Implicit eddy parameterization for Quasi-geostrophic models

Louis Thiry,
ERC STUOD Postdoc,
IRMAR, INRIA, ODISSEY Team

Joint work with Long Li, Etienne Mémin (INRIA)
and Guillaume Roullet (UBO, LOPS).

Multi-layer quasi-geostrophic model

- n stacked layer, thickness H_k and density ρ_k.
- State variables:

pressure:	$\mathbf{p} = (p_1(x,y),\dots,p_n(x,y))$
potential vorticity (PV):	$\mathbf{q} = (q_1(x,y),\dots,q_n(x,y))$

$$\partial_t \mathbf{q} + (\mathbf{u}\cdot\nabla)\mathbf{q} = 0 \qquad \text{(PV advection)}$$

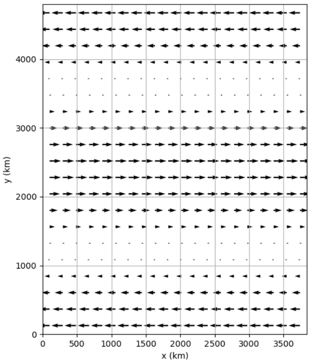
$$\Delta_H \mathbf{p} - f_0^2 A\mathbf{p} = f_0\mathbf{q} - f_0\beta y \qquad \text{(elliptic)}$$

$$-f_0\mathbf{u} = \partial_y \mathbf{p}, \qquad f_0\mathbf{v} = \partial_x \mathbf{p} \qquad \text{(geos. velocity)}$$

$$A = \begin{bmatrix} \frac{1}{H_1 g_1'} & \frac{-1}{H_1 g_1'} & . & . \\ \frac{-1}{H_2 g_1'} & \frac{1}{H_2}\left(\frac{1}{g_1'} + \frac{1}{g_2'}\right) & \frac{-1}{H_2 g_2'} & . \\ . & \dots & \dots & \dots \\ . & . & \frac{-1}{H_n g_{n-1}'} & \frac{1}{H_n g_{n-1}'} \end{bmatrix}.$$

Idealized double-gyre configuration

- 3 layers, $H_k = 350, 750, 2900$m.
- Rectangular domain, 3480×4800 km, solid boundaries, free-slip b.c.
- Baroclinic Rossby radii: 39, 22 km.
- Linear bottom drag, idealized wind stress on top, magnitude τ_0



Numerical implementation

Following Hogg et al. (2014)

- Usual 5-points laplacian discretization.
- Advection with 9-points energy-conserving Arakawa Jacobian.
- Additional hyperviscosity set with Munk rule.
- Elliptic equation solved with type-I Discrete Sine Transform.
- Heun-RK2 time stepping.

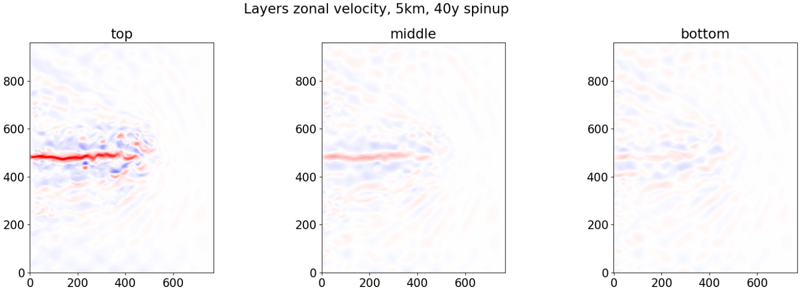
$$\partial_t \mathbf{q} = \frac{1}{f_0} J(\mathbf{q}, \mathbf{p}) - \frac{a_4}{f_0} \Delta^3 \mathbf{p} + \text{forcing} + \text{drag} \qquad \text{(PV advection)}$$

$$\left(\Delta - f_0^2 A\right)\mathbf{p} = f_0 \mathbf{q} - f_0 \beta (y - y_0), \qquad \text{(elliptic)}$$

https://github.com/louity/qgm_pytorch

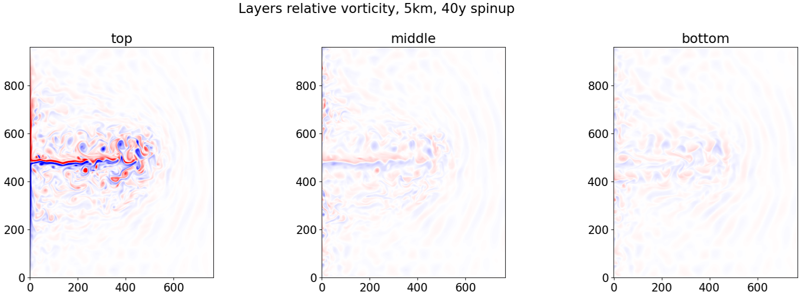
Eddy-resolving resolution

- 769×961 grid, **resolution 5km**.
- $\tau_0 = 0.02$ N/m^2
- Hyperviscosity $a_4 = 1.7 \ 10^9 \ m^4 s^{-1}$
- Apparition of proper eastward jet surrounded by eddies.
- 40 yrs spin-up animation



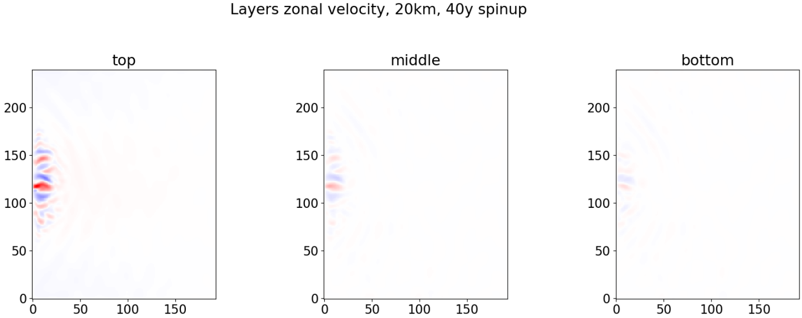
Eddy-resolving resolution

- Rich meso-scale eddies field in the recirculation zone.



