Round-Optimal Privacy-Preserving Protocols with Smooth Projective Hash Functions

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Outline

1. Blind Signatures
2. Cryptographic Tools
3. Smooth Projective Hash Functions
4. Oblivious Signature-Based Encryption

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1 Blind Signatures

2 Cryptographic Tools

3 Smooth Projective Hash Functions
- Intuition
- Applications

4 Oblivious Signature-Based Encryption

Smooth Projective Hash Functions [Cramer, Shoup, 2002]

Family of Hash Function H

Let $\{H\}$ be a family of functions:
- X, domain of these functions
- L, subset (a language) of this domain

such that, for any point x in L, $H(x)$ can be computed by using
- either a *secret* hashing key hk: $H(x) = \text{Hash}_L(\text{hk}; x)$;
- or a *public* projected key hp: $H(x) = \text{ProjHash}_L(\text{hp}; x, w)$

While the former works for all points in the domain X,
the latter works for $x \in L$ only, and requires a witness w to this fact.
There is a public mapping that converts the hashing key hk into the projected key hp: $\text{hp} = \text{ProjKG}_L(\text{hk})$

Properties

For any $x \in X$, $H(x) = \text{Hash}_L(\text{hk}; x)$
For any $x \in L$, $H(x) = \text{ProjHash}_L(\text{hp}; x, w)$ $\quad w$ witness that $x \in L$

Smoothness

For any $x \notin L$, $H(x)$ and hp are independent

Pseudo-Randomness

For any $x \in L$, $H(x)$ is pseudo-random, without a witness w

The latter property requires L to be a hard-partitioned subset of X:

Hard-Partitioned Subset

L is a hard-partitioned subset of X if it is computationally hard to distinguish a random element in L from a random element in $X \setminus L$

Element-Based Projection

Initial Definition [Cramer, Shoup, 2002]

The projected key hp depends on the hashing key hk only:
$\text{hp} = \text{ProjKG}_L(\text{hk})$

New Definition [Gennaro, Lindell, 2003]

The projected key hp depends on the hashing key hk, and x:
$\text{hp} = \text{ProjKG}_L(\text{hk}; x)$

$\rightarrow$ More applications

Examples

DH Language [Cramer, Shoup, 2002]

$L_{g,h} = \{u, v\}$ such that (g, h, u, v) is DH tuple:
there exists r such that $u = g^r$ and $v = h^r$

→ Public-key Encryption with IND-CCA Security

Commitment [Gennaro, Lindell, 2003]

$L_{pk,m} = \{c\}$ such that c is a commitment of m
using public parameter pk:
there exists r such that $c = \mathrm{com}_{pk}(m; r)$
where com is the committing algorithm

→ Password-Authenticated Key Exchange in the Standard Model

Examples (Con'd)

Labeled Encryption [Canetti, Halevi, Katz, Lindell, MacKenzie, 2005]

$L_{pk,(\ell,m)} = \{c\}$ such that c is an encryption of m
with label ℓ, under the public key pk:
there exists r such that $c = \mathcal{E}^{\ell}_{pk}(m; r)$
where $\mathcal{E}$ is the encryption algorithm

→ PAKE in the UC Framework (passive corruptions)

Extractable/Equivocable Commitment [Abdalla, Chevalier, Pointcheval, 2009]

$L_{pk,m} = \{c\}$ such that c is a equivocable/extractable commitment of m
using public parameter pk

→ PAKE in the UC Framework secure against Active Corruptions

Smooth Projective HF Family for ElGamal

The CRS: $\rho = (G, q, g, pk = h)$

Language: $L = L_{(\mathbf{EG}^+,\rho),M} = \{C = (u_1, e) = \mathrm{EG}^+_{pk}(M; r), r \stackrel{\$}{\leftarrow} \mathbb{Z}_q\}$

- L is a hard-partitioned subset of $X = G^2$, under the semantic security of the ElGamal encryption scheme (DDH assumption)
- the random r is the witness to L-membership

Algorithms

- $\mathrm{HashKG}(M) = \mathrm{hk} = (\gamma_1, \gamma_3) \stackrel{\$}{\leftarrow} \mathbb{Z}_q \times \mathbb{Z}_q$
- $\mathrm{Hash}(\mathrm{hk}; M, C) = (u_1)^{\gamma_1}(eg^{-M})^{\gamma_3}$
- $\mathrm{ProjKG}(\mathrm{hk}; M, C) = \mathrm{hp} = (g)^{\gamma_1}(h)^{\gamma_3}$
- $\mathrm{ProjHash}(\mathrm{hp}; M, C; r) = (\mathrm{hp})^r$

Certification of Public Keys [Abdalla, Chevalier, Pointcheval, 2009]

For the certification Cert of an ElGamal public key $y = g^x$, in most of the protocols, the simulator needs to be able to extract the secret key:

