Implicit eddy parameterization for Quasi-geostrophic models

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Joint work with Long Li, Etienne Mémin (INRIA)
and Guillaume Roullet (UBO, LOPS).

Multi-layer quasi-geostrophic model

- n stacked layer, thickness H_k and density ρ_k.
- State variables:

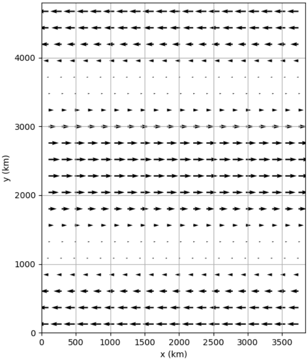
$$\begin{aligned} &\text{pressure:} && \mathbf{p} = (p_1(x,y), \dots, p_n(x,y)) \\ &\text{potential vorticity:} && \mathbf{q} = (q_1(x,y), \dots, q_n(x,y)) \end{aligned}$$

$$\begin{aligned} &\partial_t \mathbf{q} + (\mathbf{u} \cdot \nabla)\mathbf{q} = 0 \\ &\Delta_H \mathbf{p} - f_0^2 A \mathbf{p} = f_0 \mathbf{q} - f_0 \beta y \\ &-f_0 \mathbf{u} = \partial_y \mathbf{p}, \qquad f_0 \mathbf{v} = \partial_x \mathbf{p} \end{aligned}$$

$$A = \begin{bmatrix} \frac{1}{H_1 g_1'} & \frac{-1}{H_1 g_1'} & . & . \\ \frac{-1}{H_2 g_1'} & \frac{1}{H_2}\left(\frac{1}{g_1'} + \frac{1}{g_2'}\right) & \frac{-1}{H_2 g_2'} & . \\ . & \dots & \dots & \dots \\ . & . & \frac{-1}{H_n g_{n-1}'} & \frac{1}{H_n g_{n-1}'} \end{bmatrix}.$$

Idealized double-gyre configuration

- 3 layers, $H_k = 350, 750, 2900$m.
- Rectangular domain, 3480×4800 km, solid boundaries, no-slip b.c.
- Baroclinic Rossby radii: 39, 22 km.
- Linear bottom drag, idealized wind stress on top, magnitude $\tau_0 = 2e^{-5}m^2s^{-2}$



Numerical implementation

Following Hogg et al. (2014)

- Usual 5-points laplacian discretization.
- Advection with 9-points energy-conserving Arakawa Jacobian.
- Additional hyperviscosity set with Munk scale.
- Elliptic equation solved with type-I Discrete Sine Transform.
- Heun-RK2 time stepping.

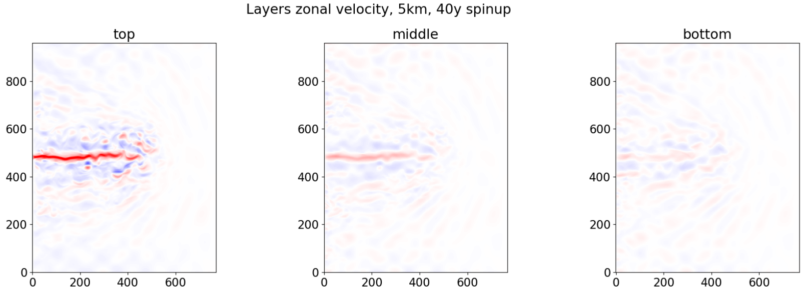
$$\partial_t \mathbf{q} = \frac{1}{f_0} J(\mathbf{q}, \mathbf{p}) - \frac{a_4}{f_0} \Delta^3 \mathbf{p} + \text{forcing} + \text{drag}$$

$$\left(\Delta - f_0^2 A\right)\mathbf{p} = f_0 \mathbf{q} - f_0 \beta (y - y_0),$$

https://github.com/louity/qgm_pytorch

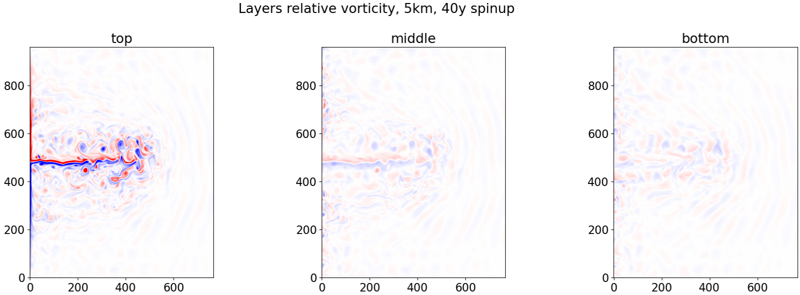
Eddy-resolving regime

- 769×961 grid, resolution 5km, dt=800s.
- Hyperviscosity $a_4 = 2\ 10^9\ m^4 s^{-1}$
- Apparition of proper eastward jet.
- Animation



