

# 8th ACM Conference on Computer and Communications Security ' 2001

5-8 November 2001

Philadelphia - Pennsylvania - USA

## *Twin Signatures: an Alternative to the Hash-and-Sign Paradigm*

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## Overview

- ◆ Introduction
- ◆ Security notions for signatures
- ◆ The twinning paradigm
- ◆ A DL-based example
- ◆ An RSA-based example
- ◆ Conclusion

# Introduction

- ◆ Digital signature = electronic version of handwritten signatures
- ⇒ authenticates the sender of a message
  - the receiver knows the identity of the sender
  - the sender cannot deny later having sent the message (non-repudiation)

## Digital signatures

Defined by two algorithms

- ◆ the signing algorithm **S**:  
private key + message  $m$   
→ signature  $\sigma$
- ◆ the verification algorithm **V**:  
public key + message  $m$   
+ alleged signature  $\sigma$   
→ agrees or not

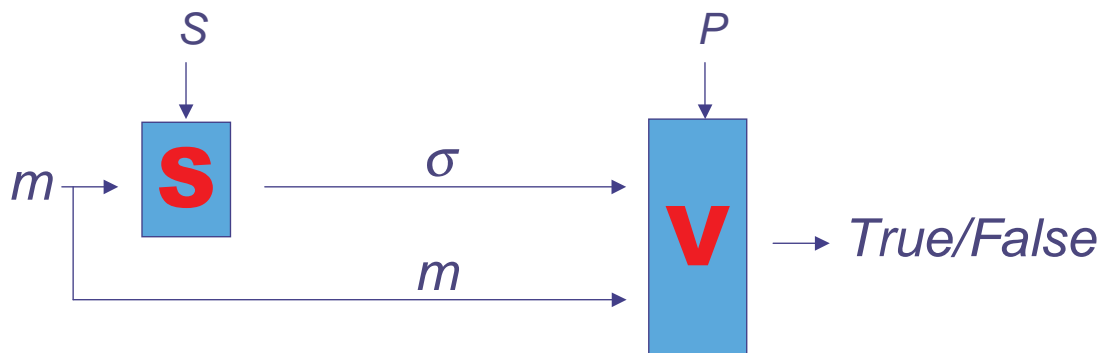
# Digital signatures

Signing algorithm **S**

Verification algorithm **V**

Private key  $S$

Public key  $P$



Security: it is impossible to produce a new valid pair  $(m, \sigma)$

## Security notions

More precisely, one considers

- ◆ total break:  
the adversary recovers the private key
- ◆ universal forgery:  
the adversary can sign any message of her choice
- ◆ existential forgery:  
the adversary can produce accepted message/signature pairs

# Adversaries

The information available to the adversary may be various, thus several attacks

- ◆ no-message attacks:  
the adversary just knows the verification algorithm (*i.e.* the public key)
- ◆ known-message attacks:  
she knows some message-signature pairs
- ◆ (adaptively) chosen-message attacks:  
she has access to a signing oracle

# Secure signature schemes

For achieving non-repudiation, the scheme must prevent existential forgeries.

Furthermore, signatures are aimed to be published, thus known-message attacks should be withstood.

## ***Secure signature scheme:***

no existential forgery even against adaptively chosen-message attacks.

# Example: RSA signature

$n = pq$  product of large primes

$e$  : **public** exponent

$d = e^{-1} \bmod \varphi(n)$  : **private** exponent

Signature of the message  $m \in \mathbf{Z}_n$

$$\sigma = m^d \bmod n$$

Verification of  $(m, \sigma)$

test whether  $m = \sigma^e \bmod n$

## RSA signature: problems

◆ Only small messages (in  $\mathbf{Z}_n$ ) can be signed

◆ Existentially forgeable

⇒ in order to solve the former problem:

use of a collision-resistant hash function  $h$

If  $h$  furthermore behaves like a truly random

function  $\{0,1\}^* \rightarrow \mathbf{Z}_n$  : FDH in the ROM

FDH-RSA, provably secure [BR96, Co00]

⇒ hash-and-sign or hash-and-decrypt

# An alternative: twinning

- ◆ Without the hash function, the RSA signature is insecure
  - even with it, the security proof only holds in the random oracle model

Insecure? Because from  $\sigma$  it is easy to compute  $m$  such that  $m = \sigma^e \bmod n$

What about considering twin-signatures  $(\sigma, \tau)$  such that  $m = \sigma^e \bmod n$  and  $m+1 = \tau^e \bmod n$  ?