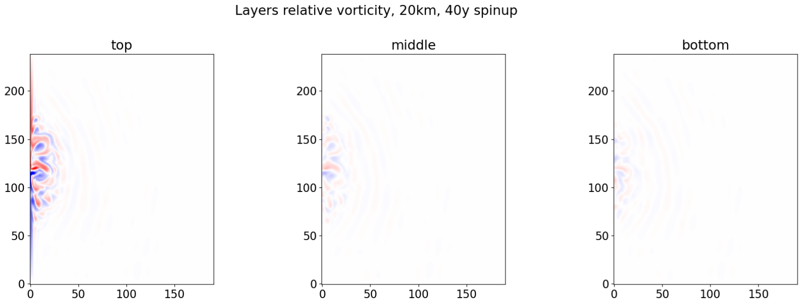
Eddy-permitting resolution

- 193×241 grid, **resolution 20km**.
- Hyperviscosity $a_4 = 5.6\ 10^{10}\ m^4 s^{-1}$
- No proper eastward jet.
- 40yrs spin-up animation



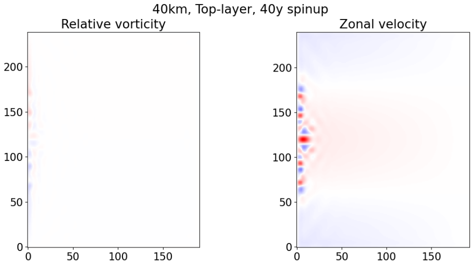
Eddy-permitting resolution

- Almost no eddies.



Non eddy-resolving resolution

- 97×121 grid, **resolution 40km**.
- Hyperviscosity $a_4 = 5\ 10^{11}\ m^4 s^{-1}$
- Tiny eastward jet without any eddy around.
- 40 yrs spin-up animation



⟶ Need for eddy parameterizations.

Existing parameterizations

Eddy parameterization for QG models tested on double-gyre configuration:

- Zanna et al. (2017): deterministic + stochastic.
- Berloff et al. (2021): deterministic data-driven.
- Li et al. (2020): stochastic + mean term (= deterministic).
- Uchida et al. (2022): deterministic.

Deterministic methods can reproduce the jet.
Stochastic ones improves variability and finer-scale structures.

$\longrightarrow$ Importance of good deterministic parameterization as basis for stochastic ones.

Implicit parameterization

Roullet and Gaillard (2022): A fast monotone discretization of the rotating shallow water equations.

"Monotone? Because what is the point of invoking an adhoc dissipation or a sophisticated SGS theory when a good numerics can do both?"
Analogous of implicit-LES for eddy parameterizations.

Implicit parameterization

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Analogous of implicit-LES for eddy parameterizations.

Ingredients:

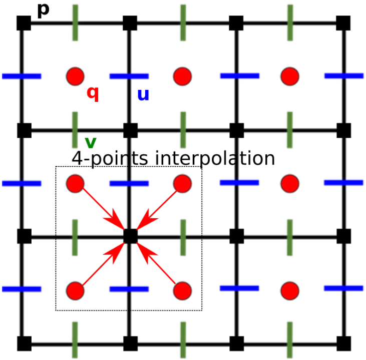
- $\mathbf{p}, \mathbf{q}$ on staggered grid
- Finite volume for PV and material conservation.
- High-order WENO (Balsara et al., 2016) interpolation for advection.
  $\implies$ implicit diffusion replaces hyper-viscosity.
- Stable strongly preverving RK3 time-stepping.

Implicit parameterization

$$\partial_t \mathbf{q} = - \underbrace{\nabla \cdot (\mathbf{uq})}_{\text{WENO5 interp.}} + \text{forcing} + \text{drag} \qquad \text{(PV advection)}$$

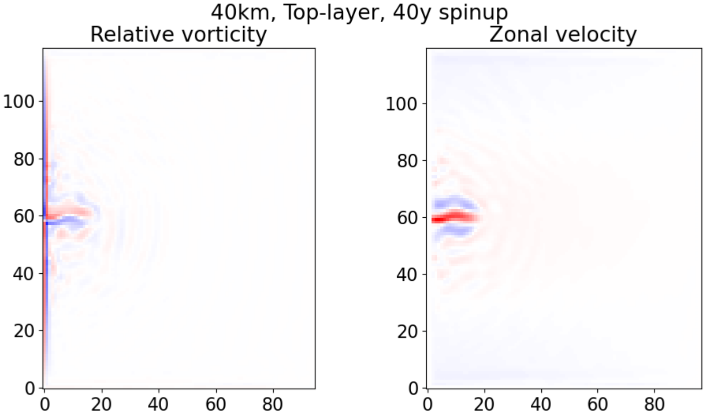
$$\left(\Delta - f_0^2 A\right)\mathbf{p} = \underbrace{f_0 \mathbf{q} - f_0 \beta (y - y_0)}_{\text{4-points interp.}} \qquad \text{(elliptic)}$$

p and **q** staggered grids:



