Authenticated Key Exchange

passwords, groups, low-power devices

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Summary

- Provable Security
- Authenticated Key Exchange
  - Security Model
  - Examples
  - Authentication
  - Password-based
- Group Key Exchange
  - Security Model
  - Example
  - Dynamic groups

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Algorithmic Assumptions *necessary*

- $n=pq$: **public modulus**
- e : **public exponent**
- $d=e^{-1} \bmod \varphi(n)$: **private**

RSA Encryption

- $\mathbf{E}(m) = m^e \bmod n$
- $\mathbf{D}(c) = c^d \bmod n$

If the RSA problem is easy,
secrecy is not satisfied:
anybody may recover m from c

Algorithmic Assumptions *sufficient?*

Security proofs give the guarantee that the assumption is **enough** for secrecy:

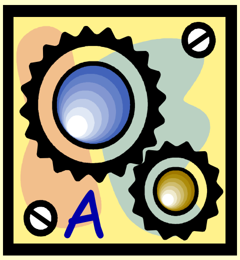
- if an adversary can break the secrecy
- one can break the assumption

⇒ “reductionist” proof

Proof by Reduction

Reduction of a problem **P** to an attack *Atk*:

- Let *A* be an adversary that breaks the scheme
- Then *A* can be used to solve **P**



Proof by Reduction

Reduction of a problem **P** to an attack *Atk*:

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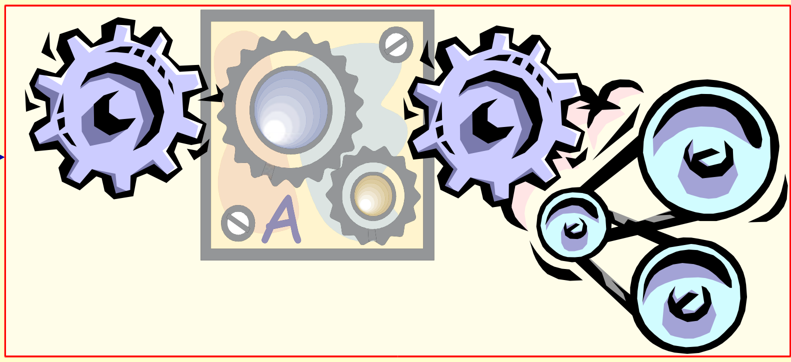
Instance
I of **P**



Solution
of **I**

P intractable ⇒ scheme unbreakable

Provably Secure Scheme

To prove the security of a cryptographic scheme, one has to make precise

- the algorithmic assumptions
  - the RSA problem, the Diffie-Hellman problems, ...
- the security notions to be guaranteed
  - depends on the scheme
- a reduction
  - an adversary can help to break the assumption
  - simulation of the « view » of the adversary

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Authenticated Key Exchange

Two parties (Alice and Bob) agree on a ***common*** secret key sk, in order to establish a secret channel

- Intuitive goal: ***implicit authentication***
  - only the intended partners can compute the session key
- Formally: ***semantic security***
  - the session key sk is indistinguishable from a random string r, to anybody else

Further Properties

- ***Mutual authentication***
  - They are both sure to **actually** share the secret with the people they think they do
- ***Forward-secrecy***
  - Even if a long-term secret data is corrupted, previously shared secrets are **still** semantically secure

Semantic Security

- For breaking the semantic security, the adversary asks one **test**-query which is answered, according to a random bit b, by
  - the actual secret data sk (if $b=0$)
  - a random string r (if $b=1$)

⇒ the adversary has to guess this bit b

The Leakage of Information

- The protocol is run over a public network, then the transcripts are public:
  - an **execute**-query provides such a transcript to the adversary
- The secret data sk may be misused (with a weak encryption scheme, ...):
  - the **reveal**-query is answered by this secret data sk

