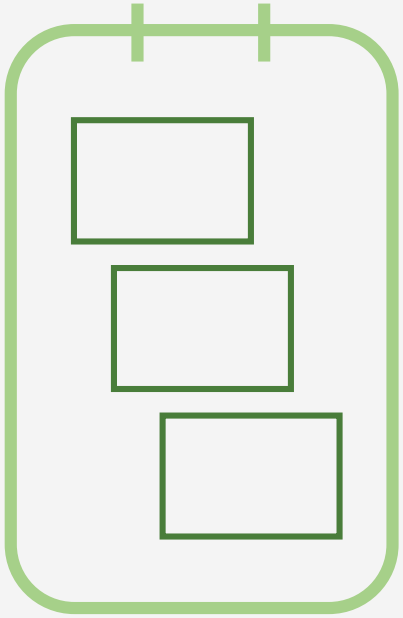
Linearly-Homomorphic Signatures and Scalable Mix-Nets

Chloé Hébant, Duong Hieu Phan and David Pointcheval

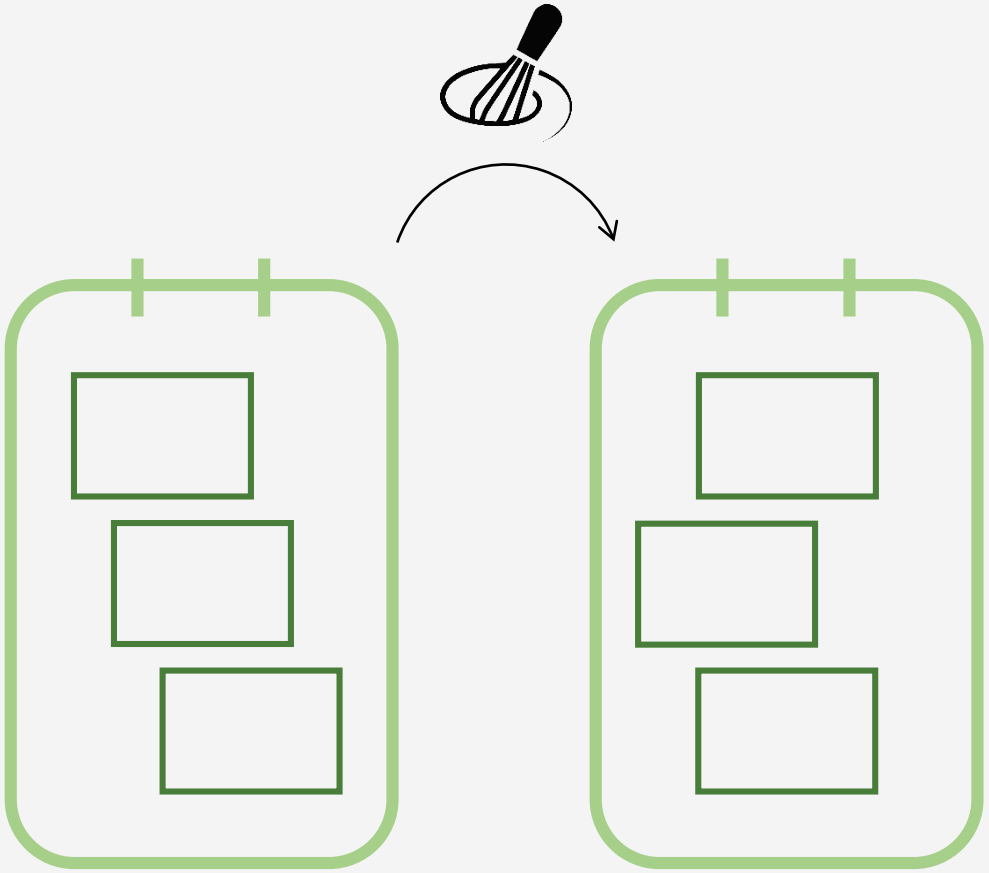
Outline

1. Mix-Nets in drawings
2. Building blocks
3. Spirit of our scheme
4. Difficulties

