# A Review of Database Reconstruction

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*joint work with:*
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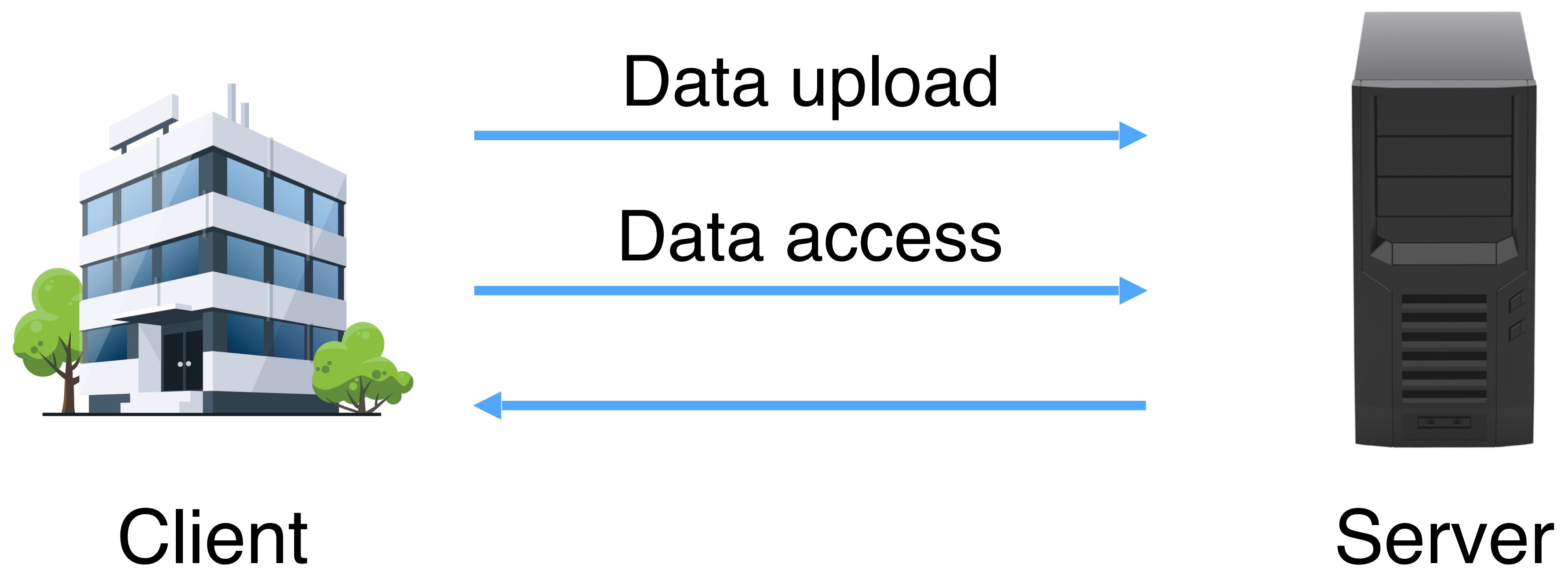
[LMP18] (S&P 2018), [GLMP18] (CCS 2018), [GLMP19] (S&P 2019)

ICERM workshop, Brown University, 2019

# Outsourcing Data

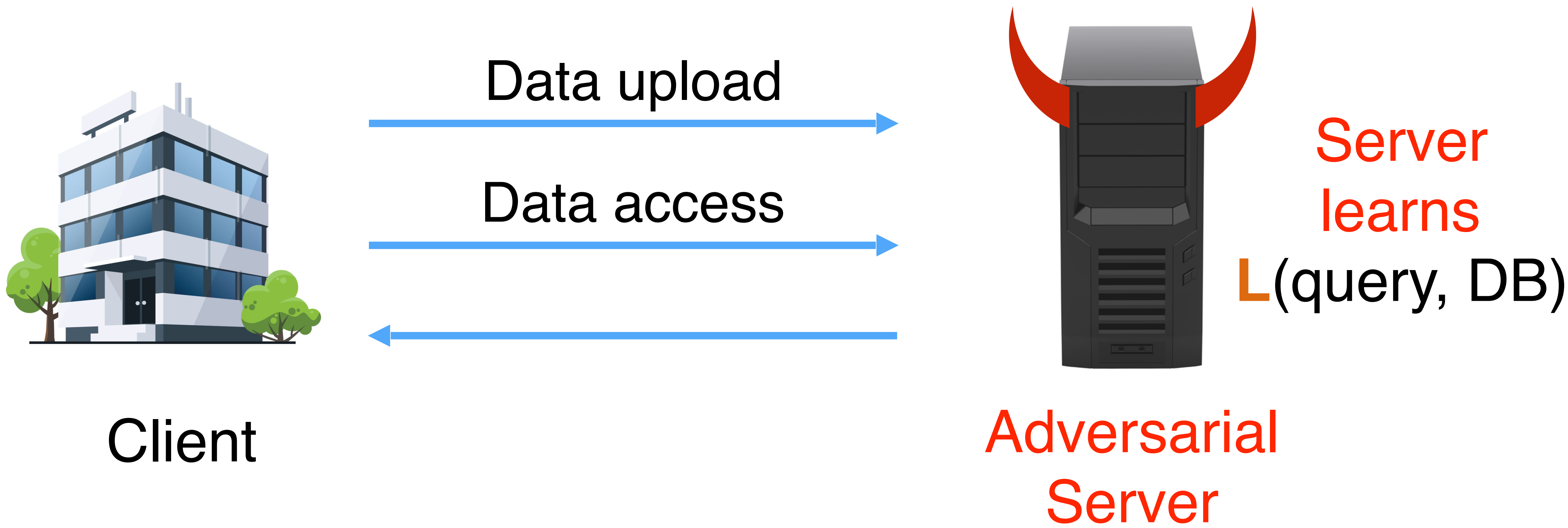



**Searchable Encryption**: **encrypted database** allowing search queries. In the static case: no updates.

**Adversary**: **honest-but-curious** host server.

**Security goal**: **confidentiality** of **data** and **queries**.

# Security Model

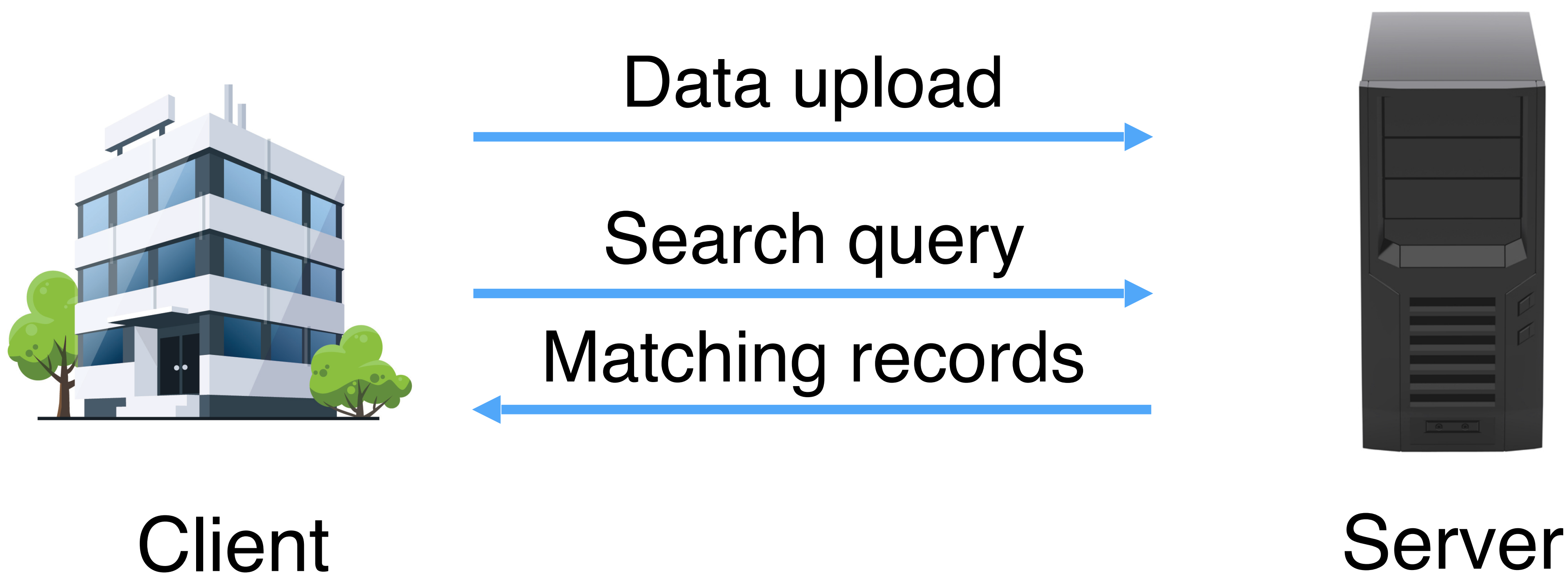



Generic solutions (FHE) are infeasible at scale → for efficiency reasons, some **leakage** is allowed.

**Security model**: parametrized by a **leakage function L**.

Server learns **nothing** except for the output of the leakage function.