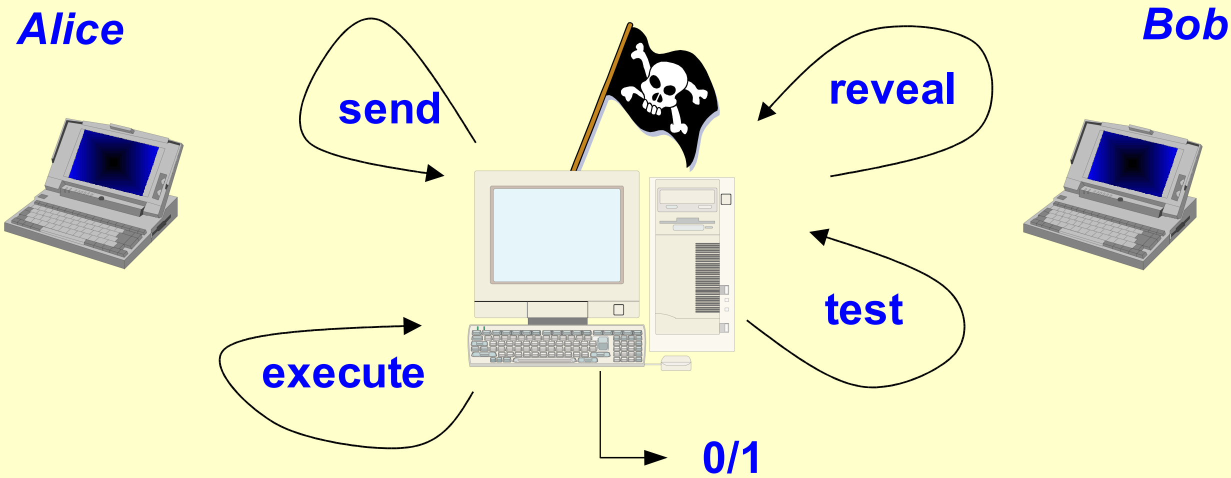
Passive/Active Adversaries

- ***Passive adversary***: history built using
  - the **execute**-queries → transcripts
  - the **reveal**-queries → session keys
- ***Active adversary***: entire control of the network
  - the **send**-queries

    active, adaptive adversary on concurrent executions
    - to send message to Alice or Bob (in place of Bob or Alice respectively)
    - to intercept, forward and/or modify messages

Security Model

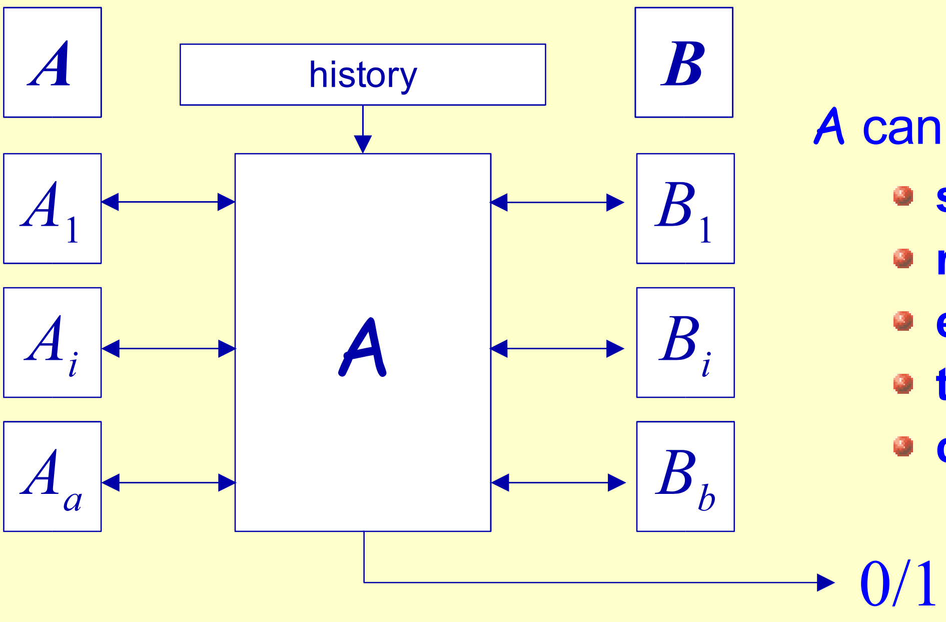
As many **execute**, **send** and **reveal** queries as the adversary wants



But one **test**-query, with b to be guessed...

