Systèmes réactifs synchrones MPRI 2-23-1
Synchronous Reactive Systems
A formally verified compiler for Lustre

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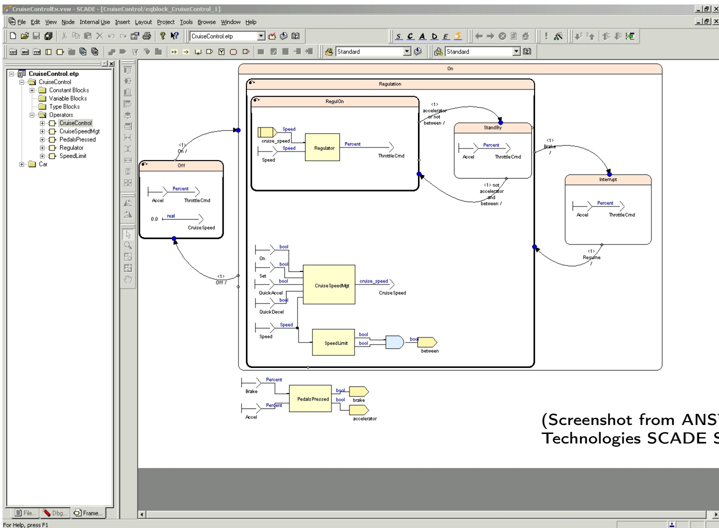
This course describes an ongoing research collaboration with Lélio Brun, Pierre-Évariste Dagand, Paul Jeanmaire, Xavier Leroy, Basile Pesin, Marc Pouzet, and Lionel Rieg.

10 October 2023



(Screenshot from ANSYS/Esterel Technologies SCADE Suite)

- Widely used to program safety-critical software:
  - Aerospace, Defense, Rail Transportation, Heavy Equipment, Energy, Nuclear.
  - Airbus (A340, A380), Comac, EADS Astrium, Embraer, Eurocopter, PIAGGIO Aerospace, Pratt & Whitney, Sukhoi, Turbomeca, Siemens, ...
- DO-178B level A certified development tool.

Certification for Safety-Critical Systems

- DO-178C: "Software Considerations in Airborne Systems and Equipment Certification"
  - » Defines a software development process.
  - » Manual review of artifacts related to design, code, and test cases.

  [Rushby (2011): New Challenges in Certification for Aircraft Software]

- The Scade KCG code generator is *certified*.
  - » No need for code reviews of generated code.
  - » Can test based on High-Level Requirements.
  - » Evaluate testing coverage on models.

- Different approach: formal specification and *verification* of a code generator.
  - » Not directly applicable to certification processes

    [Wagner, Cofer, Slind, Tinelli, and Mebsout (2017): Formal Methods Tool Qualification]
  - » Can it complement existing approaches in other ways?

We are interested in the scientific questions and the technical challenges.

Overview

Lustre: syntax and semantics

Lustre: normalization

Translation: from dataflow programs to imperative code

Optimization: control structure fusion

Generation: from Obc to Clight

Conclusion

What are we doing?

- Implement a Lustre compiler in the Coq Interactive Theorem Prover.
  - Following a previous attempt [Auger, Colaço, Hamon, and Pouzet (2014): A Formalization and Proof of a Modular Lustre Code Generator].
- Prove that the generated code implements the dataflow semantics.

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- Prove that the generated code implements the dataflow semantics.
- Coq? [The Coq Development Team (2016): The Coq proof assistant reference manual]
  - A functional programming language;
  - 'Extraction' to OCaml programs;
  - A specification language (a form of higher-order logic);
  - Tactic-based interactive proof.

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  CompCert: a formal model and compiler for a subset of C
  - A generic machine-level model of execution and memory
  - A verified path to assembly code output (PowerPC, ARM, x86)
  - No need for a garbage-collected runtime.

  [Blazy, Dargaye, and Leroy (2006): Formal Verification of a C Compiler Front-End] [Leroy (2009): Formal verification of a realistic compiler]

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- Computer assistance is all but essential for such detailed models.

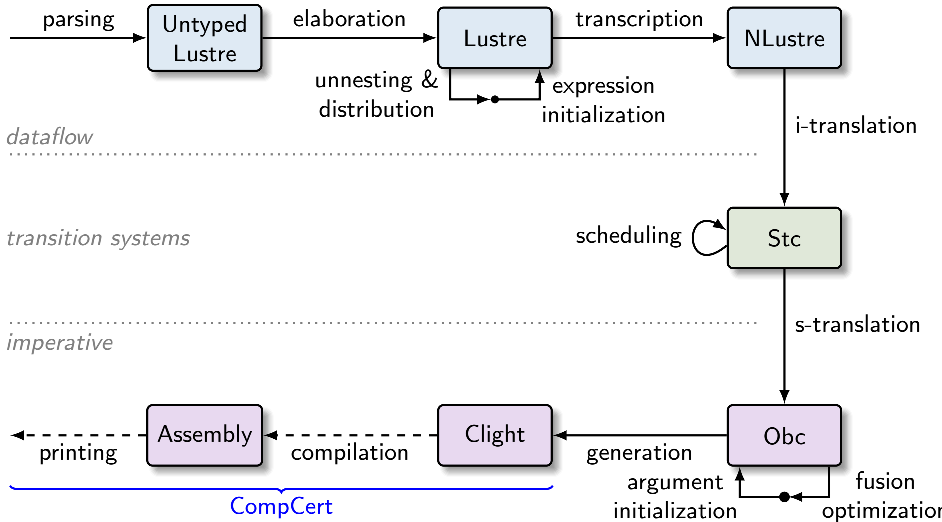
Why do it?

- Challenge: Never done before: how to do it?
  (as methodically and as simply as possible)

- Vision: Verify Lustre programs;
  Get guarantees about assembly code.
  - » Treat the details of real processors
    (memory representations and arithmetic).
  - » Treat external functions called by Lustre programs.
  - » Verify abstract ('almost executable') models of reactive-systems
    (static parameters, real-time properties).

- 'Digitized' formal models of Lustre.
  - » Machine-manipulable specification of language, type systems, compilation.
  - » Starting point for other projects:
    - Semantics of modular reset and state machines.
    - Detailed treatment of side-effects and host language.
    - Analysis of optimization passes. An aid to industrial certification?

The Vélus Lustre Compiler



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Lustre: syntax

```
Definition ann : Type := (type * nclock).
Definition lann : Type := (list type * nclock).

Inductive exp : Type :=
| Econst : const → exp
| Evar   : ident → ann → exp
| Eunop  : unop → exp → ann → exp
| Ebinop : binop → exp → exp → ann → exp

| Ewhen  : list exp → ident → bool → lann → exp
| Emerge : ident → list exp → list exp → lann → exp
| Eite   : exp → list exp → list exp → lann → exp
| Efby   : list exp → list exp → list ann → exp

| Eapp   : ident → list exp → option exp → list ann → exp.

Definition equation : Type := (list ident * list exp).

Record node : Type :=
  mk_node {
    n_name    : ident;
    n_in      : list (ident * (type * clock));
    n_out     : list (ident * (type * clock));
    n_vars    : list (ident * (type * clock));
    n_eqs     : list equation;

    n_ingt0   : 0 < length n_in;
    n_outgt0  : 0 < length n_out;
    n_defd    : Permutation (vars_defined n_eqs)
                            (map fst (n_vars ++ n_out));
    n_nodup   : NoDupMembers (n_in ++ n_vars ++ n_out);
    n_good    : Forall NotReserved (n_in ++ n_vars ++ n_out)
  }.
```

```
(* No tuples. `Lists of flows' are flattened: *)
node shuffle (a, b, c, d : bool)
returns (w, x, y, z : bool);

(w, x, y, z) = shuffle(((a, (b, (c)), d)));
```

(e,...,e) when x / (e,...,e) when not x

merge x (e,...,e) (e,...,e)

(e,...,e) fby (e,...,e)

f(e, ..., e)

x,...,x = e,...,e

node f(x,...,x) returns (y,...,y);
var w, ..., w;
let x = ...; ... tel

Lustre: semantics

- The usual story: associate Abstract Syntax Trees (programs) to models
- Sequential programs:
  - A state associates each variable to a value
  - Expressions are evaluated in the state
  - The state is updated by successive commands
  - Possibly include a sequence of interleaved ‘external events’
- Dataflow programs:
  - Associate each variable with a stream of values
  - Track synchronization by making absence explicit
  - Models the ‘grid’ that associates variables to synchronized streams
  - Each equation constrains the set of possible models

Semantic model: design constraints

- Judging the (informal) ‘correctness’ of the development.
- Proving properties of programs.
- Proving correctness of compilation.
- Proving properties of the language
  - Existence of semantics
  - Determinism of language
  - Correctness of the clocking system

Pen-and-paper Kahn semantics [Colaço and Pouzet (2003): Clocks as First Class Abstract Types]

$$
\begin{aligned}
i^{\#} &= i.i^{\#} \\
op^{\#}(s_1, s_2) &= \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon \\
op^{\#}(v_1.s_1, v_2.s_2) &= (v_1 \; op \; v_2).op^{\#}(s_1, s_2) \\
\\
\texttt{fby}^{\#}(\epsilon, ys) &= \epsilon \\
\texttt{fby}^{\#}(v.xs, ys) &= v.ys \\
\\
\texttt{when}^{\#}(s_1, s_2) &= \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon \\
\texttt{when}^{\#}(v_1.s_1, \mathit{true}.s_2) &= v_1.(\texttt{when}^{\#}(s_1, s_2)) \\
\texttt{when}^{\#}(v_1.s_1, \mathit{false}.s_2) &= \texttt{when}^{\#}(s_1, s_2) \\
\\
\texttt{merge}^{\#}(s_1, s_2, s_3) &= \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon \text{ or } s_3 = \epsilon \\
\texttt{merge}^{\#}(\mathit{true}.s_1, v_2.s_2, s_3) &= v_2.\texttt{merge}^{\#}(s_1, s_2, s_3) \\
\texttt{merge}^{\#}(\mathit{false}.s_1, s_2, v_3.s_3) &= v_3.\texttt{merge}^{\#}(s_1, s_2, s_3)
\end{aligned}
$$

