6. Conclusion

It has been shown that the existence of a \(\sigma\)-coloration of a particular graph is a necessary and sufficient condition for the existence of a solution to the class-teacher timetable problem with unavailability constraints and preassigned meetings. The knowledge of this necessary and sufficient condition does not provide an efficient algorithm which can be applied to an arbitrary timetable problem in order to determine the existence of a solution. The necessary and sufficient condition does, however, show that existing graph coloring algorithms [1, 6, 14, 15, 16] may be applied to timetable problems with unavailability constraints and preassigned meetings.

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A New Solution of Dijkstra's Concurrent Programming Problem

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A simple solution to the mutual exclusion problem is presented which allows the system to continue to operate despite the failure of any individual component.

Key Words and Phrases: critical section, concurrent programming, multiprocessing, semaphores

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The Algorithm

Consider $N$ asynchronous computers communicating with each other only via shared memory. Each computer runs a cyclic program with two parts—a critical section and a noncritical section. Dijkstra's problem, as extended by Knuth, is to write the programs so that the following conditions are satisfied:

1. At any time, at most one computer may be in its critical section.
2. Each computer must eventually be able to enter its critical section (unless it halts).
3. Any computer may halt in its noncritical section.

Moreover, no assumptions can be made about the running speeds of the computers.

The solutions of [1-4] had all $N$ processors set and test the value of a single variable $k$. Failure of the memory unit containing $k$ would halt the system. The use of semaphores also implies reliance upon a single hardware component.

Our solution assumes $N$ processors, each containing its own memory unit. A processor may read from any other processor's memory, but it need only write into its own memory. The algorithm has the remarkable property that if a read and a write operation to a single memory location occur simultaneously, then only the write operation must be performed correctly. The read may return any arbitrary value!

A processor may fail at any time. We assume that when it fails, it immediately goes to its noncritical section and halts. There may then be a period when reading from its memory gives arbitrary values. Eventually, any read from its memory must give a value of zero. (In practice, a failed computer might be detected by its failure to respond to a read request within a specified length of time.)

Unlike the solutions of [1-4], ours is a first-come-first-served method in the following sense. When a processor wants to enter its critical section, it first executes a loop-free block of code—i.e. one with a fixed number of execution steps. It is then guaranteed to enter its critical section before any other processor which later requests service.

The algorithm is quite simple. It is based upon one commonly used in bakeries, in which a customer receives a number upon entering the store. The holder of the lowest number is the next one served. In our algorithm, each processor chooses its own number. The processors are named $1, \ldots, N$. If two processors choose the same number, then the one with the lowest name goes first.

The common store consists of

\begin{verbatim}
integer array choosing[1:N], number[1:N]
\end{verbatim}

Words choosing $i$ and number $i$ are in the memory of processor $i$, and are initially zero. The range of values of number $i$ is unbounded. This will be discussed below.

The following is the program for processor $i$. Execution must begin inside the noncritical section. The arguments of the maximum function can be read in any order. The relation "less than" on ordered pairs of integers is defined by $(a,b) < (c,d)$ if $a < c$, or if $a = c$ and $b < d$.

\begin{verbatim}
begin integer j;
L1: choosing[i] := 1;
    number[i] := maximum (number[1], \ldots, number[N]);
    choosing[i] := 0;
    for j = 1 step 1 until N do
        if choosing[j] = 0 then goto L2;
L2: if number[j] = 0 and (number[j], j) < (number[i], i) then goto L3;
    end;
end
\end{verbatim}

We allow processor $i$ to fail at any time, and then to be restarted in its noncritical section (with choosing $i = number [i] = 0$). However, if a processor keeps failing and restarting, then it can deadlock the system.

Proof of Correctness

To prove the correctness of the algorithm, we first make the following definitions. Processor $i$ is said to be in the doorway while choosing $i = 1$. It is said to be in the bakery from the time it resets choosing ($i$) to zero until it either fails or leaves its critical section. The correctness of the algorithm is deduced from the following assertions. Note that the proofs make no assumptions about the value read during an overlapping read and write to the same memory location.

\textbf{Assertion 1.} If processors $i$ and $k$ are in the bakery and $i$ entered the bakery before $k$ entered the doorway, then number $i < number k$.

\textbf{Proof.} By hypothesis, number $i$ had its current value while $k$ was choosing the current value of number $k$. Hence, $k$ must have chosen number $k \geq 1 + number [i]$.

\textbf{Assertion 2.} If processor $i$ is in its critical section, processor $k$ is in the bakery, and $k \neq i$, then (number $i$, $i$) < (number $k$, $k$).

\textbf{Proof.} Since choosing $k$ has essentially just two values—zero and nonzero—we can assume that from processor $i$'s point of view, reading or writing it is done instantaneously, and a simultaneous read and write does not occur. For example, if choosing $k$ is being changed from zero to one while it is also being read by processor $i$, then the read is considered to happen first if it obtains a value of zero; otherwise the write is said to happen first. All times defined in the proof are from processor $i$'s viewpoint.

Let $t_{L2}$ be the time at which processor $i$ read choosing $k$ during its last execution of $L2$ for $j = k$, and let $t_{L3}$ be the time at which $i$ began its last execution of $L3$ for $j = k$, so $t_{L2} < t_{L3}$. When processor $k$ was choosing its
current value of \( \text{number} [k] \), let \( t_w \) be the time at which it entered the doorway, \( t_f \) the time at which it finished writing the value of \( \text{number} [k] \), and \( t_e \) the time at which it left the doorway. Then \( t_w < t_f < t_e \).

Since choosing \( [k] \) was equal to zero at time \( t_{L2} \), we have either (a) \( t_{L2} < t_w \) or (b) \( t_f < t_{L2} \). In case (a), Assertion 1 implies that \( \text{number} [i] < \text{number} [k] \), so the assertion holds.

In case (b), we have \( t_w < t_f < t_{L2} < t_{L3} \), so \( t_w < t_{L2} \). Hence, during the execution of statement \( L3 \) begun at time \( t_{L2} \), processor \( i \) read the current value of \( \text{number} [k] \). Since \( i \) did not execute \( L3 \) again for \( j = k \), it must have found \( (\text{number} [i], i) < (\text{number} [k], k) \). Hence, the assertion holds in this case, too.  

**Assertion 3.** Assume that only a bounded number of processor failures may occur. If no processor is in its critical section and there is a processor in the bakery which does not fail, then some processor must eventually enter its critical section.

**Proof.** Assume that no processor ever enters its critical section. Then there will be some time after which no more processors enter or leave the bakery. At this time, assume that processor \( i \) has the minimum value of \( \text{number} [i], i \) among all processors in the bakery. Then processor \( i \) must eventually complete the for loop and enter its critical section. This is the required contradiction.  

Assertion 2 implies that at most one processor can

The algorithm can be generalized in two ways: (i) under certain circumstances, to allow two processors simultaneously to be in their critical sections; and (ii) to modify the first-come-first-served property so that higher priority processors are served first. This will be described in a future paper.

**Conclusion**

Our algorithm provides a new, simple solution to the mutual exclusion problem. Since it does not depend upon any form of central control, it is less sensitive to component failure than previous solutions.

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**References**