize: 4
Group: C_{2v}



Nr.62 $v = 16$
$p_6 = 3 :$ 0,2,2,2
$p_5 = 3 :$ 1,0,2,2
$p_4 = 3 :$ 0,2,0,2
$p_3 = 1 :$ 0,0,3,0
Groupsize: 6
Group: C_{3v}

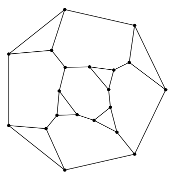


Nr.63 $v = 16$
$p_6 = 3 :$ 1,2,1,2
$p_5 = 3 :$ 0,2,2,1
$p_4 = 3 :$ 0,0,2,2
$p_3 = 1 :$ 0,0,0,3
Groupsize: 6
Group: C_{3v}

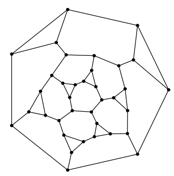


Nr.64 $v = 16$
$p_6 = 4 :$ 1,1,2,2
$p_5 = 2 :$ 0,1,0,4
$p_4 = 2 :$ 1,0,1,2
$p_3 = 2 :$ 0,1,0,2
Groupsize: 4
Group: C_{2h}

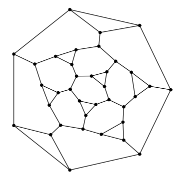
3 List 2: all 160 face-regular simple polyhedra with $b = 7$ and up to 24 faces



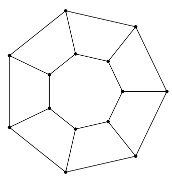
Nr.1 $v = 20$
$p_7 = 6 :$ 3,0,0,0,4
$p_3 = 6 :$ 0,0,0,0,3
Groupsize: 12



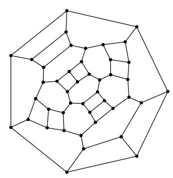
Nr.2 $v = 36$
$p_7 = 12 :$ 2,0,0,0,5
$p_3 = 8 :$ 0,0,0,0,3
Groupsize: 6



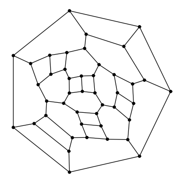
Nr.3 $v = 36$
$p_7 = 12 :$ 2,0,0,0,5
$p_3 = 8 :$ 0,0,0,0,3
Groupsize: 24



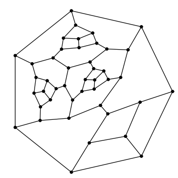
Nr.4 $v = 14$
$p_7 = 2 :$ 0,7,0,0,0
$p_4 = 7 :$ 0,2,0,0,2
Groupsize: 28



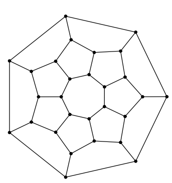
Nr.5 $v = 44$
$p_7 = 12 :$ 0,3,0,0,4
$p_4 = 12 :$ 0,1,0,0,3
Groupsize: 24



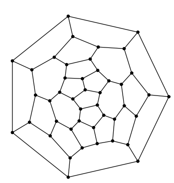
Nr.6 $v = 44$
$p_7 = 12 :$ 0,3,0,0,4
$p_4 = 12 :$ 0,1,0,0,3
Groupsize: 6



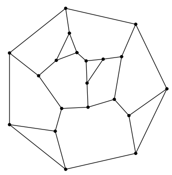
Nr.7 $v = 44$
$p_7 = 12 :$ 0,2,0,0,5
$p_4 = 12 :$ 0,2,0,0,2
Groupsize: 12



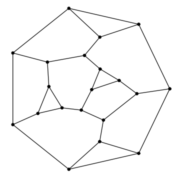
Nr.8 $v = 28$
$p_7 = 2 :$ 0,0,7,0,0
$p_5 = 14 :$ 0,0,4,0,1
Groupsize: 28



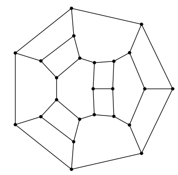
Nr.9 $v = 44$
$p_7 = 6 :$ 0,0,6,0,1
$p_5 = 18 :$ 0,0,3,0,2
Groupsize: 12



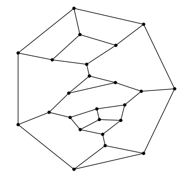
Nr.10 $v = 20$
$p_7 = 4 :$ 2,0,3,0,2
$p_5 = 4 :$ 1,0,1,0,3
$p_3 = 4 :$ 0,0,1,0,2
Groupsize: 4



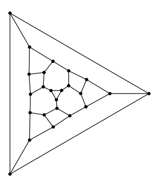
Nr.11 $v = 20$
$p_7 = 4 :$ 2,0,3,0,2
$p_5 = 4 :$ 1,0,1,0,3
$p_3 = 4 :$ 0,0,1,0,2
Groupsize: 4



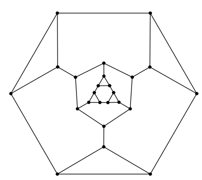
Nr.12 $v = 24$
$p_7 = 4 :$ 0,4,0,1,2
$p_6 = 2 :$ 0,4,0,0,2
$p_4 = 8 :$ 0,1,0,1,2
Groupsize: 8



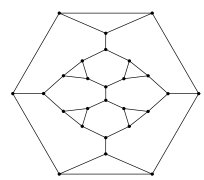
Nr.13 $v = 24$
$p_7 = 6 :$ 1,2,0,0,4
$p_4 = 6 :$ 0,2,0,0,2
$p_3 = 2 :$ 0,0,0,0,3
Groupsize: 4



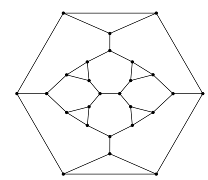
Nr.14 $v = 24$
$p_7 = 6 :$ 1,3,0,0,3
$p_4 = 6 :$ 0,1,0,0,3
$p_3 = 2 :$ 0,0,0,0,3
Groupsize: 6



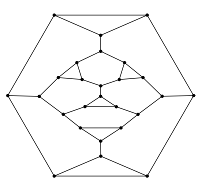
Nr.15 $v = 24$
$p_7 = 6 :$ 2,0,0,1,4
$p_6 = 2 :$ 3,0,0,0,3
$p_3 = 6 :$ 0,0,0,1,2
Groupsize: 12



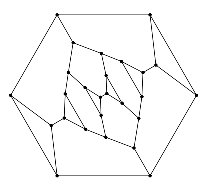
Nr.16 $v = 26$
$p_7 = 6 :$ 2,0,0,2,3
$p_6 = 3 :$ 2,0,0,0,4
$p_3 = 6 :$ 0,0,0,1,2
Groupsize: 4



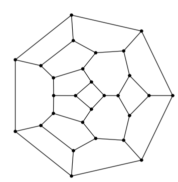
Nr.17 $v = 26$
$p_7 = 6 :$ 2,0,0,2,3
$p_6 = 3 :$ 2,0,0,0,4
$p_3 = 6 :$ 0,0,0,1,2
Groupsize: 12



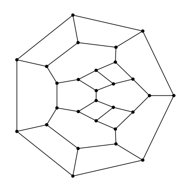
Nr.18 $v = 26$
$p_7 = 6 :$ 2,0,0,2,3
$p_6 = 3 :$ 2,0,0,0,4
$p_3 = 6 :$ 0,0,0,1,2
Groupsize: 4



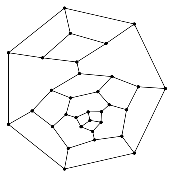
Nr.19 $v = 26$
$p_7 = 6 :$ 2,0,0,2,3
$p_6 = 3 :$ 2,0,0,0,4
$p_3 = 6 :$ 0,0,0,1,2
Groupsize: 12



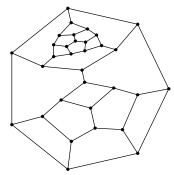
Nr.20 $v = 28$
$p_7 = 4 :$ 0,2,4,0,1
$p_5 = 8 :$ 0,1,2,0,2
$p_4 = 4 :$ 0,0,2,0,2
Groupsize: 8



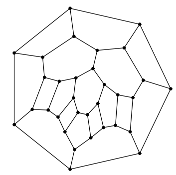
Nr.21 $v = 28$
$p_7 = 4 :$ 0,4,0,3,0
$p_6 = 4 :$ 0,2,0,1,3
$p_4 = 8 :$ 0,1,0,1,2
Groupsize: 8



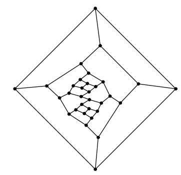
Nr.22 $v = 32$
$p_7 = 6:$ 0,2,3,0,2
$p_5 = 6:$ 0,0,2,0,3
$p_4 = 6:$ 0,2,0,0,2
Groupsize: 12



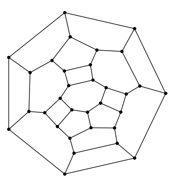
Nr.23 $v = 32$
$p_7 = 6:$ 0,2,1,0,4
$p_5 = 6:$ 0,2,2,0,1
$p_4 = 6:$ 0,0,2,0,2
Groupsize: 12



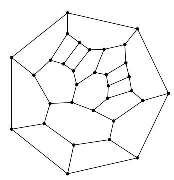
Nr.24 $v = 32$
$p_7 = 6:$ 0,2,3,0,2
$p_5 = 6:$ 0,2,0,0,3
$p_4 = 6:$ 0,0,2,0,2
Groupsize: 12



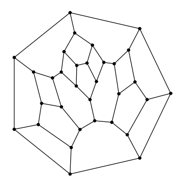
Nr.25 $v = 32$
$p_7 = 6:$ 0,1,3,0,3
$p_5 = 6:$ 0,2,0,0,3
$p_4 = 6:$ 0,1,2,0,1
Groupsize: 6



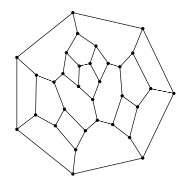
Nr.26 $v = 32$
$p_7 = 6:$ 0,3,2,0,2
$p_5 = 6:$ 0,1,2,0,2
$p_4 = 6:$ 0,0,1,0,3
Groupsize: 12



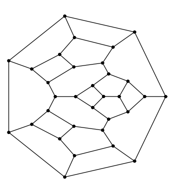
Nr.27 $v = 32$
$p_7 = 6:$ 0,2,3,0,2
$p_5 = 6:$ 0,1,1,0,3
$p_4 = 6:$ 0,1,1,0,2
Groupsize: 6



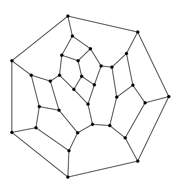
Nr.28 $v = 32$
$p_7 = 6:$ 0,2,2,0,3
$p_5 = 6:$ 0,2,1,0,2
$p_4 = 6:$ 0,0,2,0,2
Groupsize: 12



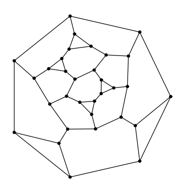
Nr.29 $v = 32$
$p_7 = 6:$ 0,2,2,0,3
$p_5 = 6:$ 0,2,1,0,2
$p_4 = 6:$ 0,0,2,0,2
Groupsize: 4



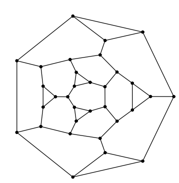
Nr.30 $v = 32$
$p_7 = 6:$ 0,2,2,0,3
$p_5 = 6:$ 0,2,1,0,2
$p_4 = 6:$ 0,0,2,0,2
Groupsize: 12



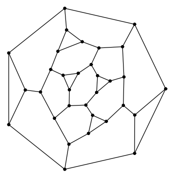
Nr.31 $v = 32$
$p_7 = 6:$ 0,2,2,0,3
$p_5 = 6:$ 0,2,1,0,2
$p_4 = 6:$ 0,0,2,0,2
Groupsize: 4



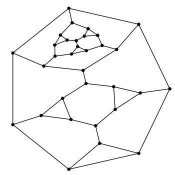
Nr.32 $v = 32$
$p_7 = 6:$ 2,0,0,3,2
$p_6 = 6:$ 1,0,0,2,3
$p_3 = 6:$ 0,0,0,1,2
Groupsize: 6