- Ignoring synchronization gives a relatively simple model based on fixed points of mutually recursive stream transformers.
- 'Machines' reading and writing through unbounded FIFOs
  [Kahn (1974): The Semantics of a Simple Language for Parallel Programming]

Pen-and-paper synchronous semantics [Colaço and Pouzet (2003): Clocks as First Class Abstract Types]

$$
\begin{aligned}
i^{\#}[\epsilon] &= \epsilon \\
i^{\#}[true.cl] &= v.i^{\#}[cl] \\
i^{\#}[false.cl] &= abs.i^{\#}[cl] \\[1em]
op^{\#}(s_1, s_2) &= \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon \\
op^{\#}(abs.s_1, abs.s_2) &= abs.op^{\#}(s_1, s_2) \\
op^{\#}(v_1.s_1, v_2.s_2) &= (v_1\ op\ v_2).op^{\#}(s_1, s_2) \\[1em]
\texttt{fby}^{\#}(\epsilon, ys) &= \epsilon \\
\texttt{fby}^{\#}(abs.xs, abs.ys) &= abs.\texttt{fby}^{\#}(xs, ys) \\
\texttt{fby}^{\#}(x.xs, y.ys) &= x.\texttt{fby1}^{\#}(y, xs, ys) \\
\texttt{fby1}^{\#}(v, \epsilon, ys) &= \epsilon \\
\texttt{fby1}^{\#}(v, abs.xs, abs.ys) &= abs.\texttt{fby1}^{\#}(v, xs, ys) \\
\texttt{fby1}^{\#}(v, w.xs, s.ys) &= v.\texttt{fby1}^{\#}(s, xs, ys) \\[1em]
\texttt{when}^{\#}(s_1, s_2) &= \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon \\
\texttt{when}^{\#}(abs.xs, abs.cs) &= abs.\texttt{when}^{\#}(xs, cs) \\
\texttt{when}^{\#}(x.xs, true.cs) &= x.\texttt{when}^{\#}(xs, cs) \\
\texttt{when}^{\#}(x.xs, false.cs) &= abs.\texttt{when}^{\#}(xs, cs) \\[1em]
\texttt{merge}^{\#}(s_1, s_2, s_3) &= \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon \text{ or } s_3 = \epsilon \\
\texttt{merge}^{\#}(abs.cs, abs.xs, abs.ys) &= abs.\texttt{merge}^{\#}(cs, xs, ys) \\
\texttt{merge}^{\#}(true.cs, x.xs, abs.ys) &= x.\texttt{merge}^{\#}(cs, xs, ys) \\
\texttt{merge}^{\#}(false.cs, abs.xs, y.ys) &= y.\texttt{merge}^{\#}(cs, xs, ys)
\end{aligned}
$$

- Synchronous semantics: explicitly represent absence
- The definitions become more technical
- Defined for fewer programs
- The link to discrete time is explicit
- The model is closer to an implementation
- Unbounded FIFOs are not required

Dataflow semantic model

sec	F	F	F	T	F	T	T	...	base
r	0	1	3	4	6	9	9	...	base
r when sec				4		9	9	...	base on (sec = T)
t				1		2	3	...	base on (sec = T)
(0 fby v) when not sec	0	0	0		4			...	base on (sec = F)

- History environment maps identifiers to streams.

```
Definition history := PM.t (Stream value)
```

- Model absence

```
Inductive value := absent | present (v : const).
```

- Lists: $1 :: (2 :: (3 :: (4 :: [])))$ *or* $(((\epsilon \cdot 1) \cdot 2) \cdot 3) \cdot 4$
- Coinductive streams

```
CoInductive Stream (A : Type) :=
  Cons : A → Stream A → Stream A.
```

- Functions from natural numbers to values:

```
Notation stream A := (nat → A).
```

Lustre: semantics 1/4

```
Definition history := PM.t (Stream value).

Notation sem_var H := (fun (x: ident) (s: Stream value) ⇒ PM.MapsTo x s H).

Inductive sem_exp : history → Stream bool → exp → list (Stream value) → Prop :=
| Sconst:
    sem_exp H b (Econst c) [const c b]

| Svar:
    sem_var H x s →
    sem_exp H b (Evar x ann) [s]

| Swhen:
    Forall2 (sem_exp H b) es ss →
    sem_var H x s →
    Forall2 (when k s) (concat ss) os →
    sem_exp H b (Ewhen es x k lann) os

| ...
```

```
CoFixpoint const (c: const) (b: Stream bool) : Stream value :=
  match b with
  | true  ::: b' ⇒ present (sem_const c) ::: const c b'
  | false ::: b' ⇒ absent ::: const c b'
  end.
```

$$i^{\#}[\epsilon] = \epsilon$$
$$i^{\#}[true.cl] = v.i^{\#}[cl]$$
$$i^{\#}[false.cl] = abs.i^{\#}[cl]$$

```
CoInductive when (k: bool)
   : Stream value → Stream value → Stream value → Prop :=
| WhenA:
     when k xs cs rs →
     when k (absent ::: cs) (absent ::: xs) (absent ::: rs)

| WhenPA:
     when k xs cs rs →
     val_to_bool c = Some (negb k) →
     when k (present c ::: cs) (present x ::: xs) (absent ::: rs)

| WhenPP:
     when k xs cs rs →
     val_to_bool c = Some k →
     when k (present c ::: cs) (present x ::: xs) (present x ::: rs)
```

$$\mathtt{when}^{\#}(s_1, s_2) = \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon$$
$$\mathtt{when}^{\#}(abs.xs, abs.cs) = abs.\mathtt{when}^{\#}(xs, cs)$$
$$\mathtt{when}^{\#}(x.xs, true.cs) = x.\mathtt{when}^{\#}(xs, cs)$$
$$\mathtt{when}^{\#}(x.xs, false.cs) = abs.\mathtt{when}^{\#}(xs, cs)$$

Lustre: semantics 2/4

```
Inductive sem_exp : history → Stream bool → exp → list (Stream value) → Prop :=
| Sconst:
    sem_exp H b (Econst c) [const c b]

| ...

| Smerge:
    sem_var H x s →
    Forall2 (sem_exp H b) ets ts →
    Forall2 (sem_exp H b) efs fs →
    Forall3 (merge s) (concat ts) (concat fs) os →
    sem_exp H b (Emerge x ets efs lann) os
```

$$
\begin{aligned}
\texttt{merge}^{\#}(s_1, s_2, s_3) &= \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon \text{ or } s_3 = \epsilon \\
\texttt{merge}^{\#}(abs.cs, abs.xs, abs.ys) &= abs.\texttt{merge}^{\#}(cs, xs, ys) \\
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\texttt{merge}^{\#}(false.cs, abs.xs, y.ys) &= y.\texttt{merge}^{\#}(cs, xs, ys)
\end{aligned}
$$

```
CoInductive merge
| MergeA:
    merge xs ts fs rs →
    merge (           absent ::: xs) (   absent ::: ts) (   absent ::: fs) (   absent ::: rs)
| MergeT:
    merge xs ts fs rs →
    merge (present  true_val ::: xs) (present t ::: ts) (   absent ::: fs) (present t ::: rs)
| MergeF:
    merge xs ts fs rs →
    merge (present false_val ::: xs) (   absent ::: ts) (present f ::: fs) (present f ::: rs).
```

$$\frac{s \equiv \text{const } bs \, [\![\text{c}]\!]}{G, H, bs \vdash \text{c} \Downarrow [s]}$$

$$\text{const } (\text{T} \cdot bs) \; c = \langle c \rangle \cdot \text{const } bs \; c$$

$$\text{const } (\text{F} \cdot bs) \; c = \langle\rangle \cdot \text{const } bs \; c$$

(a) Constants

$$\frac{G, H, bs \vdash \boldsymbol{e} \Downarrow H(\boldsymbol{x})}{G, H, bs \vdash \boldsymbol{x} = \boldsymbol{e}}$$

(b) Equations

$$\frac{H(x) = s \qquad G, H, bs \vdash \boldsymbol{e_t} \Downarrow \boldsymbol{ts} \qquad G, H, bs \vdash \boldsymbol{e_f} \Downarrow \boldsymbol{fs} \qquad \text{merge } s \; \boldsymbol{ts} \; \boldsymbol{fs} \doteq \boldsymbol{vs}}{G, H, bs \vdash \texttt{merge(} \; x; \; \boldsymbol{e_t}; \; \boldsymbol{e_f}) \Downarrow \boldsymbol{vs}}$$

$$\frac{\text{merge } cs \; ts \; fs \doteq vs}{\text{merge } (\langle\rangle \cdot cs) \; (\langle\rangle \cdot ts) \; (\langle\rangle \cdot fs) \doteq \langle\rangle \cdot vs}$$

$$\frac{\text{merge } cs \; ts \; fs \doteq vs}{\text{merge } (\langle \text{T} \rangle \cdot cs) \; (\langle t \rangle \cdot ts) \; (\langle\rangle \cdot fs) \doteq \langle t \rangle \cdot vs}$$

$$\frac{\text{merge } cs \; ts \; fs \doteq vs}{\text{merge } (\langle \text{F} \rangle \cdot cs) \; (\langle\rangle \cdot ts) \; (\langle f \rangle \cdot fs) \doteq \langle f \rangle \cdot vs}$$