Classical Process

- the user U sends his public key $y = g^x$;
- U and the authority A run a ZK proof of knowledge of x
- if convinced, A generates and sends the certificate Cert for y

For extracting x, the reduction requires a rewinding
(that is not always allowed: *e.g.*, in the UC Framework)

Certification of Public Keys

[Abdalla, Chevalier, Pointcheval, 2009]

For the certification Cert of an ElGamal public key $y = g^x$, in most of the protocols, the simulator needs to be able to extract the secret key:

New Process

The user U and the authority A use a smooth projective hash system for L: $y = g^x$ and $C = \mathcal{E}_{pk}(x; r)$ contain the same x

- U sends $y = g^x$, with $C = \mathcal{E}_{pk}(x; r)$, for a random r;
- A generates
  - a hashing key $\mathsf{hk} \overset{\$}{\leftarrow} \mathsf{HashKG}()$,
  - the corresponding projected key on (y, C),
  - the hash value $\mathsf{Hash} = \mathsf{Hash}(\mathsf{hk}; (y, C))$

  and sends hp along with $\mathsf{Cert} \oplus \mathsf{Hash}$;
- U computes $\mathsf{Hash} = \mathsf{ProjHash}(\mathsf{hp}; (y, C), r)$, and gets Cert.

Blind Signature

[Blazy, Fuchsbauer, Pointcheval, Vergnaud, 2011]

In order to get M blindly signed under a Waters' signature:

Previous Process

- the user U encrypts M into C_1, and $\mathcal{F}(M)$ into C_2;
- U produces a Groth-Sahai NIZK that
  C_1 and C_2 contain the same M
- if convinced, A generates a signature on C_2
- granted the commutativity, U decrypts it
  into a Waters' signature of M,
  and eventually re-randomizes the signature

Such a NIZK requires $9\ell + 24$ group elements

Blind Signature

[Blazy, Pointcheval, Vergnaud, 2012]

In order to get M blindly signed under a Waters' signature:

Previous Process

The user U and the authority A use a smooth projective hash system for L: $C_1 = \mathcal{E}_{pk_1}(M; r)$ and $C_2 = \mathcal{E}_{pk_2}(\mathcal{F}(M); s)$ contain the same M

- U sends encryptions of M, into C_1, and $\mathcal{F}(M)$, into C_2;
- A generates
  - a signature σ on C_2,
  - masks it using $\mathsf{Hash} = \mathsf{Hash}(\mathsf{hk}; (C_1, C_2))$
- U computes $\mathsf{Hash} = \mathsf{ProjHash}(\mathsf{hp}; (C_1, C_2), (r, s))$, and gets σ. Granted the commutativity, U decrypts it into a Waters' signature of M, and eventually re-randomizes it

Such a protocol requires $8\ell + 12$ group elements in total

Outline

1 Blind Signatures
2 Cryptographic Tools
3 Smooth Projective Hash Functions
4 Oblivious Signature-Based Encryption
- Example
- Security Notions

Linear Encryption

In a group $\mathbb{G}$ of order p, with a generator g,
and a bilinear map $e : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$

Linear Encryption

[Boneh, Boyen, Shacham, 2004]

- *EKeyGen*: $dk = (x_1, x_2) \stackrel{\$}{\leftarrow} \mathbb{Z}_p^2$, $pk = (X_1 = g^{x_1}, X_2 = g^{x_2})$;
- *Encrypt*$(pk = (X_1, X_2), m; (r_1, r_2))$, for $m \in \mathbb{G}$ and $(r_1, r_2) \stackrel{\$}{\leftarrow} \mathbb{Z}_p^2$
  $\to \quad c = (c_1 = X_1^{r_1}, c_2 = X_2^{r_2}, c_3 = g^{r_1+r_2} \cdot m)$;
- *Decrypt*$(dk = (x_1, x_2), c = (c_1, c_2, c_3)) \quad \to \quad m = c_3 / c_1^{1/x_1} c_2^{1/x_2}$.

Re-Randomization

- *Random*$_{\mathcal{E}}(pk = (X_1, X_2), c = (c_1, c_2, c_3); (r_1', r_2'))$, for $(r_1', r_2') \stackrel{\$}{\leftarrow} \mathbb{Z}_p^2$
  $\to \quad c' = (c_1' = c_1 \cdot X_1^{r_1'}, c_2' = c_2 \cdot X_2^{r_2'}, c_3' = c_3 \cdot g^{r_1'+r_2'})$.

Waters Signature

In a group $\mathbb{G}$ of order p, with a generator g,
and a bilinear map $e : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$

Waters Signature

[Waters, 2005]

For a message $M = (M_1, \ldots, M_k) \in \{0,1\}^k$,
we define $F = \mathcal{F}(M) = u_0 \prod_{i=1}^k u_i^{M_i}$, where $\vec{u} = (u_0, \ldots, u_k) \stackrel{\$}{\leftarrow} \mathbb{G}^{k+1}$.
For an additional generator $h \stackrel{\$}{\leftarrow} \mathbb{G}$.