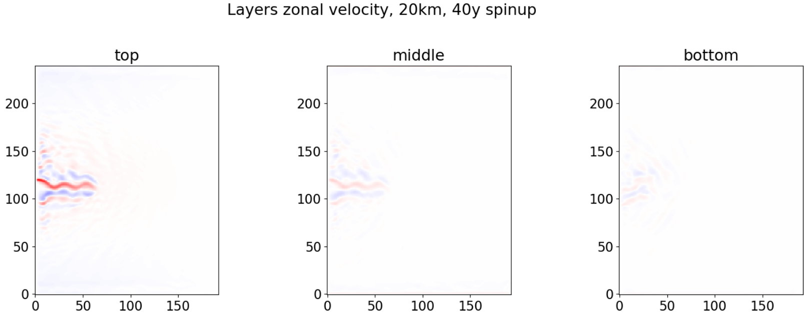
Results in non-eddy-resolving resolution

- 97×121 grid, resolution 40km.
- **No hyper-viscosity**.
- Small eastward jet pushing.
- 40 yrs spin-up animation



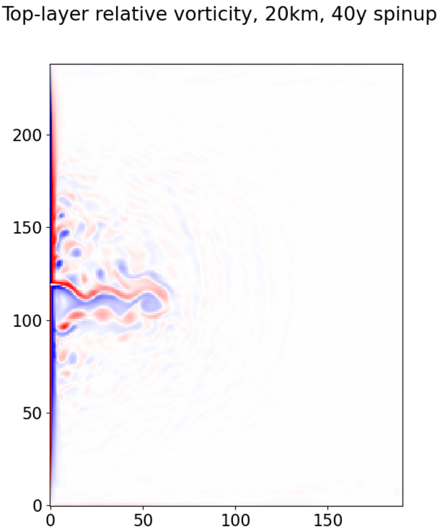
Results in eddy-permitting resolution

- 193×241 grid, resolution 20km.
- **No hyper-viscosity**.
- Half-length eastward jet.
- 40 yrs spin-up animation



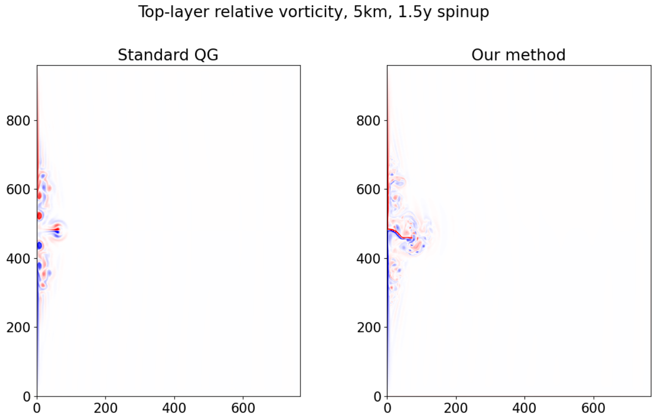
Results in eddy-permitting resolution

- Large meso-scale eddies in the recirculation zone.



Results in eddy-resolving resolution

- 769×961 grid, resolution 5km.
- Symmetry breaking and effective resolution



Results in eddy-resolving resolution

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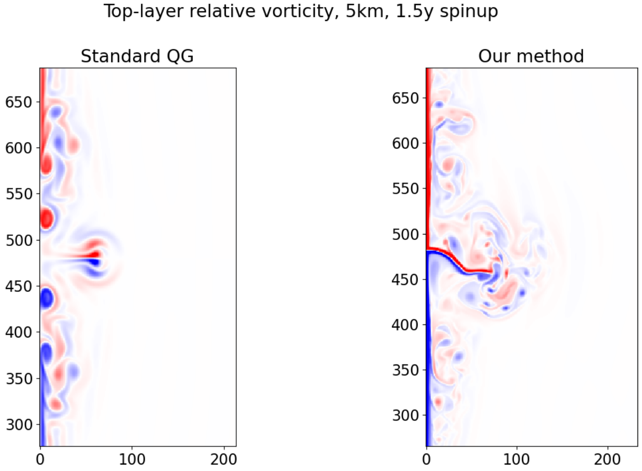
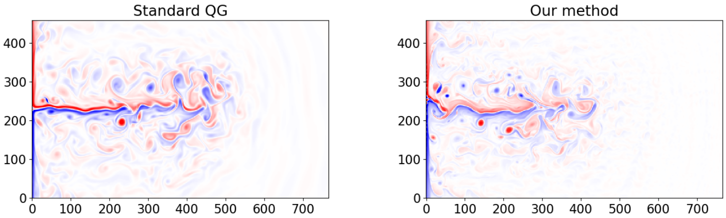