Formal Model

Bellare-Rogaway model revisited by Shoup



A can ask

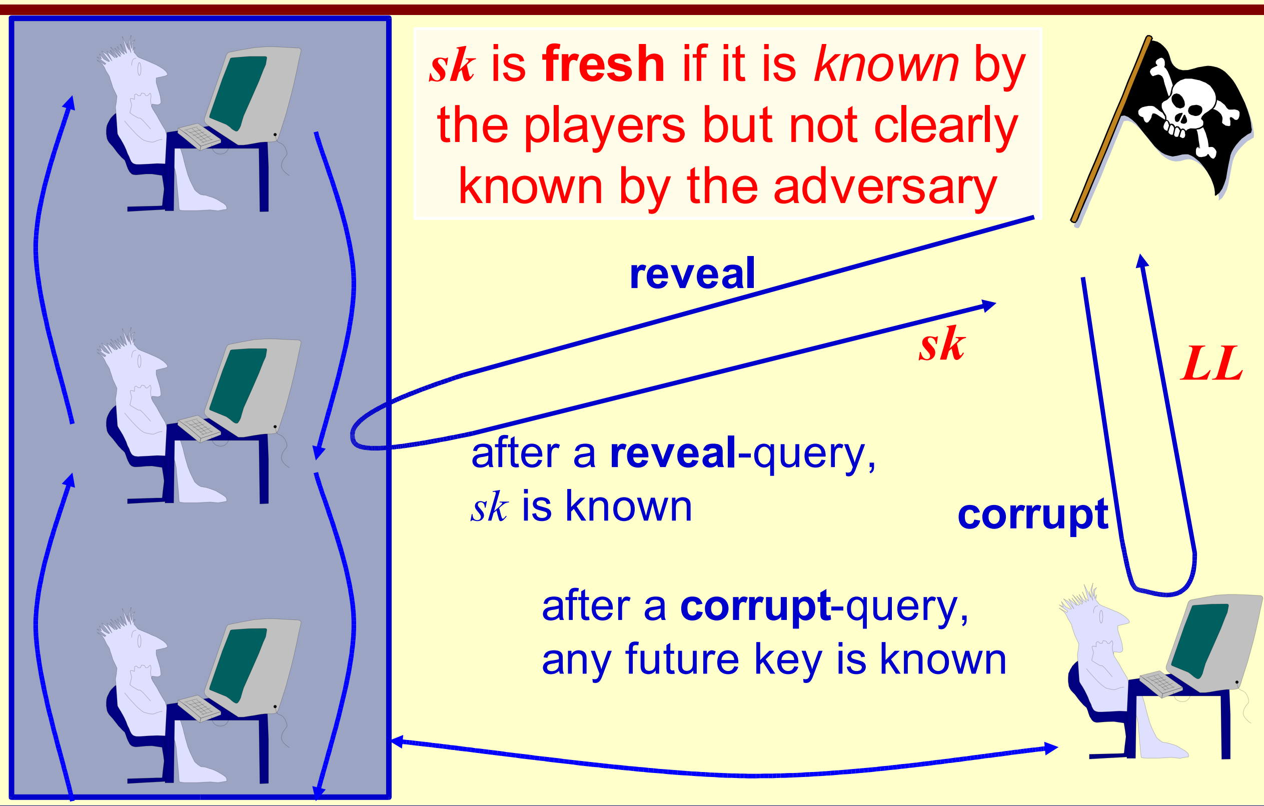
- **send**-queries
- **reveal**-queries
- **execute**-queries
- **test**-query
- **corrupt**-queries

Forward Secrecy

Forward secrecy means that the adversary cannot distinguish a session key established ***before*** any corruption of the long-term private keys:

- the **corrupt**-query is answered by the long-term private key of the corrupted party
- then the **test**-query must be asked on a session key established ***before*** any **corrupt**-query

Freshness



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Diffie-Hellman Key Exchange

The most classical key exchange scheme has been proposed by Diffie and Hellman:

$\mathbf{G} = <g>$, cyclic group of prime order q

- Alice chooses a random $x \in \mathbf{Z}_q$,
  computes and sends $X = g^x$
- Bob chooses a random $y \in \mathbf{Z}_q$,
  computes and sends $Y = g^y$
- They can both compute the value

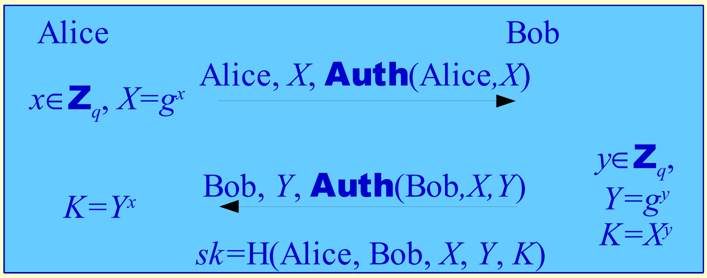
$$K = Y^x = X^y$$

Properties

- Without any authentication, no security is possible: man-in-the-middle attack

  ⇒ some authentication is required
- If flows are **Strongly Authenticated** (ie. MAC or Signature), it provides the semantic security of the session key under the **DDH Problem**
- If one derives the session key as $sk = \mathrm{H}(K)$, in the random oracle model, semantic security is relative to the **CDH Problem**