Mix-Net



Mix-Net



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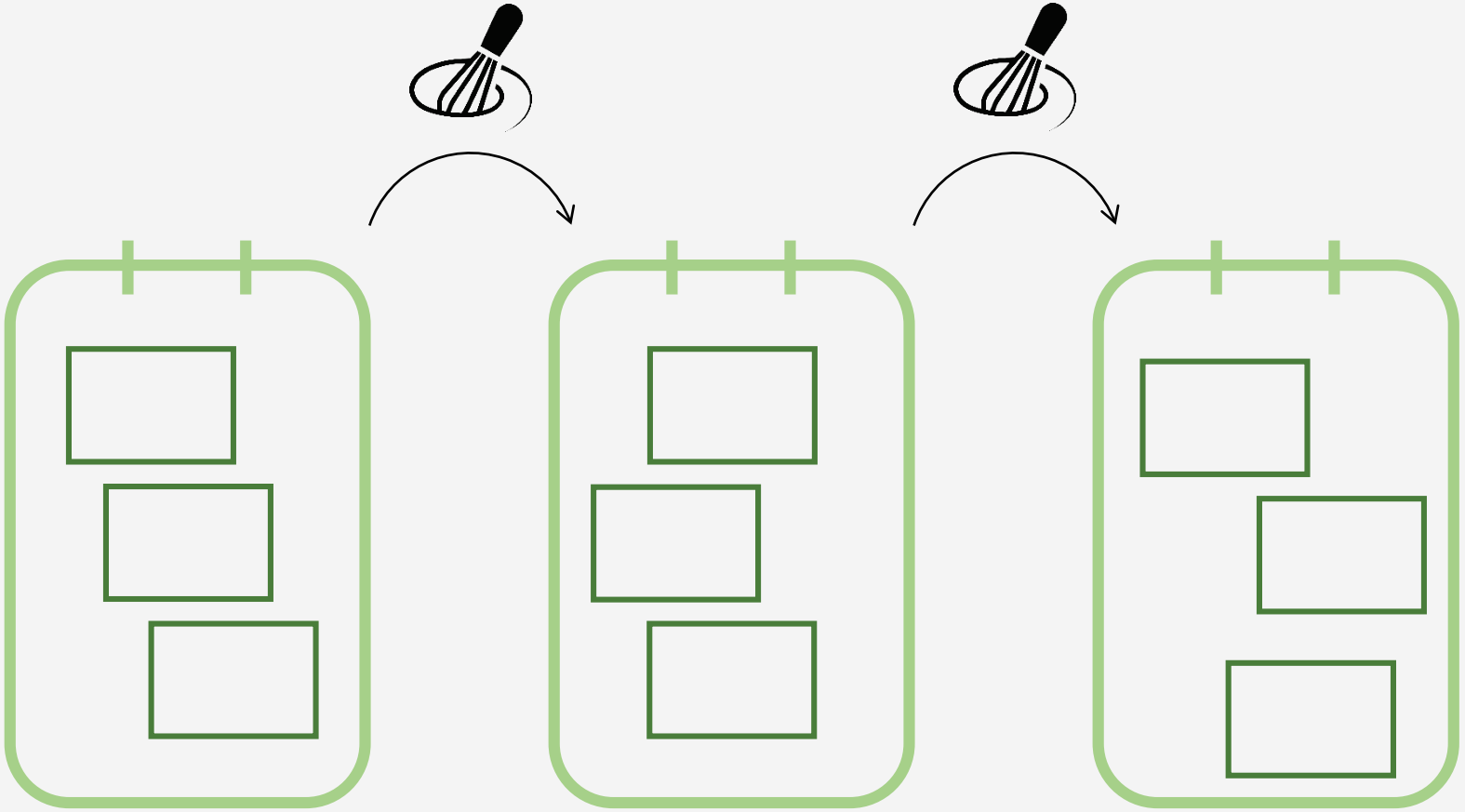
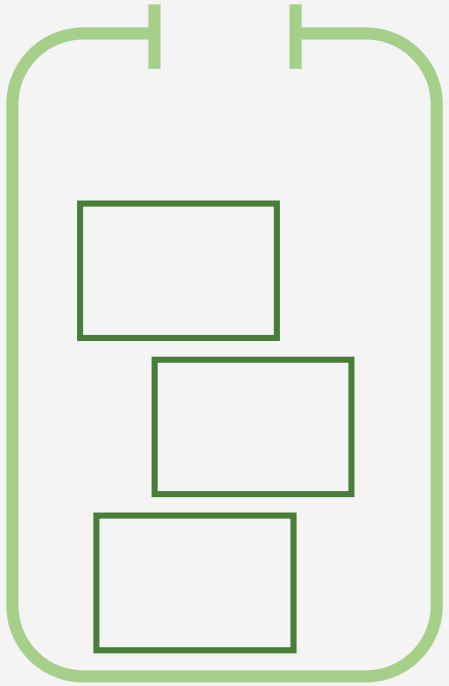


Electronic Voting



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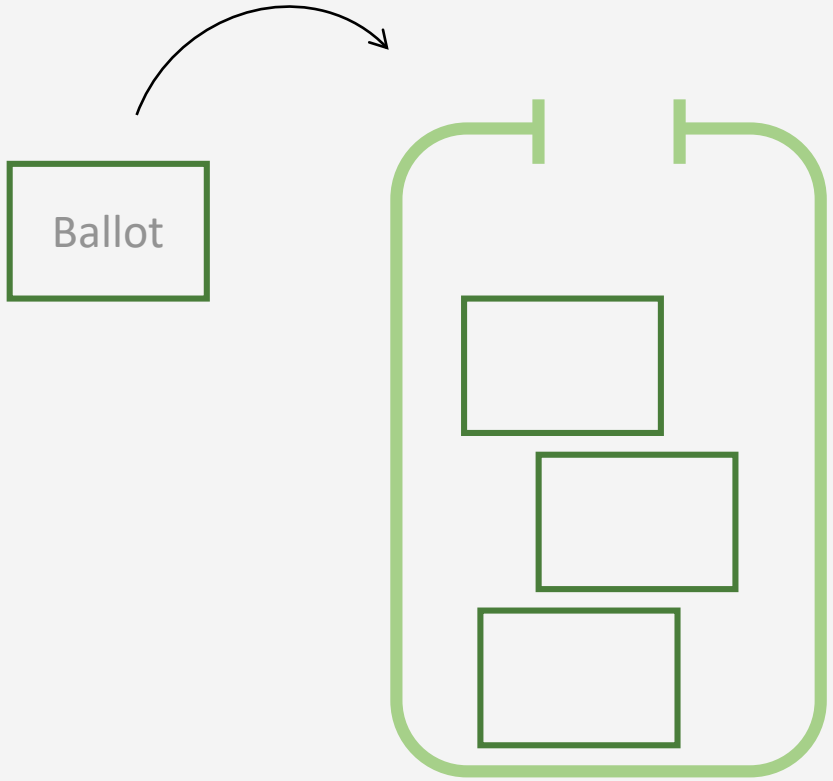

