The Unreasonable Effectiveness of Patches in Deep Convolutional Kernels Methods.

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Plan

1. Introduction
2. Convolutional kernel methods
3. Our method
4. Results

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- Key ingredient: **whitening**.

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- Comparable accuracies on CIFAR-10 with shallow classifier.
- Scalable to ImageNet: S.O.T.A. as non-learned visual representation.

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Convolutional kernel methods

x, y images.

$$K_{k,\Phi,\mathcal{X}}(x,y) = k(\Phi_{\mathcal{X}} L_{\mathcal{X}} x, \Phi_{\mathcal{X}} L_{\mathcal{X}} y)$$

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- Predefined (e.g. Linear, Gaussian, Neural Tangent) kernel

$$k(x, y)$$

Data-driven convolutional kernel methods

$K(x, y)$ is **data-driven** if Φ or L depend on the training set $\mathcal{X}$,
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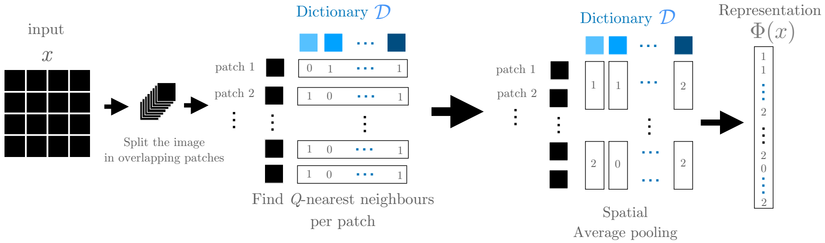
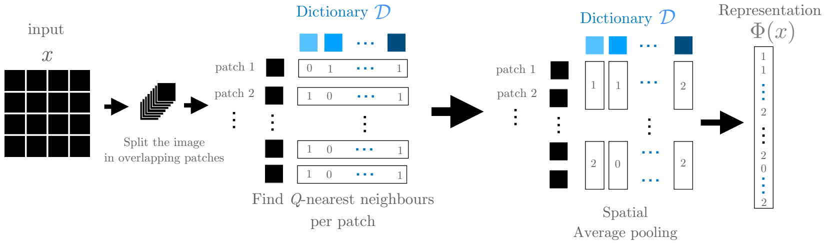
  - L: whitening of the image
  - k: Custom *Neural Kernel*

Plan

Our method


input
x
Split the image
in overlapping patches
Dictionary $\mathcal{D}$
patch 1
patch 2
0 1 ⋯ 1
1 0 ⋯ 1
1 0 ⋯ 1
1 0 ⋯ 1
Find Q-nearest neighbours
per patch
Dictionary $\mathcal{D}$
patch 1
patch 2
1 1 ⋯ 2
2 0 ⋯ 2
Spatial
Average pooling
Representation
$\Phi(x)$
1
1
2
2
0
2

Our method



- x: image viewed as a collection of overlapping patches.

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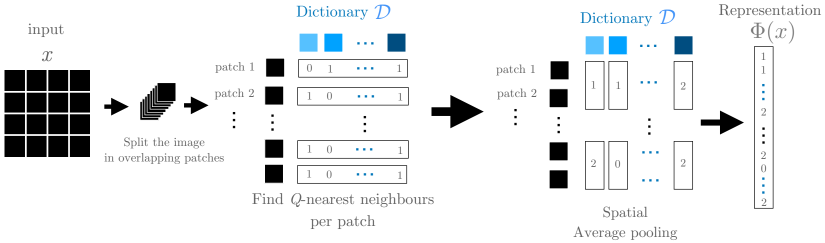

- x: image viewed as a collection of overlapping patches.
- L: whitening operator

$$L : x \mapsto (\Sigma + \lambda I)^{-1}(x - \mu)$$

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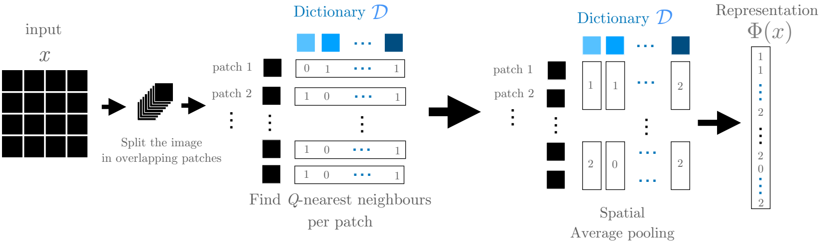

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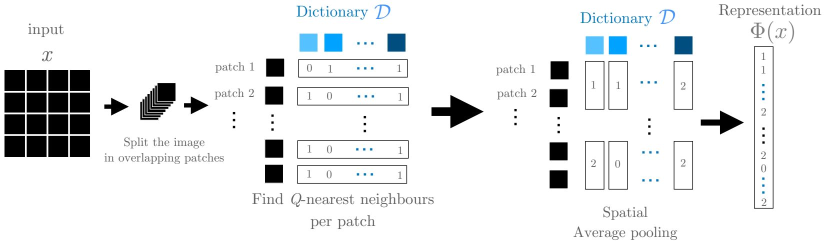