- *SKeyGen*: $vk = X = g^x$, $sk = Y = h^x$, for $x \stackrel{\$}{\leftarrow} \mathbb{Z}_p$;
- *Sign*$(sk = Y, F; s)$, for $M \in \{0,1\}^k$, $F = \mathcal{F}(M)$, and $s \stackrel{\$}{\leftarrow} \mathbb{Z}_p$
  $\to \quad \sigma = (\sigma_1 = Y \cdot F^s, \sigma_2 = g^{-s})$;
- *Verif*$(vk = X, M, \sigma = (\sigma_1, \sigma_2))$ checks whether
  $$e(g, \sigma_1) \cdot e(F, \sigma_2) = e(X, h).$$

Waters Signature on a Linear Ciphertext: Idea

We define $F = \mathcal{F}(M) = u_0 \prod_{i=1}^k u_i^{M_i}$, and encrypt it

$$c = (c_1 = X_1^{r_1}, c_2 = X_2^{r_2}, c_3 = g^{r_1+r_2} \cdot F)$$

- *KeyGen*: $vk = X = g^x$, $sk = Y = h^x$, for $x \stackrel{\$}{\leftarrow} \mathbb{Z}_p$
  $dk = (x_1, x_2) \stackrel{\$}{\leftarrow} \mathbb{Z}_p^2$, $pk = (X_1 = g^{x_1}, X_2 = g^{x_2})$
- *Sign*$((X_1, X_2), Y, c; s)$, for $c = (c_1, c_2, c_3)$
  $\to \quad \sigma = (\sigma_1 = Y \cdot c_3^s, \sigma_2 = (c_1^s, c_2^s), \sigma_3 = (g^s, X_1^s, X_2^s))$
- *Verif*$((X_1, X_2), X, c, \sigma)$ checks $\quad e(g, \sigma_1) = e(X, h) \cdot e(\sigma_{3,0}, c_3)$
  $$e(\sigma_{2,0}, g) = e(c_1, \sigma_{3,0}) \qquad e(\sigma_{2,1}, g) = e(c_2, \sigma_{3,0})$$
  $$e(\sigma_{3,1}, g) = e(X_1, \sigma_{3,0}) \qquad e(\sigma_{3,2}, g) = e(X_2, \sigma_{3,0})$$

σ_3 is needed for ciphertext re-randomization

Re-Randomization of Ciphertext

$$c = (c_1 = X_1^{r_1}, \quad c_2 = X_2^{r_2}, \quad c_3 = g^{r_1+r_2} \cdot F \quad)$$
$$\sigma = (\sigma_1 = Y \cdot c_3^s, \quad \sigma_2 = (c_1^s, c_2^s), \quad \sigma_3 = (g^s, X_1^s, X_2^s) \quad)$$

after re-randomization by (r_1', r_2')

$$c' = (c_1' = c_1 \cdot X_1^{r_1'}, \quad c_2' = c_2' \cdot X_2^{r_2'}, \quad c_3' = c_3 \cdot g^{r_1'+r_2'} \quad)$$
$$\sigma' = (\sigma_1' = \sigma_1 \cdot \sigma_{3,0}^{r_1'+r_2'}, \quad \sigma_2' = (\sigma_{2,0} \cdot \sigma_{3,1}^{r_1'}, \sigma_{2,1} \cdot \sigma_{3,2}^{r_2'}), \quad \sigma_3' = \sigma_3 \quad)$$

Anybody can publicly re-randomize c into c'
with additional random coins (r_1', r_2'),
and adapt the signature σ of c into σ' of c'

Unforgeability under Chosen-Ciphertext Attacks

Chosen-Ciphertext Attacks

The adversary is allowed to ask any valid ciphertext of his choice to the signing oracle

Because of the re-randomizability of the ciphertext-signature, we cannot expect resistance to existential forgeries, but we should allow a restricted malleability only:

Forgery

A valid ciphertext-signature pair, so that the plaintext is different from all the plaintexts in the ciphertexts sent to the signing oracle

Unforgeability

From a valid ciphertext-signature pair:

$$c = (c_1 = X_1^{r_1}, c_2 = X_2^{r_2}, c_3 = g^{r_1+r_2} \cdot F)$$
$$\sigma = (\sigma_1 = Y \cdot c_3^s, \sigma_2 = (c_1^s, c_2^s), \sigma_3 = (g^s, X_1^s, X_2^s))$$

and the decryption key (x_1, x_2), one extracts

$$\begin{array}{llll} F = & c_3/(c_1^{1/x_1} c_2^{1/x_2}) & & \\ \Sigma = (& \Sigma_1 = \sigma_1/(\sigma_{2,0}^{1/x_1}\sigma_{2,1}^{1/x_2}), & \Sigma_2 = \sigma_{3,0}) \\ = (& = Y \cdot F^s & = g^s) \end{array}$$