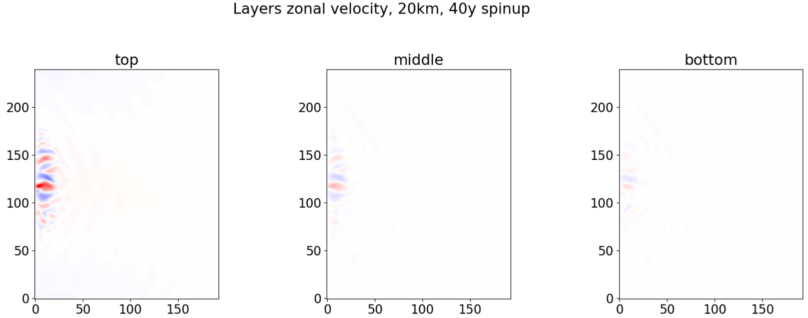
Eddy-resolving regime

- Rich meso-scale eddies field in the recirculation zone.



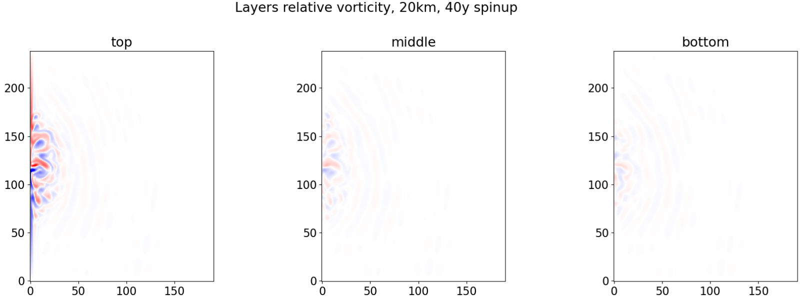
Eddy-permitting regime

- 193×241 grid, resolution 20km, dt=1200s.
- Hyperviscosity $a_4 = 8\ 10^{10}\ m^4 s^{-1}$
- No proper eastward jet.
- Animation



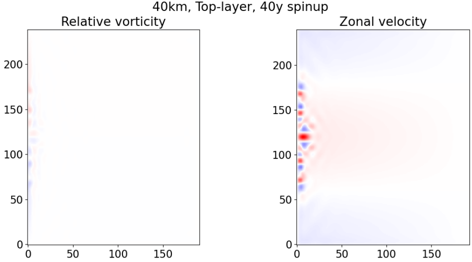
Eddy-permitting regime

- Almost no eddies.



Non eddy-resolving regime

- 97×121 grid, resolution 40km, dt=1400s.
- Hyperviscosity $a_4 = 5\ 10^{11}\ m^4 s^{-1}$
- Tiny eastward jet without any eddy around.
- Animation



$\longrightarrow$ Need for eddy parameterizations.

Existing parameterizations

Eddy parameterization for QG models tested on double-gyre configuration:

- Zanna et al. (2017): deterministic + stochastic.
- Berloff et al. (2021): deterministic data-driven.
- Li et al. (2020): stochastic + mean term (= deterministic).
- Uchida et al. (2022): deterministic.

Deterministic parameterizations help producing large scale structures (jet).
Stochasticity improves variability, finer-scale structures and ensemble spread.

$\longrightarrow$ Importance of good deterministic parameterization as basis for stochastic ones.

Implicit parameterization

Roullet and Gaillard (2022): A fast monotone discretization of the rotating shallow water equations.

"Monotone? Because what is the point of invoking an adhoc dissipation or a sophisticated SGS theory when a good numerics can do both?"
Analogous of implicit-LES for eddy parameterizations.

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Analogous of implicit-LES for eddy parameterizations.

