6. Conclusion

It has been shown that the existence of a $\sigma$-coloration of a particular graph is a necessary and sufficient condition for the existence of a solution to the class-teacher timetable problem with unavailability constraints and preassigned meetings. The knowledge of this necessary and sufficient condition does not provide an efficient algorithm which can be applied to an arbitrary timetable problem in order to determine the existence of a solution. The necessary and sufficient condition does, however, show that existing graph coloring algorithms [1, 6, 14, 15, 16] may be applied to timetable problems with unavailability constraints and preassigned meetings.
The Algorithm

Consider $N$ asynchronous computers communicating with each other only via shared memory. Each computer runs a cyclic program with two parts—a critical section and a noncritical section. Dijkstra's problem, as extended by Knuth, is to write the programs so that the following conditions are satisfied:

1. At any time, at most one computer may be in its critical section.
2. Each computer must eventually be able to enter its critical section (unless it halts).
3. Any computer may halt in its noncritical section.

Moreover, no assumptions can be made about the running speeds of the computers.

The solutions of [1-4] had all $N$ processors set and test the value of a single variable $k$. Failure of the memory unit containing $k$ would halt the system. The use of semaphores also implies reliance upon a single hardware component.

Our solution assumes $N$ processors, each containing its own memory unit. A processor may read from any other processor's memory, but it need only write into its own memory. The algorithm has the remarkable property that if a read and a write operation to a single memory location occur simultaneously, then only the write operation must be performed correctly. The read may return any arbitrary value!

A processor may fail at any time. We assume that when it fails, it immediately goes to its noncritical section and halts. There may then be a period when reading from its memory gives arbitrary values. Eventually, any read from its memory must give a value of zero. (In practice, a failed computer might be detected by its failure to respond to a read request within a specified length of time.)

Unlike the solutions of [1-4], ours is a first-come-first-served method in the following sense. When a processor wants to enter its critical section, it first executes a loop-free block of code—i.e., one with a fixed length of time. It is then guaranteed to enter its critical section before any other processor which later requests service.

We allow processor $i$ to fail at any time, and then to be restarted in its noncritical section (with choosing $[i] = number[i] = 0$). However, if a processor keeps failing and restarting, then it can deadlock the system.

Proof of Correctness

To prove the correctness of the algorithm, we first make the following definitions. Processor $i$ is said to be in the doorway while choosing $[i] = 1$. It is said to be in the bakery from the time it resets choosing $(i)$ to zero until it either fails or leaves its critical section. The correctness of the algorithm is deduced from the following assertions. Note that the proofs make no assumptions about the value read during an overlapping read and write to the same memory location.

Assertion 1. If processors $i$ and $k$ are in the bakery and $i$ entered the bakery before $k$ entered the doorway, then $number[i] \leq number[k]$.
current value of $number\ [k]$, let $t_o$ be the time at which it entered the doorway, $t_w$ the time at which it finished writing the value of $number\ [k]$, and $t_c$ the time at which it left the doorway. Then $t_o < t_w < t_c$.

Since $choosing\ [k]$ was equal to zero at time $t_{12}$, we have either (a) $t_{12} < t_o$ or (b) $t_o < t_{12}$, In case (a), Assertion 1 implies that $number\ [i] < number\ [k]$, so the assertion holds.

In case (b), we have $t_w < t_o < t_{12} < t_{13}$, so $t_w < t_{12}$. Hence, during the execution of statement $L3$ begun at time $t_{14}$, processor $i$ read the current value of $number\ [k]$. Since $i$ did not execute $L3$ again for $j = k$, it must have found $(number\ [i], i) < (number\ [k], k)$. Hence, the assertion holds in this case, too. □

Assertion 3. Assume that only a bounded number of processor failures may occur. If no processor is in its critical section and there is a processor in the bakery which does not fail, then some processor must eventually enter its critical section.

Proof. Assume that no processor ever enters its critical section. Then there will be some time after which no more processors enter or leave the bakery. At this time, assume that processor $i$ has the minimum value of $(number\ [i], i)$ among all processors in the bakery. Then processor $i$ must eventually complete the for loop and enter its critical section. This is the required contradiction. □

Assertion 2 implies that at most one processor can be in its critical section at any time. Assertions 1 and 2 prove that processors enter their critical sections on a first-come-first-served basis. Hence, an individual processor cannot be blocked unless the entire system is dead-locked. Assertion 3 implies that the system can only be deadlocked by a processor halting in its critical section, or by an unbounded sequence of processor failures and re-entries. The latter can tie up the system as follows. If processor $j$ continually fails and restarts, then with bad luck processor $i$ could always find $choosing\ [j] = 1$, and loop forever at $L2$.

Further Remarks

If there is at least one processor in the bakery, then the value of $number\ [i]$ can become arbitrarily large. This problem cannot be solved by any simple scheme of cycling through a finite set of integers. For example, given any numbers $r$ and $s$, if $N \geq 4$, then it is possible to have simultaneously $number\ (i) = r$ and $number\ (j) = s$ for some $i$ and $j$.

Fortunately, practical considerations will place an upper bound on the value of $number\ [i]$ in any real application. For example, if processors enter the doorway at the rate of at most one per m/sec, then after a year of operation we will have $number\ [i] < 2^{20}$—assuming that a read of $number\ [i]$ can never obtain a value larger than one which has been written there.

The unboundedness of $number\ [i]$ does raise an interesting theoretical question: can one find an algorithm for finite processors such that processors enter their critical sections on a first-come-first-served basis, and no processor may write into another processor's memory? The answer is not known.  

The algorithm can be generalized in two ways: (i) under certain circumstances, to allow two processors simultaneously to be in their critical sections; and (ii) to modify the first-come-first-served property so that higher priority processors are served first. This will be described in a future paper.

Conclusion

Our algorithm provides a new, simple solution to the mutual exclusion problem. Since it does not depend upon any form of central control, it is less sensitive to component failure than previous solutions.

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References


Computer Systems

Erratum

In "A Note on Subexpression Ordering in the Evaluation of Arithmetic Expressions" by Peter J. Denning and G. Scott Graham, Comm. ACM 16, 11 (Nov. 1973), 700-702, the following erratum has been submitted by Denning.

The first two sentences in the first full paragraph on p. 701 should read as follows:

Hu shows that an optimal list $L_0$ for any $m$ and any tree (of equal-execution-time tasks) can be constructed by taking a first appearance of each task in the sequence $M_{11}, M_{22}, \ldots, M_{KK}$. Ramamoorthy and Gonzales order the tasks of each $M_{ij}$ according to decreasing execution time, then construct a list $L$ by taking the first appearance of each task in the sequence $M_{11}, \ldots, M_{1K}, M_{22}, \ldots, M_{2K}, M_{33}, \ldots, M_{3K}, \ldots, M_{KK}$; they claim that $L$ is optimal for any tree and any $m$.

It should be noted that even for equal-execution-time tasks, a list constructed from the latter sequence above need not be consistent with the former sequence above and, hence, need not be optimal for that reason alone.

We are grateful to Dr. Shimon Even for calling our unfortunately incorrect wording to our attention.