An Introduction to Lustre

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May 4, 2017
The language Lustre

- Invented by Paul Caspi and Nicolas Halbwachs around 1984, in Grenoble (France).
- A program is a set of equations. An equation defines an infinite sequence of values.
- Boolean operators applied point-wise, a unit-delay, and sampling operators.
- Equivalent graphical representation by block diagrams.
- Feedback loops must cross a unit delay.
- Time is synchronous: at every tick of a global clock, every operation does a step.
- Code generation to sequential code and formal verification techniques.
- An industrial success: SCADE (Esterel-Technologies company) is used for programming critical control software (e.g., planes, nuclear plants).
**Lustre**

Program by writing stream equations.

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<tr>
<td>X + Y</td>
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<td>X + 1</td>
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Equation $Z = X + Y$ means that at any instant $n \in \mathbb{N}$, $Z_n = X_n + Y_n$.

Time is logical: inputs $X$ and $Y$ arrive **at the same time**; the output $Z$ is produced at the same time.

Synchrony means that at instant $n$, all streams take their $n$-th value.

In practice, check that the current output is produced before the next input arrives.
Example: 1-bit adder

```plaintext
node full_add(a, b, c:bool) returns (s, co:bool);
  let
    s = (a xor b) xor c;
    co = (a and b) or (b and c) or (a and c);
  tel;

or:

node full_add(a, b, c:bool) returns (s, co:bool);
  let
    co = if a then b or c else b and c;
    s = (a xor b) xor c;
  tel;
```
Full Adder

Compose two “half adder”

```plaintext
node half_add(a,b:bool)
    returns (s, co:bool);
    let s = a xor b;
    co = a and b;
    tel;
```

Instanciate it twice:

```plaintext
node full_add_h(a,b,c:bool)
    returns (s, co:bool);
    var s1,c1,c2:bool;
    let
        (s1, c1) = half_add(a,b);
        (s, c2) = half_add(c, s1);
        co = c1 or c2;
    tel;
```
Verify properties

How to be sure that full_add and full_add_h are equivalent?

\[ \forall a, b, c : \text{bool}. \text{full_add}(a, b, c) = \text{full_add_h}(a, b, c) \]

Write the following program and prove that it returns true at every instant!

```luss
def node equivalence(a,b,c:bool) returns (ok:bool);
    var o1, c1, o2, c2: bool;
    let
        (o1, c1) = full_add(a,b,c);
        (o2, c2) = full_add_h(a,b,c);
        ok = (o1 = o2) and (c1 = c2);
    tel;

Then, use the model-checking tool lesar:

% lesar prog.lus equivalence
--Pollux Version 2.2

TRUE PROPERTY
```
The Unit Delay

One can refer to the value of a input at the “previous” step.

|     | 1  | 2  | 1  | 4  | 5  | 6  | ...
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<tr>
<td>pre X</td>
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<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>...</td>
</tr>
<tr>
<td>Y</td>
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<td>4</td>
<td>2</td>
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<td>...</td>
</tr>
<tr>
<td>Y -&gt; pre X</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>...</td>
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<tr>
<td>S</td>
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<td>4</td>
<td>8</td>
<td>13</td>
<td>19</td>
<td>...</td>
</tr>
</tbody>
</table>

The stream \((S_n)_{n \in \mathbb{N}}\) with \(S_0 = X_0\) and \(S_n = S_{n-1} + X_n\), for \(n > 0\) is written:

\[
S = X \rightarrow \text{pre } S + X
\]

Introducing intermediate equations does not change the meaning of programs:

\[
S = X \rightarrow I; I = \text{pre } S + X
\]
Example: convolution

Define the sequence:

\[ Y_0 = X_0 / 2 \quad \land \quad \forall n > 0. Y_n = (X_n + X_{n-1}) / 2 \]

```
node convolution(X:real) returns (Y:real);
let Y = (X + (0 -> pre X)) / 2.0;
tel;
```

or:

```
node convolution(X:real) returns (Y:real);
var pY:int;
let Y = (X + pY) / 2;
pY = 0 -> pre X;
tel;
```
Linear filters

FIR (Finite Impulse Response)

\[ y(n) = \sum_{m=0}^{L-1} x(n - m)b(m) \]

IIR (Infinite Impulse Response) or recursive filter

\[ y(n) = \sum_{m=0}^{L-1} x(n - m)b(m) + \sum_{m=1}^{M-1} y(n - m)a(m) \]
FIR

Build a block-diagram with three operators: a gain (multiplication by a constant), a sum and a unit delay (register).