Ingredients:

- $\mathbf{p}, \mathbf{q}$ staggered grid
- Finite volume for PV and material conservation.
- High-order WENO (Balsara et al., 2016) interpolation for advection. $\implies$ implicit diffusion replaces hyper-viscosity.
- Stable strongly preverving RK3 time-stepping.

Numerical implementation

$$\partial_t \mathbf{q} = - \underbrace{\nabla \cdot (\mathbf{u}\mathbf{q})}_{\text{WENO5 interp.}} + \text{forcing} + \text{drag}$$

$$\left(\Delta - f_0^2 A\right)\mathbf{p} = \underbrace{f_0 \mathbf{q} - f_0 \beta (y - y_0)}_{\text{4-points interp.}}$$

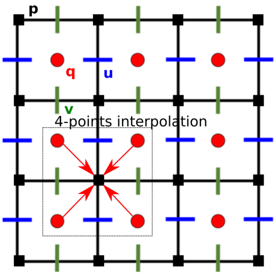
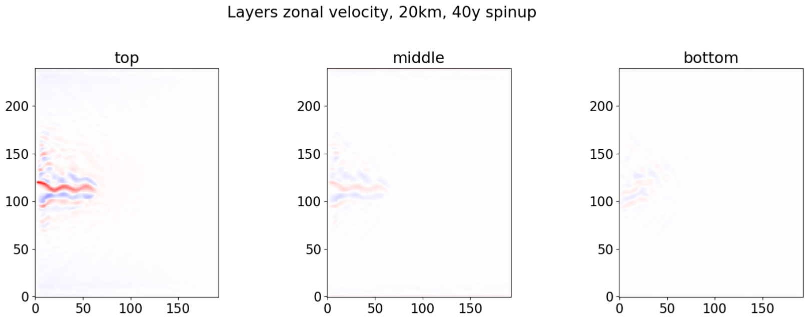


Figure: Staggered grid discretization

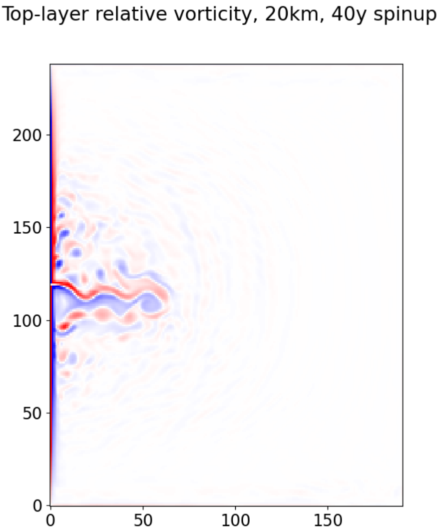
Results in eddy-permitting regime

- 193×241 grid, resolution 20km, **dt=2000**s.
- **No hyper-viscosity**.
- **Half-length eastward jet**.
- Animation



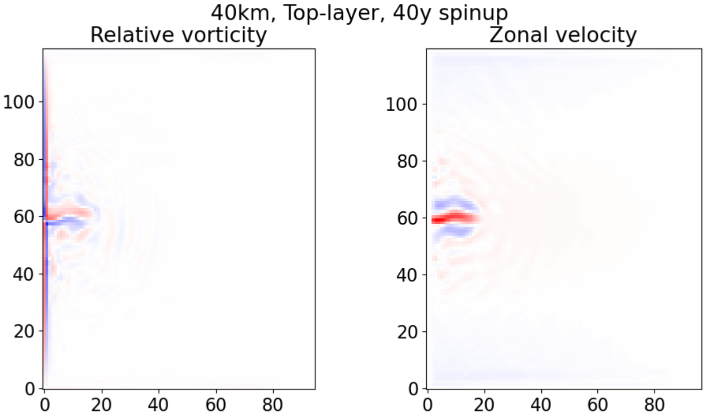
Results in eddy-permitting regime

- Large meso-scale eddies in the recirculation zone.