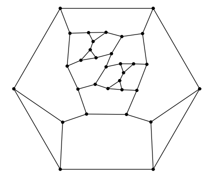
Nr.33 $v = 32$
$p_7 = 6:$ 2,0,0,3,2
$p_6 = 6:$ 1,0,0,2,3
$p_3 = 6:$ 0,0,0,1,2
Groupsize: 12



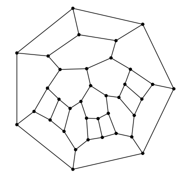
Nr.34 $v = 32$
$p_7 = 6 :$ 2,0,0,3,2
$p_6 = 6 :$ 1,0,0,2,3
$p_3 = 6 :$ 0,0,0,1,2
Groupsize: 6



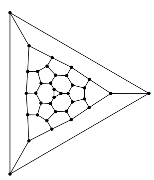
Nr.35 $v = 32$
$p_7 = 6 :$ 1,0,0,2,4
$p_6 = 6 :$ 2,0,0,2,2
$p_3 = 6 :$ 0,0,0,2,1
Groupsize: 12



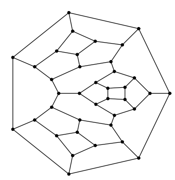
Nr.36 $v = 32$
$p_7 = 6 :$ 1,0,0,3,3
$p_6 = 6 :$ 2,0,0,1,3
$p_3 = 6 :$ 0,0,0,2,1
Groupsize: 6



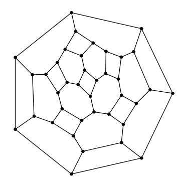
Nr.37 $v = 36$
$p_7 = 4 :$ 0,2,0,4,1
$p_6 = 8 :$ 0,2,0,2,2
$p_4 = 8 :$ 0,1,0,2,1
Groupsize: 8



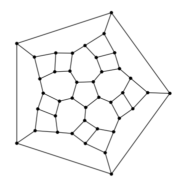
Nr.38 $v = 36$
$p_7 = 6 :$ 1,0,4,0,2
$p_5 = 12 :$ 0,0,3,0,2
$p_3 = 2 :$ 0,0,0,0,3
Groupsize: 12



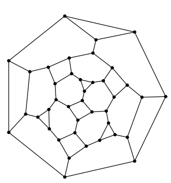
Nr.39 $v = 38$
$p_7 = 6 :$ 0,0,4,0,3
$p_5 = 12 :$ 0,1,2,0,2
$p_4 = 3 :$ 0,0,4,0,0
Groupsize: 12



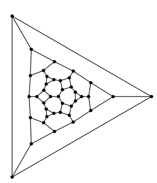
Nr.40 $v = 38$
$p_7 = 6 :$ 0,3,0,2,2
$p_6 = 6 :$ 0,3,0,1,2
$p_4 = 9 :$ 0,0,0,2,2
Groupsize: 12



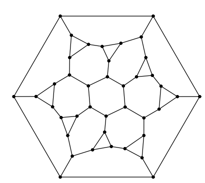
Nr.41 $v = 40$
$p_7 = 10 :$ 0,3,1,0,3
$p_5 = 2 :$ 0,0,0,0,5
$p_4 = 10 :$ 0,1,0,0,3
Groupsize: 10



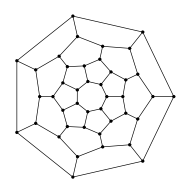
Nr.42 $v = 40$
$p_7 = 12 :$ 1,2,0,0,4
$p_4 = 6 :$ 0,0,0,0,4
$p_3 = 4 :$ 0,0,0,0,3
Groupsize: 6



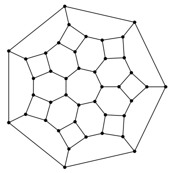
Nr.43 $v = 40$
$p_7 = 12 :$ 1,2,0,0,4
$p_4 = 6 :$ 0,0,0,0,4
$p_3 = 4 :$ 0,0,0,0,3
Groupsize: 24



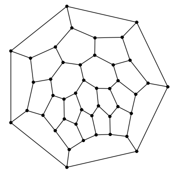
Nr.44 $v = 40$
$p_7 = 12 :$ 2,0,0,1,4
$p_6 = 2 :$ 0,0,0,0,6
$p_3 = 8 :$ 0,0,0,0,3
Groupsize: 8



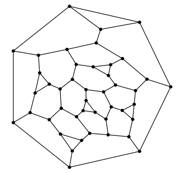
Nr.45 $v = 42$
$p_7 = 2 :$ 0,0,7,0,0
$p_6 = 7 :$ 0,0,4,2,0
$p_5 = 14 :$ 0,0,2,2,1
Groupsize: 28



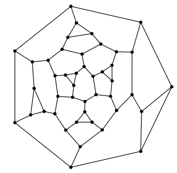
Nr.46 $v = 42$
$p_7 = 2$: 0,0,0,7,0
$p_6 = 14$: 0,2,0,3,1
$p_4 = 7$: 0,0,0,4,0
Groupsize: 28



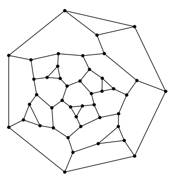
Nr.47 $v = 44$
$p_7 = 4$: 0,0,4,2,1
$p_6 = 4$: 0,0,4,0,2
$p_5 = 16$: 0,0,3,1,1
Groupsize: 8



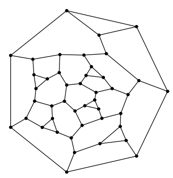
Nr.48 $v = 44$
$p_7 = 6$: 1,0,0,4,2
$p_6 = 12$: 1,0,0,3,2
$p_3 = 6$: 0,0,0,2,1
Groupsize: 6



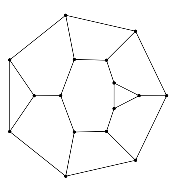
Nr.49 $v = 44$
$p_7 = 12$: 1,0,2,0,4
$p_5 = 6$: 1,0,0,0,4
$p_3 = 6$: 0,0,1,0,2
Groupsize: 6



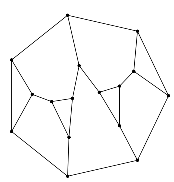
Nr.50 $v = 44$
$p_7 = 12$: 1,0,2,0,4
$p_5 = 6$: 1,0,0,0,4
$p_3 = 6$: 0,0,1,0,2
Groupsize: 6



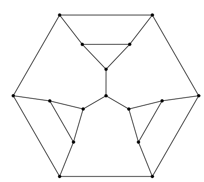
Nr.51 $v = 44$
$p_7 = 12$: 1,0,2,0,4
$p_5 = 6$: 1,0,0,0,4
$p_3 = 6$: 0,0,1,0,2
Groupsize: 2



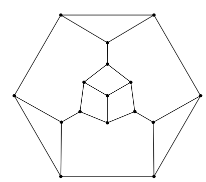
Nr.52 $v = 16$
$p_7 = 2$: 1,2,4,0,0
$p_5 = 4$: 1,1,1,0,2
$p_4 = 2$: 0,0,2,0,2
$p_3 = 2$: 0,0,2,0,1
Groupsize: 4



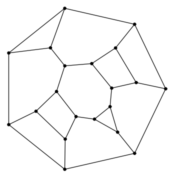
Nr.53 $v = 16$
$p_7 = 2$: 2,0,2,2,1
$p_6 = 2$: 2,0,1,1,2
$p_5 = 2$: 2,0,0,1,2
$p_3 = 4$: 0,0,1,1,1
Groupsize: 4



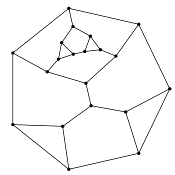
Nr.54 $v = 16$
$p_7 = 3$: 2,2,0,1,2
$p_6 = 1$: 0,3,0,0,3
$p_4 = 3$: 1,0,0,1,2
$p_3 = 3$: 0,1,0,0,2
Groupsize: 6



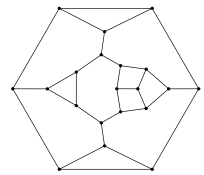
Nr.55 $v = 16$
$p_7 = 3$: 2,2,0,1,2
$p_6 = 1$: 3,0,0,0,3
$p_4 = 3$: 0,2,0,0,2
$p_3 = 3$: 0,0,0,1,2
Groupsize: 6



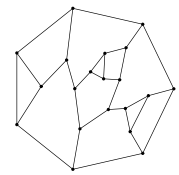
Nr.56 $v = 20$
$p_7 = 2 :$ 1,2,0,4,0
$p_6 = 4 :$ 1,2,0,1,2
$p_4 = 4 :$ 0,1,0,2,1
$p_3 = 2 :$ 0,0,0,2,1
Groupsize: 4



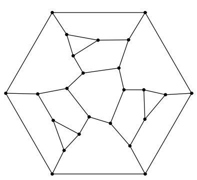
Nr.57 $v = 20$
$p_7 = 2 :$ 2,0,1,4,0
$p_6 = 4 :$ 1,0,1,2,2
$p_5 = 2 :$ 2,0,0,2,1
$p_3 = 4 :$ 0,0,1,1,1
Groupsize: 4



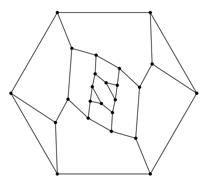
Nr.58 $v = 20$
$p_7 = 3 :$ 1,2,0,2,2
$p_6 = 3 :$ 2,0,0,2,2
$p_4 = 3 :$ 0,2,0,0,2
$p_3 = 3 :$ 0,0,0,2,1
Groupsize: 6



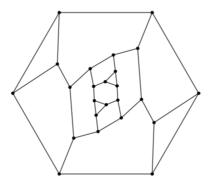
Nr.59 $v = 20$
$p_7 = 3 :$ 1,2,0,2,2
$p_6 = 3 :$ 1,1,0,2,2
$p_4 = 3 :$ 1,0,0,1,2
$p_3 = 3 :$ 0,1,0,1,1
Groupsize: 3



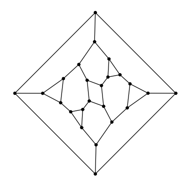
Nr.60 $v = 24$
$p_7 = 3 :$ 1,0,4,2,0
$p_6 = 2 :$ 0,0,3,0,3
$p_5 = 6 :$ 1,0,1,1,2
$p_3 = 3 :$ 0,0,2,0,1
Groupsize: 6



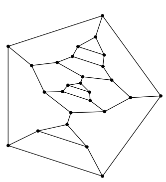
Nr.61 $v = 24$
$p_7 = 4 :$ 1,0,3,1,2
$p_6 = 2 :$ 2,0,2,0,2
$p_5 = 4 :$ 1,0,0,1,3
$p_3 = 4 :$ 0,0,1,1,1
Groupsize: 4



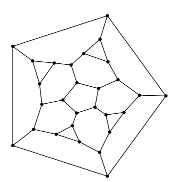
Nr.62 $v = 24$
$p_7 = 4 :$ 1,0,3,1,2
$p_6 = 2 :$ 2,0,2,0,2
$p_5 = 4 :$ 1,0,0,1,3
$p_3 = 4 :$ 0,0,1,1,1
Groupsize: 4



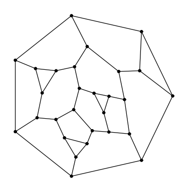
Nr.63 $v = 24$
$p_7 = 4 :$ 2,1,0,3,1
$p_6 = 4 :$ 1,1,0,1,3
$p_4 = 2 :$ 0,0,0,2,2
$p_3 = 4 :$ 0,0,0,1,2
Groupsize: 4



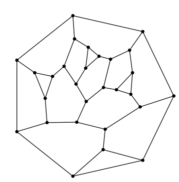
Nr.64 $v = 26$
$p_7 = 6 :$ 1,1,2,0,3
$p_5 = 3 :$ 0,1,0,0,4
$p_4 = 3 :$ 1,0,1,0,2
$p_3 = 3 :$ 0,1,0,0,2
Groupsize: 6



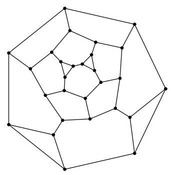
Nr.65 $v = 28$
$p_7 = 2 :$ 2,0,1,4,0
$p_6 = 8 :$ 1,0,1,3,1
$p_5 = 2 :$ 0,0,0,4,1
$p_3 = 4 :$ 0,0,0,2,1
Groupsize: 4