(c) Rule and operator for `merge`

$$\frac{\text{node}(G, f) \doteq n \qquad H(n.\mathbf{in}) = \boldsymbol{xs} \qquad H(n.\mathbf{out}) = \boldsymbol{ys} \qquad \forall eq \in n.\mathbf{eqs}, \; G, H, (\texttt{base-of } \boldsymbol{xs}) \vdash eq}{G \vdash f(\boldsymbol{xs}) \Downarrow \boldsymbol{ys}}$$

(d) Nodes

Fig. 8. Lustre: Selected semantic rules and operators

Lustre: semantics 3/4

```
Inductive sem_exp : history → Stream bool → exp → list (Stream value) → Prop :=
| Sconst:
    sem_exp H b (Econst c) [const c b]

| ...

| Sfby:
    Forall2 (sem_exp H b) e0s s0ss →
    Forall2 (sem_exp H b) es sss →
    Forall3 fby (concat s0ss) (concat sss) os →
    sem_exp H b (Efby e0s es anns) os
```

$$
\begin{aligned}
\mathtt{fby}^{\#}(\epsilon, ys) &= \epsilon \\
\mathtt{fby}^{\#}(abs.xs, abs.ys) &= abs.\mathtt{fby}^{\#}(xs, ys) \\
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\mathtt{fby1}^{\#}(v, w.xs, s.ys) &= v.\mathtt{fby1}^{\#}(s, xs, ys)
\end{aligned}
$$

```
CoInductive fby
| FbyA:
      fby xs ys rs →
      fby (absent ::: xs) (absent ::: ys) (absent ::: rs)

| FbyP:
      fby1 y xs ys rs →
      fby (present x ::: xs) (present y ::: ys) (present x ::: rs).

CoInductive fby1
| Fby1A:
      fby1 v xs ys rs →
      fby1 v (absent ::: xs) (absent ::: ys) (absent ::: rs)

| Fby1P:
      fby1 s xs ys rs →
      fby1 v (present w ::: xs) (present s ::: ys) (present v ::: rs).
```

Lustre: semantics 4/4

```
Inductive sem_exp : history → Stream bool → exp → list (Stream value) → Prop :=
| Sconst:
    sem_exp H b (Econst c) [const c b]

| …

| Sapp:
    Forall2 (sem_exp H b) es ss →
    sem_node f (concat ss) os →
    sem_exp H b (Eapp f es lann) os

| …

with sem_equation: history → Stream bool → equation → Prop :=
| Seq:
    Forall2 (sem_exp H b) es ss →
    Forall2 (sem_var H) xs (concat ss) →
    sem_equation H b (xs, es)

with sem_node: ident → list (Stream value) → list (Stream value) → Prop :=
| Snode:
    find_node f G = Some n →
    Forall2 (sem_var H) (idents n.(n_in)) ss →
    Forall2 (sem_var H) (idents n.(n_out)) os →
    Forall (sem_equation H b) n.(n_eqs) →
    b = sclocksof ss →
    sem_node f ss os.
```

```
CoFixpoint clocks_of (ss: list (Stream value)) : Stream bool :=
  ∃b (fun s⇒ hd s <>b absent) ss ::: clocks_of (List.map tl ss).
```

Semantics Summary

- Relational semantic model: a set of constraints on an environment H
- Inductive rules over syntax using coinductive operators on streams.
- Good:
  - No need to worry about fixed points
  - Easy to destruct and construct in correctness proofs
  - Works well in compiler correctness proofs
- Less good:
  - We cannot execute the model to see what it means
  - Are the constraints satisfiable? (Existence of a semantics, > 0)
  - Does a program have one model? (Determinism, $<= 1$)
  - What about proofs about programs?

Prior work on Lustre semantics in Coq

- [Boulmé and Hamon (2001): A clocked denotational semantics for Lucid-Synchrone in Coq]
  - Shallow embedding of Lucid Synchrone into Coq.
  - Embed the clocking rules into the Coq type system.
  - Use clocks (boolean streams) to control rhythms.
  - Denotational semantics using co-fixpoints.
  - Values: present, absent, and *fail*.

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- [Paulin-Mohring (2009): A constructive denotational semantics for Kahn networks in Coq]
  - Denotational semantics of Kahn process networks.
  - ```
    CoInductive Str (A:Type) : Type :=
    | Eps: Str A → Str A
    | cons: A → Str A → Str A
    ```
  - Least element: $\texttt{Eps}^{\infty}$
  - Shallow embedding of programs.
    ```

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    ```
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  - Shallow embedding of programs.

- [Auger (2013): Compilation certifiée de SCADE/LUSTRE]
  - Streams as (backward) finite lists.
  - Deep embedding of programs.
  - Relational semantics linking programs to lists.
    ```

Semantics of the Modular Reset

- The modular reset is a relatively new feature compared to the classic Lustre operators
- It introduces imperative characteristics into the dataflow language
- Coq formalization:
  - Cannot simply just encode existing pen-and-paper definitions
  - Need to invent new ones
- Early definition propagates 'reset wires' through to fbys
  [Hamon and Pouzet (2000): Modular Resetting of Synchronous Data-Flow Programs]
- In fact, the reset can be expressed as a recursive dataflow program
  [Caspi (1994): Towards recursive block diagrams] [Hamon and Pouzet (2000): Modular Resetting of Synchronous Data-Flow Programs]
- This inspired the original approach in L. Brun's thesis
  [Brun (2020): Mechanized Semantics and Verified Compilation for a Dataflow Synchronous Language with Reset] [Bourke, Brun, and Pouzet (2020): Mechanized Semantics and Verified Compilation for a Dataflow Synchronous Language with Reset]

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

A simple example with reset

```
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let
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tel
```

r	F
i	0
$nat(i)$	0
(restart nat every r)(i)	0

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

r	F	F
i	0	5
$nat(i)$	0	1
(restart nat every r)(i)	0	1

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

r	F	F	T
i	0	5	10
$nat(i)$	0	1	2
(restart nat every r)(i)	0	1	10

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

r	F	F	T	F
i	0	5	10	15
$nat(i)$	0	1	2	3
(restart nat every r)(i)	0	1	10	11

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

r	F	F	T	F	F
i	0	5	10	15	20
$nat(i)$	0	1	2	3	4
(restart nat every r)(i)	0	1	10	11	12

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

r	F	F	T	F	F	T
i	0	5	10	15	20	25
$nat(i)$	0	1	2	3	4	5
(restart *nat* every r)(i)	0	1	10	11	12	25

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

r	F	F	T	F	F	T	F
i	0	5	10	15	20	25	30
$nat(i)$	0	1	2	3	4	5	6
(restart *nat* every r)(i)	0	1	10	11	12	25	26

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

r	F	F	T	F	F	T	F	T
i	0	5	10	15	20	25	30	35
$nat(i)$	0	1	2	3	4	5	6	7
(restart nat every r)(i)	0	1	10	11	12	25	26	35

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

r	F	F	T	F	F	T	F	T	F
i	0	5	10	15	20	25	30	35	40
$nat(i)$	0	1	2	3	4	5	6	7	8
(restart nat every r)(i)	0	1	10	11	12	25	26	35	36

A simple example with reset

```
node nat(i: int) returns (n: int)
let
  n = i fby (n + 1);
tel
```

r	F	F	T	F	F	T	F	T	F	⋯
i	0	5	10	15	20	25	30	35	40	⋯
$nat(i)$	0	1	2	3	4	5	6	7	8	⋯
(restart nat every r)(i)	0	1	10	11	12	25	26	35	36	⋯

Semantics of modular reset

Recursive definition (not valid in Lustre) [Hamon and Pouzet (2000): Modular Resetting of Synchronous Data-Flow Programs]

> f(x) every r acts as f(x) until r is true and after r has been true, [it] acts as f(x') every r' where x' (resp. r') is the sub-stream of x (resp. r) starting when r is true.

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```
node true_until(r: bool) returns (c: bool)
let
  c = if r then false else (true fby c);
tel

node reset_f(x: int, r: bool) returns (y: int)
  var c: bool;
let
  c = true_until(r);
  y = merge c (f(x when c)) (reset_f((x, r) whenot c));
tel
```

Semantics of modular reset

Recursive definition (not valid in Lustre) [Hamon and Pouzet (2000): Modular Resetting of Synchronous Data-Flow Programs]

> f(x) every r acts as f(x) until r is true and after r has been true, [it] acts as f(x') every r' where x' (resp. r') is the sub-stream of x (resp. r) starting when r is true.

```
node true_until(r: bool) returns (c: bool)
let
  c = if r then false else (true fby c);
tel

node reset_f(x: int, r: bool) returns (y: int)
  var c: bool;
let
  c = true_until(r);
  y = merge c (f(x when c)) (reset_f((x, r) whenot c));
tel
```

Use an intricate co-inductive predicate or find a simpler solution?