Security of Waters signature is for a pair (M, Σ)

$\rightarrow$ needs of a proof of knowledge Π_M of M in $F = \mathcal{F}(M)$

bit-by-bit commitment of M and Groth-Sahai proof

Chosen-Message Attacks

From a valid ciphertext $c = (c_1 = X_1^{r_1}, c_2 = X_2^{r_2}, c_3 = g^{r_1+r_2} \cdot F)$, and the additional proof of knowledge of M, one extracts M and asks for a Waters signature:

$$\Sigma = (\Sigma_1 = Y \cdot F^s, \Sigma_2 = g^s)$$

In this signature, the random coins s are unknown, we thus need to know the coins in c

$\rightarrow$ needs of a proof of knowledge Π_r of r_1, r_2 in c

bit-by-bit commitment of r_1, r_2 and Groth-Sahai proof

From the random coins r_1, r_2 (and the decryption key):

$$\begin{array}{llll} \sigma = (\sigma_1 = \Sigma_1 \cdot \Sigma_2^{r_1+r_2}, & \sigma_2 = (\Sigma_2^{x_1 r_1}, \Sigma_2^{x_2 r_2}), & \sigma_3 = (\Sigma_2, \Sigma_2^{r_1}, \Sigma_2^{r_2}) &) \\ = Y \cdot c_3^s, & = (c_1^s, c_2^s), & = (g^s, X_1^s, X_2^s) & \end{array}$$

Security

Chosen-Ciphertext Attacks

A valid ciphertext $C = (c_1, c_2, c_3, \Pi_M, \Pi_r)$ is a

- ciphertext $c = (c_1, c_2, c_3)$
- a proof of knowledge Π_M of the plaintext M in $F = \mathcal{F}(M)$
- a proof of knowledge Π_r of the random coins r_1, r_2

From such a ciphertext and the decryption key (x_1, x_2), and a Waters signing oracle, one can generate a signature on C

Forgery

From a valid ciphertext-signature pair (C, σ), where C encrypts M, one can generate a Waters signature on M

Security

- From the Waters signing oracle,
  we answer Chosen-Ciphertext Signing queries
- From a Forgery, we build a Waters Existential Forgery

Security Level

Since the Waters signature is EF-CMA under the *CDH* assumption, our signature on randomizable ciphertext is Unforgeable
against Chosen-Ciphertext Attacks
under the *CDH* assumption

Properties

Proofs

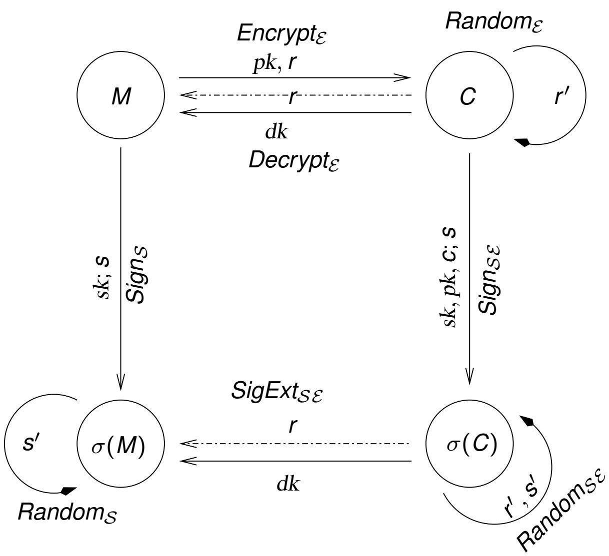
Since we use the Groth-Sahai methodology for the proofs Π_M and Π_r

- in case of re-randomization of c, one can adapt Π_M and Π_r
- because of the need of M, but also r_1 and r_2 in the simulation, we need bit-by-bit commitments:
  - M can be short (ℓ bit-long)
  - r_1 and r_2 are random in $\mathbb{Z}_p$
  - $\rightarrow$ C is large!

Efficiency

We can improve efficiency: shorter signatures

Randomizable Commutative Signature/Encryption



Conclusion

Randomizable Commutative Signature/Encryption

Various Applications

- non-interactive receipt-free electronic voting scheme
- (fair) blind signature

Security relies on the *CDH* and the *DLin* assumptions
For an ℓ-bit message, ciphertext-signature:
$9\ell + 24$ group elements

A more efficient variant with asymmetric pairing
on the CDH^* and the *SXDH* assumptions
Ciphertext-signature: $6\ell + 7$ group elements in $\mathbb{G}_1$
and $6\ell + 5$ group elements in $\mathbb{G}_2$