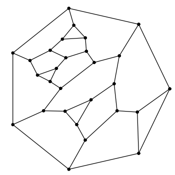
Nr.66 $v = 28$
$p_7 = 4 :$ 1,0,2,3,1
$p_6 = 4 :$ 1,0,2,0,3
$p_5 = 4 :$ 1,0,0,2,2
$p_3 = 4 :$ 0,0,1,1,1
Groupsize: 4



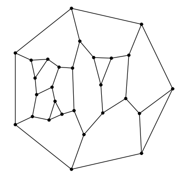
Nr.67 $v = 28$
$p_7 = 4 :$ 1,0,2,3,1
$p_6 = 4 :$ 1,0,2,0,3
$p_5 = 4 :$ 1,0,0,2,2
$p_3 = 4 :$ 0,0,1,1,1
Groupsize: 4



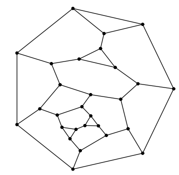
Nr.68 $v = 28$
$p_7 = 4$: 2,0,2,2,1
$p_6 = 4$: 1,0,2,1,2
$p_5 = 4$: 0,0,1,2,2
$p_3 = 4$: 0,0,0,1,2
Groupsize: 8



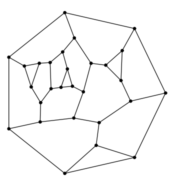
Nr.69 $v = 28$
$p_7 = 4$: 1,0,1,3,2
$p_6 = 4$: 1,0,2,0,3
$p_5 = 4$: 1,0,1,2,1
$p_3 = 4$: 0,0,1,1,1
Groupsize: 4



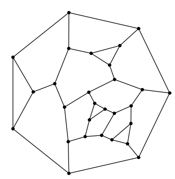
Nr.70 $v = 28$
$p_7 = 4$: 1,0,1,3,2
$p_6 = 4$: 1,0,2,0,3
$p_5 = 4$: 1,0,1,2,1
$p_3 = 4$: 0,0,1,1,1
Groupsize: 4



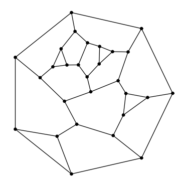
Nr.71 $v = 28$
$p_7 = 4$: 1,0,3,2,1
$p_6 = 4$: 2,0,1,1,2
$p_5 = 4$: 0,0,1,1,3
$p_3 = 4$: 0,0,0,2,1
Groupsize: 4



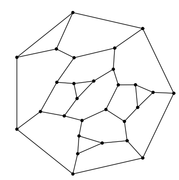
Nr.72 $v = 28$
$p_7 = 4$: 1,0,2,3,1
$p_6 = 4$: 1,0,1,1,3
$p_5 = 4$: 1,0,1,1,2
$p_3 = 4$: 0,0,1,1,1
Groupsize: 2



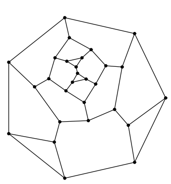
Nr.73 $v = 28$
$p_7 = 4$: 1,0,2,3,1
$p_6 = 4$: 1,0,1,1,3
$p_5 = 4$: 1,0,1,1,2
$p_3 = 4$: 0,0,1,1,1
Groupsize: 4



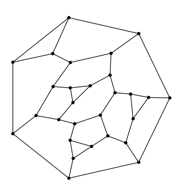
Nr.74 $v = 28$
$p_7 = 4$: 1,0,2,3,1
$p_6 = 4$: 1,0,1,1,3
$p_5 = 4$: 1,0,1,1,2
$p_3 = 4$: 0,0,1,1,1
Groupsize: 4



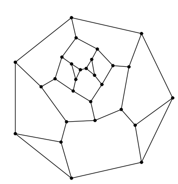
Nr.75 $v = 28$
$p_7 = 4$: 1,0,2,2,2
$p_6 = 4$: 1,0,2,1,2
$p_5 = 4$: 1,0,0,2,2
$p_3 = 4$: 0,0,1,1,1
Groupsize: 4



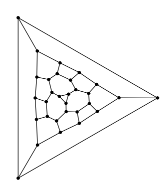
Nr.76 $v = 28$
$p_7 = 4$: 2,0,2,2,1
$p_6 = 4$: 0,0,2,2,2
$p_5 = 4$: 1,0,0,2,2
$p_3 = 4$: 0,0,1,0,2
Groupsize: 8



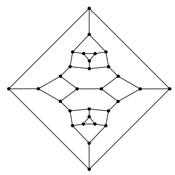
Nr.77 $v = 28$
$p_7 = 4$: 1,0,2,2,2
$p_6 = 4$: 1,0,2,1,2
$p_5 = 4$: 1,0,0,2,2
$p_3 = 4$: 0,0,1,1,1
Groupsize: 4



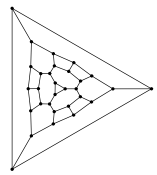
Nr.78 $v = 28$
$p_7 = 4$: 2,0,2,2,1
$p_6 = 4$: 0,0,2,2,2
$p_5 = 4$: 1,0,0,2,2
$p_3 = 4$: 0,0,1,0,2
Groupsize: 8



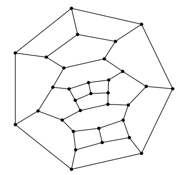
Nr.79 $v = 30$
$p_7 = 6$: 1,1,3,0,2
$p_5 = 6$: 0,1,1,0,3
$p_4 = 3$: 0,0,2,0,2
$p_3 = 2$: 0,0,0,0,3
Groupsize: 6



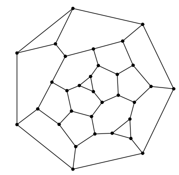
Nr.80 $v = 30$
$p_7 = 6:$ 0,2,2,0,3
$p_5 = 6:$ 1,0,2,0,2
$p_4 = 3:$ 0,0,0,0,4
$p_3 = 2:$ 0,0,3,0,0
Groupsize: 12



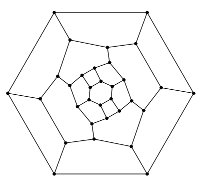
Nr.81 $v = 30$
$p_7 = 6:$ 1,2,0,2,2
$p_6 = 3:$ 0,2,0,0,4
$p_4 = 6:$ 0,1,0,1,2
$p_3 = 2:$ 0,0,0,0,3
Groupsize: 12



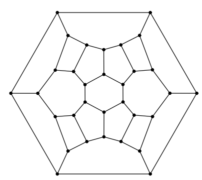
Nr.82 $v = 32$
$p_7 = 3:$ 0,2,1,4,0
$p_6 = 6:$ 0,1,1,2,2
$p_5 = 3:$ 0,2,0,2,1
$p_4 = 6:$ 0,1,1,1,1
Groupsize: 6



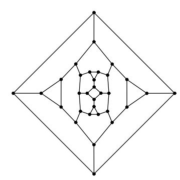
Nr.83 $v = 32$
$p_7 = 3:$ 1,0,2,4,0
$p_6 = 6:$ 1,0,2,1,2
$p_5 = 6:$ 0,0,2,2,1
$p_3 = 3:$ 0,0,0,2,1
Groupsize: 6



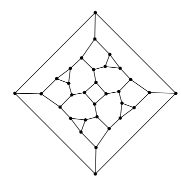
Nr.84 $v = 32$
$p_7 = 4:$ 0,1,4,0,2
$p_6 = 2:$ 0,2,4,0,0
$p_5 = 8:$ 0,1,1,1,2
$p_4 = 4:$ 0,0,2,1,1
Groupsize: 8



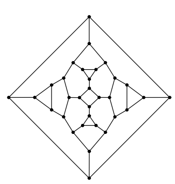
Nr.85 $v = 32$
$p_7 = 4:$ 0,2,4,1,0
$p_6 = 2:$ 0,0,4,0,2
$p_5 = 8:$ 0,1,1,1,2
$p_4 = 4:$ 0,0,2,0,2
Groupsize: 8



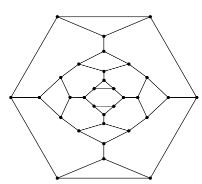
Nr.86 $v = 32$
$p_7 = 4:$ 1,0,0,4,2
$p_6 = 8:$ 1,1,0,2,2
$p_4 = 2:$ 0,0,0,4,0
$p_3 = 4:$ 0,0,0,2,1
Groupsize: 8



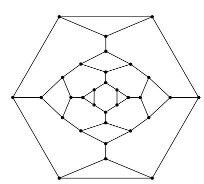
Nr.87 $v = 32$
$p_7 = 8:$ 1,1,2,0,3
$p_5 = 4:$ 1,0,0,0,4
$p_4 = 2:$ 0,0,0,0,4
$p_3 = 4:$ 0,0,1,0,2
Groupsize: 8



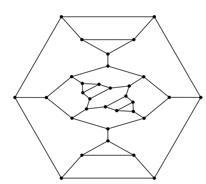
Nr.88 $v = 32$
$p_7 = 8:$ 1,1,2,0,3
$p_5 = 4:$ 1,0,0,0,4
$p_4 = 2:$ 0,0,0,0,4
$p_3 = 4:$ 0,0,1,0,2
Groupsize: 8



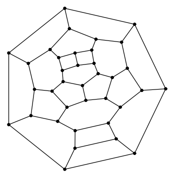
Nr.89 $v = 32$
$p_7 = 8:$ 1,2,0,1,3
$p_6 = 2:$ 2,0,0,0,4
$p_4 = 4:$ 0,0,0,0,4
$p_3 = 4:$ 0,0,0,1,2
Groupsize: 8



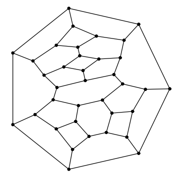
Nr.90 $v = 32$
$p_7 = 8:$ 1,2,0,1,3
$p_6 = 2:$ 2,0,0,0,4
$p_4 = 4:$ 0,0,0,0,4
$p_3 = 4:$ 0,0,0,1,2
Groupsize: 8



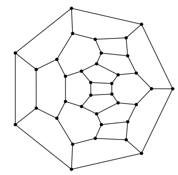
Nr.91 $v = 32$
$p_7 = 8:$ 1,1,0,1,4
$p_6 = 2:$ 0,2,0,0,4
$p_4 = 4:$ 1,0,0,1,2
$p_3 = 4:$ 0,1,0,0,2
Groupsize: 4



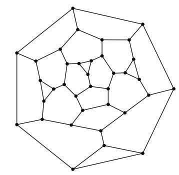
Nr.92 $v = 36$
$p_7 = 4:$ 0,1,4,2,0
$p_6 = 4:$ 0,2,2,0,2
$p_5 = 8:$ 0,0,2,1,2
$p_4 = 4:$ 0,1,0,2,1
Groupsize: 8



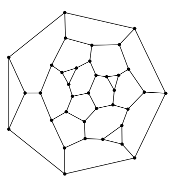
Nr.93 $v = 36$
$p_7 = 4:$ 0,0,4,1,2
$p_6 = 4:$ 0,2,2,1,1
$p_5 = 8:$ 0,1,1,1,2
$p_4 = 4:$ 0,0,2,2,0
Groupsize: 8



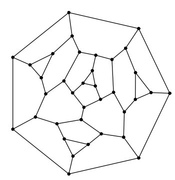
Nr.94 $v = 36$
$p_7 = 4:$ 0,1,4,2,0
$p_6 = 4:$ 0,1,2,1,2
$p_5 = 8:$ 0,1,1,1,2
$p_4 = 4:$ 0,0,2,1,1
Groupsize: 8



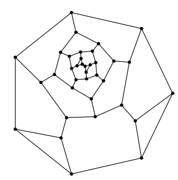
Nr.95 $v = 36$
$p_7 = 4:$ 1,0,2,4,0
$p_6 = 8:$ 1,0,1,2,2
$p_5 = 4:$ 0,0,1,2,2
$p_3 = 4:$ 0,0,0,2,1
Groupsize: 8