Semantics of modular reset: unrolling the recursion

r	F	F	T	F	F	T	F	T	F	$\cdots$
i	0	5	10	15	20	25	30	35	40	$\cdots$

Semantics of modular reset: unrolling the recursion

r	F	F	T	F	F	T	F	T	F	$\cdots$
i	0	5	10	15	20	25	30	35	40	$\cdots$
mask 0 r i	0	5	–	–	–	–	–	–	–	$\cdots$
nat(mask 0 r i)	0	1	–	–	–	–	–	–	–	$\cdots$

Semantics of modular reset: unrolling the recursion

r	F	F	T	F	F	T	F	T	F	$\cdots$
i	0	5	10	15	20	25	30	35	40	$\cdots$
mask 0 r i	0	5	–	–	–	–	–	–	–	$\cdots$
nat(mask 0 r i)	0	1	–	–	–	–	–	–	–	$\cdots$
mask 1 r i	–	–	10	15	20	–	–	–	–	$\cdots$
nat(mask 1 r i)	–	–	10	11	12	–	–	–	–	$\cdots$

Semantics of modular reset: unrolling the recursion

r	F	F	T	F	F	T	F	T	F	⋯
i	0	5	10	15	20	25	30	35	40	⋯
mask 0 r i	0	5	–	–	–	–	–	–	–	⋯
nat(mask 0 r i)	0	1	–	–	–	–	–	–	–	⋯
mask 1 r i	–	–	10	15	20	–	–	–	–	⋯
nat(mask 1 r i)	–	–	10	11	12	–	–	–	–	⋯
mask 2 r i	–	–	–	–	–	25	30	–	–	⋯
nat(mask 2 r i)	–	–	–	–	–	25	26	–	–	⋯

Semantics of modular reset: unrolling the recursion

r	F	F	T	F	F	T	F	T	F	⋯
i	0	5	10	15	20	25	30	35	40	⋯
mask 0 r i	0	5	–	–	–	–	–	–	–	⋯
nat(mask 0 r i)	0	1	–	–	–	–	–	–	–	⋯
mask 1 r i	–	–	10	15	20	–	–	–	–	⋯
nat(mask 1 r i)	–	–	10	11	12	–	–	–	–	⋯
mask 2 r i	–	–	–	–	–	25	30	–	–	⋯
nat(mask 2 r i)	–	–	–	–	–	25	26	–	–	⋯
mask 3 r i	–	–	–	–	–	–	–	35	40	⋯
nat(mask 3 r i)	–	–	–	–	–	–	–	35	36	⋯

Semantics of modular reset: unrolling the recursion

r	F	F	T	F	F	T	F	T	F	$\cdots$
i	0	5	10	15	20	25	30	35	40	$\cdots$
mask 0 r i	0	5	–	–	–	–	–	–	–	$\cdots$
nat(mask 0 r i)	0	1	–	–	–	–	–	–	–	$\cdots$
mask 1 r i	–	–	10	15	20	–	–	–	–	$\cdots$
nat(mask 1 r i)	–	–	10	11	12	–	–	–	–	$\cdots$
mask 2 r i	–	–	–	–	–	25	30	–	–	$\cdots$
nat(mask 2 r i)	–	–	–	–	–	25	26	–	–	$\cdots$
mask 3 r i	–	–	–	–	–	–	–	35	40	$\cdots$
nat(mask 3 r i)	–	–	–	–	–	–	–	35	36	$\cdots$
$\vdots$										
$nat(i)$ every r	0	1	10	11	12	25	26	35	36	$\cdots$

Semantics of modular reset in LaTeX

$$\mathsf{mask}^k_{k'}\ (\mathtt{F} \cdot rs)\ (sv \cdot xs) \triangleq (\text{if } k' = k \text{ then } sv \text{ else } \langle\rangle) \cdot \mathsf{mask}^k_{k'}\ rs\ xs$$
$$\mathsf{mask}^k_{k'}\ (\mathtt{T} \cdot rs)\ (sv \cdot xs) \triangleq (\text{if } k' + 1 = k \text{ then } sv \text{ else } \langle\rangle) \cdot \mathsf{mask}^k_{k'+1}\ rs\ xs$$

(a) mask CoindStreams.v:2414

xs	0	1	2	3	4	5	6	7	8	9	...
rs	F	F	**T**	F	F	F	**T**	F	**T**	F	...
$\mathsf{mask}^0\ rs\ xs$	0	1	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	...
$\mathsf{mask}^1\ rs\ xs$	$\langle\rangle$	$\langle\rangle$	2	3	4	5	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	...
$\mathsf{mask}^2\ rs\ xs$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	6	7	$\langle\rangle$	$\langle\rangle$	...
$\mathsf{mask}^3\ rs\ xs$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	$\langle\rangle$	8	9	...

(b) Example execution of mask

$$\mathsf{bools\text{-}of}\ (\langle\mathtt{T}\rangle \cdot xs) \triangleq \mathtt{T} \cdot \mathsf{bools\text{-}of}\ xs$$
$$\mathsf{bools\text{-}of}\ (\langle\mathtt{F}\rangle \cdot xs) \triangleq \mathtt{F} \cdot \mathsf{bools\text{-}of}\ xs$$
$$\mathsf{bools\text{-}of}\ (\langle\rangle \cdot xs) \triangleq \mathtt{F} \cdot \mathsf{bools\text{-}of}\ xs$$

(c) bools_of CoindStreams.v:1214

$$(\mathsf{mask}^k\ rs\ H)(x) = \mathsf{mask}^k\ rs\ (H(x))$$
$$\mathsf{mask}^k\ rs\ (H, bs) = (\mathsf{mask}^k\ rs\ H, \mathsf{maskb}^k\ rs\ bs)$$

(d) mask_hist CoindStreams.v:2421

$$\frac{G, H, bs \vdash es \Downarrow xss \quad G, H, bs \vdash e \Downarrow [ys] \quad \mathsf{bools\text{-}of}\ ys \equiv rs \quad \forall k,\ G \vdash f(\mathsf{mask}^k\ rs\ xss) \Downarrow (\mathsf{mask}^k\ rs\ yss)}{G, H, bs \vdash (\mathtt{reset}\ f\ \mathtt{every}\ e)(es) \Downarrow yss}$$

(e) Sapp Lustre/LSemantics.v:246

$$\frac{G, H, bs \vdash e \Downarrow [ys] \quad \mathsf{bools\text{-}of}\ ys \equiv rs \quad \forall k,\ G, \mathsf{mask}^k\ rs\ (H, bs) \vdash blks}{G, H, bs \vdash \mathtt{reset}\ blks\ \mathtt{every}\ e}$$

(f) Sreset Lustre/LSemantics.v:286

Figure 2.16: Semantics of reset

Semantics of modular reset in Coq 1/2

```
Inductive sem_exp : history → Stream bool → exp → list (Stream value) → Prop :=
...
| Sapp:
    Forall2 (sem_exp H b) es ss →
    sem_node f (concat ss) os →
    sem_exp H b (Eapp f es None lann) os

| Sreset:
    Forall2 (sem_exp H b) es ss →
    sem_exp H b r [rs] →
    bools_of rs bs →
    (∀ k, sem_node f (map (mask k bs) (concat ss)) (map (mask k bs) os)) →
    sem_exp H b (Eapp f es (Some r) lann) os



with sem_equation: history → Stream bool → equation → Prop :=
| Seq:
    Forall2 (sem_exp H b) es ss →
    Forall2 (sem_var H) xs (concat ss) →
    sem_equation H b (xs, es)



with sem_node: ident → list (Stream value) → list (Stream value) → Prop :=
  Snode:
    find_node f G = Some n →
    Forall2 (sem_var H) (idents n.(n_in)) ss →
    Forall2 (sem_var H) (idents n.(n_out)) os →
    Forall (sem_equation H b) n.(n_eqs) →
    b = clocks_of ss →
    sem_node f ss os.
```

Semantics of modular reset in Coq 2/2

```
CoFixpoint mask' (k k': nat) (rs: Stream bool) (xs: Stream svalue) : Stream svalue:=
    let mask' k' := mask' k k' (tl rs) (tl xs) in
    match hd rs with
    | false ⇒ (if k' =? k then hd xs else absent) ::: mask' k'
    | true ⇒ (if S k' =? k then hd xs else absent) ::: mask' (S k')
    end.

Definition mask k rs xs := mask' k 0 rs xs.
```

Overview

Lustre: syntax and semantics

Lustre: normalization

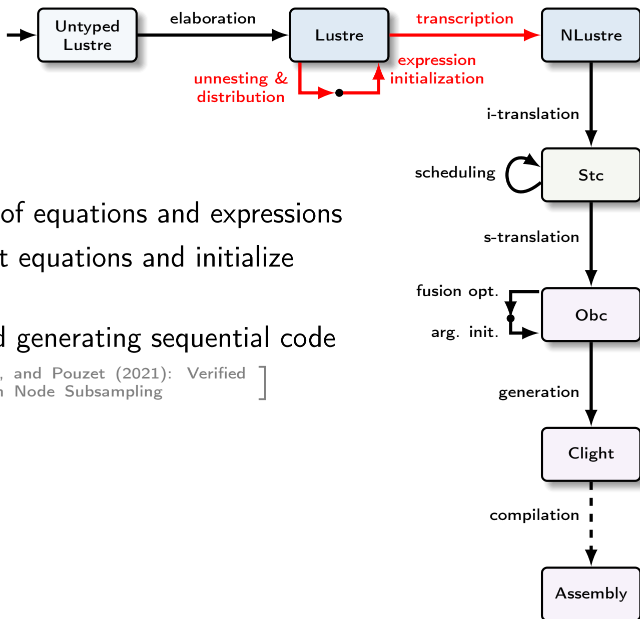
Translation: from dataflow programs to imperative code