Replay Attack



No explicit authentication

⇒ replay attacks

- The adversary intercepts "Alice, X, **Auth**(Alice, X)"
- It can initiate a new session with it

Bob believes it comes from Alice

- Bob accepts the key, but does not share it with Alice

  ⇒ **no mutual authentication**
- The adversary does not know the key either

  ⇒ **still semantic security**

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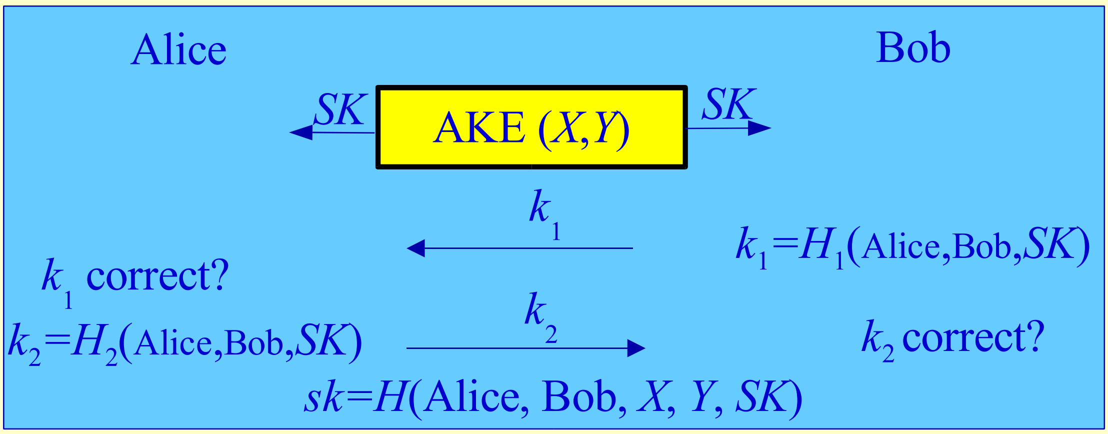
Mutual Authentication

Adding key confirmation rounds:
mutual authentication

[Bellare-P.-Rogaway Eurocrypt ‘00]



David Pointcheval – CNRS - ENS
Authenticated Key Exchange

Authentication

- **Asymmetric**: (sk_A, pk_A) and possibly (sk_B, pk_B)
  - they authenticate to each other using the knowledge of the private key associated to the certified public key
- **Symmetric**: common (long – high-entropy) secret
  - they use the long term secret to derive a secure and authenticated ephemeral key sk
- **Password**: common (short - low-entropy) secret
  - let us assume a 20-bit password

Asymmetric

- the most classical authentication mode is the signature (*cf*. SIGMA)
- the ability to decrypt (with an asymmetric encryption scheme) is also a way to provide authentication

Mutual Authentication for Low-Power Devices
[Jakobsson-P. - FC 01]

  - Client: Schnorr signature with off-line pre-processing
    - very efficient signing process (for the client)
  - Server: RSA decryption
    - efficient encryption process (for the client)
  - Fixed random for the Server: precomputation

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Password-based Authentication

Password (short – low-entropy secret – say 20 bits)

- exhaustive search is possible
- basic attack: on-line exhaustive search
  - the adversary guesses a password
  - tries to play the protocol with this guess
  - failure ⇒ it erases the password from the list
  - and restarts…
- after 1,000,000 attempts, the adversary wins

cannot be avoided

Dictionary Attack

- The on-line exhaustive search
  - cannot be prevented
  - can be made less serious (delay, limitations, …)
- We want it to be the **best attack**…
- The off-line exhaustive search
  - a few passive or active attacks
  - failure ⇒ erasure of MANY passwords from the list
  - this is called **dictionary attack**

Security

One wants to prevent dictionary attacks:

- a passive trial (**execute** + **reveal**)
  - does not reveal any information about the password
- an active trial (**send**)
  - allows to erase at most one password from the list of possible passwords
    - (or maybe 2 or 3 for technical reasons in the proof)