Previous example
\[ \forall n \geq 0. y(n) = \frac{1}{2} (x(n) + x(n - 1)) \]
Example: follow $x$ with a 20% gain.

$$\forall n \geq 0. y(n) = 0.2(x(n) - y(n - 1)) + y(n - 1)$$

node filter(x: real) returns (y:real);
    let y = 0.0 -> 0.2 * (x - pre y) + pre y; tel;

Retiming:
Optimise by moving unit delays arround combinatorial operators.

**DEMO:** type luciole filter.lus filter
Counting events

Count the number of instants where the input signal tick is true between two top.

node counter(tick, top:bool) returns (cpt:int);
let
    cpt = if top then 0
        else if tick then (0 -> pre cpt) + 1 else pre cpt;
tel;

Is this program well defined? Is-it deterministic? No: initialization issue.

<table>
<thead>
<tr>
<th>tick</th>
<th>f</th>
<th>f</th>
<th>t</th>
<th>t</th>
<th>t</th>
<th>t</th>
<th>t</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>top</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>t</td>
<td>f</td>
<td>f</td>
</tr>
<tr>
<td>cpt</td>
<td>nil</td>
<td>nil</td>
<td>nil</td>
<td>nil</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>...</td>
</tr>
</tbody>
</table>

Write instead:

    cpt = if top then 0
        else if tick then (0 -> pre cpt) + 1 else 0 -> pre cpt;
An explicit Euler integrator

node integrator(const step: real; x0, x’:real) returns (x:real);
let
   x = x0 -> pre(x) + pre(x’) * step;
tel;

step is a constant stream computed at compile-time.

Sinus/cosine functions

node sinus_cosinus(theta:real) returns (sin,cos:real);
let sin = theta * integrator(0.01, 0.0, cos);
   cos = theta * integrator(0.01, 1.0, 0.0 -> pre sin);
tel;
Initial Value Problem (IVP)

$f$ is a combinatorial function with $y$ of type $ty$. $t$ is the current time. $x(t)$ be defined by the IVP:

\[ \dot{x} = f(y, t, x) \quad \text{with} \quad x(0) = x_0 \]

```plaintext
def node ivp(const step: real; y: ty; read) returns (x: real)
    var t: real;
    let
        x = integr(step, x0, f(y, t, x));
        t = 0.0 -> pre t + step;
    tel;
end
```

Exercice

- Program a classical explicit Runge Kutta method (e.g., order 4).
- More difficult: program a variable step Runge Kutta method (RK45). Hint: use a control bit `error-too-large` to shrink the step dynamically.
Counting Beacons

Counting beacons and seconds to decide whether a train is on time.

Use an **hysteresis** with a low and high threshold to reduce oscillations.

```plaintext
node beacon(sec, bea: bool) returns (ontime, late, early: bool);
var diff, pdiff: int; pontime: bool;
let
    pdiff = 0 -> pre diff;
    diff = pdiff + (if bea then 1 else 0) +
        (if sec then -1 else 0);
    early = pontime and (diff > 3) or
        (false -> pre early) and (diff > 1);
    late = pontime and (diff < -3) or
        (false -> pre late) and (diff < -1);
    ontime = not (early or late);
pontime = true -> pre ontime;
tel;
```

¹This example is due to Pascal Raymond
Two types of properties

Safety property
“Something wrong never happen”, i.e., a property is invariant and true in any accessible state. E.g.:

- “The train is never both early and late”, it is either on time, late or early;
- “The train never passes immediately from late to early”; “It is impossible to stay late only a single instant”.

Liveness property
“Something good with eventually happen.”, i.e., any execution will reach a state verifying the property. E.g., “If the trains stop, it will eventually be late.”