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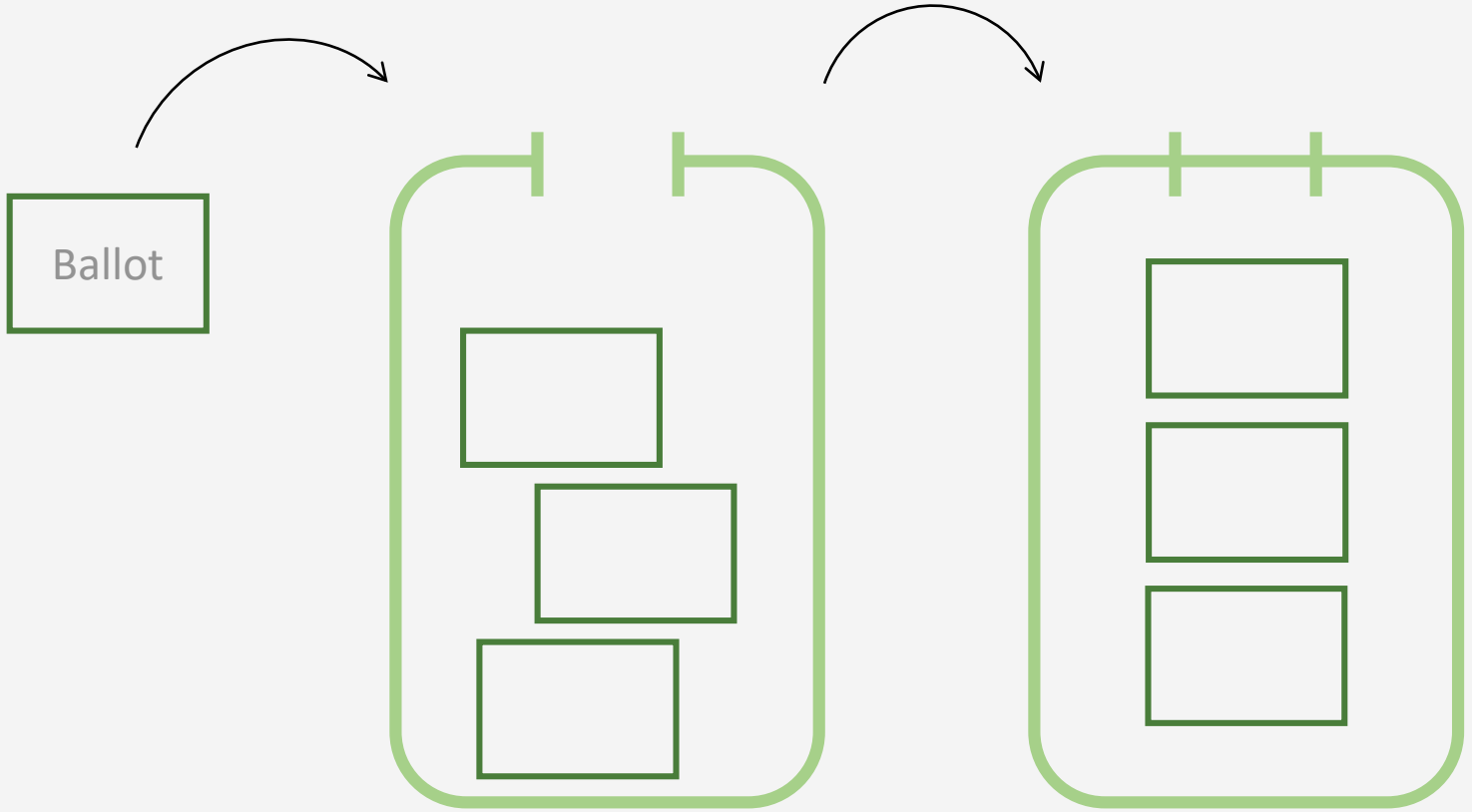