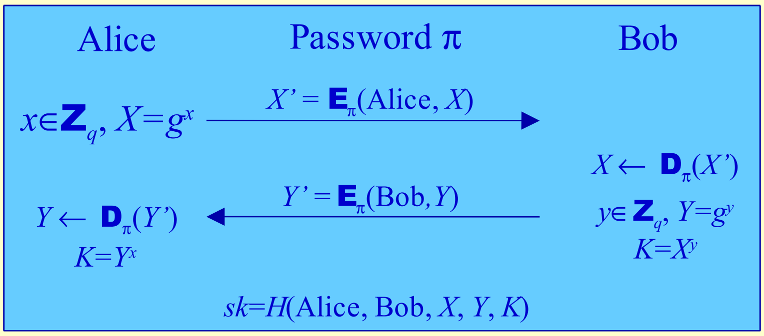
Example: EKE

The most famous scheme EKE:

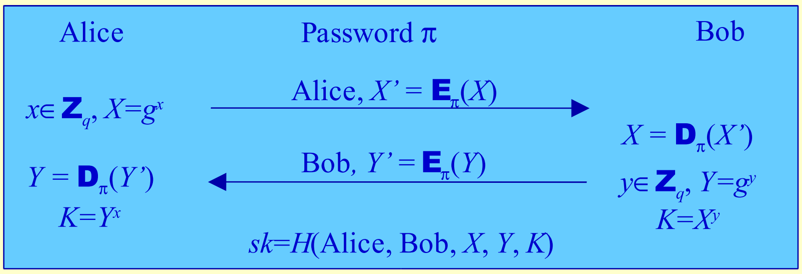
Encrypted Key Exchange

- Flows are encrypted with the password.
- Must be done carefully: no redundancy
- From X',
  for any password π
  - decrypt X'
  - check whether
    it begins with “Alice”

bad one



EKE - AuthA



EKE

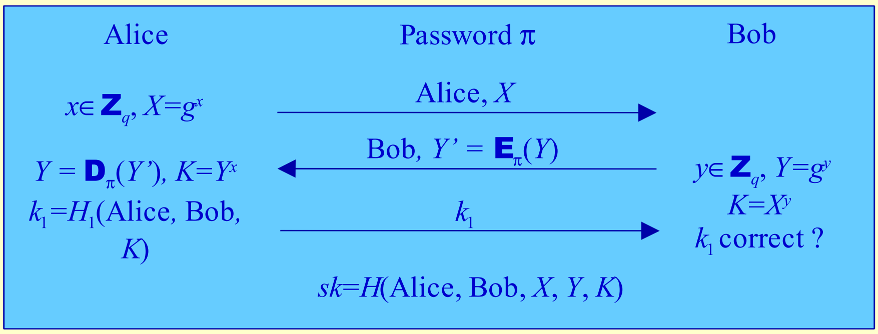
Bellovin-Merritt 1992

Two-flow Encrypted Key Exchange

AuthA

Bellare-Rogaway 2000

One-flow Encrypted Key Exchange



- **EKE**: security claimed, but never fully proved
- **AuthA**: security = open problem

Security Results

[BCP - ACM-CCS '03]

- Assumptions
  - the ideal-cipher model – for (**E**,**D**)
  - the random-oracle model – for H and H_1
- Semantic security of **AuthA**:
  - Advantage $\geq 3\ q_{\text{send}}/\mathrm{N} + \varepsilon$,
    $\Rightarrow$ CDH problem : probability $\geq \varepsilon/8q_{\text{hash}}$
    (within almost the same time)
- Similar (but less efficient) results for **EKE**

New Security Results

[BCP - PKC '04]

- Assumptions
  - the random-oracle model
- Symmetric encryption = one-time pad:
  - $\mathbf{E}_\pi(X) = X \times G(\pi)$
- Semantic security of **AuthA**:
  - Advantage $\geq 12\ q_{\text{send}}/\mathrm{N} + \varepsilon$,
    $\Rightarrow$ CDH problem : probability $\geq \varepsilon\ /\ 12q_{\text{hash}}{}^2$
- Similar (but less efficient) results for **EKE**