Optimization: control structure fusion

Generation: from Obc to Clight

Conclusion

Normalization: Lustre to NLustre



- Simplify the form of equations and expressions
- Put fbys in distinct equations and initialize with a constant
- A first step toward generating sequential code
- [Bourke, Jeanmaire, Pesin, and Pouzet (2021): Verified Lustre Normalization with Node Subsampling]

Ultimate goal: transform Lustre AST into NLustre AST

$$
\begin{array}{rcl}
e & ::= & c \\
& | & x \\
& | & \diamond\, e \\
& | & e \oplus e \\
& | & e^{+}\ \texttt{fby}\ e^{+} \\
& | & e^{+}\ \texttt{when}\ x \\
& | & \texttt{merge(}\ x\ ;\ e^{+}\ ;\ e^{+}\ \texttt{)} \\
& | & \texttt{if}\ e\ \texttt{then}\ e^{+}\ \texttt{else}\ e^{+} \\
& | & f\ (\ e^{+}\ ) \\
eq & ::= & x^{+} = e^{+}\ ;
\end{array}
$$

Fig. 5. Lustre: equations

$$
\begin{array}{rcl}
e & ::= & c \\
& | & x \\
& | & \diamond\, e \\
& | & e \oplus e \\
& | & e\ \texttt{when}\ x \\
ce & ::= & e \\
& | & \texttt{merge(}\ x\ ;\ ce\ ;\ ce\ \texttt{)} \\
& | & \texttt{if}\ e\ \texttt{then}\ ce\ \texttt{else}\ ce \\
eq & ::= & x = ce \\
& | & x = c\ \texttt{fby}\ e \\
& | & x^{+} = f\ (\ e^{+}\ )
\end{array}
$$

Fig. 6. NLustre: equations

$$
\begin{array}{rcl}
n & ::= & \texttt{node}\ f\ (\ d^{+}\ )\ \texttt{returns}\ (\ d^{+}\ ) \\
& & (\texttt{var}\ d^{+}\ ;)^{?} \\
& & \texttt{let}\ eq^{*}\ \texttt{tel} \\
d & ::= & x_{ty}^{ck} \\
G & ::= & n^{+}
\end{array}
$$

Fig. 7. Nodes and Programs

NLustre

- Distinguish *expressions* (*e*) from *control expressions* (*ce*)
- Eliminate most expression lists (still allowed in node instances)
- Each fby in a separate equation and initialized by a constant
- Each node instance in a separate equation

N-Lustre: syntax

```
Inductive lexp : Type :=
| Econst : const → lexp
| Evar   : ident → type → lexp
| Ewhen  : lexp → ident → bool → lexp
| Eunop  : unop → lexp → type → lexp
| Ebinop : binop → lexp → lexp → type → lexp.
```

le when x / le when not x

```
Inductive cexp : Type :=
| Emerge : ident → cexp → cexp → cexp
| Eite   : lexp → cexp → cexp → cexp
| Eexp   : lexp → cexp.
```

merge x ce1 ce2

```
Inductive equation : Type :=
| EqDef : ident → clock → cexp → equation
| EqApp : list ident → clock → ident → list lexp → equation
| EqFby : ident → clock → const → lexp → equation.
```

$x = (ce)^{ck}$

$y,\ldots,y = (f(le, \ldots, le))^{ck}$

$x = (c \text{ fby } le)^{ck}$

```
Record node : Type :=
  mk_node {
    n_name : ident;
    n_in   : list (ident * (type * clock));
    n_out  : list (ident * (type * clock));
    n_vars : list (ident * (type * clock));
    n_eqs  : list equation;

    n_ingt0 : 0 < length n_in;
    n_outgt0 : 0 < length n_out;
    n_defd  : Permutation (vars_defined n_eqs)
          (map fst (n_vars ++ n_out));
    n_vout  : ∀ out, In out (map fst n_out) →
          ¬ In out (vars_defined (filter is_fby n_eqs));
    n_nodup : NoDupMembers (n_in ++ n_vars ++ n_out);
    n_good  : Forall NotReserved (n_in ++ n_vars ++ n_out)
  }.
```

node f(x,...,x) returns (y,...,y);
var w, ..., w;
let x = ...; ... tel

Example: Rising-Edge Retrigger [Pouzet (2006): Lucid Synchrone, v. 3. Tutorial and reference manual]

"Every rising edge on input i, hold the output o true for n cycles."

Example: Rising-Edge Retrigger [Pouzet (2006): Lucid Synchrone, v. 3. Tutorial and reference manual]

"Every rising edge on input *i*, hold the output *o* true for *n* cycles."

```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
let
  ...
tel
```

i	F	T	F	F	F	F	F	T	F	...
n	3	3	3	3	3	3	3	3	3	...

Example: Rising-Edge Retrigger [Pouzet (2006): Lucid Synchrone, v. 3. Tutorial and reference manual]

"Every rising edge on input i, hold the output o true for n cycles."

```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
var edge : bool;
let
  edge = i and (false fby (not i));
  ...
tel
```

i	F	T	F	F	F	F	F	T	F	···
n	3	3	3	3	3	3	3	3	3	···
edge	F	T	F	F	F	F	F	T	F	···

Example: Rising-Edge Retrigger [Pouzet (2006): Lucid Synchrone, v. 3. Tutorial and reference manual]

"Every rising edge on input *i*, hold the output *o* true for *n* cycles."

```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
var edge : bool; v : int;
let
  edge = i and (false fby (not i));
  v = /* count down from n whenever edge is true */
  o = v > 0;
tel
```

i	F	T	F	F	F	F	F	T	F	⋯
n	3	3	3	3	3	3	3	3	3	⋯
edge	F	T	F	F	F	F	F	T	F	⋯
v	0	3	2	1	0	0	0	3	2	⋯
o	F	T	T	T	F	F	F	T	T	⋯

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"Every rising edge on input *i*, hold the output *o* true for *n* cycles."

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var edge : bool; v : int;
let
  edge = i and (false fby (not i));
  v = /* count down from n whenever edge is true */
  o = v > 0;
tel
```

```
node dcount(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n
        else (n fby (cpt − 1));
tel
```

i	F	T	F	F	F	F	F	T	F	···
n	3	3	3	3	3	3	3	3	3	···
edge	F	T	F	F	F	F	F	T	F	···
v	0	3	2	1	0	0	0	3	2	···
o	F	T	T	T	F	F	F	T	T	···

Example: Rising-Edge Retrigger [Pouzet (2006): Lucid Synchrone, v. 3. Tutorial and reference manual]

"Every rising edge on input *i*, hold the output *o* true for *n* cycles."

```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
var edge, ck : bool; v : int;
let
  edge = i and (false fby (not i));
  ck = edge or (false fby o);
  v = dcount((false, n) when ck);
  o = v > 0;
tel
```

```
node dcount(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n
        else (n fby (cpt − 1));
tel
```

i	F	T	F	F	F	F	F	T	F	...
n	3	3	3	3	3	3	3	3	3	...
edge	F	T	F	F	F	F	F	T	F	...
v	0	3	2	1	0	0	0	3	2	...
ck	F	T	T	T	T	F	F	T	T	...
n when ck		3	3	3	3			3	3	...
dcount((false, n) when ck)		3	2	1	0			-1	-2	...
o	F	T	T	T	F	F	F	T	T	...

Example: Rising-Edge Retrigger [Pouzet (2006): Lucid Synchrone, v. 3. Tutorial and reference manual]

"Every rising edge on input *i*, hold the output *o* true for *n* cycles."

```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
var edge, ck : bool; v : int;
let
  edge = i and (false fby (not i));
  ck = edge or (false fby o);
  v = merge ck
        (true -> dcount((false, n) when ck))
        (false -> 0);
  o = v > 0;
tel
```

```
node dcount(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n
          else (n fby (cpt - 1));
tel
```

i	F	T	F	F	F	F	F	T	F	⋯
n	3	3	3	3	3	3	3	3	3	⋯
edge	F	T	F	F	F	F	F	T	F	⋯
v	0	3	2	1	0	0	0	3	2	⋯
ck	F	T	T	T	T	F	F	T	T	⋯
n when ck		3	3	3	3			3	3	⋯
dcount((false, n) when ck)		3	2	1	0			-1	-2	⋯
o	F	T	T	T	F	F	F	T	T	⋯

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"Every rising edge on input i, hold the output o true for n cycles."