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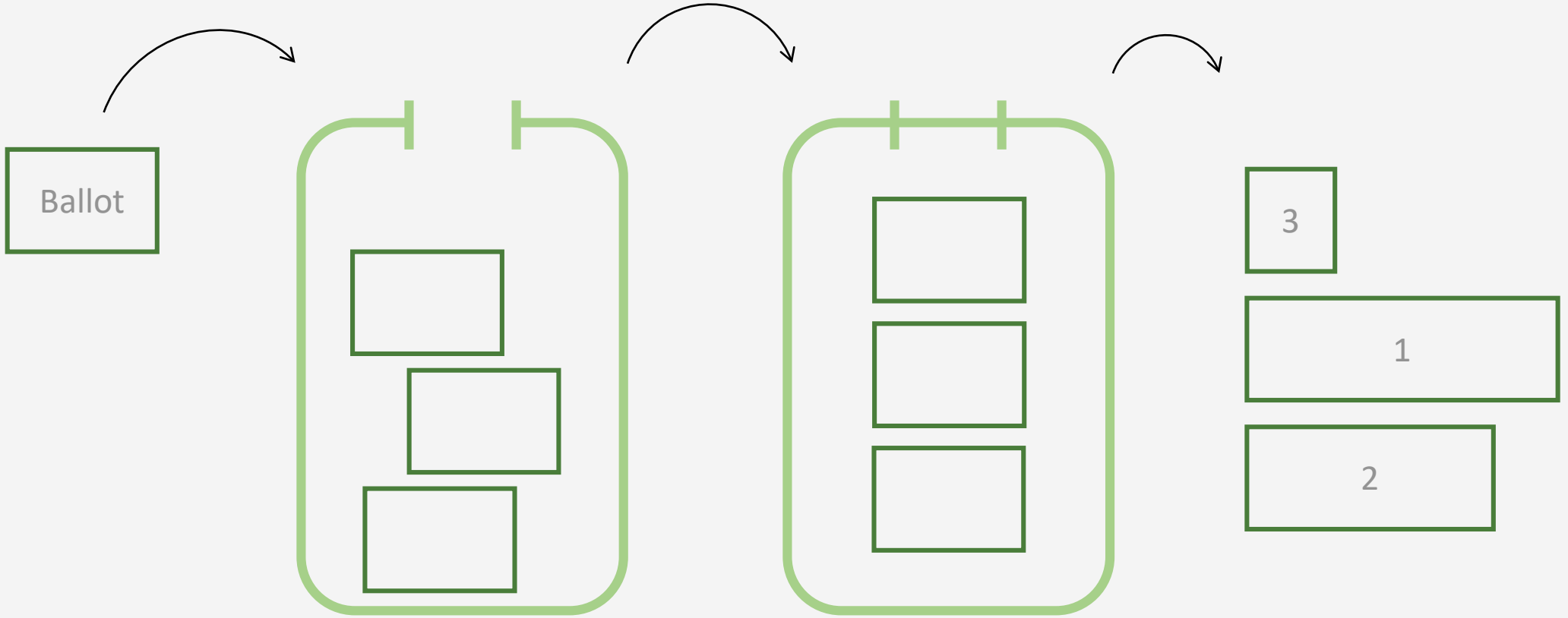

Some Guarantees

Authorized voters

Ballot

Unforgeability

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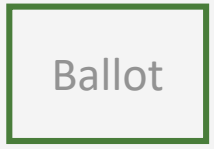
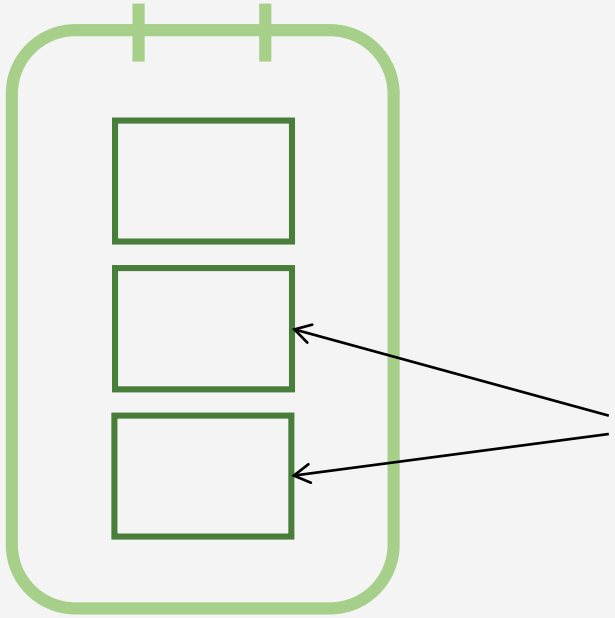
Authorized voters



Unforgeability



Indistinguishable

Some Guarantees

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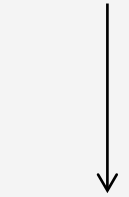

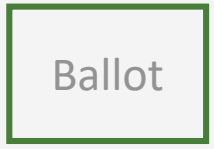

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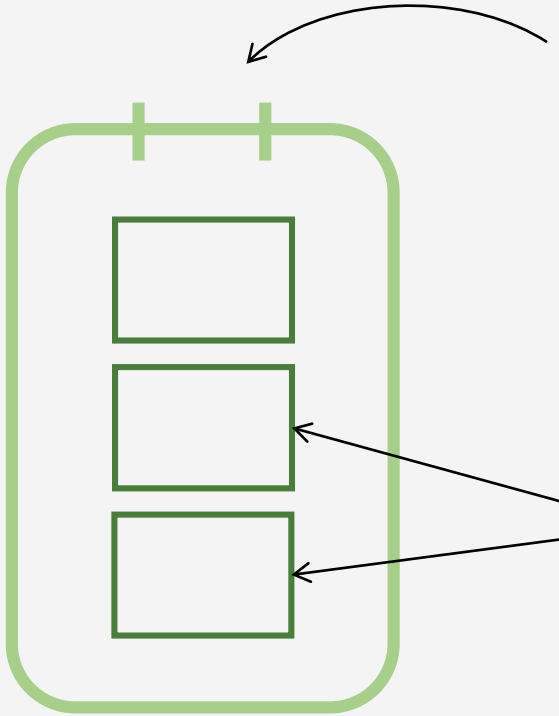

Sealed

- No added ballot
- No deleted ballot
- No modified ballot

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Some Guarantees

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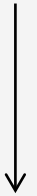