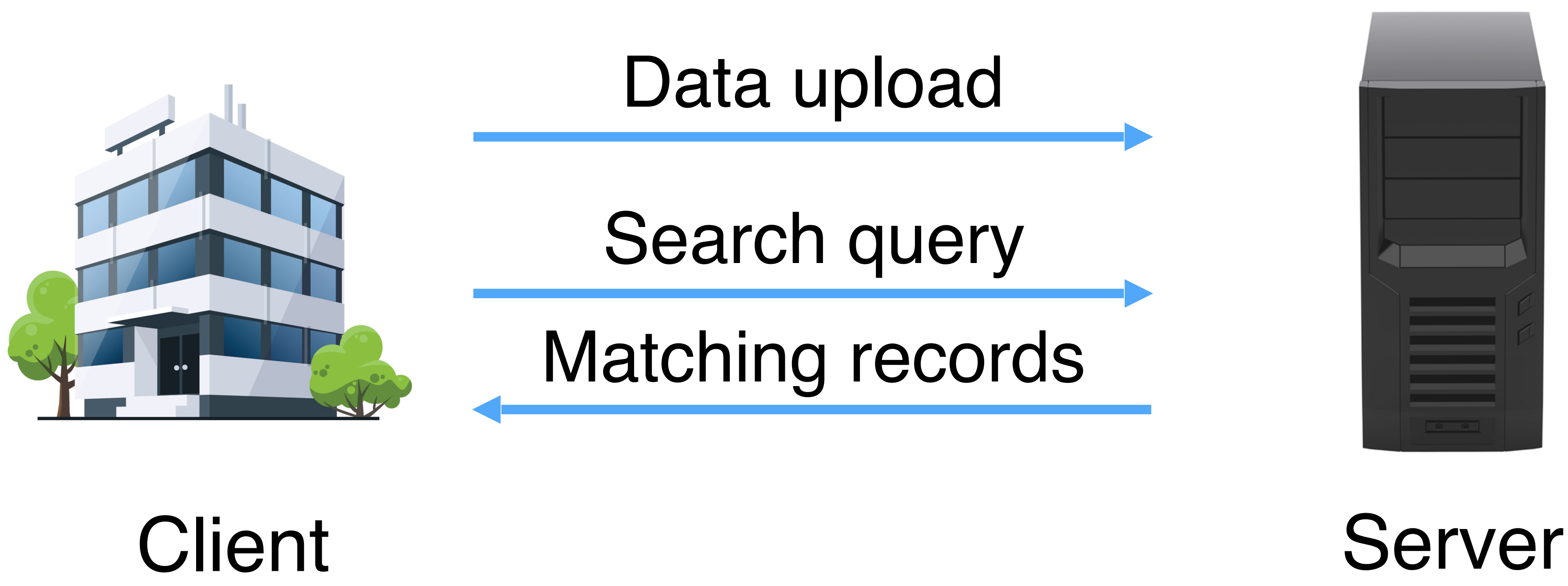
# Keyword Search




**Symmetric Searchable Encryption** (SSE) = keyword search:

- Data = collection of documents. *e.g. messages.*
- Serch query = find documents containing given keyword(s).

# Beyond Keyword Search




For an **encrypted database management system**:

- Data = collection of records. *e.g. health records.*
- Basic query examples:
  - find records with given value. *e.g. patients aged 57.*
  - find records within a given range. *e.g. patients aged 55-65.*

# Range Queries

In this talk: **range queries**.

- Fundamental for any encrypted DB system.
- Many constructions out there.
- Simplest type of query that can't “just” be handled by an index.

Natural solutions:

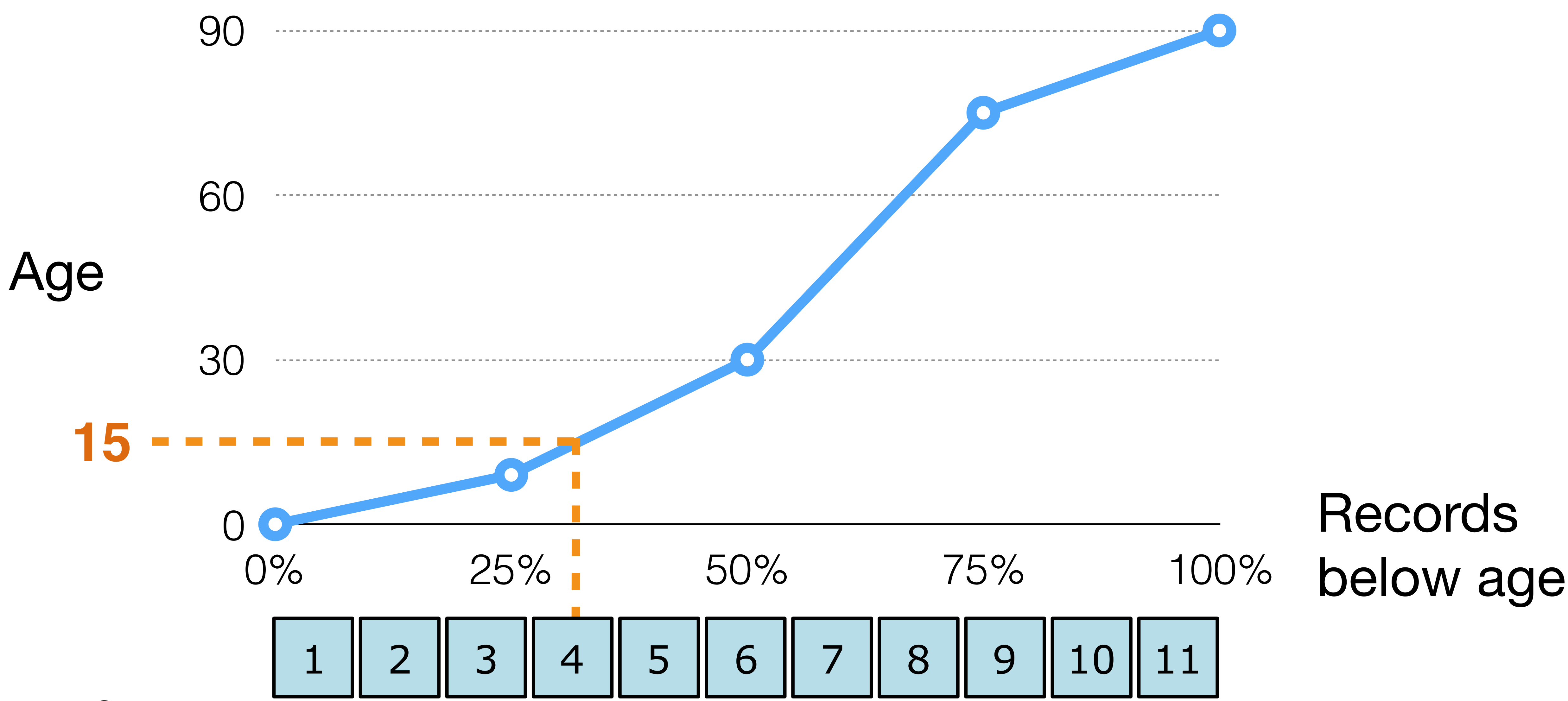
**Order-Preserving, Order-Revealing Encryption**.

- Plaintexts are **ordered**, ciphertexts are **ordered**.
- The encryption map **preserves order**.

# Attacks Exploiting ORE*

- **"Sorting" attack**: if every possible value appears in the DB... Just sort the ciphertexts and you learn their value!

- **"CDF-matching" attack**: say the attacker has an approximation of the **Cumulative Distribution Function** of DB values...




*not L/R ORE.

# Leakage-Abuse Attacks

**“Leakage-abuse attacks”** (coined by Cash et al. CCS'15):

- Do not contradict security proofs.
- Can be devastating in practice.

**ORE:** order information can be used to infer (approximate) values.
**Leaking order is too revealing.**

→ **“Second-generation” schemes** enable range queries *without* relying on OPE/ORE.

# Cryptanalysis and Leakage Abuse

*What is the point of these attacks?*

- Understand concrete security implications of leakage.
- “Impossibility results” → help guide design.

**Approach**: consider *general* settings. Pioneered by [KKNO16].

**Here**:

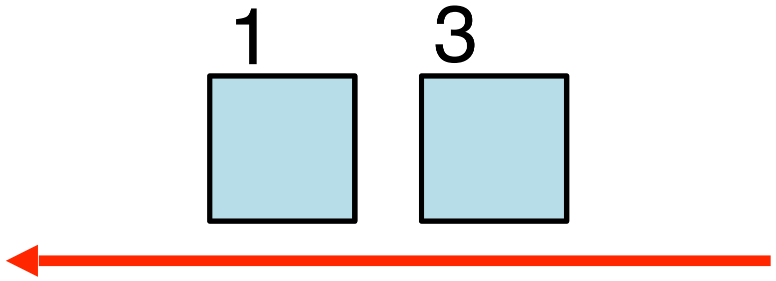
- Range queries.
- Passive, persistent adversary. No injections, no chosen queries.

# Roadmap

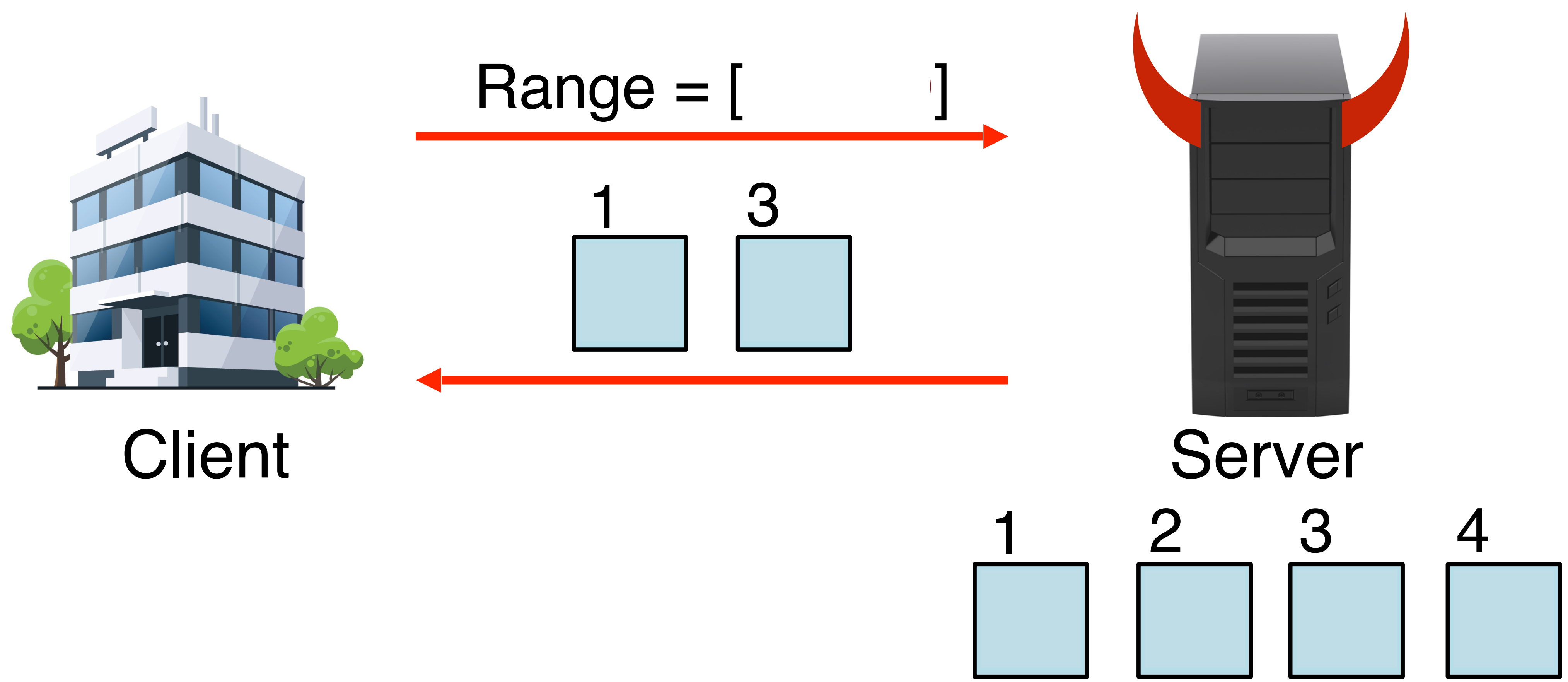
1. Access pattern leakage.