```
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returns (o : bool)
var edge, ck : bool; v : int;
let
  edge = i and (false fby (not i));
  ck = edge or (false fby o);
  v = merge ck
        (true -> dcount((edge, n) when ck))
        (false -> 0);
  o = v > 0;
tel
```

```
node dcount(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n
          else (n fby (cpt - 1));
tel
```

i	F	T	F	F	F	F	F	T	F	···
n	3	3	3	3	3	3	3	3	3	···
edge	F	T	F	F	F	F	F	T	F	···
v	0	3	2	1	0	0	0	3	2	···
ck	F	T	T	T	T	F	F	T	T	···
n when ck		3	3	3	3			3	3	···
dcount((false, n) when ck)		3	2	1	0			3	2	···
o	F	T	T	T	F	F	F	T	T	···

Example: Rising-Edge Retrigger [Pouzet (2006): Lucid Synchrone, v. 3. Tutorial and reference manual]

```
node dcount(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n else (n fby (cpt − 1));
tel
```

```
node dcount(res : bool; n : int)
returns (cpt : int)
var norm1$1, norm2$2 : int; norm2$1 : bool;
let
  norm2$1 = true fby false;
  norm2$2 = 0 fby (cpt − 1);
  norm1$1 = if norm2$1 then n else norm2$2;
  cpt = if res then n else norm1$1;
tel
```

Example: Rising-Edge Retrigger [Pouzet (2006): Lucid Synchrone, v. 3. Tutorial and reference manual]

```
node dcount(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n else (n fby (cpt − 1));
tel
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```
node dcount(res : bool; n : int)
returns (cpt : int)
var norm1$1, norm2$2 : int; norm2$1 : bool;
let
  norm2$1 = true fby false;
  norm2$2 = 0 fby (cpt − 1);
  norm1$1 = if norm2$1 then n else norm2$2;
  cpt = if res then n else norm1$1;
tel
```

```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
var edge, ck : bool; v : int;
let
  edge = i and (false fby (not i));
  ck = edge or (false fby o);
  v = merge ck
      (true −> dcount((edge, n) when ck))
      (false −> 0);
  o = v > 0;
tel
```

```
node rising_edge_retrigger (i : bool; n : int)
returns (o : bool)
var edge, ck, norm1$1, norm1$2 : bool;
    v : int; elab$4 : int when ck;
let
  norm1$2 = false fby (not i);
  edge = i and norm1$2;
  norm1$1 = false fby o;
  ck = edge or norm1$1;
  elab$4 = dcount(edge when ck, n when ck);
  v = merge(ck; elab$4; 0 when not ck);
  o = v > 0;
tel
```

Unnesting and distribution: selected cases

$$
\begin{aligned}
\lfloor \mathsf{c} \rfloor &= ([\mathsf{c}], [\,]) \\
\lfloor \mathsf{x} \rfloor &= ([\mathsf{x}], [\,]) \\
\lfloor e_1 \oplus e_2 \rfloor &= ([e_1'], \boldsymbol{eqs}_1') \leftarrow \lfloor e_1 \rfloor \\
&\quad ([e_2'], \boldsymbol{eqs}_2') \leftarrow \lfloor e_2 \rfloor \\
&\quad ([e_1' \oplus e_2'], \boldsymbol{eqs}_1' \ \cup\ \boldsymbol{eqs}_2') \\
\lfloor (e_1, \ldots, e_n) \ \mathsf{when}\ b \rfloor &= ([e_1', \ldots, e_m'], \boldsymbol{eqs}') \leftarrow \lfloor e_1, \ldots, e_n \rfloor \\
&\quad ([e_1' \ \mathsf{when}\ b, \ldots, e_m' \ \mathsf{when}\ b], \boldsymbol{eqs}') \\
\lfloor (e_1, \ldots, e_n) \ \mathsf{fby}\ (f_1, \ldots, f_m) \rfloor &= ([e_1', \ldots, e_k'], \boldsymbol{eqs}_1') \leftarrow \lfloor e_1, \ldots, e_n \rfloor \\
&\quad ([f_1', \ldots, f_k'], \boldsymbol{eqs}_2') \leftarrow \lfloor f_1, \ldots, f_m \rfloor \\
&\quad ([x_1, \ldots, x_k], [x_1 = e_1' \ \mathsf{fby}\ f_1', \ldots, x_k = e_k' \ \mathsf{fby}\ f_k'] \ \cup\ \boldsymbol{eqs}_1' \ \cup\ \boldsymbol{eqs}_2') \\
\lfloor f(e_1, \ldots, e_n) \rfloor &= ([e_1', \ldots, e_m'], \boldsymbol{eqs}') \leftarrow \lfloor e_1, \ldots, e_n \rfloor \\
&\quad ([x_1, \ldots, x_k], [(x_1, \ldots x_k) = f(e_1', \ldots, e_m')] \ \cup\ \boldsymbol{eqs}')
\end{aligned}
$$

Expression initialization

$$\left\lfloor \mathsf{x} = (\mathrm{e0}\ \mathsf{fby}\ \mathrm{e})^{ck} \right\rfloor_{\mathsf{fby}} = \begin{cases} \mathsf{x} = \mathsf{if}\ \mathsf{xinit}\ \mathsf{then}\ \mathrm{e0}\ \mathsf{else}\ \mathsf{px}; \\ \mathsf{xinit} = \mathsf{true}^{ck}\ \mathsf{fby}\ \mathsf{false}^{ck}; \\ \mathsf{px} = \mathtt{def}_{ty}^{ck}\ \mathsf{fby}\ \mathrm{e}; \end{cases}$$

Transcription

- Convert between normalized Lustre AST and NLustre AST
- Difficulty: alignment clause in NLustre semantics

Indexed or coinductive: fby

```
CoFixpoint fby (c: val) (xs: Stream value) : Stream value :=
  match xs with
  | absent ::: xs ⇒ absent ::: fby c xs
  | present x ::: xs ⇒ present c ::: fby x xs
  end.
```

Indexed or coinductive: fby

```
CoFixpoint fby (c: val) (xs: Stream value) : Stream value :=
  match xs with
  | absent ::: xs ⇒ absent ::: fby c xs
  | present x ::: xs ⇒ present c ::: fby x xs
  end.


Fixpoint hold (v0: val) (xs: stream value) (n: nat) : val :=
  match n with
  | 0 ⇒ v0
  | S m ⇒ match xs m with
          | absent ⇒ hold v0 xs m
          | present hv ⇒ hv
          end
  end.

Definition fby (v0: val) (xs: stream value) : nat → value :=
  fun n ⇒
    match xs n with
    | absent ⇒ absent
    | _ ⇒ present (hold v0 xs n)
    end.
```

Indexed or coinductive (NLustre)

Indexed streams (nat $\mapsto$ value)

Coinductive streams

```
fun n ⇒ n
```

```
(cofix f n := n ::: f (S n)) 0
```

Indexed or coinductive (NLustre)

Indexed streams (nat ↦ value)

Coinductive streams

```
CoFixpoint idx_to_coind (n: nat) (xs: nat → A) : Stream A :=
  xs n ::: idx_to_coind (S n) xs.
```

```
fun n ⇒ n
```

```
(cofix f n := n ::: f (S n)) 0
```

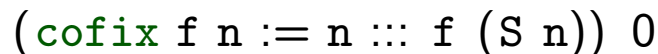
Indexed or coinductive (NLustre)

Indexed streams (nat ↦ value)

Coinductive streams

```
CoFixpoint idx_to_coind (n: nat) (xs: nat → A) : Stream A :=
  xs n ::: idx_to_coind (S n) xs.
```

```
fun n ⇒ n
```



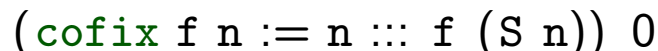

```
Definition coind_to_idx (xs: Stream A) : nat → A :=
  fun n ⇒ hd (Str_nth_tl n xs).
```

Indexed or coinductive (NLustre)

Indexed streams (nat ↦ value) Coinductive streams

```
CoFixpoint idx_to_coind (n: nat) (xs: nat → A) : Stream A :=
  xs n ::: idx_to_coind (S n) xs.
```

`fun n ⇒ n`




```
Definition coind_to_idx (xs: Stream A) : nat → A :=
  fun n ⇒ hd (Str_nth_tl n xs).
```

```
idx_to_coind n (Idx.fby c xs)
== CoInd.fby (Idx.hold c xs n) (idx_to_coind n xs).
```

```
coind_to_idx (CoInd.fby c xs) =∗= Idx.fby c (coind_to_idx xs).
```

Indexed or coinductive (NLustre)

Indexed streams (nat ↦ value) | Coinductive streams

```
CoFixpoint idx_to_coind (n: nat) (xs: nat → A) : Stream A :=
  xs n ::: idx_to_coind (S n) xs.
```

fun n ⇒ n ⟶ (cofix f n := n ::: f (S n)) 0

(cofix f n := n ::: f (S n)) 0 ⟶ fun n ⇒ n

```
Definition coind_to_idx (xs: Stream A) : nat → A :=
  fun n ⇒ hd (Str_nth_tl n xs).
```

```
idx_to_coind n (Idx.fby c xs)
== CoInd.fby (Idx.hold c xs n) (idx_to_coind n xs).
```

```
coind_to_idx (CoInd.fby c xs) =*= Idx.fby c (coind_to_idx xs).
```

Extends to NLustre semantics (proof: L. Brun)

N-Lustre: semantics 1/3

```
Notation stream A := (nat → A).

Definition history := PM.t (stream value).
Definition R := PM.t value.

Section InstantSemantics.

  Variable base : bool.
  Variable R : R.

  Inductive sem_var_instant (x: ident) (v: value): Prop :=
  | Sv:
      PM.find x R = Some v →
      sem_var_instant x v.

End InstantSemantics.

Section LiftSemantics.

  Variable bk : stream bool.

  Definition restr H (n: nat): R :=
    PM.map (fun xs ⇒ xs n) H.

  Definition lift (sem: bool → R → A → B → Prop) H x (ys: stream B): Prop :=
    ∀ n, sem (bk n) (restr H n) x (ys n).

  Definition sem_var H (x: ident)(xs: stream value): Prop :=
    lift (fun base ⇒ sem_var_instant) H x xs.