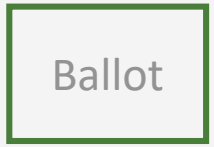

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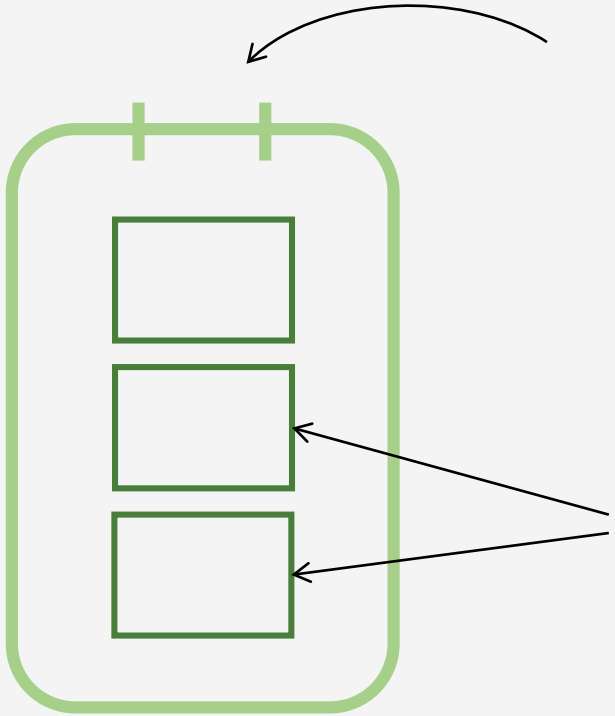

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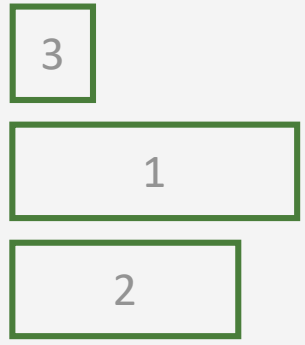
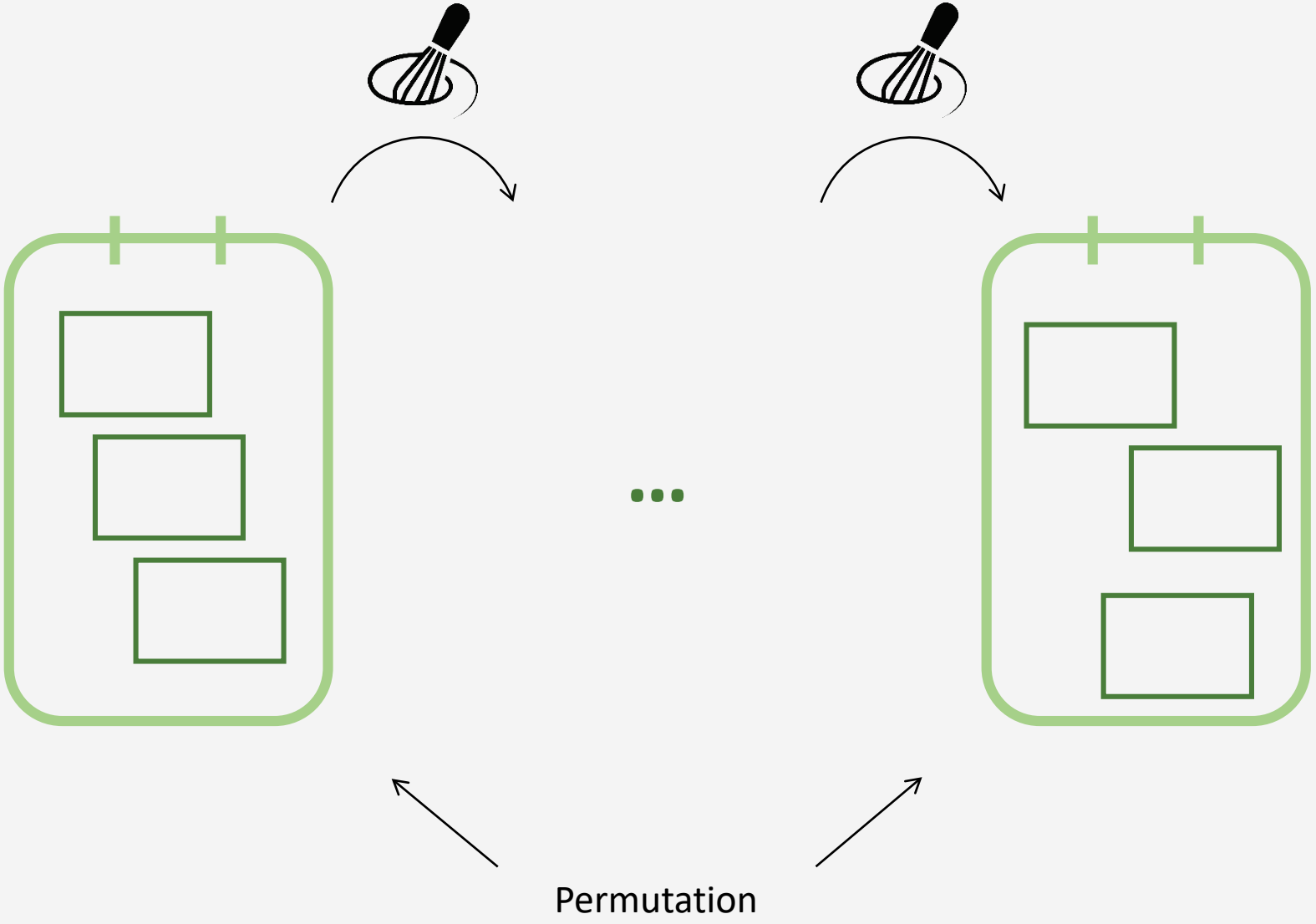
Publicly verifiable



