A Simpler Variant of Universally Composable Security for Standard Multi Party Computation

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Context

Protocol

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Proof of security

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Adversary model
$\rightarrow$ who?
$\rightarrow$ capabilities?
$\rightarrow$ goals?

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→ Find-then-Guess
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Simulation
→ Classical Simulation
→ Universal Composability

Definition

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Ensure that an **environment** $\mathcal{Z}$ can't distinguish between both worlds

Definition

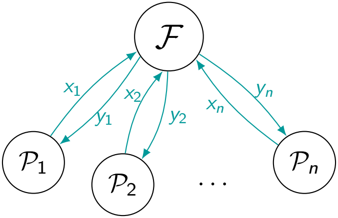

Figure 1: Ideal World

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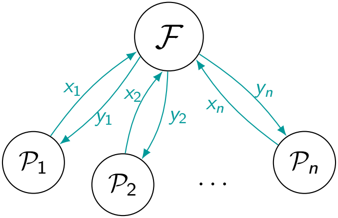

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Construction of UC protocols:

- Define the ideal Functionality $\mathcal{F}$
- Construct a protocol Π that realises $\mathcal{F}$
- Make the proof: construct a simulator $\mathcal{S}$

Interest 1: $\mathcal{A}$ can choose a distribution for the inputs

In the UC model, no description of:

- what are the possible actions of the adversary
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$\Rightarrow$ Model attacks where the **inputs are not uniform**

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Example 2:

UC-secure authenticated key exchange + secure symmetric encryption
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$\Rightarrow$ Because of these 2 points, the **UC model is more secure** than the Find-then-Guess or Real-or-Random models

Difficulty to define the ideal functionality

Ideal Functionality for Secure Message Transfer

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$\mathcal{F}^{l}_{\mathsf{STM}}$ proceeds as follows:
parameterized by leakage function $l : \{0,1\}^\star \to \{0,1\}^\star$,

Upon receiving an input $(\mathsf{Send}, \mathsf{sid}, m)$ from S, verify that $\mathsf{sid} = (S, R, \mathsf{sid}')$ for some R, else ignore the input. Next, send $(\mathsf{Sent}, \mathsf{sid}, l(m), m)$ to R.

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For example: leaking $l(m) = \mathsf{length}(m)$ is important because no cryptosystem can fully hide the size of the information being encrypted

Difficulties in proofs

In UC model, proofs more complex than in game based security:

- no rewind, need extractable inputs $\Rightarrow$ protocol more complex
- no end when the adversary wins $\Rightarrow$ proofs more complex

Communication model and rules

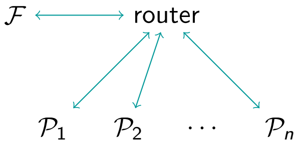

Figure 2: SUC communication model

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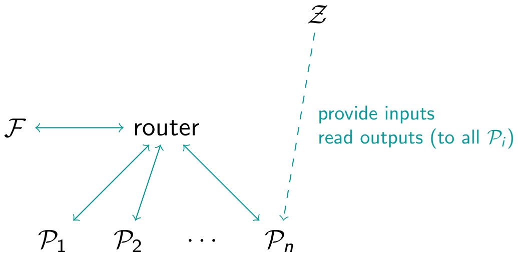

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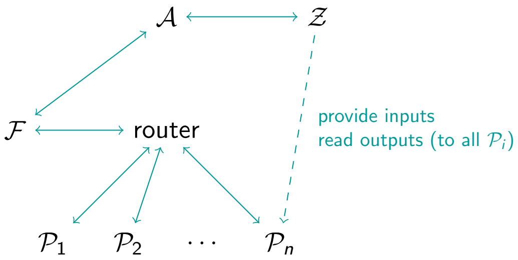

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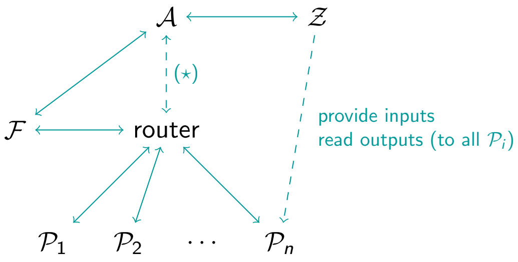

Figure 2: SUC communication model

$(\star)$ Router sends all messages to $\mathcal{A}$ and delivers them when instructed by $\mathcal{A}$

- Messages are of the format (sender,receiver;content)
- Router only sends public header of messages to and from $\mathcal{F}$ to $\mathcal{A}$ (so $\mathcal{A}$ does not see the private content)
- $\mathcal{A}$ notifies the router when to deliver messages but has no influence beyond that

π SUC-securely computes $\mathcal{F}$

Definition

Let π be a protocol for up to m parties and let $\mathcal{F}$ be an ideal functionality.