3. Volume leakage.

# Access Pattern Leakage



# Range Queries

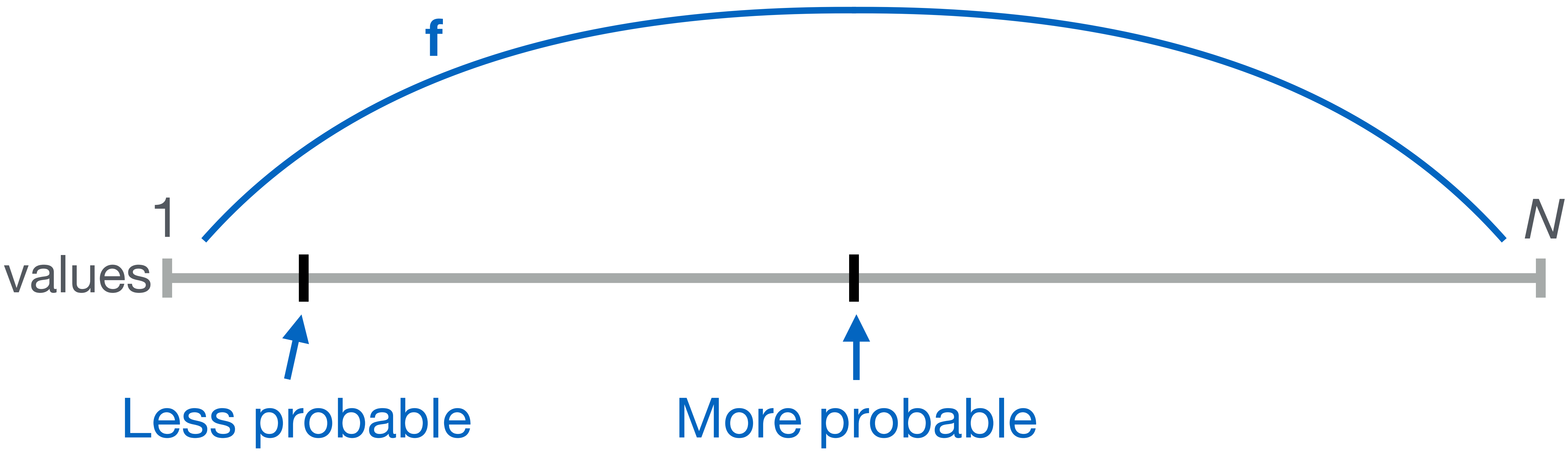



SE schemes supporting range queries are proven secure w.r.t. a leakage function including **access pattern leakage**.

*What can the server learn from the above leakage?*

Let ***N*** = number of possible values.

# KKNO16 Attack




Assume a **uniform distribution** on range queries.

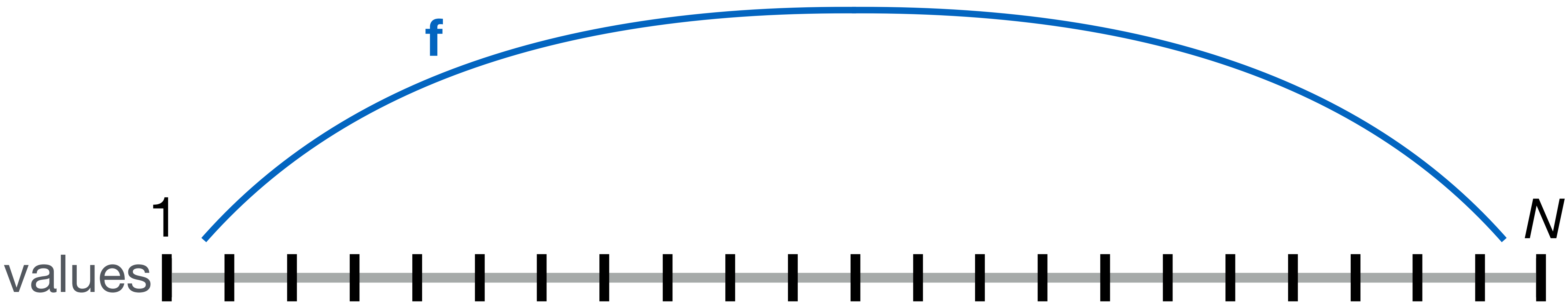
Induces a distribution **f** on the prob. that a given value is hit.

**Idea**: for each record...

**1.** Count frequency at which the record is hit.
→ gives estimate of probability it's hit by uniform query.

**2.** deduce estimate of its value by "inverting" **f**.

# KKNO16 Attack




**Step 1**: for **every** record, estimate prob of the record being hit.

**Step 2**: "invert" **f**.

**Step 3**: break the symmetry, i.e. reconcile which values are on the same side of N/2.

After $O(N^4 \log N)$ uniform queries, previous alg. recovers the *exact* value of *all* records.

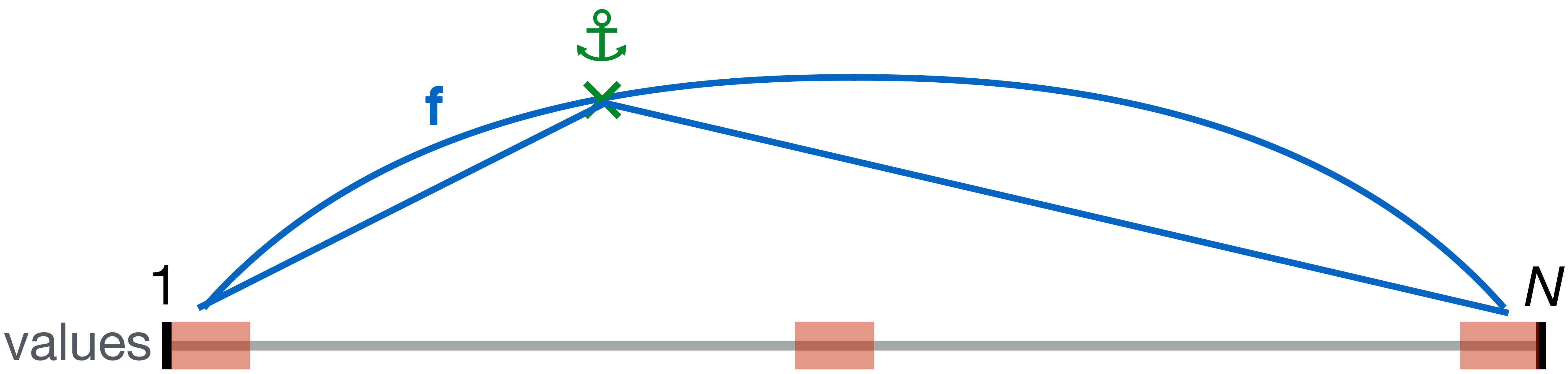
# KKNO16 Attack

After $O(N^4 \log N)$ uniform queries, previous alg. recovers the *exact* value of *all* records.

Remarks:

- Requires **uniform** distribution.
- **Expensive**. In fact, uses up *all possible* leakage information!
- Lower bound of $\Omega(N^4)$.

# Revisiting the Analysis, Part I [GLMP19]




**Step 0**: find suitable "anchor" record.

**Step 1**: for **every** record, estimate distance to anchor.

**Step 2**: "invert" **f**. ← costs a **constant** factor!

**Step 3**: break the symmetry, i.e. reconcile which values are on the same side of N/2.

After $O(N^2 \log N)$ uniform queries, previous alg. recovers the *exact* value of *all* records.

# Cheaper KKNO16 attack

After $O(N^2 \log N)$ uniform queries, previous alg. recovers the *exact* value of *all* records.

Remarks:

- Requires **uniform** distribution.
- Requires existence of a favorably placed record.
- **Still fairly expensive**.
- Lower bound of $\Omega(N^2)$. Can't hope to get below.

# Approximate Reconstruction

**Strongest goal**: **full** database reconstruction = recovering the exact value of every record.

**More general**: **approximate** database reconstruction = recovering all values within $\varepsilon N$.

$\varepsilon = 0.05$ is recovery within 5%. $\varepsilon = 1/N$ is full recovery.

(“Sacrificial” recovery: values very close to 1 and *N* are excluded.)