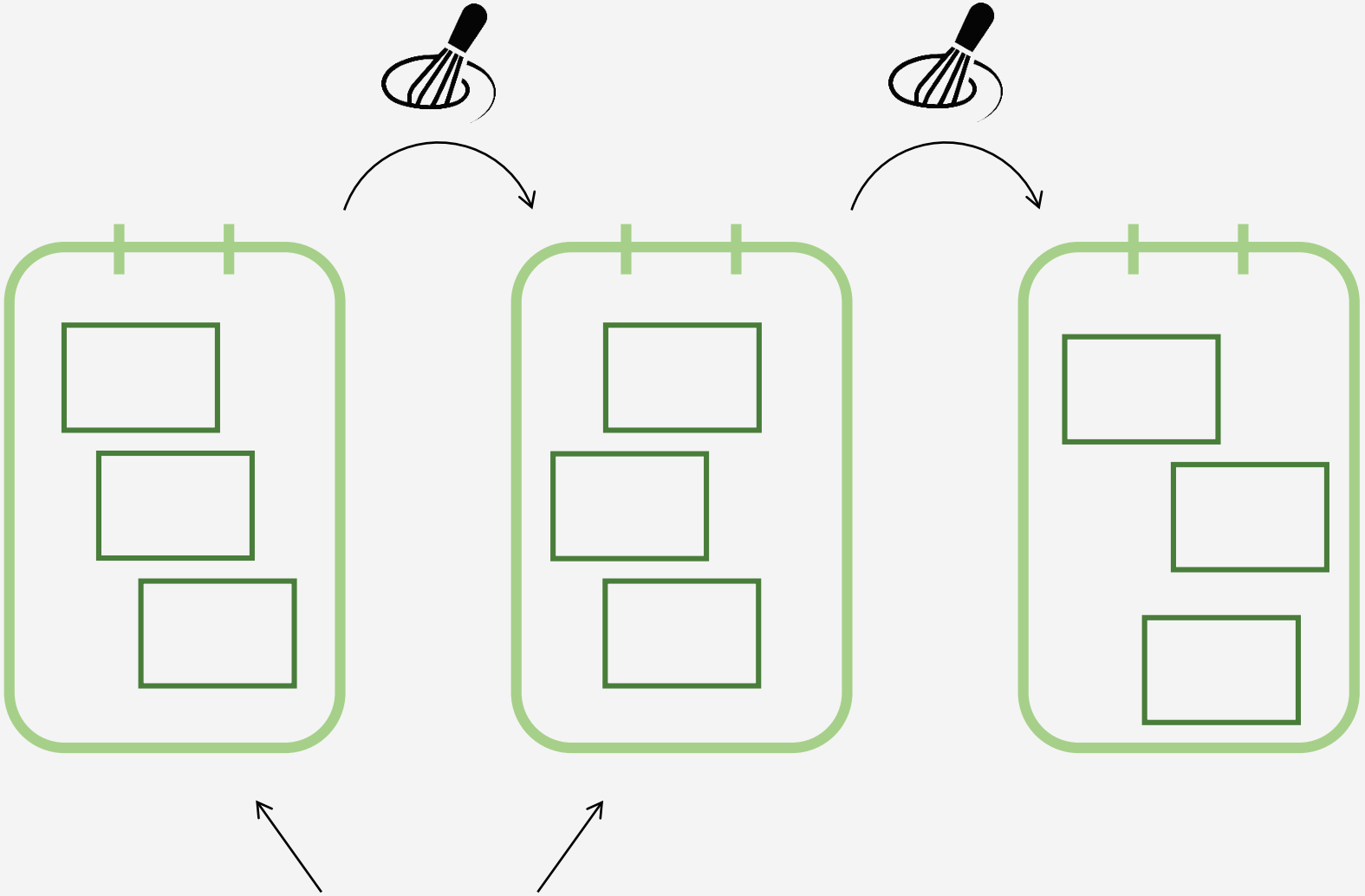
Mix-Net: Security Notions

1. Soundness
2. Unlinkability

Mix-Net: Soundness



Mix-Net: Unlinkability



Building Blocks

Key Ingredients

- Ciphertext randomization
- “Signature randomization”

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Signature

$\mathsf{Setup}(1^\kappa) \to \mathsf{param}$

$\mathsf{Keygen}(\mathsf{param}, n) \to (\mathsf{sk}, \mathsf{vk})$

$\mathsf{Sign}(\mathsf{sk}, \quad \vec{M} = (M_i)_i \in \mathbb{G}^n) \to \sigma$

$\mathsf{Verif}(\mathsf{vk}, \quad \vec{M}, \sigma) \to 1 \text{ if } \sigma \text{ valid relative to } \mathsf{vk}, 0 \text{ otherwise}$

One-Time Linearly Homomorphic Signature (OT-LH-Sig)

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$\mathsf{RandTag}(\mathsf{vk}, \tau, m, \sigma)$: $(m, \sigma', \mathsf{vk}, \tau')$