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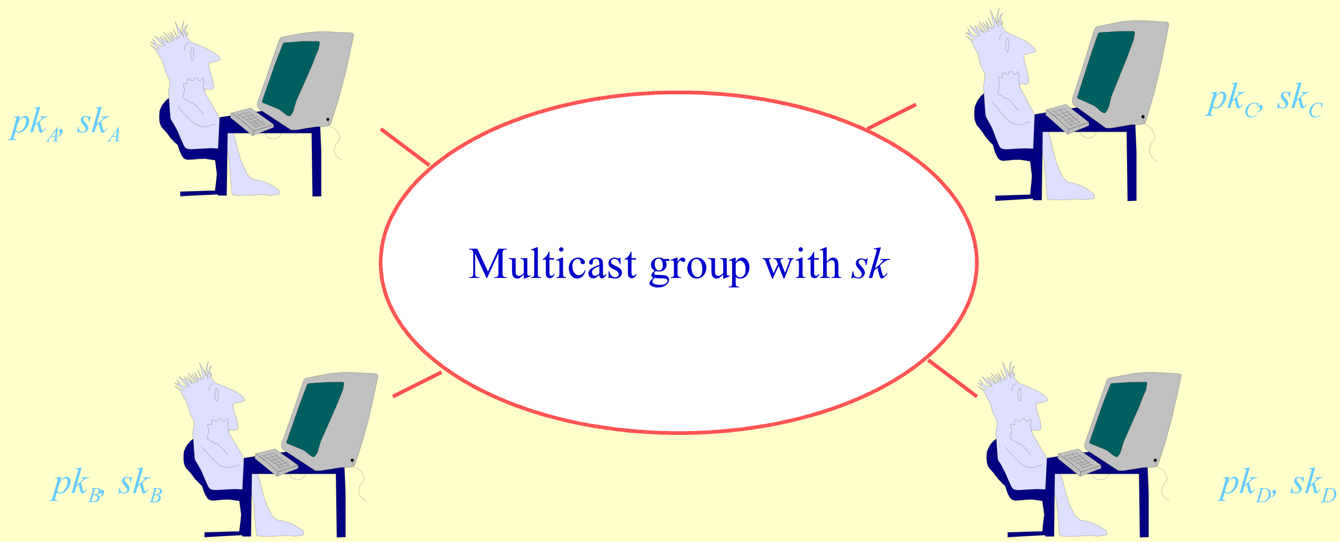
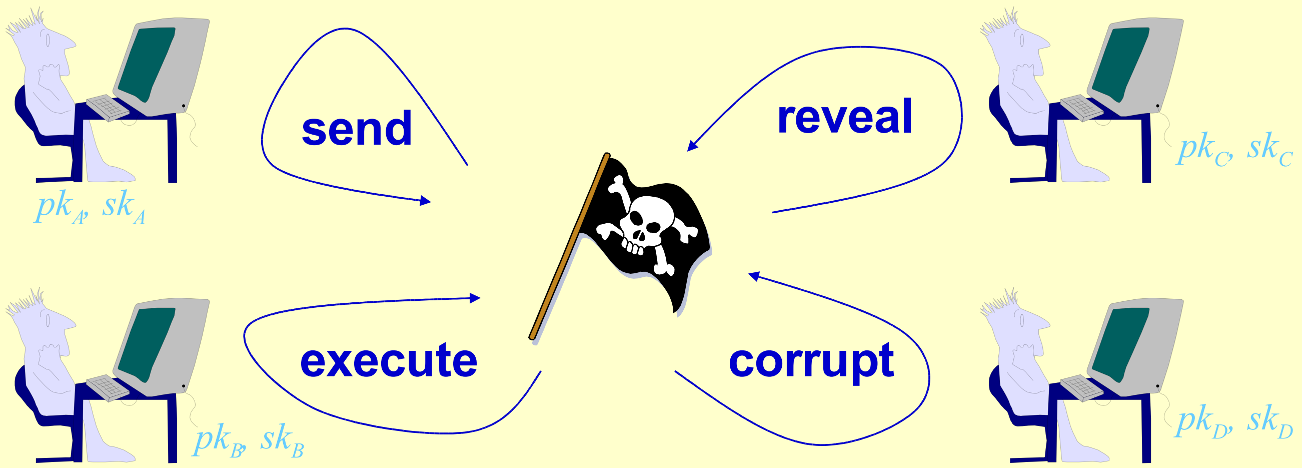
Model of Communication

- A set of n players, modelled by oracles
- A multicast group consisting of a set of players



Modelling the Adversary

- **send**: send messages to instances
- **execute**: obtain honest executions of the protocol
- **reveal**: obtain an instance's session key
- **corrupt**: obtain the value of the authentication secret