# Database Reconstruction

**[KKNO16]**: full reconstruction in $O(N^4 \log N)$ queries.

recovers

[GLMP19]:

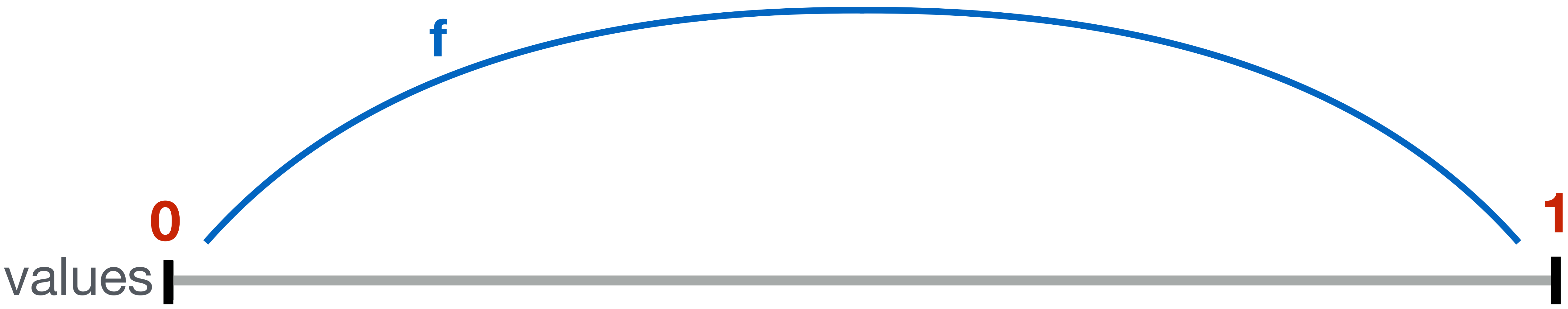
| | Full. Rec. | Lower Bound |
|---|---|---|
| ‣ $O(\varepsilon^{-4} \log \varepsilon^{-1})$ for approx. reconstruction. | $O(N^4 \log N)$ | $\Omega(\varepsilon^{-4})$ |
| ‣ $O(\varepsilon^{-2} \log \varepsilon^{-1})$ with mild hypothesis. | $O(N^2 \log N)$ | $\Omega(\varepsilon^{-2})$ |

**Scale-free**: does not depend on size of DB or number of possible values.

→ Recovering all values in DB within 5% costs O(1) queries!

**Analysis**: uses VC theory + draws connection to machine learning. See Paul's talk!

# Intuition for Scale-Freeness




**Step 1**: for **every** record, estimate prob of the record being hit.

**Step 2**: "invert" **f**.

Instead of support = integers 1 to $N$, take reals [0,1].

...so "$N = \infty$" !

**The previous algorithm still works!**

# On the i.i.d. Assumption

**+ Scale-freeness**. *N* and DB size irrelevant for query complexity.

**-** We are assuming **uniformly distributed** queries.

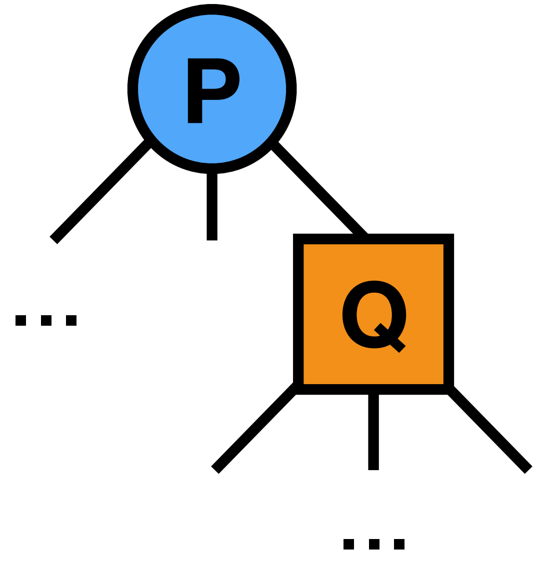
In reality we are assuming:

- Queries are **uniform**.
- The **adversary knows** the query distribution.
- Queries are **independent and identically distributed**.

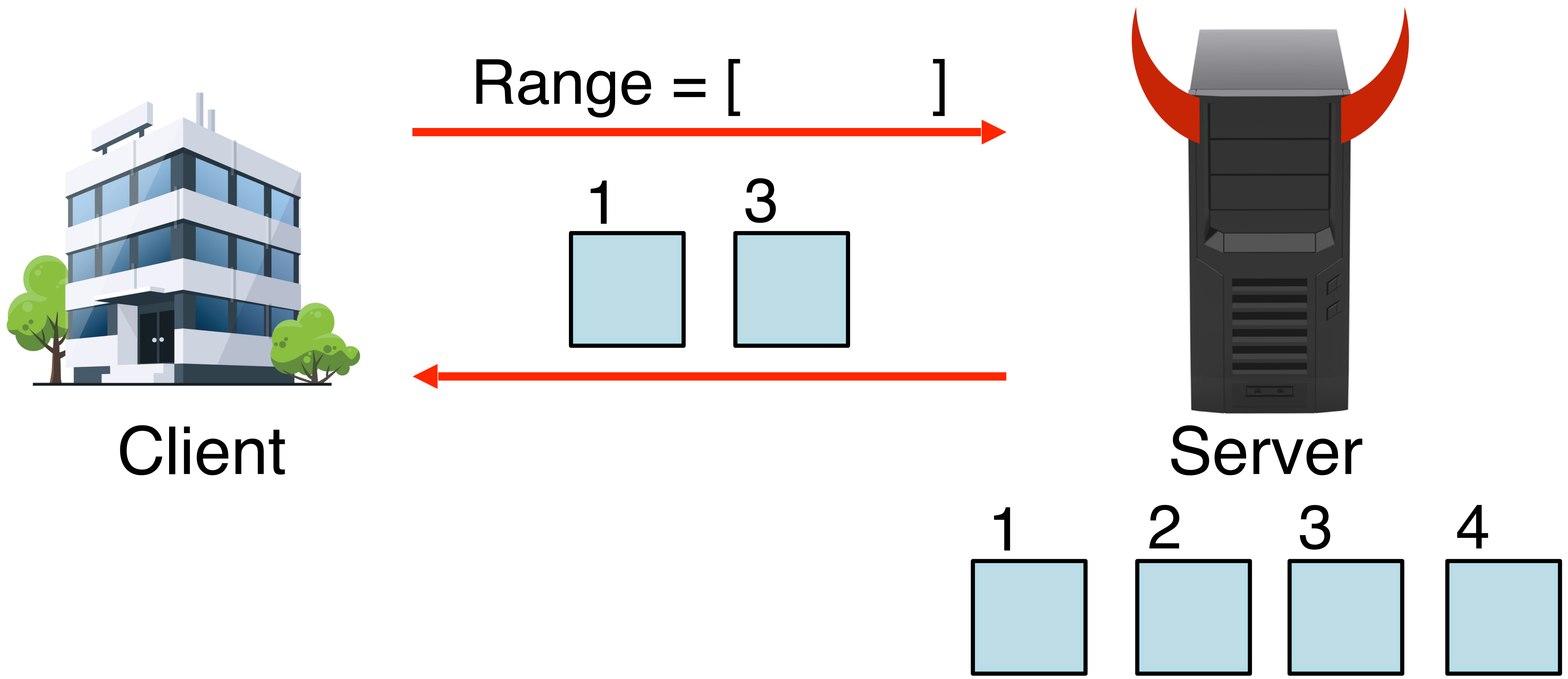
This is not realistic.

*What can we learn without that hypothesis?*

# Order Reconstruction



# Problem Statement



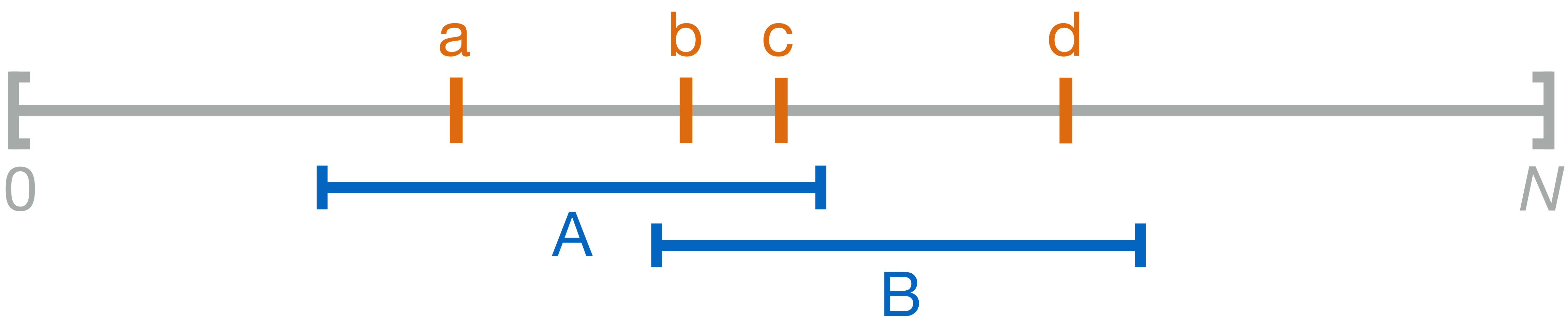

*What can the server learn from the above leakage?*

This time we **don't assume** i.i.d. queries, or knowledge of their distribution.

# Range Query Leakage

Query A matches records a, b, c.

Query B matches records b, c, d.




Then this is the only configuration (up to symmetry)!

→ we learn that records b, c are *between* a and d.