Remark:
“If the train is on time and stops for ten seconds, it will be eventually late” is a safety property.
Safety properties are critical ones in practice.
Formal verification and modeling of systems

A safety property ("something bad will never happen") is a boolean proved to be true at every instant.

Example: the alternating bit protocol
A transmitter $A$; a receiver $B$. Two unreliable lines $A2B$ and $B2A$ that may loose messages.

- $A$ asks for one input. It re-emits the data with $bit = true$ until it receives $ack = true$.
- It asks for an other input and emits the data with $bit = false$ until it receives $ack = false$.
- $B$ sends $ack = false$ until it receives $bit = true$; it sends $ack = true$ until it receives $bit = false$;
- initialization: send anything with $bit = true$. The first message arriving with $bit = false$ is valid.
Objective:

Model and prove the protocol is correct, i.e., the network is the identity function (input sequence = output sequence) with two unreliable lines.

Model the asynchronous communication by adding a “presence” bit to every data: a pair \((data, enable)\) is meaningful when \(enable = true\).
The Sender

- A asks for one input. It re-emits the data with \( bit = true \) until it receives \( ack = true \).
- It asks for an other input and emits the data with \( bit = false \) until it receives \( ack = false \).

node A(dataIn: int; recB: bool; ack: bool)
returns (reqData: bool; send: bool; data: int; bit: bool);

var
  buff: int; chstate : bool;

let
  buff = if reqData then dataIn else (0 -> pre buff);
  chstate = recB and (bit = ack);
  reqData, send, bit =
    (false, true, true) ->
    pre (if chstate then (true, true, not bit)
     else (false, send, bit));
  data = buff;

tel
The Receiver

- $B$ sends $ack = false$ until it receives $bit = true$; it sends $ack = true$ until it receives $bit = false$;

```plaintext
node B(recA : bool; data: int; bit: bool;)
returns (sendOut: bool; dataOut: int; send2A: bool; ack: bool);

var chstate : bool;

let
    chstate = recA and (ack xor bit);

    sendOut, send2A, ack =
        (false, true, true) ->
            pre (if chstate then (true, true, not ack)
                else (false, true, ack));

    dataOut = data;

tel
```
Modeling the channel and the main property

node unreliable(loose: bool; presIn: bool) returns (presOut: bool);
let
  presOut = presIn and not loose;
tel

-- The property that two signals [r] and [s] alternate.
node altern(r,s: bool) returns (ok: bool);
var
  s0, s1 : bool;
  ps0, ps1 : bool;
let
  ps0 = true -> pre s0;
  ps1 = false -> pre s1;
  s0 = ps0 and (r = s) or ps1 and s and not r;
  s1 = ps0 and r and not s or ps1 and not r and not s;
  ok = (true -> pre ok) and (s0 or s1);
tel
The main system

node obs(dataIn: int; looseA2B, looseB2A : bool;) returns (ok : bool; reqData: bool; sendOut: bool);

var
dataOut: int;
sendA2B: bool; data: int; bit: bool;
recA2B, recB2A : bool;
sendB2A: bool; ack: bool;

let
ok = altern(reqData, sendOut);

recA2B = unreliable(looseA2B, sendA2B);
recB2A = unreliable(looseB2A, sendB2A);

reqData, sendA2B, data, bit = A(dataIn, recB2A, ack);
sendOut, dataOut, sendB2A, ack = B(recA2B, data, bit);
tel

%aneto.local: lesar ba.lus obs
TRUE PROPERTY
Clocks: mixing slow and fast processes

A slow process is made by sampling its inputs; a fast one by oversampling its inputs.