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A Group Key Exchange

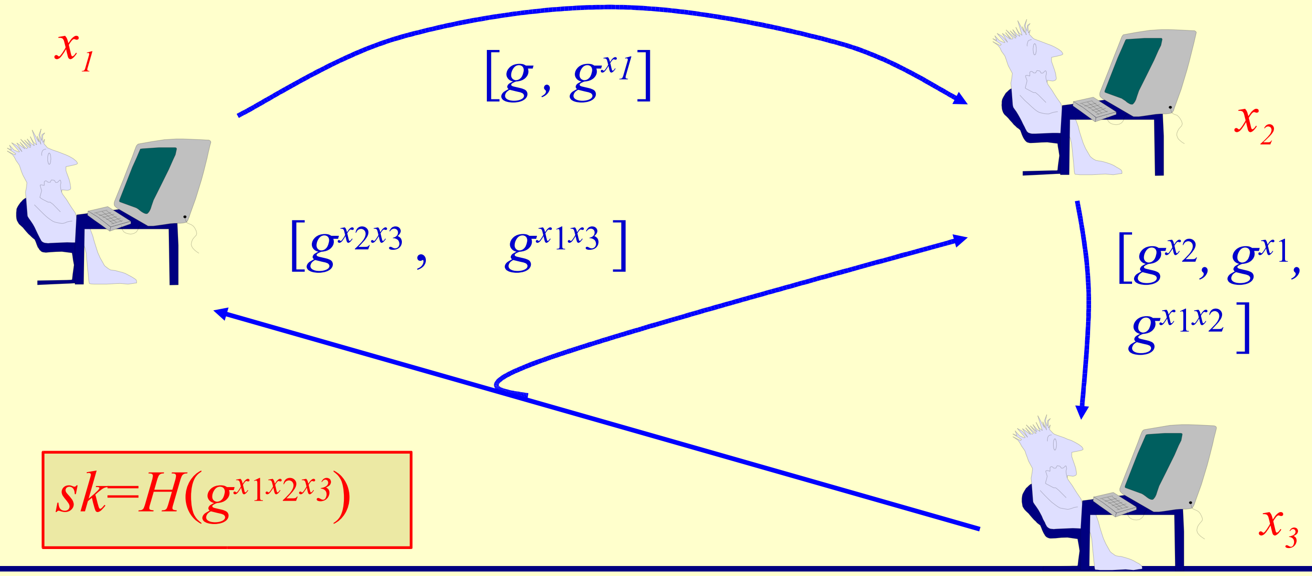
- Generalization of the 2-party DH,
  the session key is $sk = H(g^{x_1 x_2 \dots x_n})$
- Ring-based algorithm
  - **up-flow**: the contributions of each instance are gathered
  - **down-flow**: the last instance broadcasts the result
  - **end**: instances compute the session key

x_1	g				
x_2	g	g^{x1}			
x_3	g^{x2}	g^{x1}	g^{x1x2}		
x_4	g^{x2x3}	g^{x1x3}	g^{x1x2}	g^{x1x2x3}	
	g^{x2x3x4}	g^{x1x3x4}	g^{x1x2x4}	g^{x1x2x3}	$g^{x1x2x3x4}$

The Algorithm

- **Up-flow**: U_i raises received values to the power x_i
- **Down-flow**: U_n broadcasts (except $g^{x1x2\dots xn}$)

Everything is authenticated (Signature/MAC)



Group CDH

- The CDH generalized to the multi-party case
  - given the values $g^{\Pi x_i}$ for some choice of proper subset of $\{1, \ldots, n\}$
  - one has to compute the value $g^{x_1 x_2 \ldots x_n}$
- Example (n=3 and I={1,2,3})
  - given the set of the blue values
  - compute the red value

$$
\begin{array}{ccc}
 & g, & g^{x1} \\
g^{x1}, & g^{x2}, & g^{x1x2} \\
g^{x1x3}, & g^{x2x3}, & g^{x_1x_2x_3}
\end{array}
$$

GCDH ≥ DDH or CDH

[BCP - SAC '02]

Security Result

- Theorem (in the random-oracle model)

[BCPQ – ACM CCS '01]

$$\mathrm{Adv}^{\mathrm{ake}} \leq 2{q_{\mathsf{send}}}^n q_{\mathrm{hash}} \cdot \mathrm{Succ}^{\mathrm{gcdh}}(n,T) + 2n \cdot \mathrm{Succ}^{\mathrm{sign}}(q_s,T)$$

- Idea:
  - we introduce a Group Diffie-Hellman instance in the **test**ed session

    ⇒ we have to guess in which **send**-queries: factor ${q_{\mathsf{send}}}^n$
  - When the adversary has broken the scheme, the Group Diffie-Hellman solution is in the list of the queries to H

    ⇒ we have to guess it: factor q_{hash}

Improvements

- Security result: exponential in n
- Improvements

[BCP – Eurocrypt '02]