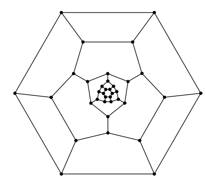
Nr.96 $v = 36$
$p_7 = 4:$ 1,0,2,4,0
$p_6 = 8:$ 1,0,1,2,2
$p_5 = 4:$ 0,0,1,2,2
$p_3 = 4:$ 0,0,0,2,1
Groupsize: 4



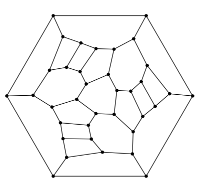
Nr.97 $v = 36$
$p_7 = 4:$ 0,0,2,4,1
$p_6 = 8:$ 1,0,1,2,2
$p_5 = 4:$ 1,0,0,2,2
$p_3 = 4:$ 0,0,1,2,0
Groupsize: 8



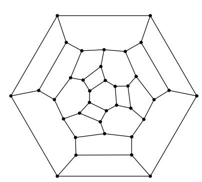
Nr.98 $v = 36$
$p_7 = 4:$ 2,0,2,2,1
$p_6 = 8:$ 0,0,1,4,1
$p_5 = 4:$ 1,0,0,2,2
$p_3 = 4:$ 0,0,1,0,2
Groupsize: 4



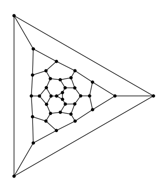
Nr.99 $v = 36$
$p_7 = 6:$ 0,1,2,0,4
$p_6 = 2:$ 0,3,3,0,0
$p_5 = 6:$ 0,2,0,1,2
$p_4 = 6:$ 0,0,2,1,1
Groupsize: 12



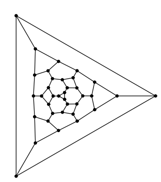
Nr.100 $v = 36$
$p_7 = 6:$ 0,2,3,1,1
$p_6 = 2:$ 0,0,3,0,3
$p_5 = 6:$ 0,1,0,1,3
$p_4 = 6:$ 0,1,1,0,2
Groupsize: 6



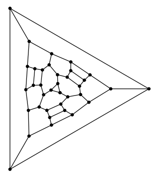
Nr.101 $v = 36$
$p_7 = 6:$ 0,2,3,1,1
$p_6 = 2:$ 0,3,0,0,3
$p_5 = 6:$ 0,1,1,0,3
$p_4 = 6:$ 0,0,1,1,2
Groupsize: 6



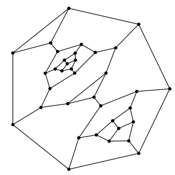
Nr.102 $v = 36$
$p_7 = 6:$ 1,2,0,2,2
$p_6 = 6:$ 0,2,0,2,2
$p_4 = 6:$ 0,0,0,2,2
$p_3 = 2:$ 0,0,0,0,3
Groupsize: 12



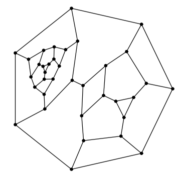
Nr.103 $v = 36$
$p_7 = 6:$ 1,2,0,2,2
$p_6 = 6:$ 0,2,0,2,2
$p_4 = 6:$ 0,0,0,2,2
$p_3 = 2:$ 0,0,0,0,3
Groupsize: 12



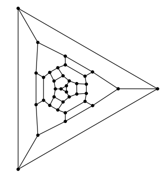
Nr.104 $v = 36$
$p_7 = 6$: 1,1,0,3,2
$p_6 = 6$: 0,2,0,1,3
$p_4 = 6$: 0,1,0,2,1
$p_3 = 2$: 0,0,0,0,3
Groupsize: 6



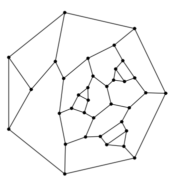
Nr.105 $v = 36$
$p_7 = 6$: 1,0,0,2,4
$p_6 = 6$: 0,2,0,2,2
$p_4 = 6$: 0,2,0,2,0
$p_3 = 2$: 0,0,0,0,3
Groupsize: 4



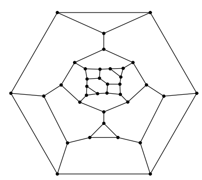
Nr.106 $v = 36$
$p_7 = 6$: 0,2,0,1,4
$p_6 = 6$: 1,2,0,2,1
$p_4 = 6$: 0,0,0,2,2
$p_3 = 2$: 0,0,0,3,0
Groupsize: 12



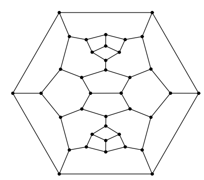
Nr.107 $v = 36$
$p_7 = 6$: 0,3,0,2,2
$p_6 = 6$: 1,1,0,2,2
$p_4 = 6$: 0,0,0,1,3
$p_3 = 2$: 0,0,0,3,0
Groupsize: 12



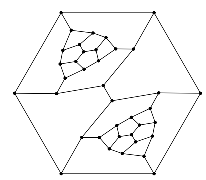
Nr.108 $v = 36$
$p_7 = 8$: 1,1,0,2,3
$p_6 = 4$: 0,1,0,1,4
$p_4 = 4$: 1,0,0,1,2
$p_3 = 4$: 0,1,0,0,2
Groupsize: 4



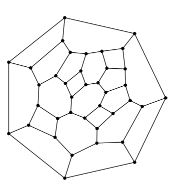
Nr.109 $v = 36$
$p_7 = 8$: 1,1,0,1,4
$p_6 = 4$: 1,2,0,1,2
$p_4 = 4$: 0,0,0,2,2
$p_3 = 4$: 0,0,0,1,2
Groupsize: 4



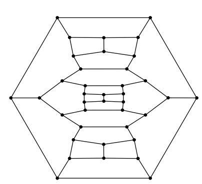
Nr.110 $v = 38$
$p_7 = 6$: 0,2,2,1,2
$p_6 = 3$: 0,0,4,0,2
$p_5 = 6$: 0,0,1,2,2
$p_4 = 6$: 0,2,0,0,2
Groupsize: 12



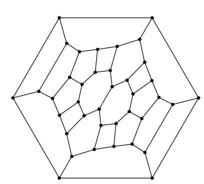
Nr.111 $v = 38$
$p_7 = 6$: 0,2,1,2,2
$p_6 = 3$: 0,0,0,2,4
$p_5 = 6$: 0,2,2,0,1
$p_4 = 6$: 0,0,2,0,2
Groupsize: 12



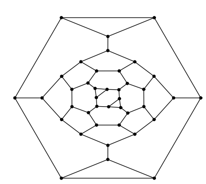
Nr.112 $v = 38$
$p_7 = 6$: 0,2,2,1,2
$p_6 = 3$: 0,2,2,0,2
$p_5 = 6$: 0,1,1,1,2
$p_4 = 6$: 0,0,1,1,2
Groupsize: 6



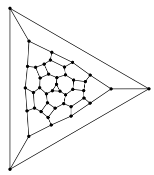
Nr.113 $v = 38$
$p_7 = 6$: 0,1,2,2,2
$p_6 = 3$: 0,0,2,0,4
$p_5 = 6$: 0,2,0,1,2
$p_4 = 6$: 0,1,2,0,1
Groupsize: 12



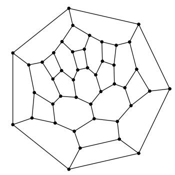
Nr.114 $v = 38$
$p_7 = 6$: 0,2,2,2,1
$p_6 = 3$: 0,2,0,0,4
$p_5 = 6$: 0,1,2,0,2
$p_4 = 6$: 0,0,1,1,2
Groupsize: 12



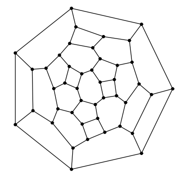
Nr.115 $v = 40$
$p_7 = 8$: 1,0,3,1,2
$p_6 = 2$: 2,0,0,0,4
$p_5 = 8$: 0,0,2,0,3
$p_3 = 4$: 0,0,0,1,2
Groupsize: 4



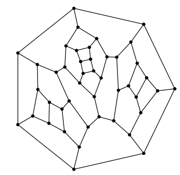
Nr.116 $v = 42$
$p_7 = 6$: 1,1,0,3,2
$p_6 = 9$: 0,2,0,2,2
$p_4 = 6$: 0,0,0,3,1
$p_3 = 2$: 0,0,0,0,3
Groupsize: 6



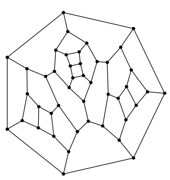
Nr.117 $v = 44$
$p_7 = 6$: 0,1,2,2,2
$p_6 = 6$: 0,2,2,0,2
$p_5 = 6$: 0,1,0,2,2
$p_4 = 6$: 0,0,1,2,1
Groupsize: 12



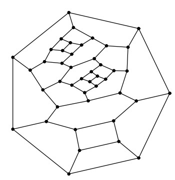
Nr.118 $v = 44$
$p_7 = 6$: 0,2,2,1,2
$p_6 = 6$: 0,1,2,2,1
$p_5 = 6$: 0,1,0,2,2
$p_4 = 6$: 0,0,1,1,2
Groupsize: 6



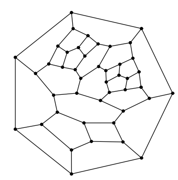
Nr.119 $v = 44$
$p_7 = 6$: 0,0,2,2,3
$p_6 = 6$: 0,2,2,0,2
$p_5 = 6$: 0,1,0,2,2
$p_4 = 6$: 0,1,1,2,0
Groupsize: 4



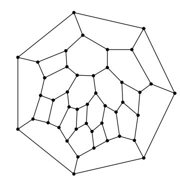
Nr.120 $v = 44$
$p_7 = 6$: 0,0,2,2,3
$p_6 = 6$: 0,2,2,0,2
$p_5 = 6$: 0,1,0,2,2
$p_4 = 6$: 0,1,1,2,0
Groupsize: 12



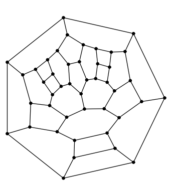
Nr.121 $v = 44$
$p_7 = 6$: 0,1,2,3,1
$p_6 = 6$: 0,0,1,2,3
$p_5 = 6$: 0,2,0,1,2
$p_4 = 6$: 0,1,2,0,1
Groupsize: 6



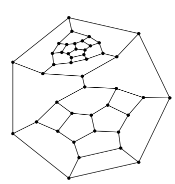
Nr.122 $v = 44$
$p_7 = 6$: 0,1,1,3,2
$p_6 = 6$: 0,1,1,1,3
$p_5 = 6$: 0,2,1,1,1
$p_4 = 6$: 0,0,2,1,1
Groupsize: 6



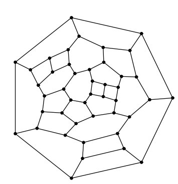
Nr.123 $v = 44$
$p_7 = 6$: 0,1,2,2,2
$p_6 = 6$: 0,2,1,1,2
$p_5 = 6$: 0,1,1,1,2
$p_4 = 6$: 0,0,1,2,1
Groupsize: 6



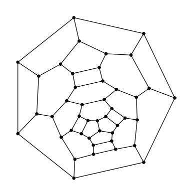
Nr.124 $v = 44$
$p_7 = 6$: 0,1,2,2,2
$p_6 = 6$: 0,2,2,0,2
$p_5 = 6$: 0,0,1,2,2
$p_4 = 6$: 0,1,0,2,1
Groupsize: 12



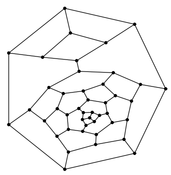
Nr.125 $v = 44$
$p_7 = 6$: 0,1,0,2,4
$p_6 = 6$: 0,2,2,0,2
$p_5 = 6$: 0,1,2,2,0
$p_4 = 6$: 0,0,1,2,1
Groupsize: 12



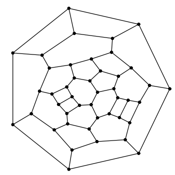
Nr.126 $v = 44$
$p_7 = 6$: 0,1,2,3,1
$p_6 = 6$: 0,2,1,0,3
$p_5 = 6$: 0,0,2,1,2
$p_4 = 6$: 0,1,0,2,1
Groupsize: 6