Summary

- Randomizable Ciphertexts: ElGamal
- Linearly Homomorphic Signatures
  - 3 properties:
    - Message Homomorphism
    - Key Homomorphism
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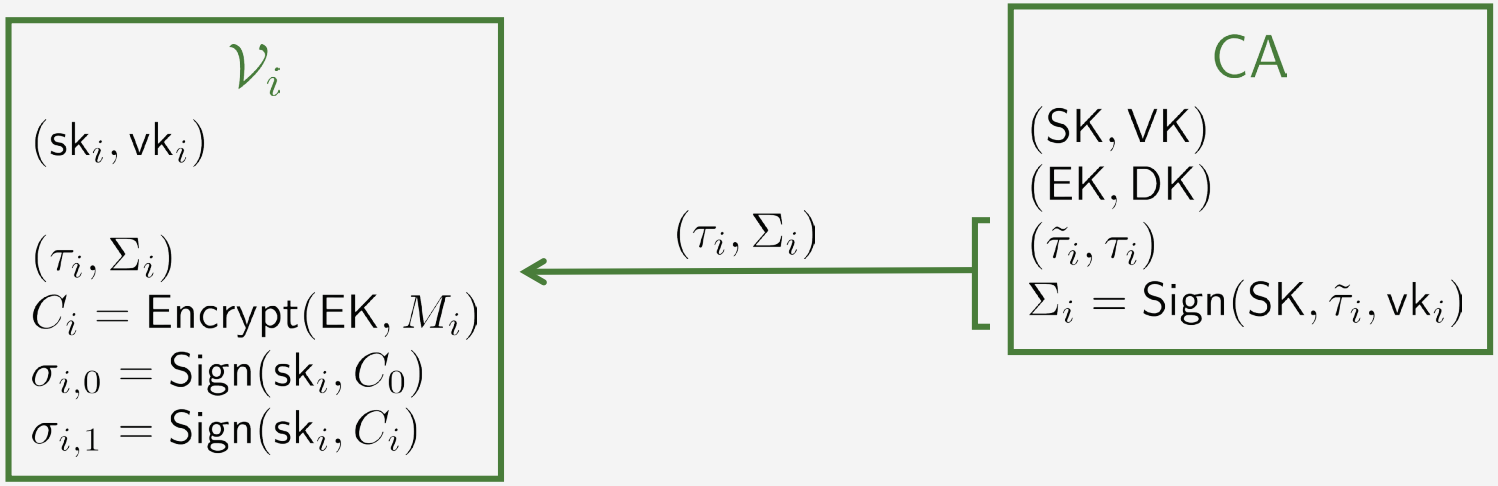
- Randomizable Ciphertexts: ElGamal
- Linearly Homomorphic Signatures
  - 3 properties:
    - Message Homomorphism
    - Key Homomorphism
    - Tag Randomizability
  - 2 schemes:
    - One-Time Linearly Homomorphic Signature (Keygen,Sign,Verif,…)
    - Linearly Homomorphic Signature (Keygen*,Sign*,Verif*,…)

Spirit of our Mix-Net

Mix-Nets

$C_0 = \mathsf{Encrypt}(\mathsf{EK}, 1)$



$$\mathcal{BB}\mathsf{ox}^{(0)} = (C_i, \sigma_{i,0}, \sigma_{i,1}, \mathsf{vk}_i, \Sigma_i, \tau_i)_i$$

Mix-Nets

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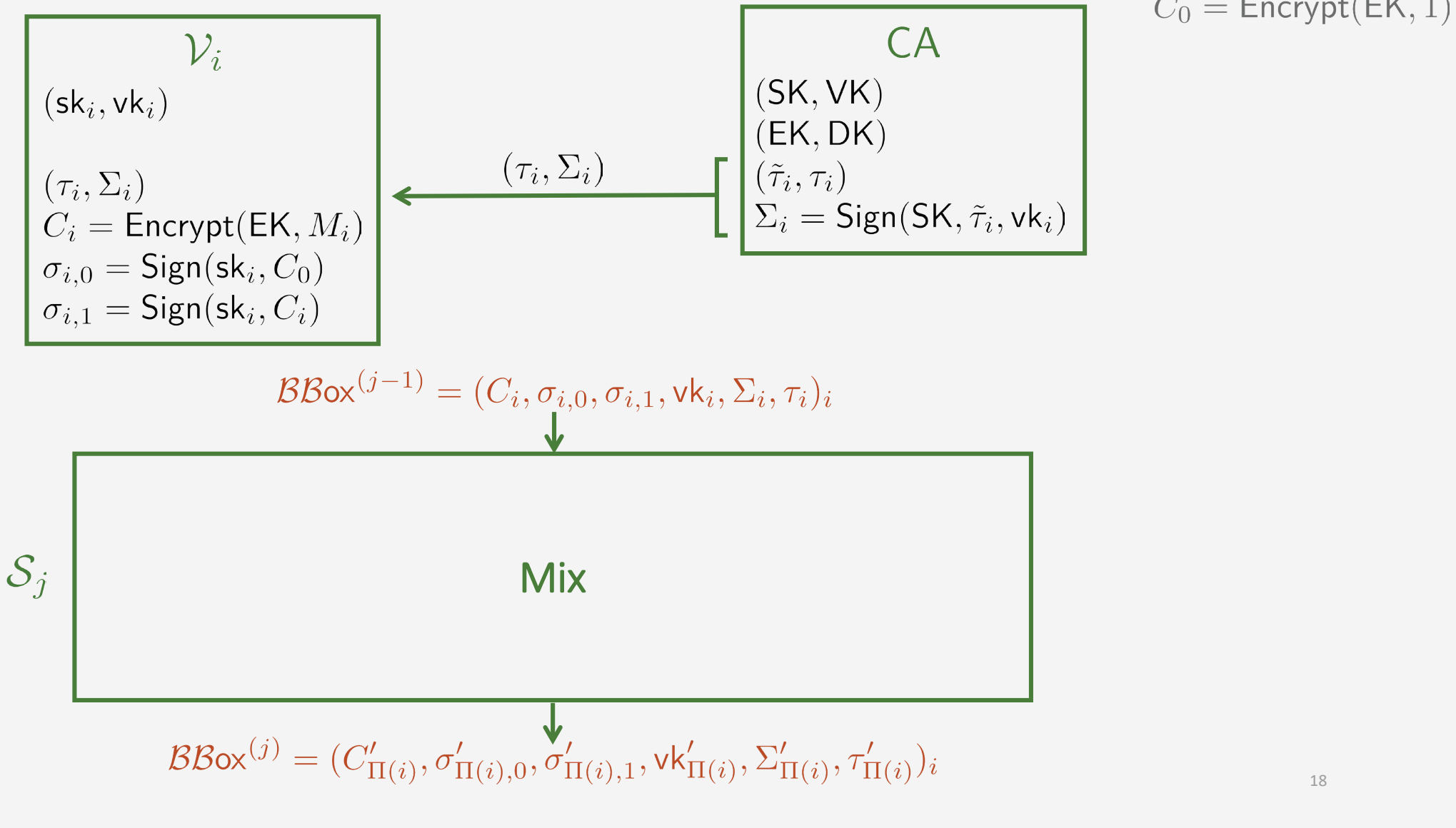