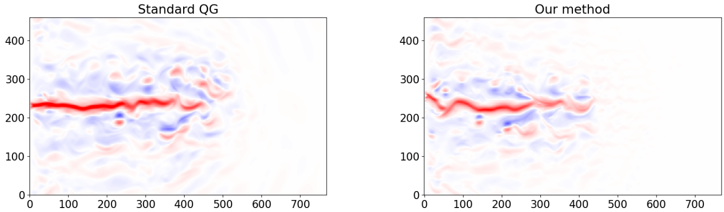
Results in eddy-resolving resolution

- Symmetry breaking and effective resolution

Top-layer relative vorticity, jet-region, 5km, 40y spinup



Top-layer zonal velocity, jet-region, 5km, 40y spinup



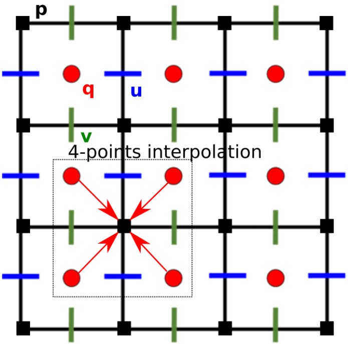
WENO5 implicit parameterization

- Accelerates symmetry breaking in the eddy-resolving resolution.
- Produces a (small) jet in non-eddy-resolving and eddy-permitting resolutions.
- Allows removing ad-hoc hyperviscosity.
- Removes the viscosity CFL condition: possibly larger dt.
- Still too much dissipating...

$\mathbf{q}$ to $\mathbf{p}$ interpolation

$$\left(\Delta - f_0^2 A\right)\mathbf{p} = \mathrm{Interp}_{q\rightarrow p}\left(f_0\mathbf{q} - f_0\beta(y - y_0)\right) \qquad \text{(elliptic)}$$

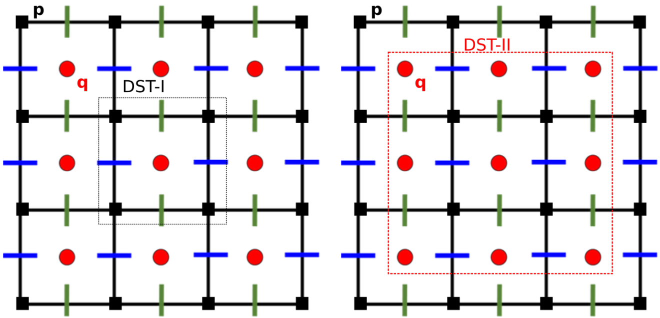
- Interpolation needed to solve elliptic equation
- 4-points interpolation has bad frequency response
  $\Longrightarrow$ high-frequency are discarded before solving elliptic equation



q to p interpolation with DST-II

$$(\Delta - f_0^2 A)\mathbf{p} = \text{Interp}_{q\to p}\left(f_0\mathbf{q} - f_0\beta(y - y_0)\right)$$

- Elliptic equation solved with DST-I
- DST-II$(\cdot)$ = DST-I$\big($Spectral-Interp$_{q\to p}(\cdot)\ \big)$
  Spectral interpolation $\implies$ highest possible order

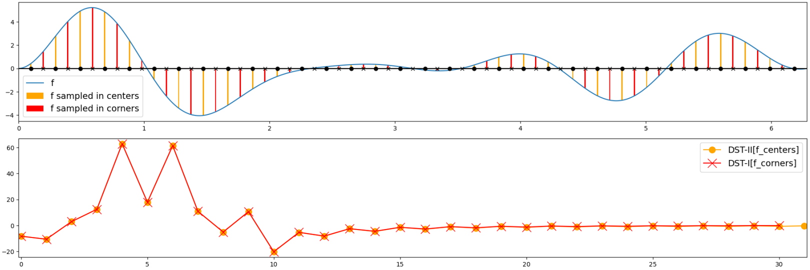


DST-I and DST-II

(x_n), $n = 0 \dots N-1$ is a real vector:

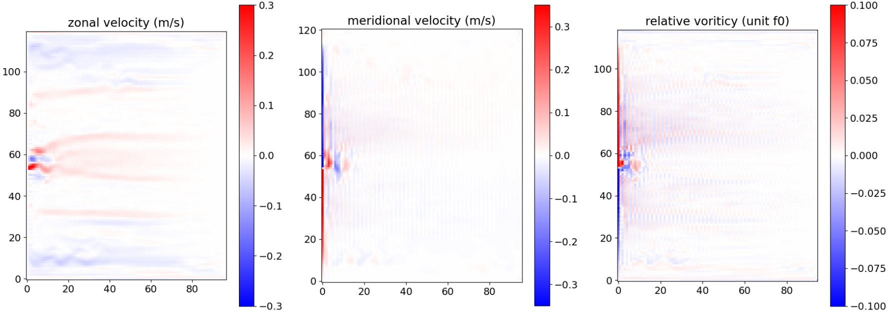
$$\text{DST-I}[x]_k = \sum_{n=0}^{N-1} x_n \sin\left[\frac{\pi}{N+1}(n+1)(k+1)\right] \qquad k = 0, \dots, N-1$$

$$\text{DST-II}[x]_k = \sum_{n=0}^{N-1} x_n \sin\left[\frac{\pi}{N}\left(n+\frac{1}{2}\right)(k+1)\right] \qquad k = 0, \dots, N-1.$$



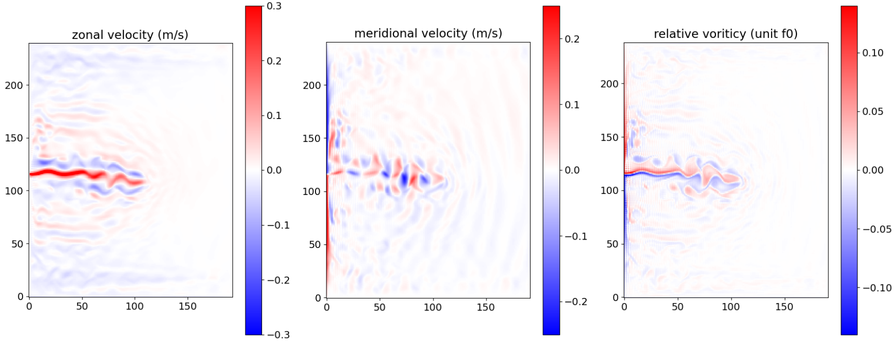
WENO+DST-II, non-eddy-resolving resolution