End LiftSemantics.
```

N-Lustre: semantics 2/3

```
Inductive sem_lexp_instant: lexp → value → Prop :=
| Sconst:
    v = (if base then present (sem_const c) else absent) →
    sem_lexp_instant (Econst c) v

| Svar:
    sem_var_instant x v →
    sem_lexp_instant (Evar x ty) v

| Swhen_abs:
    sem_lexp_instant s absent →
    sem_var_instant x absent →
    sem_lexp_instant (Ewhen s x b) absent

| Swhen_eq:
    sem_lexp_instant s (present sc) →
    sem_var_instant x (present xc) →
    val_to_bool xc = Some b →
    sem_lexp_instant (Ewhen s x b) (present sc)

| Swhen_abs1:
    sem_lexp_instant s (present sc) →
    sem_var_instant x (present xc) →
    val_to_bool xc = Some b →
    sem_lexp_instant (Ewhen s x (negb b)) absent

...

Definition sem_lexp H (e: lexp) (xs: stream value) : Prop :=
  lift sem_lexp_instant H e xs.
```

$$
\begin{aligned}
i^{\#}[\epsilon] &= \epsilon \\
i^{\#}[true.cl] &= v.i^{\#}[cl] \\
i^{\#}[false.cl] &= abs.i^{\#}[cl]
\end{aligned}
$$

$$
\begin{aligned}
\texttt{when}^{\#}(s_1, s_2) &= \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon \\
\texttt{when}^{\#}(abs.xs, abs.cs) &= abs.\texttt{when}^{\#}(xs, cs) \\
\texttt{when}^{\#}(x.xs, true.cs) &= x.\texttt{when}^{\#}(xs, cs) \\
\texttt{when}^{\#}(x.xs, false.cs) &= abs.\texttt{when}^{\#}(xs, cs)
\end{aligned}
$$

[Colaço and Pouzet (2003): Clocks as First Class Abstract Types]

N-Lustre: semantics 2/3

```
Inductive sem_lexp_instant: lexp → value → Prop :=
| (Sconst:)
    v = (if base then present (sem_const c) else absent) →
    ─────────────────────────────────────────────────────
    sem_lexp_instant (Econst c) v

| (Svar:)
    sem_var_instant x v →
    ─────────────────────────────
    sem_lexp_instant (Evar x ty) v

| (Swhen_abs:)
    sem_lexp_instant s absent →
    sem_var_instant x absent →
    ───────────────────────────────────────
    sem_lexp_instant (Ewhen s x b) absent

| (Swhen_eq:)
    sem_lexp_instant s (present sc) →
    sem_var_instant x (present xc) →
    val_to_bool xc = Some b →
    ───────────────────────────────────────────
    sem_lexp_instant (Ewhen s x b) (present sc)

| (Swhen_abs1:)
    sem_lexp_instant s (present sc) →
    sem_var_instant x (present xc) →
    val_to_bool xc = Some b →
    ────────────────────────────────────────────
    sem_lexp_instant (Ewhen s x (negb b)) absent

...

Definition sem_lexp H (e: lexp) (xs: stream value) : Prop :=
  lift sem_lexp_instant H e xs.
```

$$
\begin{aligned}
i^{\#}[\epsilon] &= \epsilon \\
i^{\#}[true.cl] &= v.i^{\#}[cl] \\
i^{\#}[false.cl] &= abs.i^{\#}[cl]
\end{aligned}
$$

$$
\begin{aligned}
\texttt{when}^{\#}(s_1, s_2) &= \epsilon \text{ if } s_1 = \epsilon \text{ or } s_2 = \epsilon \\
\texttt{when}^{\#}(abs.xs, abs.cs) &= abs.\texttt{when}^{\#}(xs, cs) \\
\texttt{when}^{\#}(x.xs, true.cs) &= x.\texttt{when}^{\#}(xs, cs) \\
\texttt{when}^{\#}(x.xs, false.cs) &= abs.\texttt{when}^{\#}(xs, cs)
\end{aligned}
$$

[Colaço and Pouzet (2003): Clocks as First Class Abstract Types]

N-Lustre: semantics 3/3

```
Inductive sem_equation G : stream bool → history → equation → Prop :=
| (SEqDef:)
    sem_var bk H x xs →
    sem_caexp bk H ck ce xs →
    ------------------------------------
    sem_equation G bk H (EqDef x ck ce)

| (SEqApp:)
    sem_lexps bk H arg ls →
    sem_vars bk H x xs →
    sem_clock bk H ck cks →
    clock_of ls cks →
    sem_node G f ls xs →
    ------------------------------------
    sem_equation G bk H (EqApp x ck f arg)

| (SEqFby:)
    sem_laexp bk H ck le ls →
    sem_var bk H x xs →
    (∀ n, xs n = fby (sem_const c0) ls n) →
    ------------------------------------
    sem_equation G bk H (EqFby x ck c0 le)

with sem_node G :
  ident → stream (list value) → stream (list value)
    → Prop :=
| (SNode:)
    find_node f G = Some n →
    clock_of xss bk →
    sem_vars bk H (map fst n.(n_in)) xss →
    sem_vars bk H (map fst n.(n_out)) yss →
    sem_clocked_vars bk H (idck n.(n_in)) →
    Forall (sem_equation G bk H) n.(n_eqs) →
    ------------------------------------
    sem_node G f xss yss.
```

```
Inductive sem_laexp_instant
  : clock → lexp → value → Prop:=
| (SLtick:)
    sem_lexp_instant ce (present c) →
    sem_clock_instant ck true →
    ------------------------------------
    sem_laexp_instant ck ce (present c)
| (SLabs:)
    sem_clock_instant ck false →
    ------------------------------------
    sem_laexp_instant ck ce absent.
```

$$\mathtt{fby}^{\#}_{v_0}(v.s) = v_0.\mathtt{fby}^{\#}_{v}(s)$$

$$\mathtt{fby}^{\#}_{v_0}(\mathbf{abs}.s) = \mathbf{abs}.\mathtt{fby}^{\#}_{v_0}(s)$$

$$\mathtt{fby}^{\#}_{v_0}(\epsilon) = \epsilon$$

```
Fixpoint hold
    (v0: val) (xs: stream value) (n: nat) : val :=
  match n with
  | 0 ⇒ v0
  | S m ⇒ match xs m with
          | absent ⇒ hold v0 xs m
          | present hv ⇒ hv
          end
  end.

Definition fby
    (v0: val) (xs: stream value) (n: nat) : value :=
  match xs n with
  | absent ⇒ absent
  | _ ⇒ present (hold v0 xs n)
  end.
```

$f : \mathrm{stream}(T^{+}) \rightarrow \mathrm{stream}(T^{+})$

Overview

Lustre: syntax and semantics

Lustre: normalization

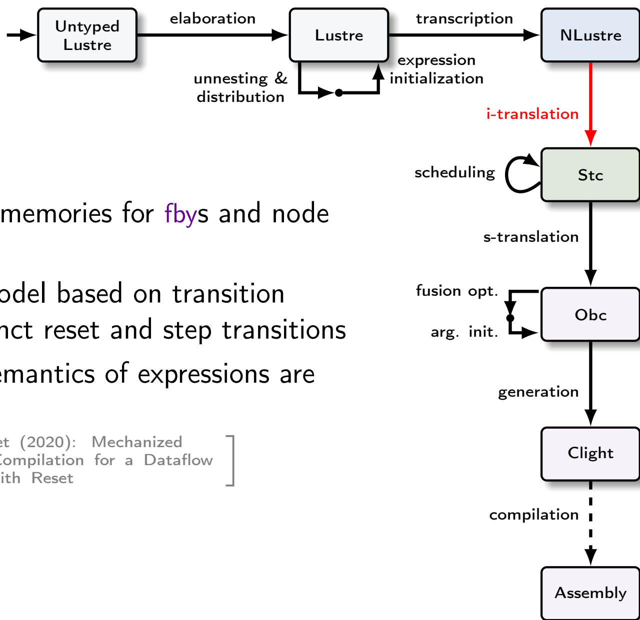
Translation: from dataflow programs to imperative code

Optimization: control structure fusion

Generation: from Obc to Clight

Conclusion

i-translation: NLustre to Stc



- Introduce explicit memories for fbys and node instances
- Use a semantic model based on transition systems with distinct reset and step transitions
- The syntax and semantics of expressions are unchanged
- [Bourke, Brun, and Pouzet (2020): Mechanized Semantics and Verified Compilation for a Dataflow Synchronous Language with Reset]

Naive compilation of modular reset

```
y = (restart f every r)(x)
```

```
if (ck_r) {
  if (r) { f(y).reset() };
};
y := f(y).step(x)
```

Naive compilation of modular reset

```
y = (restart f every r)(x)
```

```
if (ck_r) {
  if (r) { f(y).reset() };
};
y := f(y).step(x)
```

Problem: interferes with the fusion optimization

```
node main(x0, x': int; ck, r: bool)
returns (x: int) var v, w: int when ck;
let
  v = filter(x' when ck);
  w = euler((x0, v) when ck) every r;
  x = merge ck w 0;
tel
```

```
step(x0, x': int; ck, r: bool)
returns (x: int);
var v, w : int;
{
  if (ck) { v := filter(v).step(x') };
  if (r) { euler(w).reset() };
  if (ck) { w := euler(w).step(x0, v) };
  if (ck) { x := w } else { x := 0 }
}
```

Naive compilation of modular reset

```
y = (restart f every r)(x)
```

```
if (ck_r) {
  if (r) { f(y).reset() };
};
y := f(y).step(x)
```

Solution: a new intermediate language (Stc)

```
node main(x0, x': int; ck, r: bool)
returns (x: int) var v, w: int when ck;
let
  v = filter(x' when ck);
  w = euler((x0, v) when ck) every r;
  x = merge ck w 0;
tel
```