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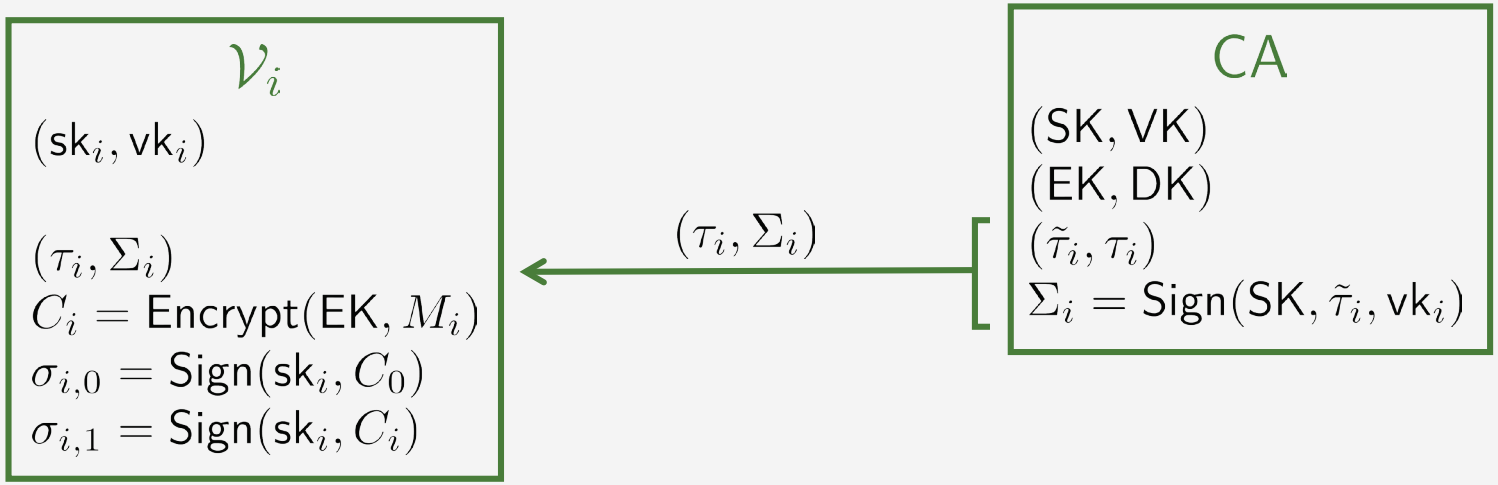

$\mathcal{BB}\mathsf{ox}^{(j-1)} = (C_i, \sigma_{i,0}, \sigma_{i,1}, \mathsf{vk}_i, \Sigma_i, \tau_i)_i$

$\mathcal{S}_j$

1. Rand C_i (and adapt $\sigma_{i,1}$) — Message Hom.
2. Rand vk_i (and adapt Σ_i)
   Rand τ_i (which change Σ_i) — Message Hom. Tag Randomizability
3. Because vk_i has changed, adapt $\sigma_{i,1}$ again and $\sigma_{i,0}$ — Key Hom.

$\mathcal{BB}\mathsf{ox}^{(j)} = (C'_{\Pi(i)}, \sigma'_{\Pi(i),0}, \sigma'_{\Pi(i),1}, \mathsf{vk}'_{\Pi(i)}, \Sigma'_{\Pi(i)}, \tau'_{\Pi(i)})_i$

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- Non-trivial transformation
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- Multiple servers

Aggregation

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- Groth-Sahai proofs 2008

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- Multi-signatures of Boneh-Drijvers-Neven 2018

Conclusion

We saw:

- (One-Time) Linearly Homomorphic Signatures schemes
- Their properties
  - Message Homomorphism
  - Key Homomorphism
  - Tag Randomizability
- A new method to construct scalable Mix-Nets