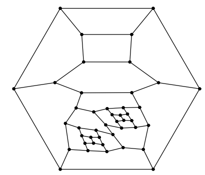
Nr.127 $v = 44$
$p_7 = 6$: 0,2,2,3,0
$p_6 = 6$: 0,1,0,2,3
$p_5 = 6$: 0,1,2,0,2
$p_4 = 6$: 0,0,1,1,2
Groupsize: 12



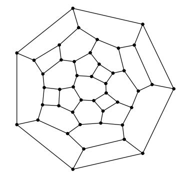
Nr.128 $v = 44$
$p_7 = 6:$ 0,2,1,2,2
$p_6 = 6:$ 0,0,3,1,2
$p_5 = 6:$ 0,0,1,3,1
$p_4 = 6:$ 0,2,0,0,2
Groupsize: 6



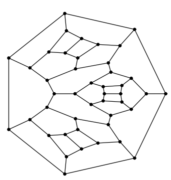
Nr.129 $v = 44$
$p_7 = 6:$ 0,2,2,3,0
$p_6 = 6:$ 0,1,1,1,3
$p_5 = 6:$ 0,0,2,1,2
$p_4 = 6:$ 0,1,0,1,2
Groupsize: 6



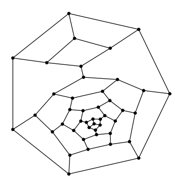
Nr.130 $v = 44$
$p_7 = 6:$ 0,0,1,3,3
$p_6 = 6:$ 0,1,2,0,3
$p_5 = 6:$ 0,2,0,2,1
$p_4 = 6:$ 0,1,2,1,0
Groupsize: 6



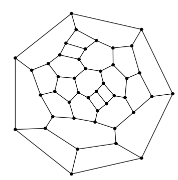
Nr.131 $v = 44$
$p_7 = 6:$ 0,2,2,1,2
$p_6 = 6:$ 0,1,2,2,1
$p_5 = 6:$ 0,1,0,2,2
$p_4 = 6:$ 0,0,1,1,2
Groupsize: 6



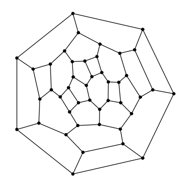
Nr.132 $v = 44$
$p_7 = 6:$ 0,0,2,2,3
$p_6 = 6:$ 0,2,2,0,2
$p_5 = 6:$ 0,1,0,2,2
$p_4 = 6:$ 0,1,1,2,0
Groupsize: 12



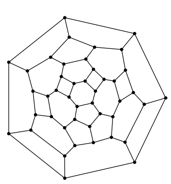
Nr.133 $v = 44$
$p_7 = 6:$ 0,2,2,1,2
$p_6 = 6:$ 0,0,3,2,1
$p_5 = 6:$ 0,0,0,3,2
$p_4 = 6:$ 0,2,0,0,2
Groupsize: 12



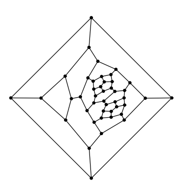
Nr.134 $v = 44$
$p_7 = 6:$ 0,2,1,3,1
$p_6 = 6:$ 0,1,2,0,3
$p_5 = 6:$ 0,0,2,2,1
$p_4 = 6:$ 0,1,0,1,2
Groupsize: 6



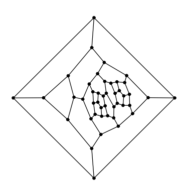
Nr.135 $v = 44$
$p_7 = 6:$ 0,2,2,2,1
$p_6 = 6:$ 0,1,1,2,2
$p_5 = 6:$ 0,1,1,1,2
$p_4 = 6:$ 0,0,1,1,2
Groupsize: 6



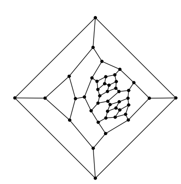
Nr.136 $v = 44$
$p_7 = 6:$ 0,2,2,2,1
$p_6 = 6:$ 0,2,1,1,2
$p_5 = 6:$ 0,0,2,1,2
$p_4 = 6:$ 0,0,0,2,2
Groupsize: 12



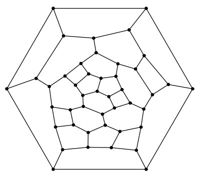
Nr.137 $v = 44$
$p_7 = 6:$ 0,1,1,3,2
$p_6 = 6:$ 0,1,1,1,3
$p_5 = 6:$ 0,2,1,1,1
$p_4 = 6:$ 0,0,2,1,1
Groupsize: 6



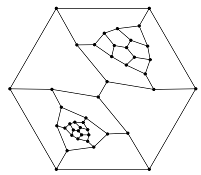
Nr.138 $v = 44$
$p_7 = 6:$ 0,1,1,3,2
$p_6 = 6:$ 0,1,1,1,3
$p_5 = 6:$ 0,2,1,1,1
$p_4 = 6:$ 0,0,2,1,1
Groupsize: 2



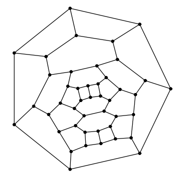
Nr.139 $v = 44$
$p_7 = 6:$ 0,1,1,3,2
$p_6 = 6:$ 0,1,1,1,3
$p_5 = 6:$ 0,2,1,1,1
$p_4 = 6:$ 0,0,2,1,1
Groupsize: 2



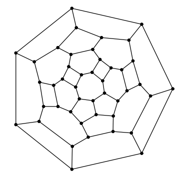
Nr.140 $v = 44$
$p_7 = 6 :$ 0,2,1,2,2
$p_6 = 6 :$ 0,1,2,1,2
$p_5 = 6 :$ 0,1,1,2,1
$p_4 = 6 :$ 0,0,1,1,2
Groupsize: 6



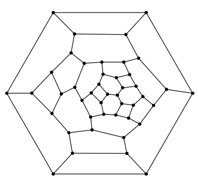
Nr.141 $v = 44$
$p_7 = 6 :$ 0,2,1,2,2
$p_6 = 6 :$ 0,0,0,4,2
$p_5 = 6 :$ 0,2,2,0,1
$p_4 = 6 :$ 0,0,2,0,2
Groupsize: 12



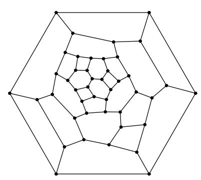
Nr.142 $v = 44$
$p_7 = 6 :$ 0,2,2,2,1
$p_6 = 6 :$ 0,0,2,2,2
$p_5 = 6 :$ 0,1,0,2,2
$p_4 = 6 :$ 0,1,1,0,2
Groupsize: 12



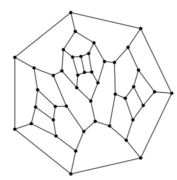
Nr.143 $v = 44$
$p_7 = 6 :$ 0,2,2,3,0
$p_6 = 6 :$ 0,2,1,0,3
$p_5 = 6 :$ 0,0,2,1,2
$p_4 = 6 :$ 0,0,0,2,2
Groupsize: 12



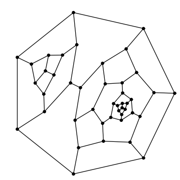
Nr.144 $v = 44$
$p_7 = 6 :$ 0,1,3,1,2
$p_6 = 6 :$ 0,2,1,2,1
$p_5 = 6 :$ 0,1,0,1,3
$p_4 = 6 :$ 0,0,1,2,1
Groupsize: 6



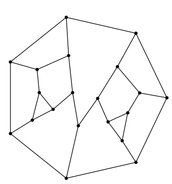
Nr.145 $v = 44$
$p_7 = 6 :$ 0,1,3,1,2
$p_6 = 6 :$ 0,2,1,2,1
$p_5 = 6 :$ 0,1,0,1,3
$p_4 = 6 :$ 0,0,1,2,1
Groupsize: 6



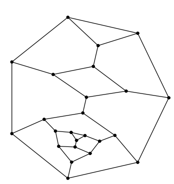
Nr.146 $v = 44$
$p_7 = 6 :$ 0,0,2,2,3
$p_6 = 6 :$ 0,2,2,0,2
$p_5 = 6 :$ 0,1,0,2,2
$p_4 = 6 :$ 0,1,1,2,0
Groupsize: 4



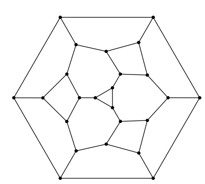
Nr.147 $v = 44$
$p_7 = 6 :$ 0,0,3,2,2
$p_6 = 6 :$ 0,2,0,2,2
$p_5 = 6 :$ 0,0,2,0,3
$p_4 = 6 :$ 0,2,0,2,0
Groupsize: 12



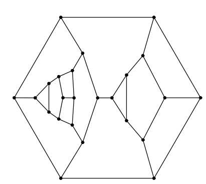
Nr.148 $v = 20$
$p_7 = 2 :$ 0,2,2,2,1
$p_6 = 2 :$ 1,0,2,1,2
$p_5 = 4 :$ 1,1,1,1,1
$p_4 = 2 :$ 0,0,2,0,2
$p_3 = 2 :$ 0,0,2,1,0
Groupsize: 4



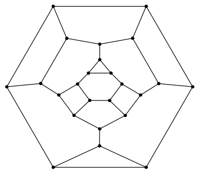
Nr.149 $v = 24$
$p_7 = 2 :$ 1,0,2,4,0
$p_6 = 4 :$ 0,1,1,2,2
$p_5 = 4 :$ 1,1,1,1,1
$p_4 = 2 :$ 0,0,2,2,0
$p_3 = 2 :$ 0,0,2,0,1
Groupsize: 4



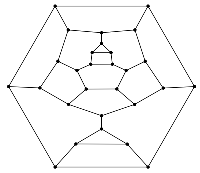
Nr.150 $v = 24$
$p_7 = 3 :$ 1,2,2,0,2
$p_6 = 1 :$ 0,0,6,0,0
$p_5 = 6 :$ 0,1,2,1,1
$p_4 = 3 :$ 0,0,2,0,2
$p_3 = 1 :$ 0,0,0,0,3
Groupsize: 6



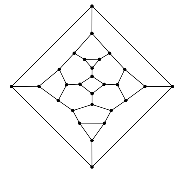
Nr.151 $v = 24$
$p_7 = 4 :$ 1,1,1,2,2
$p_6 = 2 :$ 0,2,0,0,4
$p_5 = 2 :$ 1,2,0,0,2
$p_4 = 4 :$ 0,1,1,1,1
$p_3 = 2 :$ 0,0,1,0,2
Groupsize: 4



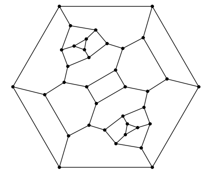
Nr.152 $v = 24$
$p_7 = 4:$ 1,2,1,1,2
$p_6 = 2:$ 1,2,1,0,2
$p_5 = 2:$ 0,2,0,1,2
$p_4 = 4:$ 0,0,1,1,2
$p_3 = 2:$ 0,0,0,1,2
Groupsize: 4



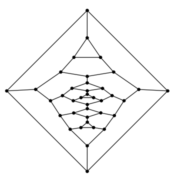
Nr.153 $v = 28$
$p_7 = 4:$ 1,1,3,1,1
$p_6 = 2:$ 0,1,3,0,2
$p_5 = 6:$ 0,0,2,1,2
$p_4 = 2:$ 1,0,0,1,2
$p_3 = 2:$ 0,1,0,0,2
Groupsize: 4



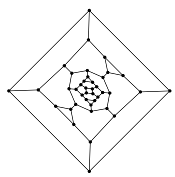
Nr.154 $v = 28$
$p_7 = 4:$ 1,2,1,2,1
$p_6 = 4:$ 0,2,1,1,2
$p_5 = 2:$ 1,0,0,2,2
$p_4 = 4:$ 0,0,0,2,2
$p_3 = 2:$ 0,0,1,0,2
Groupsize: 4



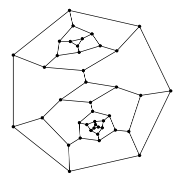
Nr.155 $v = 36$
$p_7 = 6:$ 0,1,2,2,2
$p_6 = 3:$ 0,2,0,0,4
$p_5 = 6:$ 1,0,2,0,2
$p_4 = 3:$ 0,0,0,2,2
$p_3 = 2:$ 0,0,3,0,0
Groupsize: 12