- 97×121 grid, resolution 40km.
- No hyper-viscosity.
- Jet destructed.
- Grid point oscillations in vorticity near the boundary.
- 40 yrs spin-up animation



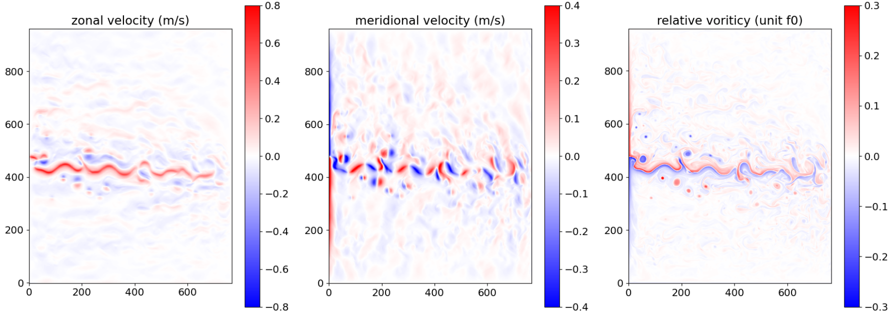
WENO+DST-II, eddy-permitting resolution

- 193×241 grid, resolution 20km.
- Pretty long jet surrounded by large eddies.
- Few grid point oscillations (vs 40km).
- 40 yrs spin-up animation



WENO+DST-II, eddy-resolving resolution

- 769×961 grid, resolution 5km.
- Longer jet, more eddies, turbulence all over the domain.
- Almost no grid point oscillations.
- 40 yrs spin-up animation



Rossby waves reflection

Vallis, 2017, chapitre 6:

- " *In mid-latitudes, the reflection at a western boundary generates Rossby waves that have a short zonal length scale [...] , which means that their meridional velocity is large.*"
- "*If the mean flow is westward so that $\overline{u}$ is negative, then very short waves will be unable to escape from the boundary: the waves will be trapped in a western boundary layer.*"
- "*Even with no mean flow, the short zonal length scale means that frictional effects will be large.*"

Need for dissipation → Munk western boundary layer with laplacian viscosity.

Implicit eddy parameterization

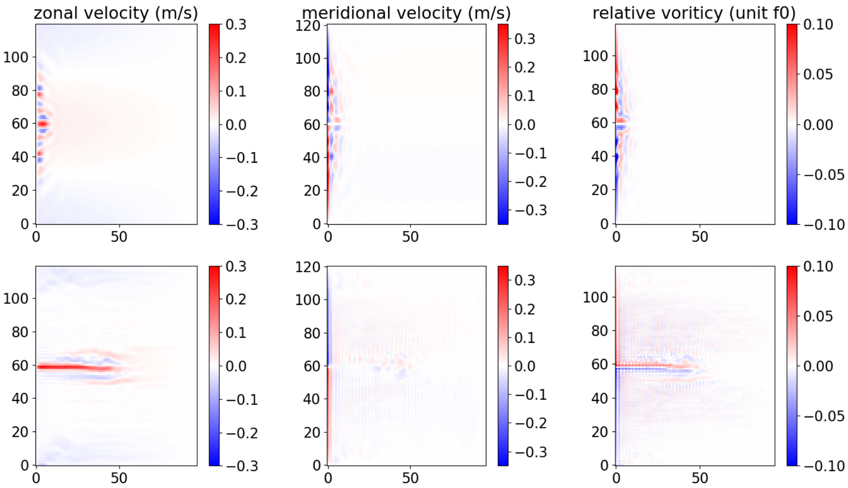
3 ingredients:

- Staggered grid and WENO-5 finite volume for PV advection: high-order, implicit dissipation.
- DST-II interpolation: preserve high-frequencies.
- Munk western boundary layer: model *large frictional effects*.

PyTorch implementation

- 500 lines of code
- seamless CPU/GPU, float32/float64.
- differentiable
- $3.9\,10^{-9} s$ per grid-point per Runge-Kutta step on NVIDIA GeForce RTX 2080 Ti GPU ($1.9\,10^{-9} s$ for Arakawa+hyperviscosity).