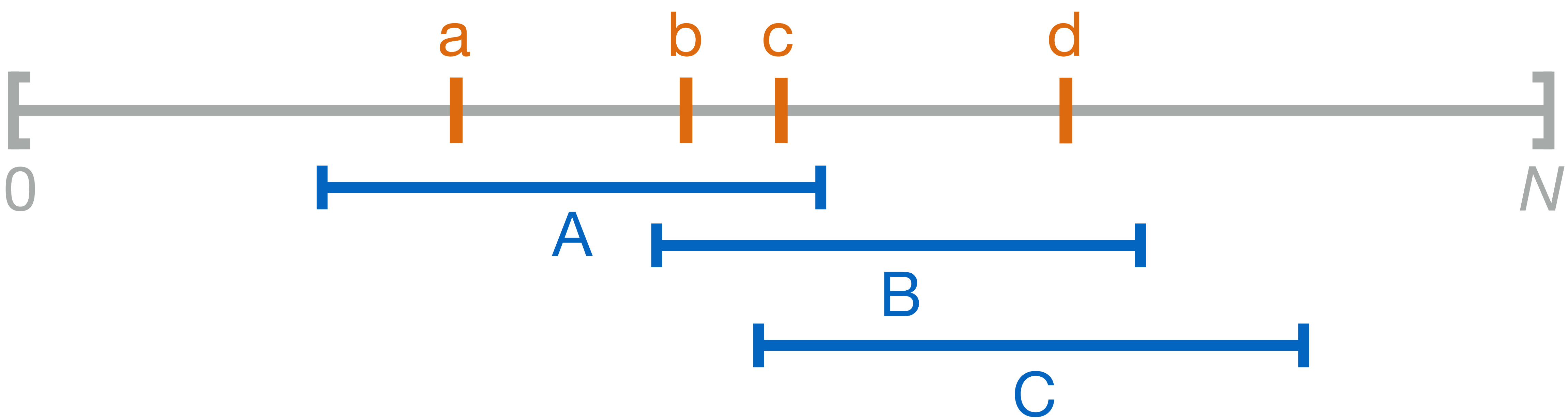
We learn something about the **order** of records.

# Range Query Leakage

Query A matches records a, b, c.

Query B matches records b, c, d.

Query C matches records c, d.




Then the only possible order is a, b, c, d (or d, c, b, a)!

**Challenges**:

- How do we extract order information? (What **algorithm**?)
- How do we **quantify** and **analyze** how fast order is learned as more queries are observed?

# Challenge 1: the Algorithm

**Short answer**: there is already an algorithm!

**Long answer**: **PQ-trees**.

*X*: linearly ordered set. Order is unknown.

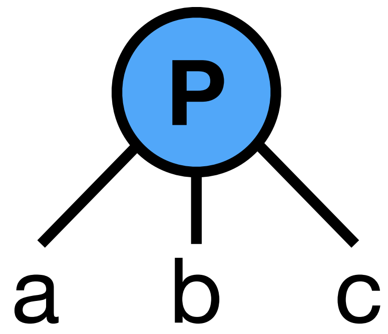
You are given a set *S* containing some intervals in *X*.

A **PQ tree** is a compact (linear in $|X|$) representation of the set of all permutations of *X* that are compatible with *S*.

Can be updated in linear time.

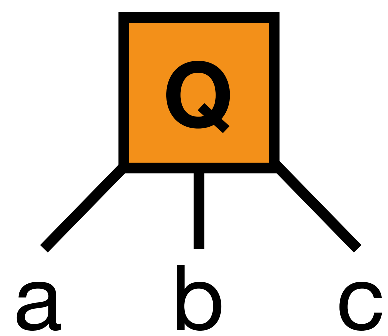
Note: was used in [DR13], didn't target reconstruction.

# PQ Trees



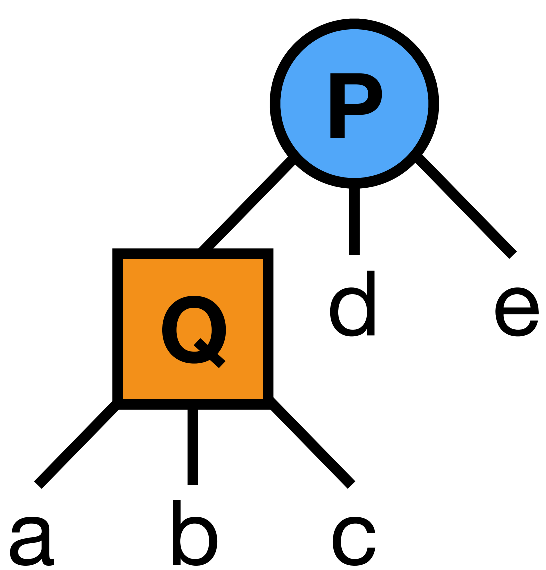

Order is completely **unknown**.

- any permutation of abc.




Order is completely **known** (up to reflection).

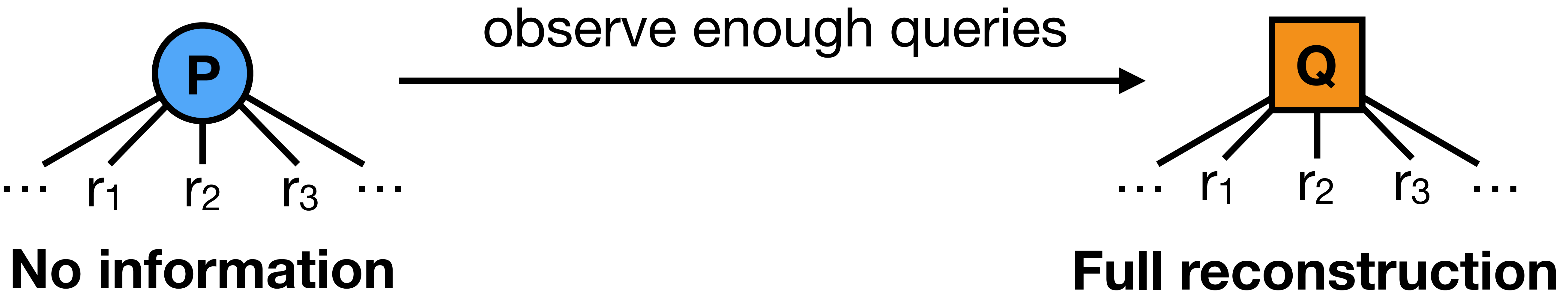
- abc’or ‘cba’.




Combines in the natural way.

- ‘abcde’, ‘abced’, ‘dabce’, ‘eabcd’, ‘deabc’, ‘edabc’, ‘cbade’ etc.

# Full Order Reconstruction




We want to **quantify** order learning...

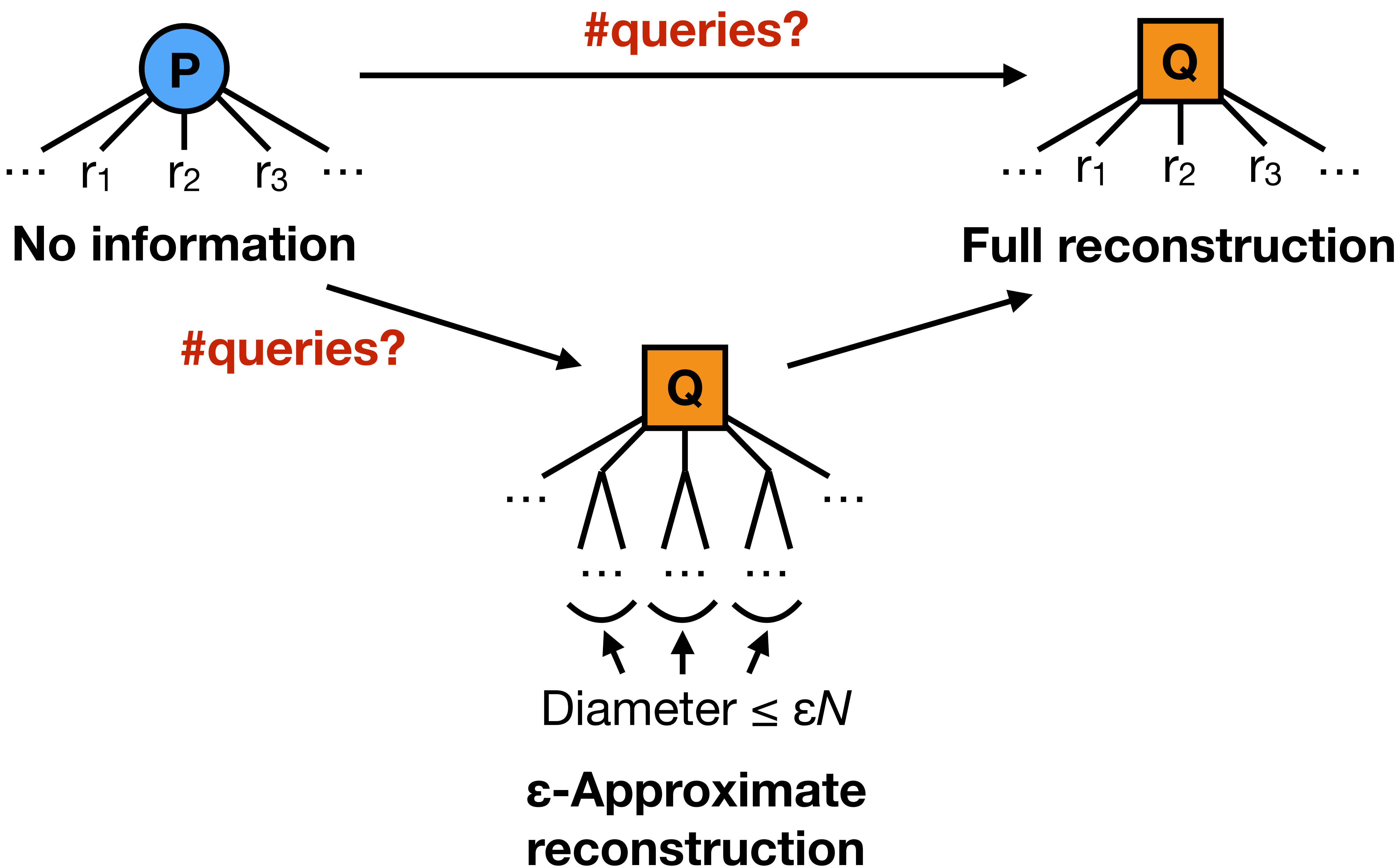
# Challenge 2a: Quantify Order Learning


ε-**Approximate order reconstruction**.