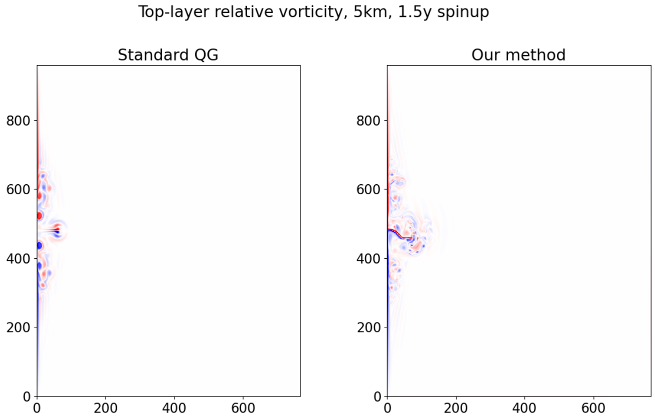
Results in non-eddy-resolving regime

- 97×121 grid, resolution 40km, **dt=4000**s.
- **No hyper-viscosity**.
- **Third-length eastward jet**.
- Animation



Results in eddy-resolving regime

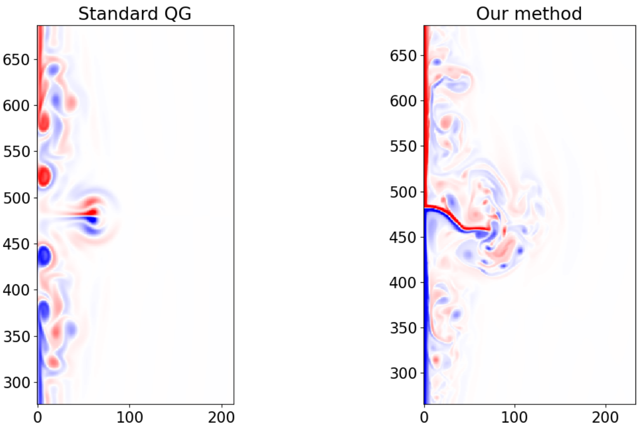
- 769×961 grid, resolution 5km, dt=800s.
- Symmetry breaking and effective resolution



Results in eddy-resolving regime

- Symmetry breaking and effective resolution

Top-layer relative vorticity, 5km, 1.5y spinup



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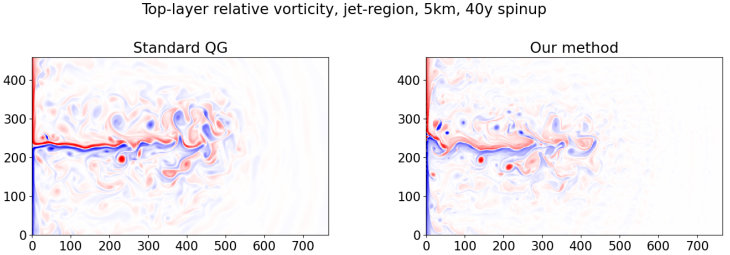
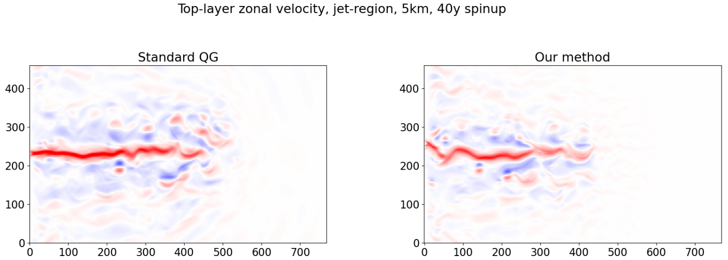
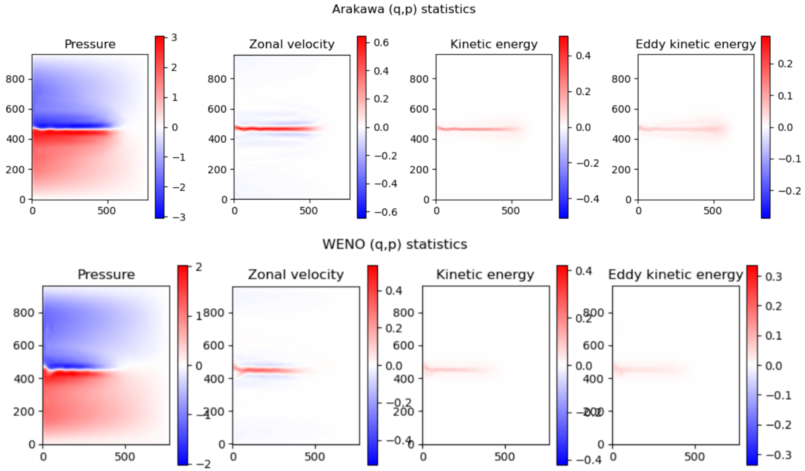




Results in eddy-resolving regime

- Statistics



WENO5 implicit parameterization

- Works in the three regimes.
- Shows different advantages.
- Removes the viscosity CFL condition: larger dt.