  - No guess of the tested pool
  - Use of the random self-reducibility of the DH problems

    $\Rightarrow$ reduction linear in n
  - Standard model (MAC and Left-Over-Hash Lemma)
- Dynamic groups

[BCP - Asiacrypt '01]

  - If one party leaves or joins the group,
    the protocol does not need to be restarted from scratch

Summary

Dynamic Groups

- **Join**: the last broadcast is sent to the new player and becomes the last up-flow
  ⇒ the new player introduces a new random
- **Remove**: the last remaining player introduces a new random x'_i in place of his x_i and broadcasts the useful values only

Remove 2 and 4

$g^{x_2x_3x_4}$ $g^{x_1x_3x_4}$ $g^{x_1x_2x_4}$ $g^{x_1x_2x_3}$ $g^{x_1x_2x_3x_4}$

$g^{x_2x'_3x_4}$ $g^{x_1x_2x_4}$ $g^{x_1x_2x'_3x_4}$

Dynamic Groups: Security Result

- Group of n people
- Tested group of size s
- Number of operations (**setup**, **join**, **remove**): Q
- Time: T

$$\mathrm{Adv}^{\mathrm{ake}} \leq 2\,Q \cdot C_n^s \cdot q_{\mathrm{hash}} \cdot \mathrm{Succ}^{\mathrm{gcdh}}(s,T) + 2n \cdot \mathrm{Succ}^{\mathrm{sign}}(q_{\mathsf{send}},T)$$

- Idea:
  - Guess the players in the **test**ed group
  - Guess the last operation before the **test**ed key
  - Guess the solution among the H queries

Improved Security Result

[BCP – Eurocrypt '02]

- Number of people involved in the group before the **test**-query (maybe removed) = s
- Number of operations (**setup**, **join**, **remove**): Q
- Time: T

$$\mathrm{Adv}^{\mathrm{ake}} \leq 2\, n\, Q \cdot \mathrm{Adv}^{\mathrm{gddh}}(s,T) + 2\, n \cdot \mathrm{Succ}^{\mathrm{sign}}(q_{\mathrm{send}},T)$$

- Idea:
  - Guess the last operation before the **test**ed key
  - Guess of the index of the player which makes the last down-flow

Details

- Given instance:

$$\begin{array}{lllll} g^{x_2} & g^{x_1} & & & \\ g^{x_2x_3} & g^{x_1x_3} & g^{x_1x_2} & & \\ g^{x_2x_3x_4} & g^{x_1x_3x_4} & g^{x_1x_2x_4} & g^{x_1x_2x_3} & g^{x_1x_2x_3x_4} \end{array}$$

- Use a new line for a new player, up to the s-1^{st}
  - For additional players: known random

  ⇒ known keys (**reveal**-queries)
  - Use the last line for the **test**ed group, introducing x_4 at the Q^{th} operation

  ⇒ **test**-query answered by the red value
  - After: back to s-1^{st} line, but **not** necessarily removing x_4

Details (Con'd)

- Extended instance:

g^{x2} g^{x1}

g^{x2x3} g^{x1x3} g^{x1x2} g^{x1x4} g^{x2x4} g^{x3x4}

g^{x2x3x4} g^{x1x3x4} g^{x1x2x4} g^{x1x2x3} $g^{x1x2x3x4}$

- In the s-1^{st} line: all the combinations of s-2 exponents
  - We remain on this line
  - We know the session key (in the s^{th} line)

Password-Based

[BCP – Eurocrypt '02]

- Generalization of the 2-party PAKE DH
- Encrypt each flow with password (in ICM)
  - Redundancy: dictionary attack
  - $\Rightarrow$ Randomization: $sk = H(g^{a_1 a_2 \ldots a_n x_1 x_2 \ldots x_n})$

g

a_1, x_1 g^{a1} g^{a1x1}

a_2, x_2 g^{a1a2x2} g^{a1a2x1} $g^{a1a2x1x2}$

a_3, x_3 $g^{a1a2a3x2x3}$ $g^{a1a2a3x1x3}$ $g^{a1a2a3x1x2}$ $g^{a1a2a3x1x2x3}$

a_4, x_4 $g^{a1a2a3a4x2x3x4}$ $g^{a1a2a3a4x1x3x4}$ $g^{a1a2a3a4x1x2x4}$ $g^{a1a2a3a4x1x2x3}$ $g^{a1a2a3a4x1x2x3x4}$