Arakawa vs IEP, non-eddy-resolving



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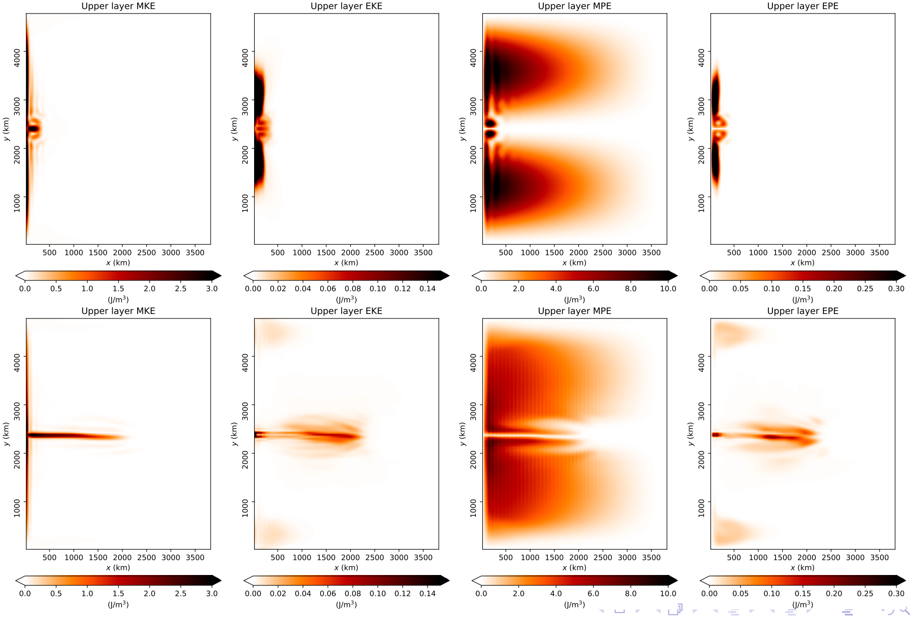

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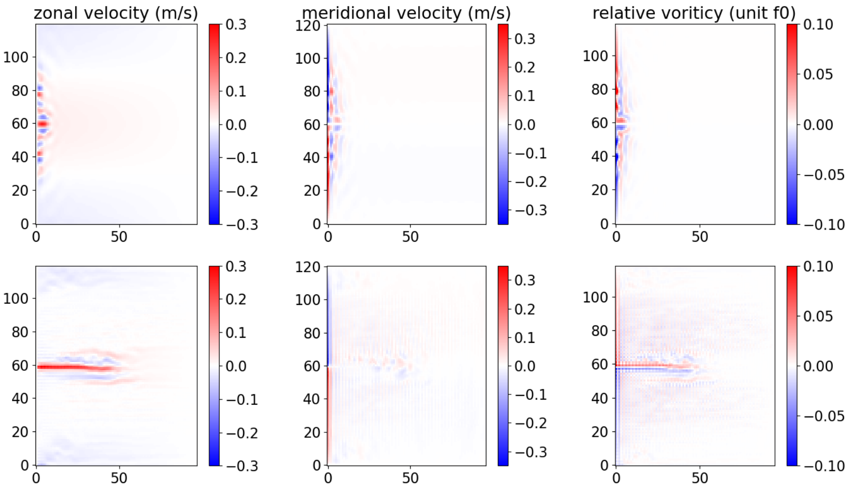

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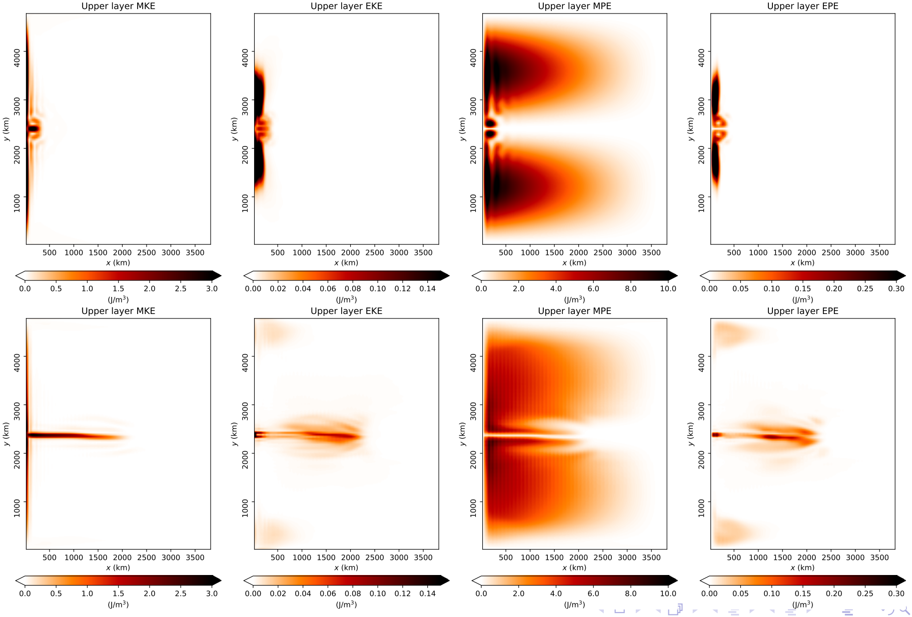
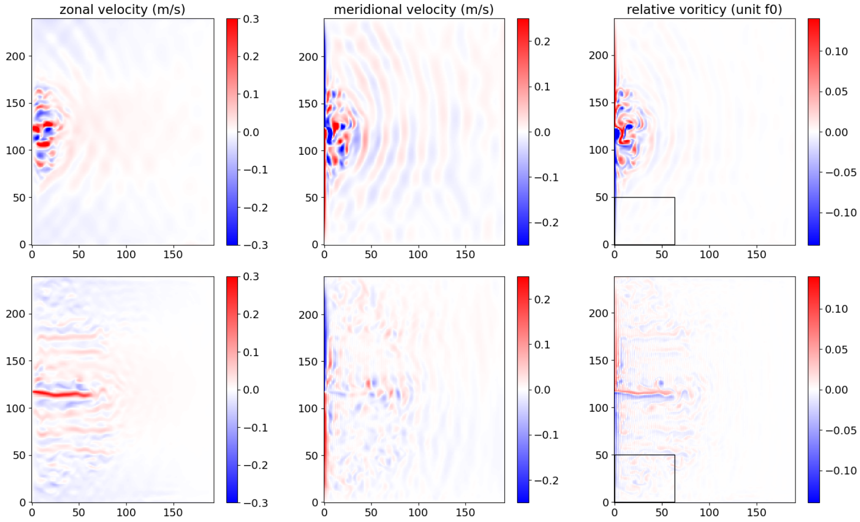
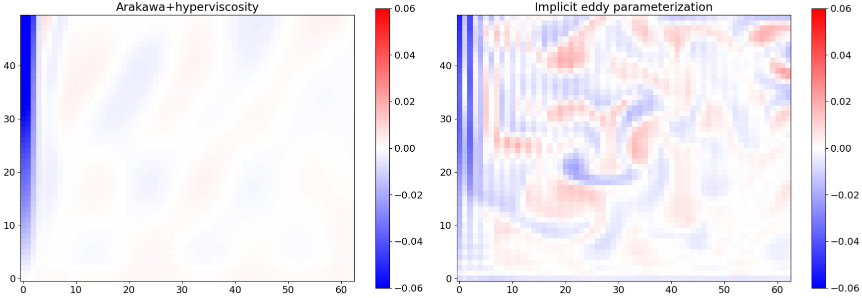


