IPAKE
Isomorphisms for Password-based Authenticated Key Exchange

Dario Catalano **David Pointcheval**

CNRS-ENS – France

Thomas Pornin

Cryptolog – France

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Summary

- Password-based
  Authenticated Key Exchange
- EKE, OKE and a generalization
  Trapdoor Hard-to-Invert Isomorphisms
- Examples

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Authenticated Key Exchange

Two parties (Alice and Bob) agree on
a ***common*** secret key SK,
in order to establish a secret channel

- Basic security requirement:

  implicit authentication

  - *only* the intended partners can compute the session key

Authentication

To prevent active attacks, some kind of authentication of the flows is required:

- **Asymmetric**: (sk_A, pk_A) and possibly (sk_B, pk_B)
- **Symmetric**: common (high-entropy) secret
- **Password**: common (low-entropy) secret
  e.g. a 20-bit password

Password-based Authentication

Password (low-entropy secret) *e.g. 20 bits*

- exhaustive search is possible
- basic attack: **on-line exhaustive search**
  - the adversary guesses a password
  - tries to play the protocol with this guess
  - failure ⇒ it erases the password from the list
  - and restarts…
- after 1,000,000 attempts, the adversary wins

cannot be avoided

We want it to be the **best attack**…

Dictionary Attack

- **Off-line exhaustive search**
  - a few passive or active attacks
  - failure/transcript ⇒ erasure of **MANY** passwords from the list: this is called **dictionary attack**
- To prevent them:
  - a passive eavesdropping
    - no *useful* information about the password
  - an active trial
    - cancels *at most one* password

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Encrypted Key Exchange

Bellovin-Merritt

Alice	Password π	Bob
sk, pk	Alice, $pk' = \mathbf{ES}_\pi(pk)$ $\longrightarrow$	$pk = \mathbf{DS}_\pi(pk')$
$c=\mathbf{DS}_\pi(c'), r=\mathbf{DA}_{sk}(c)$	$\longleftarrow$ Bob, $c' = \mathbf{ES}_\pi(c)$	$r \in M_{pk}, c=\mathbf{EA}_{pk}(r)$
	$SK=H(\text{Alice, Bob}, pk, c', r)$	

- Problems:
  - Encoding of pk not often uniformly distributed in the **ES** plaintext space
  - pk and c are rarely on the same space
  - Nice exception: ElGamal (DH-EKE) on $<g>$
  - $\Rightarrow$ Many security analyses in the ROM, ICM, ...

Open Key Exchange

Lucks

- The public key pk is sent in **clear**:

Alice	Password π	Bob
sk, pk	Alice, pk $\longrightarrow$	
$c=\mathbf{DS}_\pi(c'), r=\mathbf{DA}_{sk}(c)$	$\longleftarrow$ Bob, $c' = \mathbf{ES}_\pi(c)$	$r \in M_{pk}, c=\mathbf{EA}_{pk}(r)$
$k=H'(\text{Alice,Bob},r)$	k $\longrightarrow$	k correct ?
	$SK=H(\text{Alice, Bob}, pk, c', r)$	

- Requirements to avoid partition attacks:
  - $\mathbf{ES}_\pi$ must be a cipher from the **ciphertext space under** $\boldsymbol{pk}$
  - $\mathbf{EA}_{pk}$ must be a **surjection**

Surjection: Necessary

- If not, given c', one eliminates the π's that lead to a c which is not in the image set of $\mathbf{EA}_{pk}$: *partition attack*
- If yes, given c', any π is possible: sending the correct k means *guessing the good* π

Alice	Password π	Bob
sk, pk	Alice, pk $\longrightarrow$	
$c=\mathbf{DS}_\pi(c'), r=\mathbf{DA}_{sk}(c)$	$\longleftarrow$ Bob, $c' = \mathbf{ES}_\pi(c)$	$r \in M_{pk}, c=\mathbf{EA}_{pk}(r)$
$k=H'(\text{Alice,Bob},r)$	k $\longrightarrow$	k correct ?
	$SK=H(\text{Alice, Bob}, pk, c', r)$	

Efficient Implementation

- Using the **one-time pad**, and **bijections**

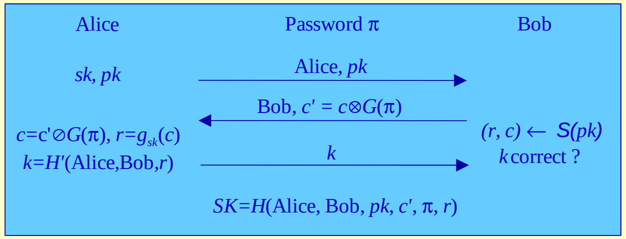
$$\mathbf{EA}_{pk} = f_{pk} \text{ and } \mathbf{DA}_{sk} = g_{sk} = f_{pk}^{-1}$$