We say that π **SUC-securely computes** $\mathcal{F}$ if for every PPT real model adversary $\mathcal{A}$ there exists a PPT ideal-model adversary $\mathcal{S}$ such that for every PPT balanced environment $\mathcal{Z}$ and every constant $d \in \mathbb{N}$, there exists a negligible function $\mu(\cdot)$ such that for every $n \in \mathbb{N}$ and every $z \in \{0,1\}^\star$ of length at most n^d,

$$|\Pr[\text{SUC-IDEAL}_{\mathcal{F},\mathcal{S},\mathcal{Z}}(n,z) = 1] - \Pr[\text{SUC-REAL}_{\pi,\mathcal{A},\mathcal{Z}}(n,z) = 1]| \leqslant \mu(n)$$

SUC composition theorem

Theorem

Let π be a protocol for the $\mathcal{F}$-hybrid model.
Let ρ be a protocol that SUC-securely computes $\mathcal{F}$ in the $\mathcal{G}$-hybrid model.

Then, for every PPT real model adversary $\mathcal{A}$ there exists a PPT ideal-model adversary $\mathcal{S}$ such that for every PPT environment $\mathcal{Z}$ there exists a negligible function $\mu(\cdot)$ such that for every $z \in \{0,1\}^\star$ and every $n \in \mathbb{N}$,

$$\left|\Pr[\text{SUC-HYBRID}^{\mathcal{G}}_{\pi^\rho,\mathcal{S},\mathcal{Z}}(n,z)=1] - \Pr[\text{SUC-HYBRID}^{\mathcal{F}}_{\pi,\mathcal{A},\mathcal{Z}}(n,z)=1]\right| \leqslant \mu(n)$$

SUC composition theorem

Corollary

Let π be a protocol that SUC-securely computes a functionality $\mathcal{H}$ in the $\mathcal{F}$-hybrid model. If protocol ρ SUC-securely computes $\mathcal{F}$ in the $\mathcal{G}$-hybrid (resp. real) model, then π^ρ SUC-securely computes $\mathcal{H}$ in the $\mathcal{G}$-hybrid (resp. real) model.

By a drawing:

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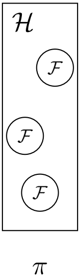

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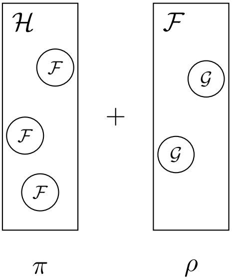

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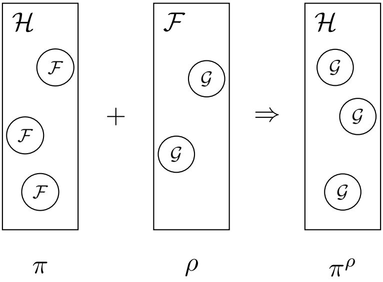

Bonus: Differences SUC - UC

In SUC, more rigid network model:

- build-in authenticated channel
- no subroutines
- set of parties a priori fixed

$\Rightarrow$ No digital signatures in SUC because no a priori polynomial bound on the number of interactions ($=$ number of signatures)

Conclusion

UC: Security model based on simulation to obtain Composition Theorem

Composition Theorem: If a protocol is UC secure then it is secure for concurrent executions

SUC: Simpler formalism for some protocols such that SUC-secure $\Rightarrow$ UC secure

$\Rightarrow$ Simpler proofs without loss of security guarantees

References

- CCL15 - A Simpler Variant of UC Security for Standard Multiparty Computation
- Che09 - Etude de protocoles cryptographiques à base de mots de passe
- Can01 - Universally Composable Security: A New Paradigm for Cryptographic Protocols