The operators when, current and merge

<table>
<thead>
<tr>
<th>B</th>
<th>false</th>
<th>true</th>
<th>false</th>
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</thead>
<tbody>
<tr>
<td>X</td>
<td>$x_0$</td>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$x_3$</td>
<td>$x_4$</td>
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<tr>
<td>Y</td>
<td>$y_0$</td>
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<tr>
<td>Z = X when B</td>
<td>$x_1$</td>
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<td>$x_3$</td>
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<tr>
<td>K = Y when not B</td>
<td>$y_0$</td>
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<td>$y_2$</td>
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<td>$y_4$</td>
<td>$y_5$</td>
</tr>
<tr>
<td>T = current Z</td>
<td>$nil$</td>
<td>$x_1$</td>
<td>$x_1$</td>
<td>$x_3$</td>
<td>$x_3$</td>
<td>$x_3$</td>
</tr>
<tr>
<td>O = merge B T K</td>
<td>$y_0$</td>
<td>$x_1$</td>
<td>$y_2$</td>
<td>$x_3$</td>
<td>$y_4$</td>
<td>$y_5$</td>
</tr>
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</table>

The operator merge is not part of Lustre. It was introduced later in Lucid Synchrone and SCADE 6.
The Gilbreath trick in SCADE 6

Take a card deck where card alternate; split it in two; shuffle them arbitrarily. Then, if you take two successives cards, their colors are different (provided bottom cards have diff. colors).

node Gilbreath_stream (clock c:bool) returns (prop: bool; o:bool);
var
  s1 : bool when c;
  s2 : bool when not c;
  half : bool;

let
  s1 = (false when c) -> not (pre s1);
  s2 = (true when not c) -> not (pre s2);
  o = merge (c; s1; s2);
  half = false -> (not pre half);

  prop = true -> not (half and (o = pre o));
tel;

The Gilbreath trick in Lustre

node Gilbreath_stream (c:bool) returns (OK: bool; o:bool);
var
    ps1, s1 : bool; ps2, s2 : bool; half : bool;
let

    s1 = if c then not ps1 else ps1;
    ps1 = false -> pre s1;
    s2 = if not c then not ps2 else ps2;
    ps2 = true -> pre s2;

    o = if c then s1 else s2;

    half = false -> (not pre half);

    OK = true -> not (half and (o = pre o));
tel;

Proved automatically using Lesar (Pascal Raymond) and KIND2 (Cesare Tinelli).
A classical use of clock: the activation condition

Run a process on a slower by sub-sampling its inputs; hold outputs.

```plaintext
node sum(i:int) returns (s:int);
  let
    s = i -> pre s + i;
  tel;
```

<table>
<thead>
<tr>
<th>cond</th>
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<tbody>
<tr>
<td>sum(1)</td>
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<td>false</td>
<td>true</td>
<td>false</td>
<td>true</td>
<td>false</td>
<td>true</td>
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<td></td>
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</tr>
<tr>
<td>sum(1 when cond)</td>
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<td>4</td>
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</tr>
<tr>
<td>(sum 1) when cond</td>
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<td>6</td>
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</table>

Sampling inputs vs sampling outputs

- current \((f(x \text{ when } c))\) is called an “activation condition”
- \(f(x \text{ when } c) \neq (f \ x) \text{ when } c\)
- \(\text{current}(x \text{ when } c) \neq x\)
Why synchrony?

let half = true -> not (pre half);
  o = x & (x when half);
tel

It defines the sequence: \( \forall n \in \mathbb{N}. o_n = x_n \& x_{2n} \)

- It cannot be computed in bounded memory.
- Its corresponding Kahn networks has unbounded buffering.
- This is forbidden, a dedicated analysis for that: clock calculus
(Intuitive) Clocking rules in Lustre

Clocks must be declared and visible from the interface of a node.

```plaintext
node stables(i:int) ← base clock (true)
returns (s:int; ncond:bool;  
        (ns:int) when ncond); ← clock declaration

var cond:bool;
(l:int) when cond; ← clock declaration

let
    cond = true -> i <> pre i;
    ncond = not cond;
    l = somme(i when cond);
    s = current(l);
    ns = somme(i when ncond);
tel;
```
Constraints

Rules

- Constants are on the base clock of the node.
- By default, variables are on the base clock of the node.
- Unless a clock is associated to the variable definition.

\[
\text{clock}(e_1 \text{ op } e_2) = \text{clock}(e_1) = \text{clock}(e_2)
\]

\[
\text{clock}(e \text{ when } c) = c
\]

\[
\text{clock(current}; e) = \text{clock(clock}(e))
\]

Implementation choices

- Clocks are declared and verified. No automatic inference.
- Two clocks are equal if expressions that define them are syntactically equal.
One hot coding of Mealy machines

