Triangulating the Klein bottle

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During the last decade, sampling and meshing surfaces in 3-space has been intensively studied, resulting in major theorical advancesand reliable and highly efficient codes [1]. Extending those results to higher dimensions is in principle possible and one may hope of approximating higher-dimensional shapes by simplicial complexes (an extension of polyhedra with triangular facets) [4]. However, this approach faces several difficulties. The most important one being the computational bottleneck caused by the fact that the size of most data structures grows exponentially with the ambient dimension. To bypass this curse of dimensionality, new types of simplicial complexes have been recently proposed, most notably the witness complex and the tangential Delaunay complex [3, 2]. Thoses simplicial complexes adapt to the intrinsic dimension of the shape one wants to approximate without relying on a subdivision of the ambient space. Their complexity thus depends mostly on the intrinsic dimension of the shape of interest rather than on the dimension of the embedding space, which is usually much bigger.

The internship will build on recent results [2] and develop a method to sample and mesh submanifolds of any codimension. The algorithm will be used to mesh the Klein bottle (a non orientable surface (2-manifold) embeddable in $\mathbb{R}^4$), the symmetry set of a 3D shape (the projection in $\mathbb{R}^3$ of a surface embeddable in $\mathbb{R}^{11}$, or the configuration space of closed kinematic chains (such as robots or molecules). The algorithm may also find applications in scientific computing for solving partial differential equations where the domain of interest has the structure of a manifold.

This work will be done as part of a european project CG-Learning (http://cglearning.eu/) whose overall goal is to design data structures and algorithms to process and analyse complex shapes in high dimensional spaces. It will also be part of an effort towards extending the CGAL library [5] to higher dimensions.

References

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