Knowledge Graphs, Description Logics and Reasoning on Data
Exam

19/03/2024

First part : Description Logics

Question 1. Build a description logic TBox that models the following statements, using the following concept and role names.

Concepts : $\{\mathsf{Pizza}, \mathsf{PizzaBase}, \mathsf{Vegetable}, \mathsf{Cheese}, \mathsf{Tomato}, \mathsf{Basil}, \mathsf{Mozzarella}, \mathsf{Gorgonzola}, \mathsf{MargheritaPizza}, \mathsf{FourCheesePizza}, \mathsf{VegetarianPizza}\}$

Roles : $\{\mathsf{hasIngredient}, \mathsf{hasTopping}\}$

(a) Mozzarella and gorgonzola are cheeses.
(b) Cheeses are not vegetables.
(c) A four cheese pizza has at least four cheese toppings.
(d) The only ingredients of a Margherita pizza are the pizza base, tomato, mozzarella and basil.
(e) A pizza which has only vegetables or cheeses as toppings is a vegetarian pizza.
(f) The base of a pizza is an ingredient of this pizza.

Question 2. Consider the following TBox :

$$\mathcal{T} = \{A \sqsubseteq \exists R.(B \sqcup D), \quad B \sqsubseteq \forall R^{-}.E, \quad A \sqcap B \sqsubseteq \bot, \quad A \sqcap D \sqsubseteq \bot, \quad E \sqcap F \sqsubseteq \bot\}$$

1. For each of the following concepts, say whether it is satisfiable w.r.t. $\mathcal{T}$. Justify your answers.

   (a) $F \sqcap \exists R.B$ (b) $F \sqcap \forall R.B$

2. For each of the following axioms α, say whether α is entailed by $\mathcal{T}$ (i.e., whether $\mathcal{T} \models \alpha$). Justify your answers.

   (a) $A \sqsubseteq \exists R.D$ (b) $A \sqcap F \sqsubseteq \exists R.D$

3. For each of the following ABoxes $\mathcal{A}$, say whether $\langle \mathcal{T}, \mathcal{A} \rangle$ is satisfiable. Justify your answers.

   (a) $\{A(a),\ A(b),\ R(a,b)\}$ (b) $\{F(a),\ B(b),\ R(a,b)\}$

Question 3. Put the following $\mathcal{ALC}$ concepts in negation normal form then use the tableau algorithm to decide whether they are satisfiable.
(a) $\forall R.(A \sqcup B) \sqcap \neg((\forall R.A) \sqcup (\forall R.B))$
(b) $\exists R.\exists S.A \sqcap \neg(\exists R.\exists S.(A \sqcap \neg B) \sqcup \exists R.\exists S.B)$

Question 4. Consider the $\mathcal{ALC}$ TBox :

$$\mathcal{T} = \{A \sqsubseteq B \sqcup C, \quad C \sqsubseteq \exists R.(B \sqcup C), \quad \exists R.B \sqsubseteq B\}.$$

Use the tableau algorithm to decide whether $\mathcal{T} \models A \sqsubseteq B$.

Question 5. Let $\mathcal{T}$ be the $\mathcal{EL}$ TBox that contains the following axioms :

$D \sqsubseteq A$	$E \sqsubseteq C$	$E \sqsubseteq \exists R.D$	$\exists R.B \sqsubseteq A$
$D \sqsubseteq C$	$A \sqcap C \sqsubseteq B$	$\exists R.\top \sqsubseteq G$	

Use the saturation algorithm to classify $\mathcal{T}$ (i.e., find all concept inclusions $X \sqsubseteq Y$ between atomic concepts such that $\mathcal{T} \models X \sqsubseteq Y$).

Question 6. Let $\langle \mathcal{T}, \mathcal{A} \rangle$ be the $\mathcal{EL}$ knowledge base such that

$$\mathcal{A} = \{R(a,b),\ A(a),\ B(b)\}$$

and $\mathcal{T}$ contains the following axioms :

$A \sqcap C \sqsubseteq D$	$\exists R.\top \sqsubseteq D$	$B \sqsubseteq \exists R.A$
$\exists R.B \sqsubseteq C$	$\exists S.\top \sqsubseteq A$	$C \sqsubseteq \exists S.\top$

(a) Construct the compact canonical model of $\langle \mathcal{T}, \mathcal{A} \rangle$.

(b) Use the compact canonical model to classify $\mathcal{T}$ (i.e., find all concept inclusions $X \sqsubseteq Y$ between atomic concepts such that $\mathcal{T} \models X \sqsubseteq Y$) and to find all assertions entailed by $\langle \mathcal{T}, \mathcal{A} \rangle$.

Second part : OBDA

Question 7. Consider the rule set $\mathcal{R}$ containing the following rules :

— $\forall x, y, z\ (r(x,y) \land r(y,z) \rightarrow r(x,z))$

— $\forall x, y\ (r(x,y) \rightarrow \exists z\ s(x,y,z))$

— $\forall x, y, z, t\ (s(x,y,z) \land q(t) \rightarrow \exists u\ s(y,z,u))$

Explain why both chase and query rewriting do not terminate with that rule set. Provide however an algorithm, which, taking a database D and a Boolean conjunctive query q, outputs yes if and only if $D, \mathcal{R} \models q$. Argue about its correction and termination.

A ruleset $\mathcal{R}$ is finitely controllable if for every Boolean conjunctive query q and every database D, $D, \mathcal{R} \models q$ if and only if every finite model of D and $\mathcal{R}$ is a model of q.

Question 8. Provide a ruleset that is not finitely controllable (hint : find a ruleset for which the query $\exists x\ r(x,x)$ is a counter example to finite controllability).

Question 9. We recall that $\mathcal{R}$ is a finite expansion set if it holds that for every database D, $\mathsf{chase}(D, \mathcal{R})$ is finite. Show that if $\mathcal{R}$ is a finite expansion set, then $\mathcal{R}$ is finitely controllable.

Question 10. We recall that $\mathcal{R}$ is a bounded treewidth set if it holds that for every database D, there exists k such that any D' obtained by $\mathcal{R}$-derivation from D is of treewidth less than k. Is that true that a bounded treewidth set is finitely controllable ? If yes, prove it. Otherwise, provide a counter example.