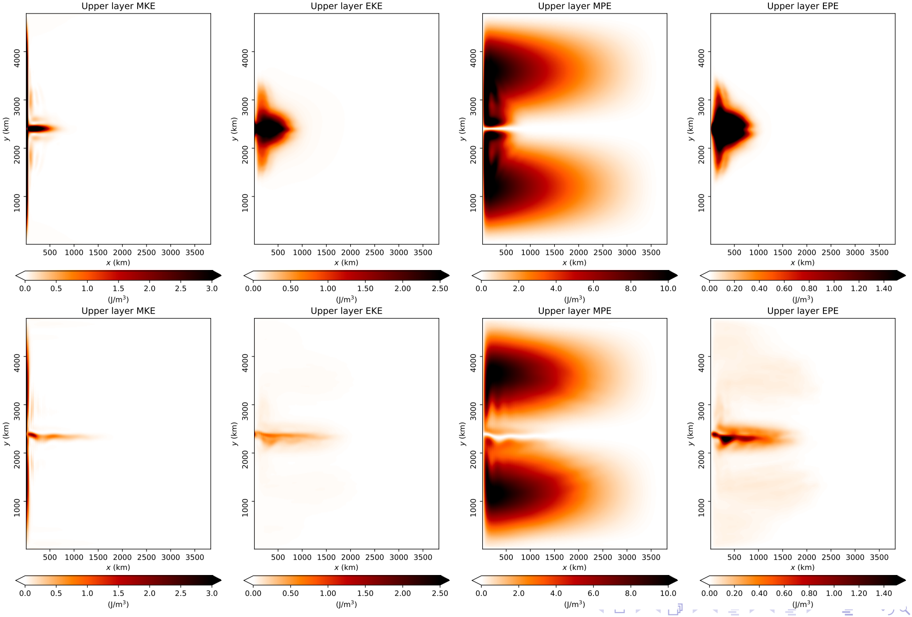
Arakawa vs IEP, eddy-permitting



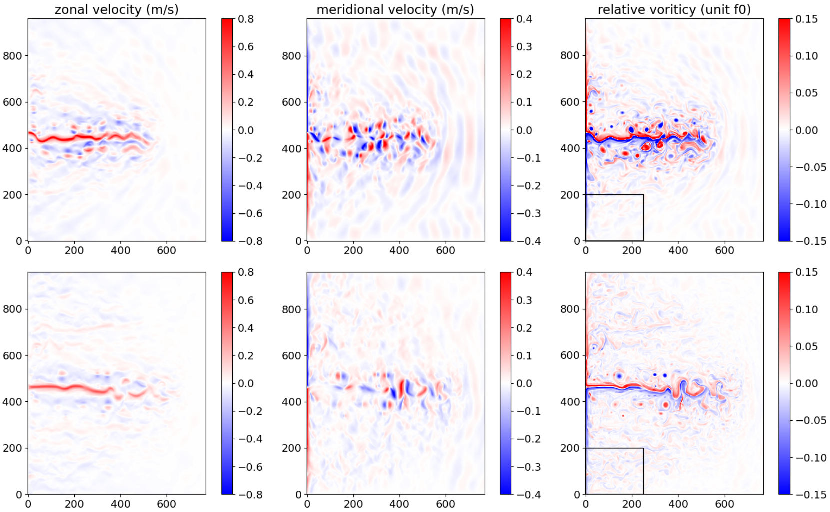
Arakawa vs IEP, eddy-permitting



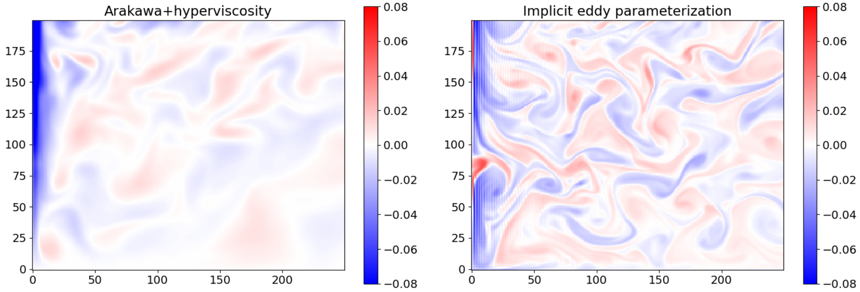
Arakawa vs IEP, eddy-permitting



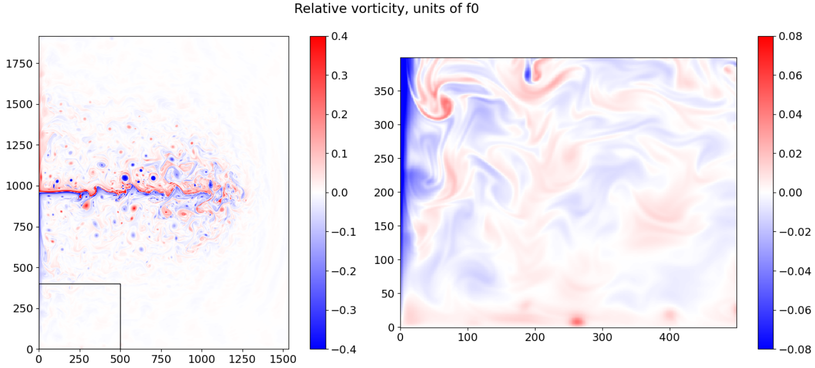
Arakawa vs IEP, eddy-resolving



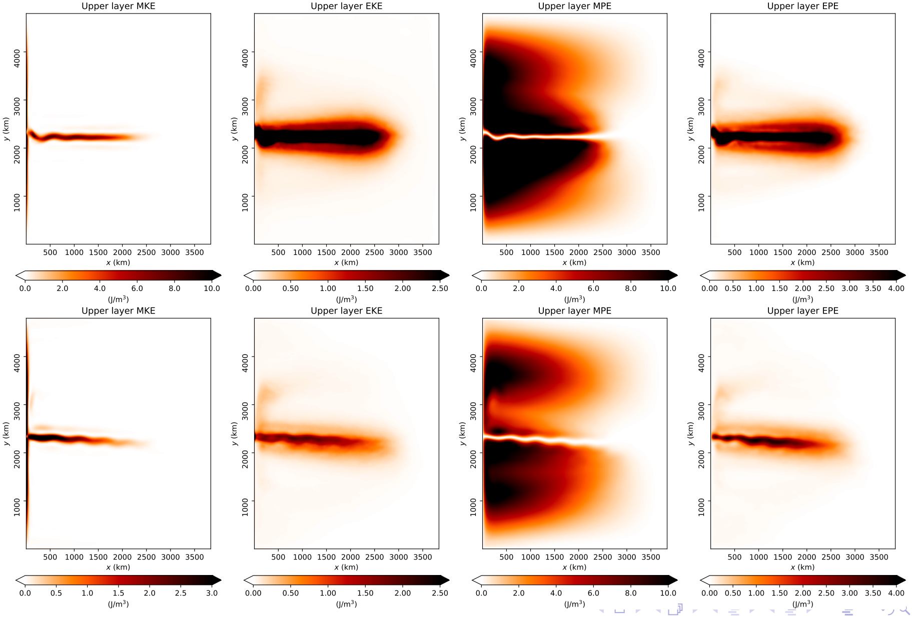
Arakawa vs IEP, eddy-resolving



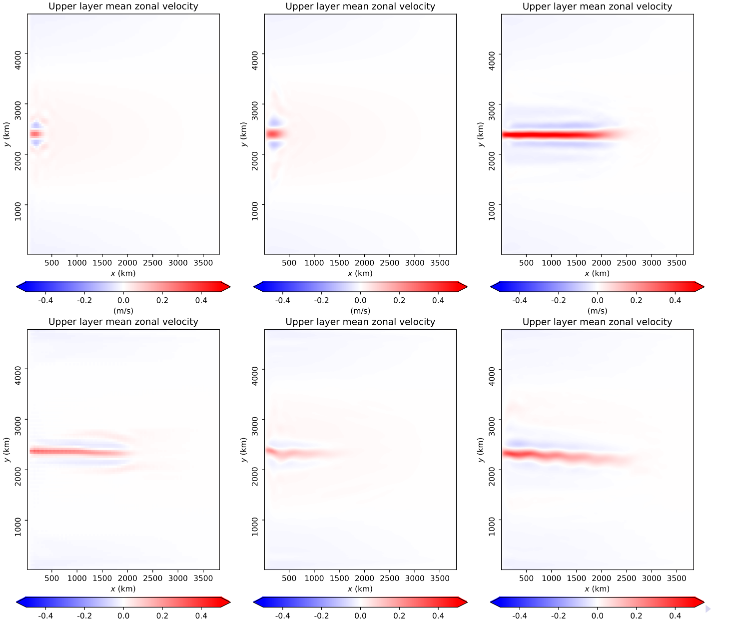
Arakawa, 2.5 km



Arakawa vs IEP, eddy-resolving



Coherence of the mean across resolution



Conclusions

Implicit eddy parameterization

- based on careful discretization choices.
- easy to implement and computationally cheap
- coherent mean states across resolutions.
- but no eddies in non-eddy-resolving resolution.
- increases effective resolution, accelerates spin up and symmetry breaking (eddy-resolving resolution).
- allows removing ad-hoc dissipation.
- is complementary to explicit parameterizations, e.g. LU.

Take home message

- Preserving high-frequencies increases significantly the effective resolution.
- Localized dissipation guided by the physics.

Questions ?

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