WENO5 implicit parameterization

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- Shows different advantages.
- Removes the viscosity CFL condition: larger dt.
- ... but can still be improved.

q to **p** interpolation

$$\left(\Delta - f_0^2 A\right)\mathbf{p} = \mathrm{Interp}_{\mathrm{q}\to\mathrm{p}}\left(f_0\mathbf{q} - f_0\beta(y - y_0)\right)$$

- Interpolation needed to solve elliptic equation
- 4-points interpolation has bad frequency response
  $\Longrightarrow$ high-frequency are discarded before solving elliptic equation

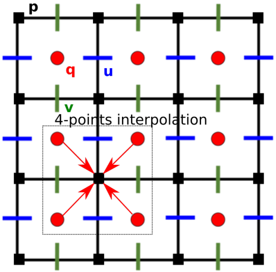

Figure: Staggered grid discretization

Interpolation with DST-II

$$(\Delta - f_0^2 A)\mathbf{p} = \mathrm{Interp}_{\mathrm{q}\to\mathrm{p}}\left(f_0\mathbf{q} - f_0\beta(y - y_0)\right)$$

- Elliptic equation solved with DST-I
- DST-II$(\cdot)$ = DST-I$\big($Spectral-Interp$_{\mathrm{q}\to\mathrm{p}}(\cdot)\ \big)$
  Spectral interpolation $\implies$ highest possible order

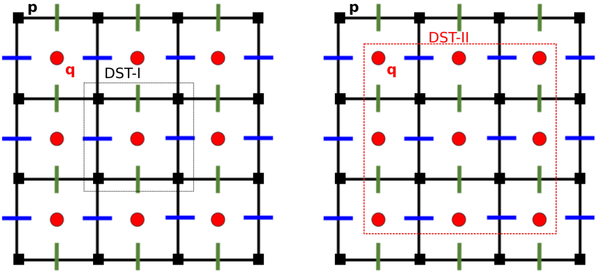

Figure: DST-I and DST-II equivalence on square grid

Results with WENO + DST-II

Eddy-resolving regime:

- 769×961 grid, resolution 5km, dt=800s.
- Animation.

Results with WENO + DST-II

Eddy-resolving regime:

- 769$\times$961 grid, resolution 5km, dt=800s.
- Animation.
- **Longer eastward jet**.

Eddy-permitting regime:

- 193$\times$241 grid, resolution 20km, dt=2000s.
- Animation

Results with WENO + DST-II

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- 193×241 grid, resolution 20km, dt=2000s.
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- **Proper eastward jet**.

Non eddy-resolving regime:

- 97×121 grid, resolution 40km, dt=4000s.
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Non eddy-resolving regime:

- 97×121 grid, resolution 40km, dt=4000s.
- Animation
- **ALIASING!**

Spectral methods need anti-aliasing treatment, *e.g.* Dedalus.

Conclusions

Our implicit eddy parameterization

- resolves large-scale structures (eddy-permitting and non-eddy-resolving regimes).
- increases effective resolution, accelerates spin up and symmetry breaking (eddy-resolving regime).
- works in a wide range of resolutions: 1.5 to 40km.
- allows removing ad-hoc dissipation, **hence the viscosity CFL condition**.
- is complementary to physical or stochastic parameterizations.

Take home message

- At low resolutions, preserving high-frequencies is crucial...

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- ...but can be tricky : Aliasing ! Rossby waves dissipation ? Badly resolved western boundary-layer ?

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Take home message

- At low resolutions, preserving high-frequencies is crucial...
- ...but can be tricky : Aliasing ! Rossby waves dissipation ? Badly resolved western boundary-layer ? (Suggestions ?)

Perspectives

- Add stochastic parameterization: Mémin (2014).
- Jump to WENO7 ? Or Semi-lagrangian ?
- Ability to resolve eddies:
  - Realistic wind
  - Inclusion of surface buoyancy: Lapeyre and Klein (2006) (animation)
- More realistic simulation :
  - Move to non-trivial geometries.
  - Beyond quasi-geostrophy: shallow-water ?
- Suitable model for data assimilation ?

Questions ?

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