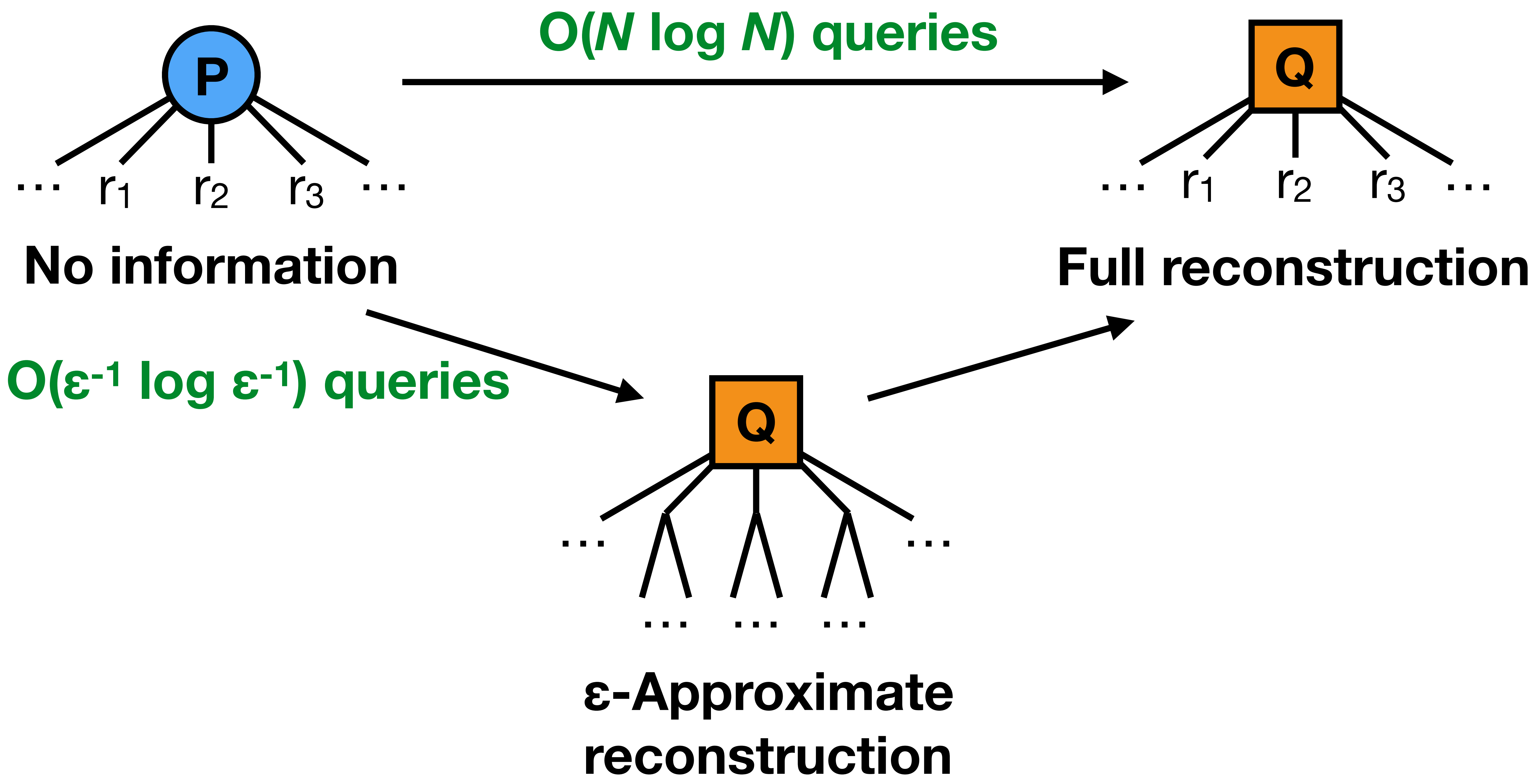
**Roughly**: we learn the order between two records as soon as their values are ≥ ε*N* apart. (ε = 1/N is full reconstruction)

**Note**: compatible with "ORE-style" CDF matching.

# Approximate Order Reconstruction



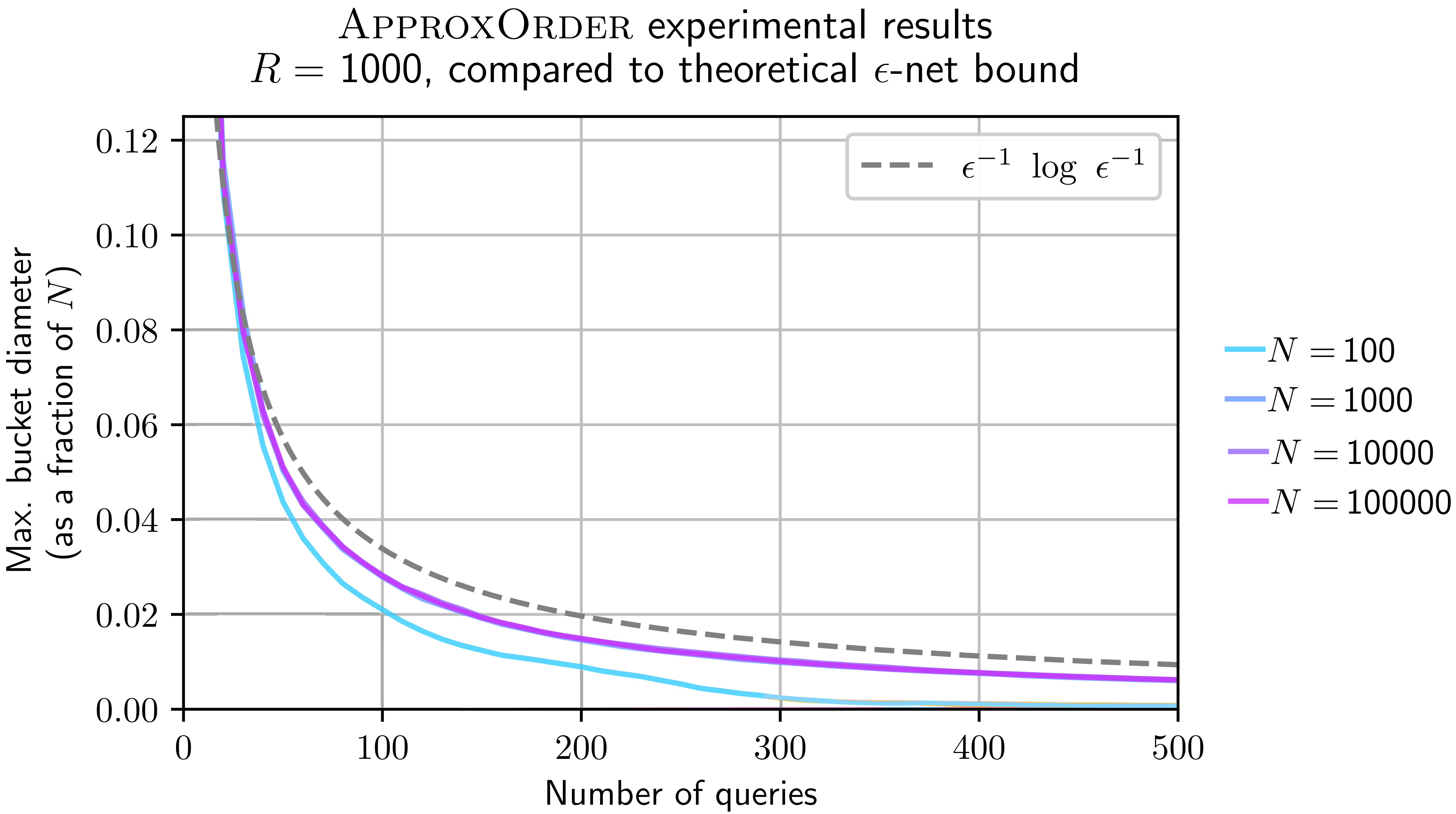
# Approximate Order Reconstruction



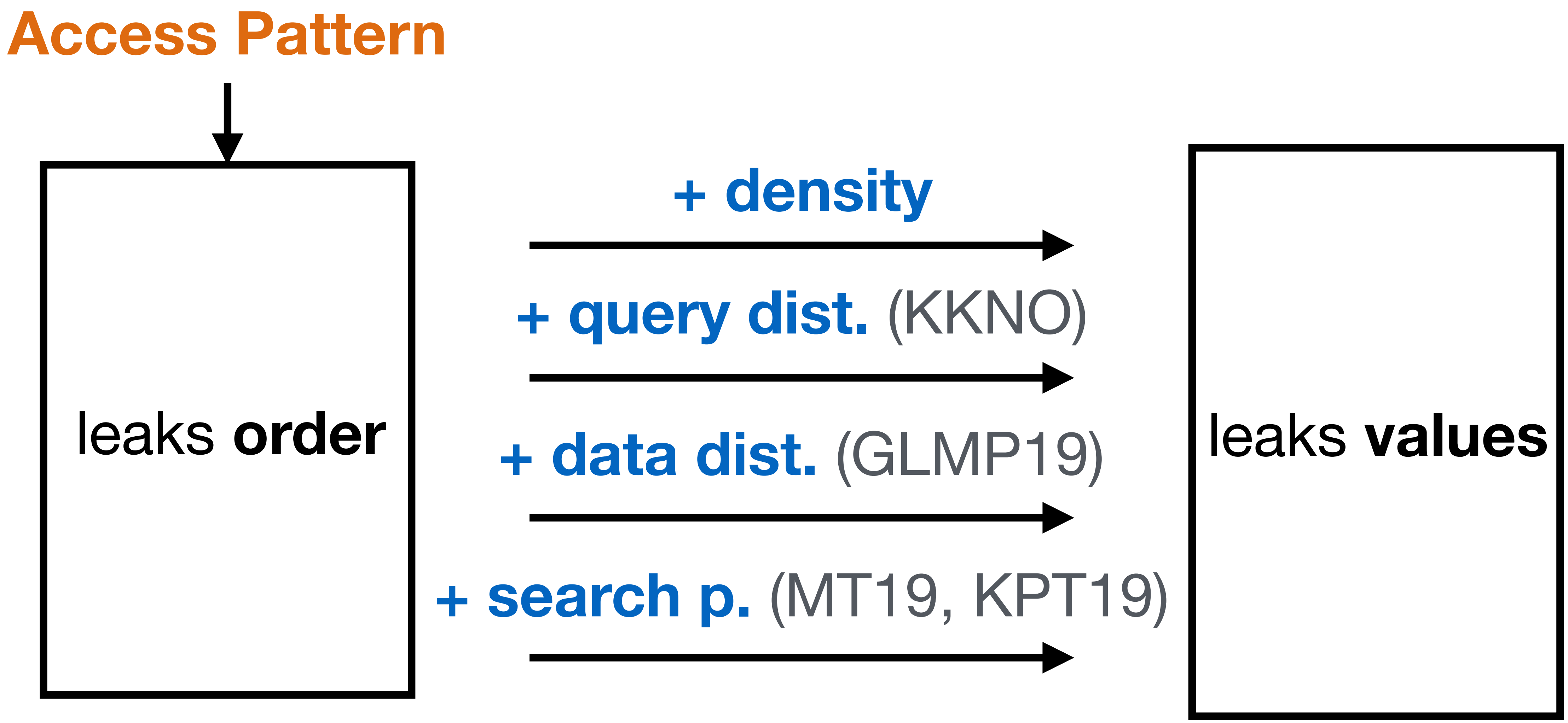

**Conclusion:** learn order very quickly.

Note: some (weak) assumptions are swept under the rug.