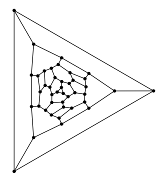
Nr.156 $v = 38$
$p_7 = 6:$ 0,1,1,2,3
$p_6 = 6:$ 1,1,1,1,2
$p_5 = 3:$ 1,0,0,2,2
$p_4 = 3:$ 0,0,0,2,2
$p_3 = 3:$ 0,0,1,2,0
Groupsize: 6



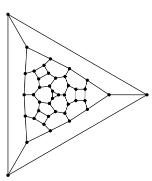
Nr.157 $v = 40$
$p_7 = 8:$ 1,0,1,1,4
$p_6 = 4:$ 1,1,2,0,2
$p_5 = 4:$ 0,1,0,2,2
$p_4 = 2:$ 0,0,2,2,0
$p_3 = 4:$ 0,0,0,1,2
Groupsize: 4



Nr.158 $v = 42$
$p_7 = 6:$ 0,2,0,2,3
$p_6 = 6:$ 0,0,2,2,2
$p_5 = 6:$ 1,0,2,2,0
$p_4 = 3:$ 0,0,0,0,4
$p_3 = 2:$ 0,0,3,0,0
Groupsize: 12

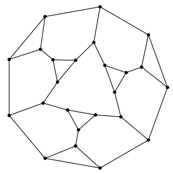


Nr.159 $v = 42$
$p_7 = 6:$ 0,1,3,2,1
$p_6 = 6:$ 1,0,1,2,2
$p_5 = 6:$ 0,1,0,1,3
$p_4 = 3:$ 0,0,2,0,2
$p_3 = 2:$ 0,0,0,3,0
Groupsize: 6

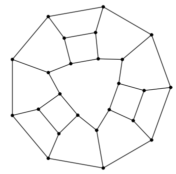


Nr.160 $v = 42$
$p_7 = 6:$ 1,0,2,2,2
$p_6 = 6:$ 0,1,2,1,2
$p_5 = 6:$ 0,1,0,2,2
$p_4 = 3:$ 0,0,2,2,0
$p_3 = 2:$ 0,0,0,0,3
Groupsize: 12

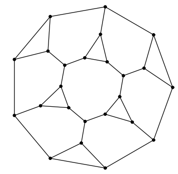
4 List 3: Selected face-regular simple polyhedra with $b \geq 8$



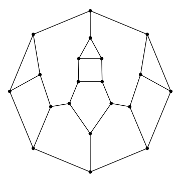
Nr.1 $v = 24$
$p_9 = 2:$ 3,0,0,6,0,0,0
$p_6 = 6:$ 2,0,0,2,0,0,2
$p_3 = 6:$ 0,0,0,2,0,0,1
Groupsize: 12



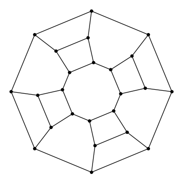
Nr.2 $v = 24$
$p_9 = 2:$ 0,3,6,0,0,0,0
$p_5 = 6:$ 0,2,1,0,0,0,2
$p_4 = 6:$ 0,1,2,0,0,0,1
Groupsize: 12



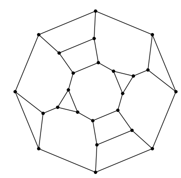
Nr.3 $v = 24$
$p_9 = 2:$ 3,0,0,6,0,0,0
$p_6 = 6:$ 2,0,0,2,0,0,2
$p_3 = 6:$ 0,0,0,2,0,0,1
Groupsize: 12



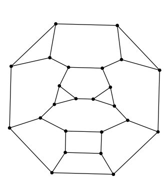
Nr.4 $v = 20$
$p_8 = 3:$ 2,2,2,0,0,2
$p_5 = 3:$ 0,1,2,0,0,2
$p_4 = 3:$ 1,0,1,0,0,2
$p_3 = 3:$ 0,1,0,0,0,2
Groupsize: 6



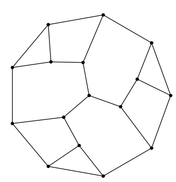
Nr.5 $v = 24$
$p_8 = 2:$ 0,4,0,4,0,0
$p_6 = 4:$ 0,4,0,0,0,2
$p_4 = 8:$ 0,1,0,2,0,1
Groupsize: 16



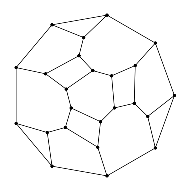
Nr.6 $v = 24$
$p_8 = 2:$ 2,2,0,0,4,0
$p_7 = 4:$ 2,2,0,0,1,2
$p_4 = 4:$ 0,1,0,0,2,1
$p_3 = 4:$ 0,0,0,0,2,1
Groupsize: 8



Nr.7 $v = 24$
$p_8 = 2:$ 2,1,2,1,2,0
$p_7 = 2:$ 2,0,1,2,0,2
$p_6 = 2:$ 2,0,0,1,2,1
$p_5 = 2:$ 0,2,0,0,1,2
$p_4 = 2:$ 0,1,2,0,0,1
$p_3 = 4:$ 0,0,0,1,1,1
Groupsize: 4



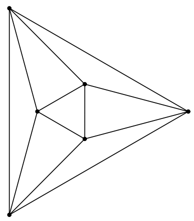
Nr.8 $v = 16$
$p_9 = 1:$ 3,3,0,3,0,0,0
$p_6 = 3:$ 1,2,0,2,0,0,1
$p_4 = 3:$ 1,0,0,2,0,0,1
$p_3 = 3:$ 0,1,0,1,0,0,1
Groupsize: 3



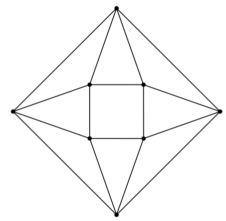
Nr.9 $v = 24$
$p_9 = 1:$ 3,0,3,0,3,0,0
$p_7 = 3:$ 1,2,2,1,0,0,1
$p_6 = 1:$ 0,3,0,0,3,0,0
$p_5 = 3:$ 1,1,0,0,2,0,1
$p_4 = 3:$ 0,0,1,1,2,0,0
$p_3 = 3:$ 0,0,1,0,1,0,1
Groupsize: 3

5 List 4: all 9 face-regular 4-valent polyhedra with $b = 4$

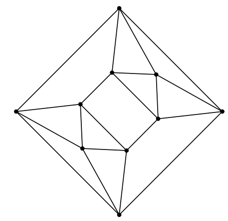
For the polyhedra in this list, the graph induced by the 4-gons is interesting: in Nrs 6,7,9 the graphs are two C_4, C_8 and the truncated Octahedron respectively.



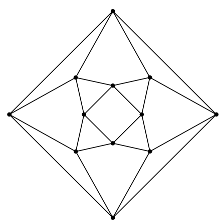
Nr.1 $v = 6$
$p_3 = 8$: 3
Groupsize: 48
Group: O_h



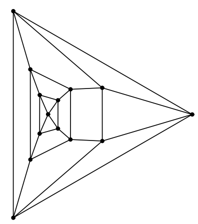
Nr.2 $v = 8$
$p_4 = 2$: 4,0
$p_3 = 8$: 2,1
Groupsize: 16
Group: D_{4d}



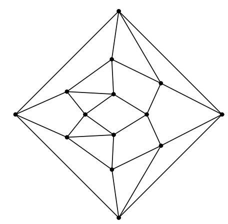
Nr.3 $v = 10$
$p_4 = 4$: 2,2
$p_3 = 8$: 2,1
Groupsize: 16
Group: D_{4h}



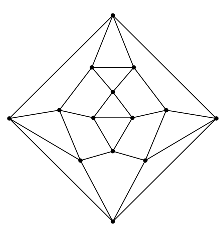
Nr.4 $v = 12$
$p_4 = 6$: 4,0
$p_3 = 8$: 0,3
Groupsize: 48
Group: O_h



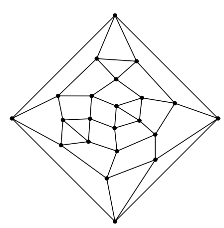
Nr.5 $v = 14$
$p_4 = 8$: 1,3
$p_3 = 8$: 2,1
Groupsize: 16
Group: D_{4h}



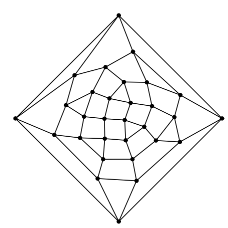
Nr.6 $v = 14$
$p_4 = 8$: 2,2
$p_3 = 8$: 1,2
Groupsize: 16
Group: D_{4h}



Nr.7 $v = 14$
$p_4 = 8$: 2,2
$p_3 = 8$: 1,2
Groupsize: 8
Group: D_{2d}



Nr.8 $v = 22$
$p_4 = 16$: 1,3
$p_3 = 8$: 1,2
Groupsize: 8
Group: D_{2d}

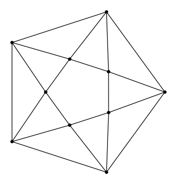


Nr.9 $v = 30$
$p_4 = 24$: 1,3
$p_3 = 8$: 0,3
Groupsize: 24
Group: O

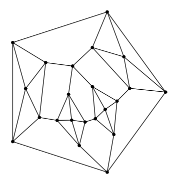
6 List 5: all face-regular 4-valent polyhedra with $b = 5$ and up to 24 faces

Among the polyhedra of Lists 4 and 5 there are: the Octahedron, three semi-regular ones (4-, 5-gonal antiprisms and the Cuboctahedron) and three regular-faced (Nr. 3 of List 4 and Nrs.4 and 6 of List 5, which are the elongated square dipyramid, the pentagonal gyrobicupola and the pentagonal orthobicupola, having number 15, 31 and 30, respectively, in the list of 92 polyhedra in [Joh66]).

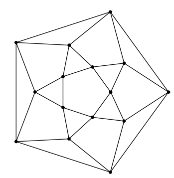
Nr.2 in list 5 is the Octahedron truncated and capped on 4 vertices of an induced C_4. Nr.3 is the elongated antiprism. Nr.5 is the dual rhombic Icosahedron (2-elongated 5-gonal antiprism).



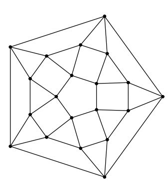
Nr.1 $v = 10$
$p_5 = 2$: 5,0,0
$p_3 = 10$: 2,0,1
Groupsize: 20
Group: D_{5d}



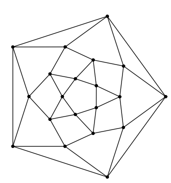
Nr.2 $v = 22$
$p_5 = 8$: 2,0,3
$p_3 = 16$: 2,0,1
Groupsize: 16
Group: D_{4h}



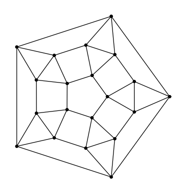
Nr.3 $v = 15$
$p_5 = 2$: 5,0,0
$p_4 = 5$: 4,0,0
$p_3 = 10$: 0,2,1
Groupsize: 20
Group: D_{5h}



Nr.4 $v = 20$
$p_5 = 2$: 0,5,0
$p_4 = 10$: 3,0,1
$p_3 = 10$: 0,3,0
Groupsize: 20
Group: D_{5d}



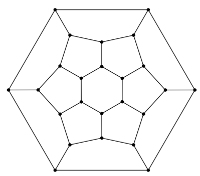
Nr.5 $v = 20$
$p_5 = 2$: 5,0,0
$p_4 = 10$: 2,2,0
$p_3 = 10$: 0,2,1
Groupsize: 20
Group: D_{5d}



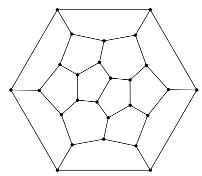
Nr.6 $v = 20$
$p_5 = 2$: 0,5,0
$p_4 = 10$: 2,1,1
$p_3 = 10$: 1,2,0
Groupsize: 20
Group: D_{5h}

7 List 6: all 26 $6R_j$ Fullerenes

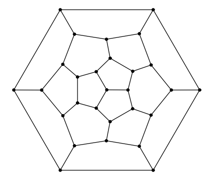
The polyhedra 1,3,6,12,16,18–21,24–26 of this list are face-regular. They are the polyhedra 23–34 of List 1, respectively.