## Twin signatures

- ◆ Let **S** be a signature scheme (maybe weakly secure)
- ◆ We consider the signature scheme which consists in computing
  - $m_1 = f(m, r)$  and  $m_2 = g(m, r)$  for some random  $r$
  - $\sigma_1 = \mathbf{S}(m_1)$  and  $\sigma = \mathbf{S}(m_2)$
- ◆ We thus sign two related messages

# A DL-based example: DSA

**G** =  $\langle g \rangle$  of prime order  $q$

$x$  : **secret** key     $y = g^x$  : **public** key

◆ For signing  $m \in \mathbf{Z}_q$ ,  $\mathbf{S}_x(m) = (c, d)$ , where

$$0 < u < q \quad c = (g^u) \bmod q \quad c \neq 0$$

$$\text{and } d = (m + x c) / u \bmod q \quad d \neq 0$$

◆ Verification,  $\mathbf{V}_y(m, c, d)$  :

$$h = 1/d \bmod q, \quad h_1 = h m \bmod q,$$

$$h_2 = h c \bmod q, \quad c' = g^{h_1} y^{h_2}$$

check whether  $0 < c, d < q$  and  $c = c' \bmod q$

## Twin-DSA

$$\text{DSA}_x(m) = \mathbf{S}_x(\text{SHA}(m))$$

◆ Unfortunately, no security result, even in the random oracle model, or the generic model.

$$\text{Twin-DSA}_x(m) = ((c, d), (c', d')),$$

where  $(c, d)$  and  $(c', d')$  are two distinct signatures of  $m$  (with different random  $u, u'$ )

**Twin-DSA is secure in the generic model**

# An RSA-based example: GHR

$n = pq$  product of large primes  
 $y \in \mathbf{Z}_n$ : **public** element

- ◆ For signing  $e$ ,  $\mathbf{S}_{p,q}(e) = s$ , where  
 $d = e^{-1} \bmod \varphi(n)$ ,  $s = y^d \bmod n$
- ◆ Verification,  $\mathbf{V}_y(e,s) : s^e = y \bmod n$
- ◆ EuroC' 99:  $\mathbf{GHR}_{p,q}(m) = \mathbf{S}_{p,q}(h(m))$   
if  $h$  is divisible-intractable + chameleon  
 $\Rightarrow$  no existential forgeries against  
adaptive chosen-message attacks

## Twin-GHR

- ◆ The chameleon property of  $h$  is required for simulating the signing oracle  
 $\Rightarrow$  without it, no security against chosen-message attacks
- ◆  $\mathbf{Twin-GHR}_{p,q}(m,a||b) = (\mathbf{S}_{p,q}(e_1), \mathbf{S}_{p,q}(e_2))$   
for  $e_i = h(m_i)$  where  
 $m_1 = (m \oplus a) || (m \oplus b)$  and  $m_2 = a || b$
- ◆ Verification: get  $m_1$  and  $m_2$ , and  $M = m_1 \oplus m_2$ ,  
check the redundancy  $M = m || m$ , output  $m$



# Twin-GHR: Security

The twinning replaces

the chameleon property:

if  $h$  simply achieves divisible-intractability  
(or injection in the primes)

Twin-GHR prevents existential forgeries even  
against adaptive chosen-message attacks

- ◆ no generic model
- ◆ no random oracle
- ◆ just the flexible RSA problem.

## Conclusion

Twinning is a new paradigm to

- ◆ prevent existential forgeries (*cf.* DSA)  
it may replace the random oracle  
model in some situations
- ◆ achieves security against adaptive  
chosen-message attacks (*cf.* GHR)  
it may replace chameleon hash function  
or the random oracle model
- ◆ this new direction should be  
more investigated.