# Experiments

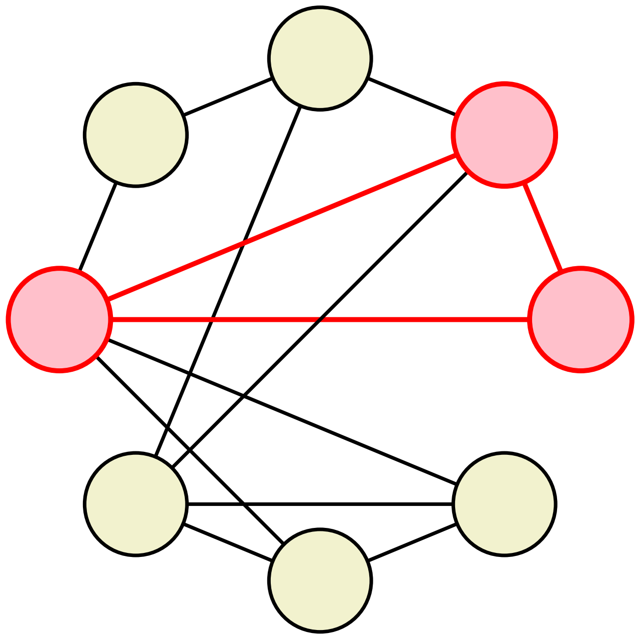


# Big Picture

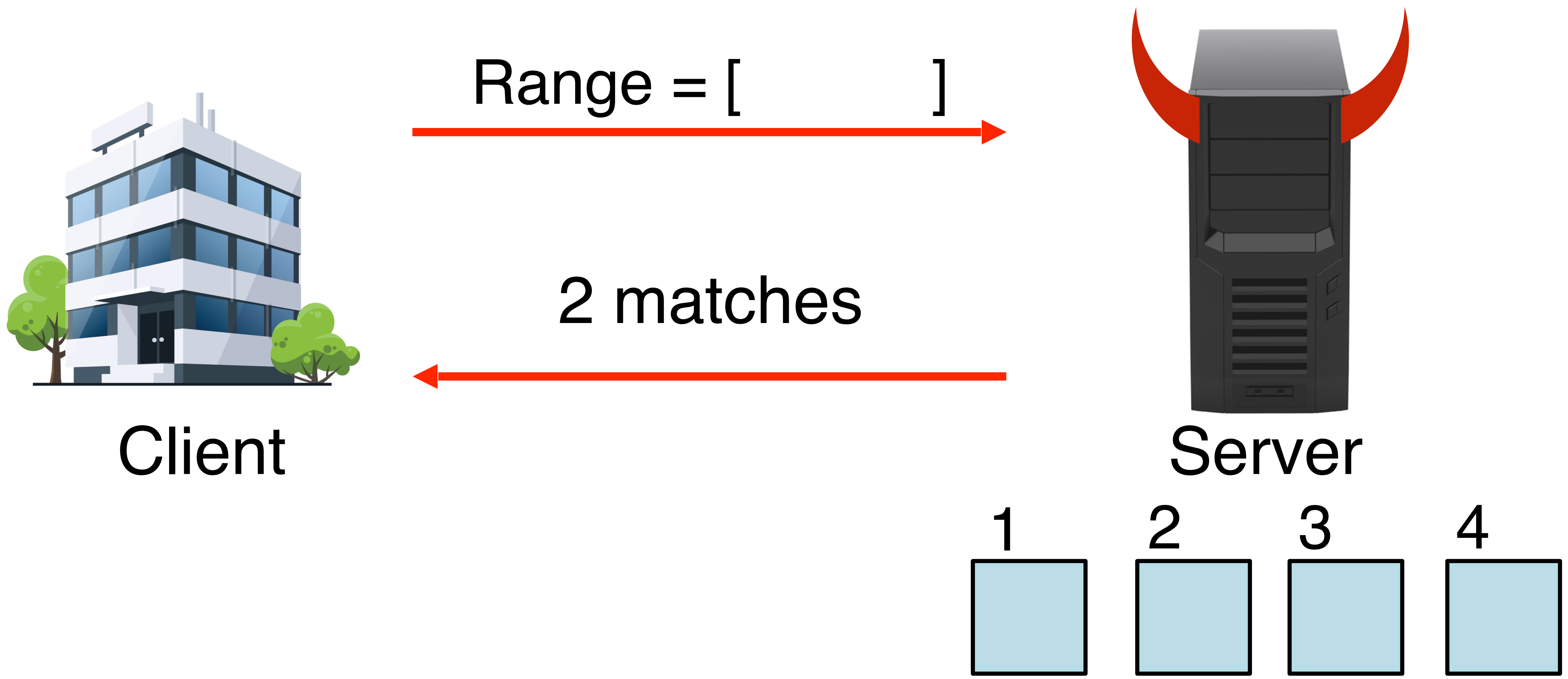



- **Resilient**, scale-free attacks.
- Effective in practice in some realistic scenarios.
- Watch out for additional leakage. E.g.:
  - Search pattern.
  - Rank information (e.g. L/R ORE). Damaging for low #queries.

# Volume Leakage



# Problem Statement

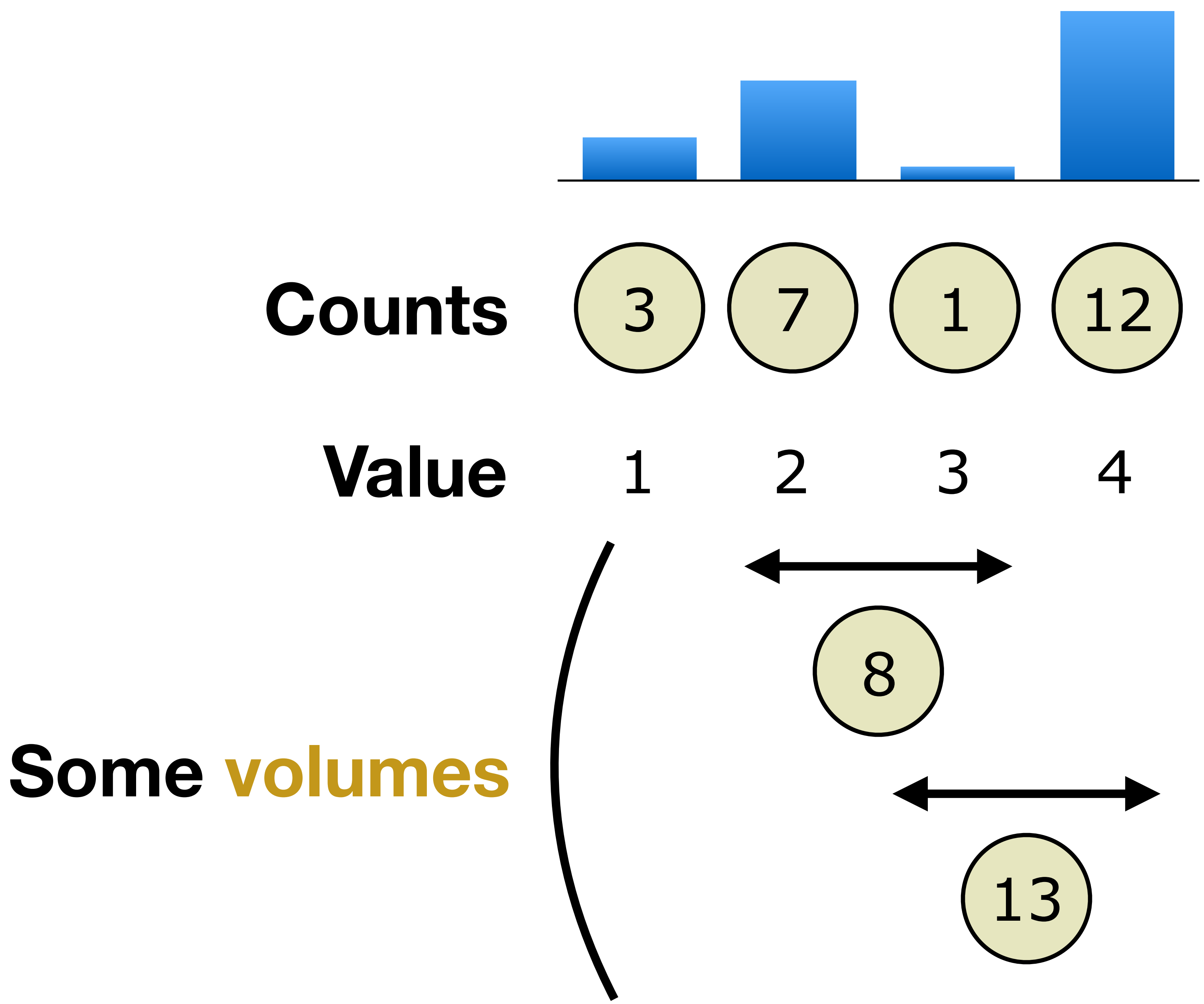



Attacker *only* sees **volumes** = **number of records** matching each query.