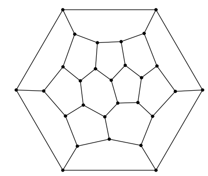
Nr.1 $v = 24$
$p_6 = 2 :$ 0,0,6,0
Groupsize: 24



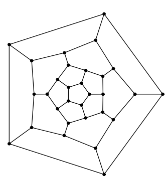
Nr.2 $v = 26$
$p_6 = 3 :$ 0,0,6,0
Groupsize: 12



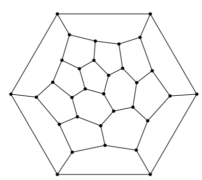
Nr.3 $v = 28$
$p_6 = 4 :$ 0,0,6,0
Groupsize: 24



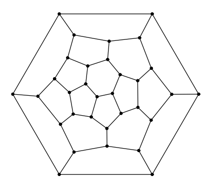
Nr.4 $v = 28$
$p_6 = 4 :$ 0,0,5,1
Groupsize: 4



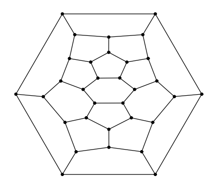
Nr.5 $v = 30$
$p_6 = 5 :$ 0,0,4,2
Groupsize: 20



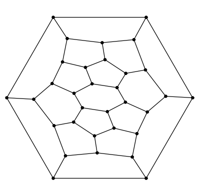
Nr.6 $v = 32$
$p_6 = 6 :$ 0,0,4,2
Groupsize: 12



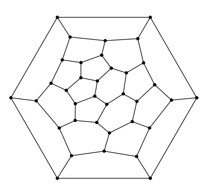
Nr.7 $v = 32$
$p_6 = 6 :$ 0,0,5,1
Groupsize: 6



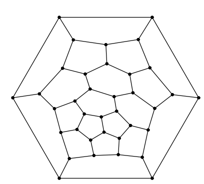
Nr.8 $v = 32$
$p_6 = 6 :$ 0,0,4,2
Groupsize: 12



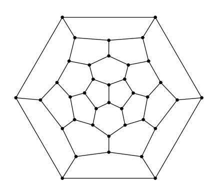
Nr.9 $v = 32$
$p_6 = 6 :$ 0,0,4,2
Groupsize: 4



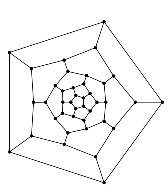
Nr.10 $v = 36$
$p_6 = 8 :$ 0,0,4,2
Groupsize: 8



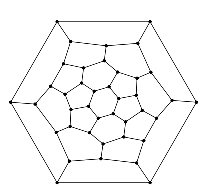
Nr.11 $v = 36$
$p_6 = 8 :$ 0,0,3,3
Groupsize: 4



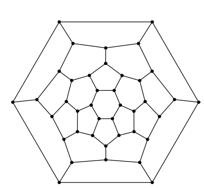
Nr.12 $v = 38$
$p_6 = 9 :$ 0,0,4,2
Groupsize: 6



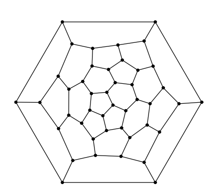
Nr.13 $v = 40$
$p_6 = 10 :$ 0,0,2,4
Groupsize: 20



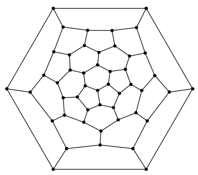
Nr.14 $v = 40$
$p_6 = 10 :$ 0,0,4,2
Groupsize: 4



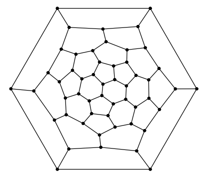
Nr.15 $v = 40$
$p_6 = 10 :$ 0,0,4,2
Groupsize: 20



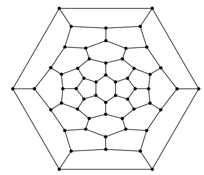
Nr.16 $v = 44$
$p_6 = 12 :$ 0,0,3,3
Groupsize: 12



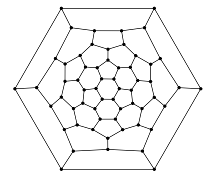
Nr.17 $v = 48$
$p_6 = 14$: 0,0,3,3
Groupsize: 6



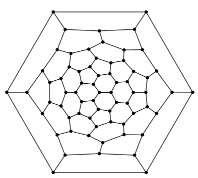
Nr.18 $v = 52$
$p_6 = 16$: 0,0,3,3
Groupsize: 12



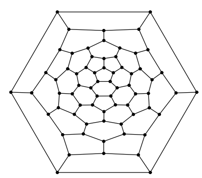
Nr.19 $v = 56$
$p_6 = 18$: 0,0,2,4
Groupsize: 24



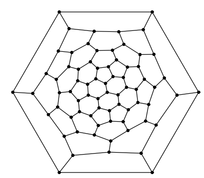
Nr.20 $v = 60$
$p_6 = 20$: 0,0,3,3
Groupsize: 120



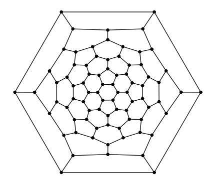
Nr.21 $v = 68$
$p_6 = 24$: 0,0,2,4
Groupsize: 24



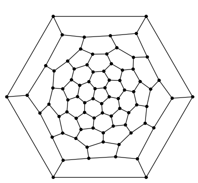
Nr.22 $v = 68$
$p_6 = 24$: 0,0,2,4
Groupsize: 12



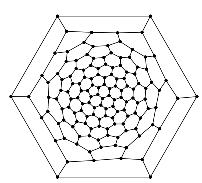
Nr.23 $v = 72$
$p_6 = 26$: 0,0,2,4
Groupsize: 8



Nr.24 $v = 80$
$p_6 = 30$: 0,0,2,4
Groupsize: 120



Nr.25 $v = 80$
$p_6 = 30$: 0,0,2,4
Groupsize: 20



Nr.26 $v = 140$
$p_6 = 60$: 0,0,1,5
Groupsize: 60

Face-regular k-valent bifaced polyhedra

In this section we will study the case where faces of exactly two sizes $a < b$ occur.

Bifaced polyhedra and similar concepts are well studied, e.g. in [Mal70], [GM66], [GZ74], [Go75], [Go77],[Za80], [JT84], [JT90], [JJ89], [Ow84], [Ow86], [JO95].

Clearly, in this case the f-graph from the proof of Theorem 1 is the K_2, so we get the equation $p_b = p_a \frac{f(a,b)}{f(b,a)}$.

By using $kv = 2e = ap_a + bp_b$ and the Euler formula $v - e + (p_a + p_b) = 2$, we get (*) $p_a = \frac{4k}{(2k+2a-ak)-(bk-2b-2k)\frac{f(a,b)}{f(b,a)}}$.

We will use the following notation for operations on polyhedra:

tetrakis: the tetrakis of a polyhedron is obtained by putting a pyramid on 4-gonal faces.

4-triakon: the 4-triakon of a polyhedron is obtained by partitioning each triangle into a ring of 3 4-gons by putting a vertex in the middle and connecting it to the midpoint of every edge in the boundary.

5-triakon: the 5-triakon of a polyhedron is obtained by partitioning each hexagon into a ring of 3 pentagons by putting a vertex in the middle and connecting it to the midpoint of every second edge in the boundary.

Theorem 2 *For $k > 3$ there is only one infinite series of face-regular (a, b)-polyhedra, that is the Antiprisms $APrism_b$ for any $b > 3$.*

Apart from this, all face-regular k-valent (a, b)-polyhedra have $(k; a) = (4; 3)$ and $b = 4, 5, 6$;

They are:

$b = 4$**:** *7 polyhedra, given as Nrs 3-9 in List 4;*

$b = 5$**:** *the Icosidodecahedron and Nr 2 in List 5 (the tetrakis of the Octahedron, truncated on all but two opposite vertices);*

$b = 6$**:** *the tetrakis of the (fully) truncated Octahedron.*

Theorem 2 will follow from the following 4 Lemmata:

Remark

Theorem 2 shows that the largest 4-valent face-regular (a, b)-polyhedra have 30 vertices (i.e. 32 faces) and $a = 3$. They have $(p_3, b) \in \{(8, 4), (20, 5), (24, 6)\}$ and are Nr 9 in List 4, the Icosidodecahedron, the tetrakis of the (fully) truncated Octahedron, respectively. The largest 3-valent face-regular $3, b)$-polyhedron also has 32 faces. It is the fully truncated Dodecahedron with $(p_3) = (12, 10)$.

The largest 3-valent face-regular $(4, b)$- and the largest known 3-valent $(5, b)$-polyhedron both have 140 vertices. They are: the 4-triakon of the truncated Dodecahedron (so $p_4 = 36$, $p_{15} = 12$) and the 5-triakon of the truncated Icosahedron (so $p_5 = 60$, $p_{10} = 12$) given below.

Lemma 1 *The only possibilities for $(k; a, b)$ are $(5; 3, 4), (4; 3, b), (3; 3, b), (3; 4, b)$ and $(3; 5, b)$.*

In all other cases the denominator in (*) will be non-positive even for $\frac{f(a,b)}{f(b,a)} = \frac{1}{b}$, the smallest possible value.

Lemma 2 *The case* $(k; a, b) = (5; 3, 4)$ *is not possible, so there is no face-regular 5-valent polyhedron.*

Proof.

The denominator in (*) is positive only for $(f(a,b), f(b,a)) = (1,4), (1,3)$.

In the first case any 4-gon is surrounded by 12 3-gons, which implies neighbouring 4-gons in the next layer – a contradiction. In the case $(1,3)$, any pair of adjacent 4-gons is surrounded by 16 3-gons, so the next layer contains a 4-gon with 2 4-gonal neighbours – again a contradiction.

□

In the following lemma we will exclude some of the theoretically possible parameters for $k = 4$.

Lemma 3 *All cases for* $(k; a) = (4; 3)$ *not being contained in the following list are impossible:*

a): $b = 4$: *the 8 polyhedra Nrs 2-9 in List 4;*

b): $b = 5$: *the Icosidodecahedron with* $f(3,5) = 3, f(5,3) = 5, v = 30$*;*

c): $b > 3$: *the infinite class of antiprisms* $APrism_b$*;*

d): $b > 3$: $(f(3,b), f(b,3) = (1, b-3), (p_3, p_b) = (8b-24, 8), v = 8b-18$*;*

e): $(f(3,b), f(b,3) = (1, b-2), (p_3, p_b) = (4b-8, 4), v = 4b-6$.

Proof. For $b > 4$ the denominator in (*) is positive only if

1) $(f(3,b), f(b,3)) \in \{(1, b-3), (1, b-2), (1, b-1), (1, b)\}$, or

2) $b \in \{5, 6, 7\}$ and $(f(3,b), f(b,3)) = (2, b)$, or

3) $b \in \{5, 6\}$ and $(f(3,b), f(b,3)) = (2, b-1)$, or

4) $b = 5$ and $f(3,b), f(b,3)) \in \{(2,3), (3,4), (3,5)\}$.

The subcase $(1, b-1)$ in case 1) is not possible, because otherwise $p_3 = \frac{8(b-1)}{3}$, $p_b = \frac{8}{3}$. The subcases $(1,b), (1,b-3), (1,b-2)$ of the case 1) are, respectively, the cases c, d and e of Lemma 3.

Cases 2 and 3 are not possible, because we get 3 3-gons on 3 consecutive edges of each b-gon; so, the 3-gonal neighbour of the 3-gon in the middle, will be adjacent to 2 3-gons, a contradiction.

The subcase $(3,4)$ of 4 is not possible, since the 3-gonal neighbours of 2 adjacent 5-gons containing a vertex of the intersection, would share an edge.

In the subcase $(2,3)$ of 4, all 3 triangles neighbouring a pentagon in a row would imply a triangle neighbouring 2 other ones, so assume we have a 5-gon and 3 neighbouring 3-gons not all in a row. But then one of the 5-gonal neighbours has all 3-gonal neighbours in a row – again a contradiction.