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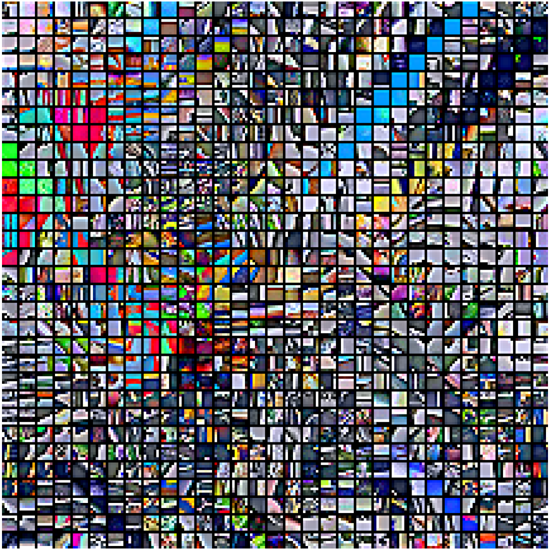
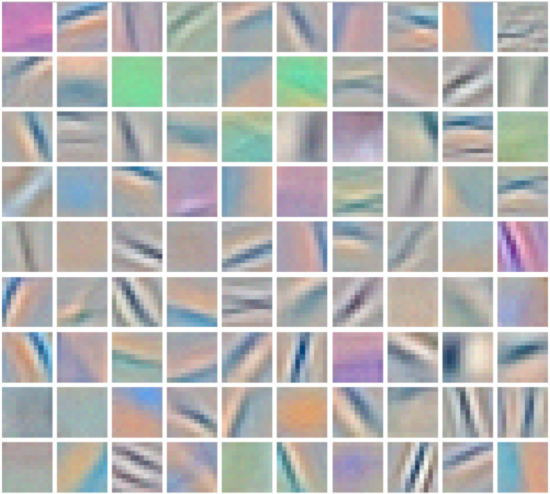
- $k(x, y)$: linear kernel.

CIFAR-10 Dictionary

ImageNet64 Dictionary



First layer of AlexNet



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CIFAR-10

Linear classification

Method	$\lvert\mathcal{D}\rvert$	**VQ**	**Online**	P	**Acc.**
Coates et al. (2011)	$1k$	✓	×	6	68.6
Wavelets (Oyallon et al. 2015)	-	×	×	8	82.2
Recht et al. (2019)	$0.2M$	×	×	6	85.6
SimplePatch (Ours)	$10k$	✓	✓	6	85.6
SimplePatch (Ours)	$60k$	×	✓	6	**86.9**

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Non-linear classification

Method	**VQ**	**Depth**	**Classifier**	**Acc.**
SimplePatch (Ours)	✓	2	1-hidden-layer	88.5
AlexNet (Krizhevsky et al. 2012)	×	5	e2e	89.1
NK (Shankar et al. 2020)	×	5	kernel	89.8
CKN (Mairal et al. 2016)	×	9	kernel	89.8

ImageNet

Method	$\|\mathcal{D}\|$	VQ	P	Depth	Res.	Top1	Top5
Random CNN	-	$\times$	-	9	224	18.9	-
Zarka et al. (19)	-	$\times$	32	2	224	26.1	44.7
Ours	$2k$	✓	12	1	128	35.9	57.4
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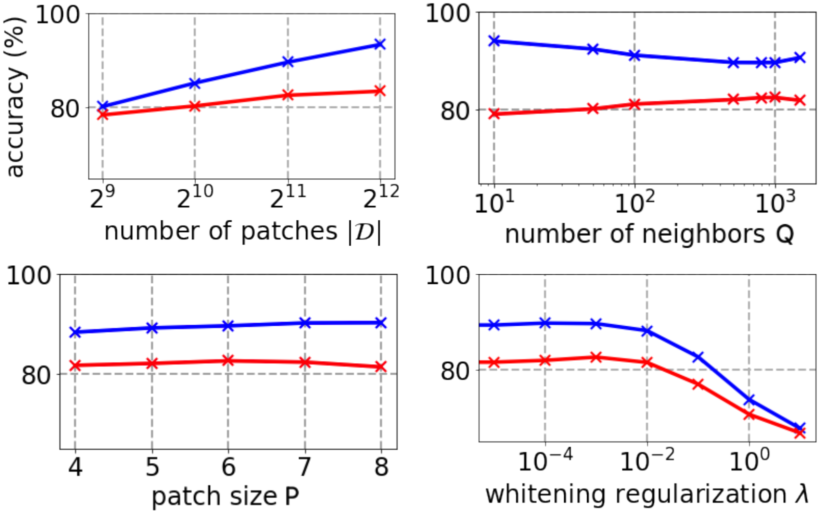
Belilov. al. (18)	$\times$	-	2	224	e2e	-	44
Ours	✓	6	2	64	1-layer	39.4	62.1
Brendel al. (19)	$\times$	9	50	224	e2e	-	70.0

Ablation Study

Train accuracies in blue, test accuracies in red.



Questions ?

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