Represent a state by a Boolean variable.

```
node switch(set,reset:bool) returns (ok :bool);
var on: bool;
let
  on = false ->
    if set and not (pre on) then true
    else if reset and (pre on) then false
    else (pre on);
  ok = on;
tel;
```

Think in term of an invariant: what is the expression defining the current value of on at every instant?
Verification with assertions

Consider a second version.

node switch2(set, reset: bool) returns (ok: bool);
  var s1, s2: bool;
let
  s1 = true -> if reset and pre s2 then true
  else if pre s1 and set then false else pre s1;
  s2 = false -> if set and pre s1 then true
  else if pre s2 and reset then false else pre s2;
  ok = s2;
tel;

node compare(set, reset: bool) returns (ok: bool);
  let ok = switch(set, reset) = switch2(set, reset); tel;

We get:

% lesar prog.lus compare
--Pollux Version 2.2

TRUE PROPERTY
Synchronous observers

Comparison is a particular case of a synchronous observer.

- Let $y = F(x)$, and $ok = P(x, y)$ for the property relating $x$ and $y$
- $\text{assert}(H(x, y))$ is an hypothesis on the environment.

\[
\text{node check}(x : t) \text{ returns (ok : bool)};
\]
\[
\text{let}
\]
\[
\text{assert } H(x, y);
\]
\[
y = F(x);
\]
\[
ok = P(x, y);
\]
\[
\text{tel};
\]

If $\text{assert}$ is (infinitely) true, then $ok$ stay infinitely true
($\text{always}(\text{assert})) \Rightarrow (\text{always}(ok))$.

Any safety temporal property can be expressed as a Lustre
program [6, 5]. No temporal logic/language is necessary.

\textbf{Safety temporal properties are regular Lustre programs}
Array are manipulated by slices with implicit point-wise extension of operations. $t[0..N]$ defines a slice of $t$ from index 0 to $N$.

const $N = 10$;

node plus(const $N$: int; a1, a2: int$^N$) returns (o: int$^N$);
  let
    o[1..N] = a1[1..N] + a2[1..N];
  tel;
Arrays

-- serial adder

node add(a1: bool^N; a2: bool^N; carry: bool)
returns (a: bool^N; new_carry: bool);
  var c: bool^N;
  let
    (a[0..N-1], c[0..N-1]) =
      bit_add(a1[0..N-1], a2[0..N-1], ([carry] | c[0..N-2]));
    new_carry = c[N-1];
  tel;

node add_short(a1: bool^N; a2: bool^N; carry: bool)
returns (a: bool^N; new_carry: bool);
  var c: bool^N;
  let
    (a, c) = bit_add(a1, a2, ([carry] | c[0..N-2]));
    new_carry = c[N-1];
  tel;
Conclusion

Compilation

- Static, compile-time checking to ensure the absence of deadlock, that the code behave deterministically.
- Execution in bounded memory and time.
- Code generation into sequential “single loop” code. More advanced methods into automata and/or modular.

Verification by Model-checking

- Synchronous observer: a safety property is a Lustre program
- Avoid to introduce an ad-hoc temporal logic.
- Tool Lesar (BDD technique) by Pascal Raymond (VERIMAG, France).
- KIND and KIND2 (k-induction, PDR based on SMT techniques) by Cesare Tinelli (Iowa State Univ., USA).
- Plug-in (k-induction based on SAT techniques) by Prover-Technologies (associated to SCADE 6).
Related languages and verification tools

Various teams have done their own variant of Lustre.

Language embedding in Haskell

- Copilot (Nasa, USA), an Embedding of Lustre.
- FRAN (images, animation), Functional Reactive Programming (FRP), Hawk (architecture), Lava (synchronous circuits).
- Based on a compilation-by-evaluation technique.

Language extensions, formal verification

- Heptagon: Extended Lustre (automata, arrays) with controller synthesis (Gwenael Delaval, Univ. Grenoble)
- Prelude: Lustre with periodic clocks and a compiler that generates tasks for a real-time OS (Julien Forget, Univ. Lille).
- Lustre compiler at Onera for verification purposes (Pierre-Loic Garoche)
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