The remaining subcase $(3,5)$ of 4 is case b of Lemma 3.

□

Lemma 4 *The cases d and e of Lemma 3 are realized only by polyhedron Nr.3 in List 4 and the tetrakis of suitably truncated Octahedra, given in Theorem 2.*

Proof.

In both cases all 3-gons are organized in 4-cycles, surrounded by b-gons, since otherwise we will get $APrism_b$. In case e of Lemma 3 the number of edges between two b-gons is $\frac{p_b(b-f(b,3))}{2} = 4$. So, the only possibility is $b = 4$ and Nr 3 of List 4 is unique realization (it is the dual of the Octahedron, truncated on 2 opposite vertices). In case d of Lemma 3, the number of $(b-b)$-edges is 12. This implies $b \in \{4,5,6\}$ and we get the tetrakis of 3 suitably truncated Octahedra, the first one being Nr 5 of List 4 (the elongated Nr 3 of the List).

□

Theorem 3 *All face-regular cubic $(3,b)$-polyhedra have $b \leq 10$.*

They are 14 special truncations of the Tetrahedron, the Cube and the Dodecahedron:

- *the 1- and 4-truncated Tetrahedron;*
- *$2(b-4)$-truncated Cubes (one for $b \in \{5,7,8\}$ and two for $b=6$);*
- *$4(b-5)$-truncated Dodecahedra (one for $b = 6,9,10$ and two for $b = 7,8$).*

Proof.

Due to the 3-connectedness of polyhedra we get $f(3,b) = 3$ for all cubic $(3,b)$-polyhedra. So each triangle is isolated and $3p_3 \leq v = p_3(2 + \frac{6}{f(b,3)}) - 4$. Together with equality (*) and $p_3 > 0$ we get $f(b,3) \geq b-5$ and $f(b,3) \leq min(5, \frac{b}{2})$. So $b \leq 10$.

Actually this is a result by Malkevitch ([Mal70]) for general (that is: not only face-regular) cubic $(3,b)$-polyhedra.

The remaining possibilities for $b > 6$ are $(b, f(b,3); v) \in \{(10,5;60), (9,4;52), (8,3;44), (7,2;36), (8,4;24)$ and $(7,3;20)$. The first 4 cases are realized by truncations of the Dodecahedron giving only 1 polyhedron in the first two cases and 2 non-isomorphic polyhedra in the others. The last 2 cases are realized by truncations of the Cube (giving a unique polyhedron in every case). For $b \leq 6$ all wanted polyhedra are Nrs 4,5 and 10-13 of List 1.

Nrs 1-3 of List 2 are the 6-truncated Cube and two 8-truncated Dodecahedra. For $b > 6$ there remain 3 (3,8)-polyhedra, one (3,9)- and one (3,10)-polyhedron.

□

Remark

If we do not require 3-connectedness in Theorem 3, more graphs exist, e.g. one for every $b \geq 9$, divisible by 3 with $p_b = 2, p_3 = \frac{2b}{3}, v = \frac{4b}{3}$.

Theorem 4 *There is only one infinite family of cubic $(4,b)$-polyhedra, that is $Prism_b$ for any $b \geq 3$;*

The finite families are:

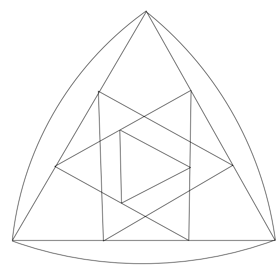
1) *Two 80-vertex $(4,7)$-polyhedra, coming as the truncation of the dual Rombicuboctahedron or dual Miller's solid (its twist) on all 18 4-valent vertices;*

2) *14 polyhedra, coming from those of Theorem 3 by the* 4-triakon *decoration; they have* $(b, v) \in \{(15, 140), (13, 116), (11, 92), (9, 68), (7, 44), (12, 56), (10, 44), (8, 32), (6, 20); (9, 28)\ (6, 14)\}$. *There are exactly 2 non-isomorphic polyhedra for the 3rd, 4th and 8th case and one for the others.*

3) *3 polyhedra (besides the prism) for* $b = 5$ *and 6 for* $b = 6$ *not covered by previous cases: Nrs 7-9 and 17-22 of List 1;*

4) *2 44-vertex (4,7)-polyhedra Nrs 5,6 of List 2 (coming by suitable doubling of all 6 isolated 4-gons in Nrs 21,20 of List 1);*

5) *the 80-vertex (4,8)-polyhedron (coming by suitable doubling of all 12 isolated 4-gons of the truncation on all 12 4-valent vertices of the dual of the unique 18-vertex 4-valent (3,4)polyhedron given below, the 3-gons of which are organized into 2 isolated ones and 3 isolated pairs);*



Proof.

Possible values for $f(4, b)$ are 2, 3 and 4. The case $f(4, b) = 2$ is possible only if the 4-gons form a ring (giving a prism) or isolated 3-rings of 4-gons, which is exactly case 2 of Theorem 4.

If $f(4, b) = 4$, all 4-gons are isolated. So $4p_4 \leq v$. The equality (*) and $p_4 > 0$ imply $2(b - 6) < f(4, b) \leq min(3, \frac{b}{2})$. So $b \leq 7$ and the only possibility for $b = 7$ is $f(b, 4) = 3, p_4 = 18, p_7 = 24; v = 80$. It is exactly case 1 of Theorem 4.

In the remaining case $f(4, b) = 3$, the 4-gons are organized into isolated adjacent pairs. So, $6\frac{p_4}{2} \leq v$ and, using (*) and $p_4 > 0$, we get $\frac{3(b-6)}{2} < f(b, 4) \leq min(5, \frac{3b}{5})$, which implies $b \leq 9$. Moreover, the only possible values for $(b, f(b, 4))$ are $(9, 5)$, $(8, 4)$, $(7, 2)$, $(7, 3)$ and $(7, 4)$. The last subcase is not possible, because it gives $p_4 = \frac{48}{5}$. The subcase $(7, 3)$ gives $v = 44$. A computer search gave that it is exactly case 4 of the theorem. The remaining subcases leave 3 possibilities: $(b, f(b, 4); p_4, p_b; v) \in \{(7, 2; 24, 36; 116), (8, 4; 24, 18; 80), (9, 5; 60, 36; 188)\}$. The first of them can easily be removed by a geometric argument. For the last one there are 8 vertices contained only in 9-gons. It is easy to show that only one out of two possible ways in which the pairs of 4-gons can neighbour a 9-gon containing such a vertex can occur. Using this, geometric arguments give a contradiction when trying to construct the polyhedron. The middle one is realized by the polyhedron of case 5 of the theorem. The uniqueness follows from its construction.

Clearly, all remaining wanted polyhedra have $b \leq 6$ and so they are covered by List 1; it gives the last entry of case 3 in Theorem 4.

□

Remark

The largest face-regular cubic $(4,6)$- and $(5,6)$-polyhedra are also (just like in case 1 of Theorem 4) the dual tetrakis of the Catalan (i.e. dual Archimedean) polyhedra: snub cube and snub dodecahedron.

Theorem 5 *There is only one infinite family of face-regular cubic $(5,b)$-polyhedra with $b \geq 6$:*

$Barrel_b$ (2 b-gons, separated by 2 rows of b 5-gons) for any b.

The finite families are

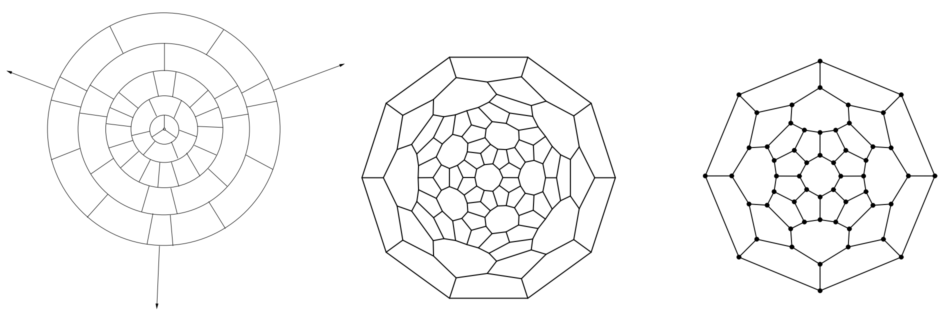
1) *12 $(5,6)$ polyhedra: Nrs 23-34 in List 1;*

2) *a unique 92-vertex (5,7)-polyhedron, organized into concentric 3-, 15- and 12-ring of 5-gons, separated by 6-, 9- and 3-ring of 7-gons (as given below);*

3) **possibly** *some 164-vertex (5,7)-polyhedra with all 48 5-gons organized into isolated rings (i.e. $f(5,7)=3$) and $f(7,5)=4$ (i.e. $p_7=36$);*

4) *a unique 44-vertex (5,7)-polyhedron (Nr 9 in List 2), a unique 56-vertex (5,8)-polyhedron, given below and unique 140-vertex (5,10)-polyhedron (also given below) (all with $f(5,b)=2$);*

5) **possibly** *other $(5,b)$-polyhedra with $f(5,b)=2$, having $(b, f(b,5); p_5, p_b; v) \in \{(7,3;36,24;116), (8,5;60,24;162), (8,6;36,12;92), (9,7;84,24;212)$.*



Proof.

The case $f(5,b)=1$ gives exactly $Barrel_b$. If $f(5,b)=5$, then all 5-gons are isolated; so $5p_5 \leq v$. So (*) and $p_5 > 0$ imply $5(b-6) < f(b,5) \leq 3$ giving $b < 7$.

If $f(5,b)=4$, then all 5-gons are organized into isolated pairs; so $8\frac{p_5}{2} \leq v$. Again we get $4(b-6) < f(b,5) \leq 3$ and $b < 7$.

The case $f(5,b)=3$ has 5-gons organized in disjoint rings. Let t denote the number of 3-rings among them, so $3p_5 + t \leq v$. The same count as above, gives $3(b-6) < f(b,5) \leq min(5, \frac{3b(t+4)-18}{t+16})$. So $b=7$ is the only possibility for $b>6$. In the case $b=7$ we have either $f(7,5)=4$ and $t \leq 20$, or $f(7,5)=5$ and $t \leq 2$. The first subcase gives $p_5=48$, $p_7=36$, given as case 3 in the theorem. The second subcase is $f(7,5)=5$, i.e. 7-gons also should be organized in isolated rings. We get $p_5=30$, $p_7=18$; $v=92$. So 5- and 7-gons should be organized in concentric alternating rings and only 2 vertices belong to 3 faces of the same type. It is easy to show that case 2 of the theorem is the unique possibility for such a polyhedron.

All possibilities with $b=6$ are covered by List 1 (it is case 1 of the theorem). The only remaining case is $f(5,b)=2, b>6$. Using (*), we get $2(b-6) < f(b,5) \leq b$.

Clearly, $p_b = p_5 \frac{f(5,b)}{f(b,5)} = \frac{24}{f(b,5)-2(b-6)}$ and $v = 2p_5 + 2p_b - 4 = \frac{8b+20f(b,5)}{f(b,5)-2(b-6)}$. So all possibilities with positive integer v are given by $f(b,5) = b - i$, $b \leq 11 - i$ for $0 \leq i \leq 4$. In subcase $f(b,5) = b$ the b-gons are all isolated. It is easy to see that b should be even and that for $b \in \{8, 10\}$ the only possibilities are the 140- and 56-vertex polyhedra in case 5 of the theorem. In subcase $f(b,5) = b-1$ an attempt to construct the structures easily gives the impossibility for $b \in \{9, 8\}$ and unicity (the 44-vertex polyhedron of case 4 of the theorem) for $b = 7$. The impossibility of cases $(b, f(b,5); v) = (7,5;52), (7,4;68)$ were checked with the help of a computer. The remaining 4 cases are exactly those given in case 5 of the theorem.

□

Remark

The polyhedron Nr 24 of List 1 and the 2-nd and 3-rd polyhedron in case 4 of Theorem 5 come by the *5-triakon* decoration of the (fully) truncated Tetrahedron, Cube and Dodecahedron, respectively.

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