Alice	Password π	Bob
sk, pk	Alice, pk $\longrightarrow$	
$c=c' \oslash G(\pi), r=g_{sk}(c)$	$\longleftarrow$ Bob, $c' = c \otimes G(\pi)$	$r \in M_{pk}, c=f_{pk}(r)$
$k=H'(\text{Alice,Bob},r)$	k $\longrightarrow$	k correct ?
	$SK=H(\text{Alice, Bob}, pk, c', \pi, r)$	

- f_{pk} must be a ***bijection*** onto a group $(G_{pk}, \otimes)$
- f_{pk} must be "***hard-to-invert***"
- G must be a random function (***RO***) onto G_{pk}

Efficiently Samplable

- f_{pk} must be *trapdoor* “hard-to-invert”,
  not necessarily “one-way”: but just ***samplable***
  - $(r, c) \leftarrow S(pk)$ such that r random in M_{pk} and $c = f_{pk}(r)$



- pk must be easy to generate
- f_{pk} must be a bijection ⇒ ***to be checked***

Hard-to-Invert: not Enough?

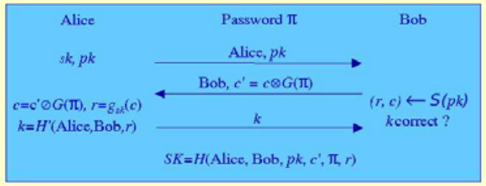
When pk is chosen by Alice

sk is unknown to the adversary

- the adversary can know only one pre-image r (for the guessed password π)
- for other π's, the “hard-to-invert” property prevents from extracting/checking other r values

This is the intuition... For the formal proof

- Hard-to-invert
- Bijection
- **Morphism**



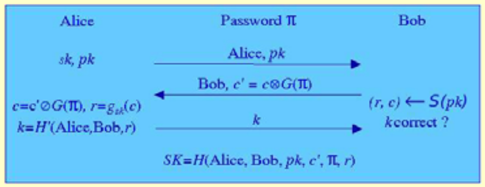
Morphism: for the Proof

For checking a password, one uses k or SK

⇒ one must compute r (appears in H-H' queries)

Passive: ≤1

- Either c' sent by Bob: from any correct (π,r) such that $c' = f_{pk}(r) \otimes G(\pi)$, one can invert f_{pk}
  - by simulating $c' = f_{pk}(a)$ for a known a
  - by embedding the challenge y in $G(\pi)$



$$y = c' \oslash f_{pk}(a) = f_{pk}(r) \oslash f_{pk}(a) = f_{pk}(r\text{-}a)$$

Active: ≤2

- Or by the adversary: from two correct pairs (π,r)

Trapdoor Hard-to-Invert Isomorphisms Family

$F = (f_{pk})_{pk}$ trapdoor hard-to-invert isomorphisms

- $(pk, sk) \leftarrow G(1^k)$: generation
  - f_{pk} is an isomorphism from M_{pk} onto G_{pk}
- $(r, c) \leftarrow S(pk)$: sample
  - such that r random in M_{pk} and $c = f_{pk}(r)$ (random in G_{pk})
- Given y and pk, check whether $y \in f_{pk}(M_{pk}) = G_{pk}$
- Given y and sk, easy to invert f_{pk} on y
- Without sk, hard to invert f_{pk}

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Candidates

Diffie-Hellman: $sk = x, pk = g^x$

$$f_{pk}(g^a) = g^{ax} = pk^a \qquad g_{sk}(b) = b^{1/x}$$

f_{pk} is not one-way, but hard-to-invert
under the ***CDH assumption***

⇒ classical DH-AKE variants (PAK or AuthA)

RSA: $sk = d, pk = (n,e)$

f_{pk} is one-way under the ***RSA assumption***,
but pk must contain a valid RSA key: NIZK proof

⇒ variant of "protected OKE"

Candidates (Cont'd)

Square root: $sk = (p,q), pk = n$

f_{pk} is an automorphism onto QR_n,
but for specific moduli only (Blum moduli)

⇒ to be checked: can be done (verified) efficiently

f_{pk} is one-way under
the ***integer factoring problem***

⇒ the first *Password-Based Authenticated Key Exchange* based on **factoring**