*What can the server learn from the above leakage?*

# Volumes

The attacker wants to learn exact **counts.**




A **volume** = number of records matching some range.

# KKNO16 Volume Attack

Assume **uniform** queries.

**Step 1**: recover exact probability of every volume ➔ number of queries that have each volume.

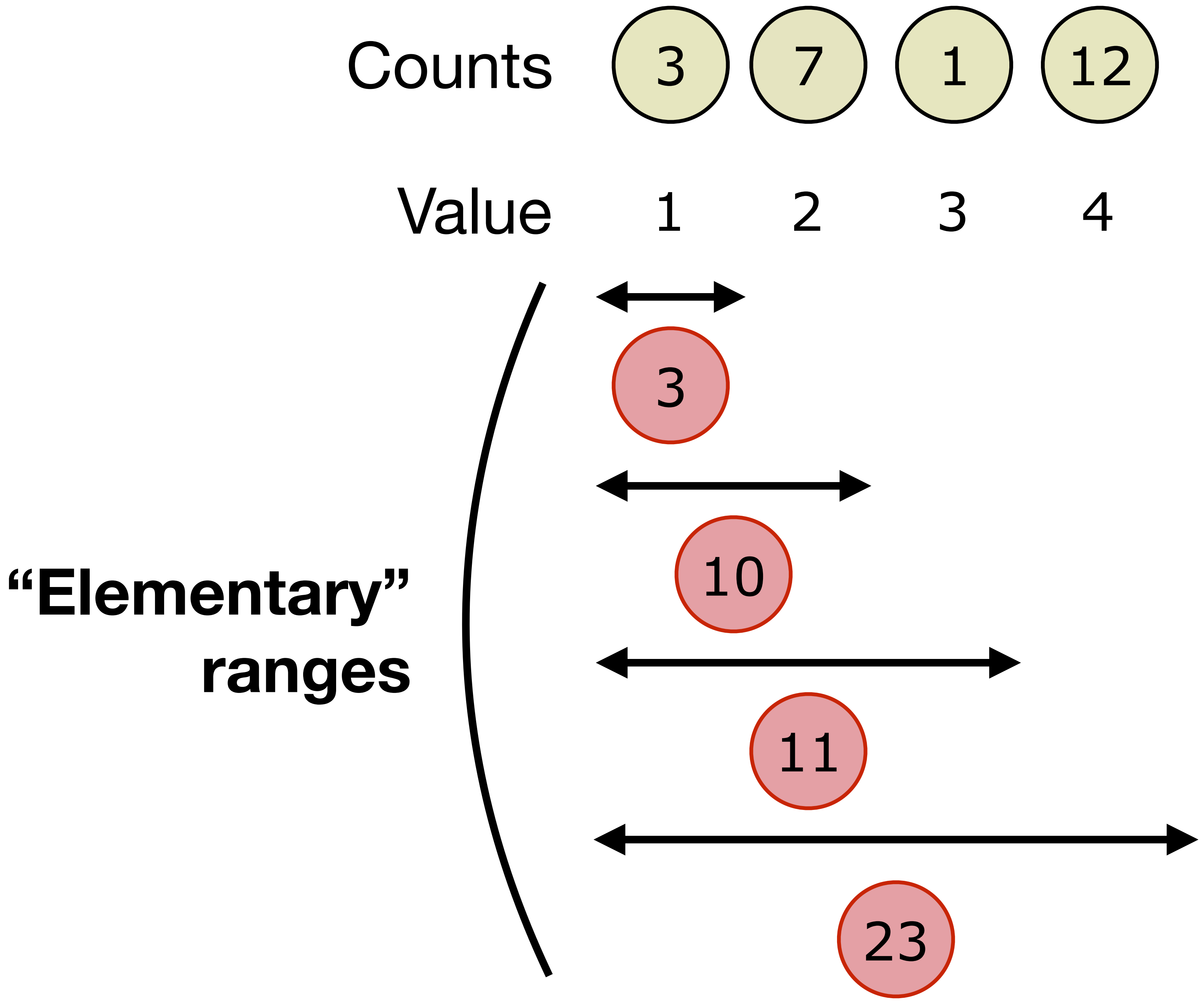
**Step 2**: express and solve equation system linking above data back to DB counts. (Ends up as polynomial factorization.)

> After $O(N^4 \log N)$ uniform queries, previous alg. recovers all DB counts.

Remarks:

- Requires **uniform** distribution.
- **Expensive**. In fact, uses up *all possible* leakage information!
- Lower bound of $\Omega(N^4)$.

# Elementary Volumes [GLMP18]




**Elementary volumes** = volumes of ranges [1,1], [1,2], [1,3]...

# Elementary Volumes

| | | | | |
|---|---|---|---|---|
| Counts | 3 | 7 | 1 | 12 |
| Value | 1 | 2 | 3 | 4 |

Fact:

$$\mathrm{vol}([a,b]) = \mathrm{vol}([1,b]) - \mathrm{vol}([1,a])$$

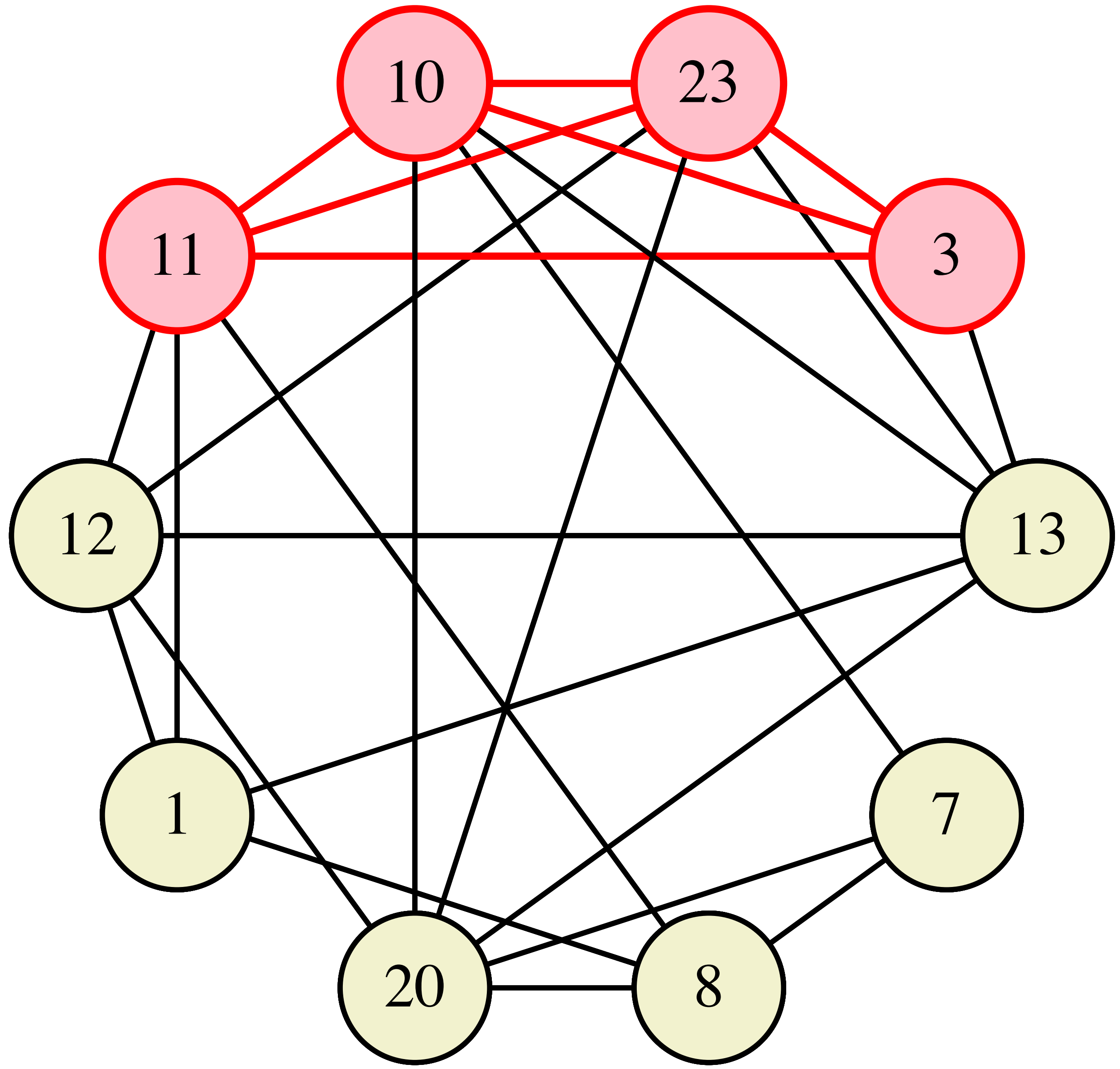
so...

- Every volume is = difference of two elementary volumes.
- Knowing set of elementary volumes ⇔ knowing counts.

**Our goal:** finding elementary volumes.

# The Attack

**Assumption:** the volumes of all queries are observed.




Draw an **edge** between volumes **a** and **b** iff **|b-a|** is a volume.

# Summary

**Attack: elementary volumes** form a clique in the volume graph → clique-finding algorithm reveals them.

For structured queries, even just volume leakage can be quite damaging. Attack requires strong assumption.

*In the article:*

- *Pre-processing to avoid clique finding.*
- *Analysis of parameters + experiments.*
- *Other attacks.*

# Conclusion

# Conclusion

**Access pattern**:

- **Resilient**, scale-free attacks.
- Effective in practice in some realistic scenarios.

➔ non-trivial countermeasures are required.

**Volume attacks**:

- **Fragile attacks**. Currently.
- Expensive query complexity $O(N^2 \log N)$.
- Unsatisfactory: limits of attacks not clear.

➔ "simple" countermeasures might be enough in most scenarios.

Some open problems: mixed queries, scale-free volumes.