```
step(x0, x': int; ck, r: bool)
returns (x: int);
var v, w : int;
{
  if (r) { euler(w).reset() };
  if (ck) {
    v := filter(v).step(x');
    w := euler(w).step(x0, v);
    x := w
  } else { x := 0 }
}
```

Stc: expose node instances, separate reset and step

$$
\begin{array}{rcl}
tc & ::= & x =_{ck} ce \\
& \mid & \mathit{next}\ x =_{ck} e \\
& \mid & \mathit{reset}\ f\langle x\rangle\ \mathit{every}\ ck \\
& \mid & x, \ldots, x =_{ck} f\langle x, k\rangle(e, \ldots, e)
\end{array}
$$

- No `fby`; use `next` to constrains successor state.
- Distinguish `reset` and `step` constraints.
- Associate each with an instance ($\langle x\rangle$).
- For `step` constraints, k is 1 if there is a `reset` on the same instance and 0 otherwise.

Make states explicit, separate reset and step transitions

```
node rising_edge_retrigger (i : bool; n : int32)
returns (o : bool)
var edge, ck, v, norm1$1, norm1$2 : bool;
    elab$3 : int32 when ck;
let
  norm1$2 = 0 fby not i;
  edge = i and norm1$2;
  norm1$1 = 0 fby o;
  ck = edge or norm1$1;
  elab$3 = (restart dcount every edge)(n when ck);
  v = merge ck
        (true -> elab$3)
        (false -> 0 when not ck);
  o = v > 0;
tel
```

Make states explicit, separate reset and step transitions

```
node rising_edge_retrigger (i : bool; n : int32)
returns (o : bool)
var edge, ck, v, norm1$1, norm1$2 : bool;
   elab$3 : int32 when ck;
let
  norm1$2 = 0 fby not i;
  edge = i and norm1$2;
  norm1$1 = 0 fby o;
  ck = edge or norm1$1;
  elab$3 = (restart dcount every edge)(n when ck);
  v = merge ck
        (true −> elab$3)
        (false −> 0 when not ck);
  o = v > 0;
tel
```

```
system rising_edge_retrigger {
  init norm1$1 = 0,
       norm1$2 = 0;
  sub <elab$3> : dcount;

  transition(i : bool; n : int32)
  returns (o : bool)
  var edge, ck : bool; v : int32;
     elab$3 : int32 when ck;
  {
    next norm1$2 = not i;
    edge = i and norm1$2;
    next norm1$1 = o;
    ck = edge or norm1$1;
    elab$3 = dcount<elab$3>(n when ck);
    reset(dcount<elab$3>) every (. on edge);
    v = merge ck
          (true −> elab$3)
          (false −> 0 when not ck);
    o = v > 0;
  }
}
```

Stc: new intermediate language

- Explicit states
- Distinct *step* and *reset* constraints
- Constraint ordering is unimportant

Make states explicit, separate reset and step transitions

```
node rising_edge_retrigger (i : bool; n : int32)
returns (o : bool)
var edge, ck, v, norm1$1, norm1$2 : bool;
    elab$3 : int32 when ck;
let
  norm1$2 = 0 fby not i;
  edge = i and norm1$2;
  norm1$1 = 0 fby o;
  ck = edge or norm1$1;
  elab$3 = (restart dcount every edge)(n when ck);
  v = merge ck
        (true -> elab$3)
        (false -> 0 when not ck);
  o = v > 0;
tel
```

i-translation

```
system rising_edge_retrigger {
  init norm1$1 = 0,
       norm1$2 = 0;
  sub <elab$3> : dcount;

  transition(i : bool; n : int32)
  returns (o : bool)
  var edge, ck : bool; v : int32;
     elab$3 : int32 when ck;
  {
    next norm1$2 = not i;
    edge = i and norm1$2;
    next norm1$1 = o;
    ck = edge or norm1$1;
    elab$3 = dcount<elab$3>(n when ck);
    reset(dcount<elab$3>) every (. on edge);
    v = merge ck
          (true -> elab$3)
          (false -> 0 when not ck);
    o = v > 0;
  }
}
```

Stc: new intermediate language

- Explicit states
- Distinct *step* and *reset* constraints
- Constraint ordering is unimportant

Make states explicit, separate reset and step transitions

```
node rising_edge_retrigger (i : bool; n : int32)
returns (o : bool)
var edge, ck, v, norm1$1, norm1$2 : bool;
   elab$3 : int32 when ck;
let
  norm1$2 = 0 fby not i;
  edge = i and norm1$2;
  norm1$1 = 0 fby o;
  ck = edge or norm1$1;
  elab$3 = (restart dcount every edge)(n when ck);
  v = merge ck
        (true -> elab$3)
        (false -> 0 when not ck);
  o = v > 0;
tel
```

i-translation

```
system rising_edge_retrigger {
  init norm1$1 = 0,
       norm1$2 = 0;
  sub <elab$3> : dcount;

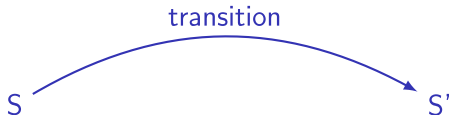
  transition(i : bool; n : int32)
  returns (o : bool)
  var edge, ck : bool; v : int32;
     elab$3 : int32 when ck;
  {
    next norm1$2 = not i;
    edge = i and norm1$2;
    next norm1$1 = o;
    ck = edge or norm1$1;
    elab$3 = dcount<elab$3>(n when ck);
    reset(dcount<elab$3>) every (. on edge);
    v = merge ck
          (true -> elab$3)
          (false -> 0 when not ck);
    o = v > 0;
  }
}
```

Stc: new intermediate language

- Explicit states
- Distinct *step* and *reset* constraints
- Constraint ordering is unimportant

Stc: Synchronous Transition Code

- Transition system: relation between states S and S'.
- next x = y + 1 means $S'[x] = S[y] + 1$.
- reset(dcount<i>) every r and dcount<i> constrain subtrees of S and S'.
- Need to introduce an intermediate state.
- And so on, recursively.
- 'Hiding' the intermediate states is key for inductive proofs.
- Composing transitions gives the columns of the dataflow semantics.



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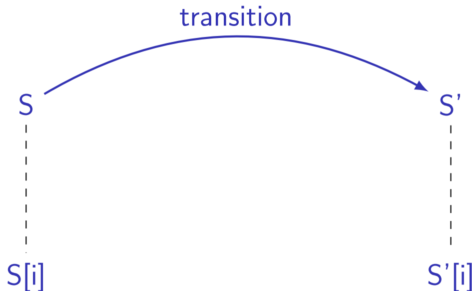
$S_0 \frown S_1 \frown S_2 \frown S_3 \frown \cdots$



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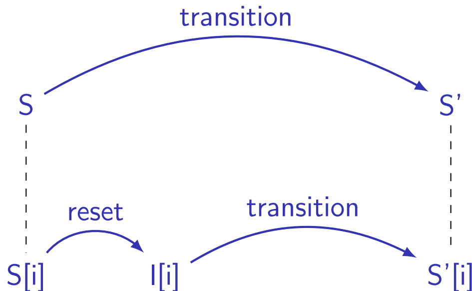

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transition
S
S’
reset
transition
S[i]
I[i]
S’[i]
reset
transition
(I[i])[j]
I’[j]
(S’[i])[j]

Stc: Synchronous Transition Code

- Transition system: relation between states S and S'.
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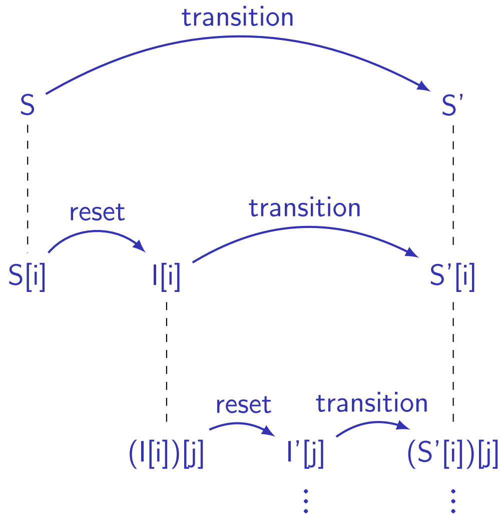
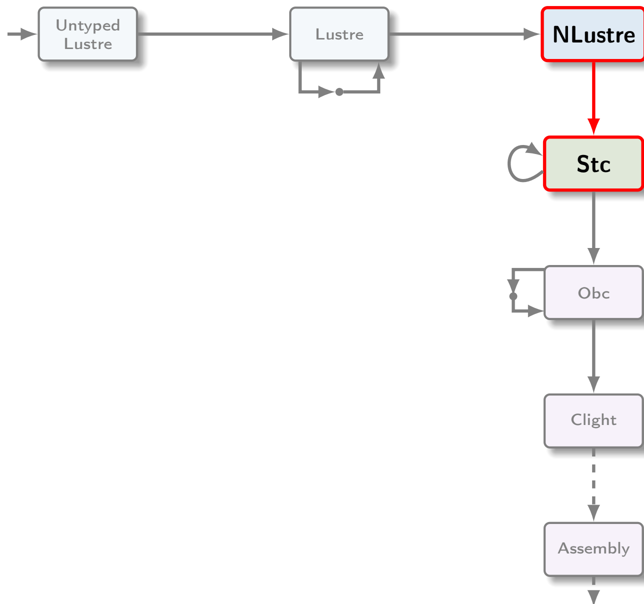
$S_0 \curvearrowright S_1 \curvearrowright S_2 \curvearrowright S_3 \curvearrowright \cdots$



Correctness of NLustre to Stc translation



Correctness of NLustre to Stc translation

`sem_node G f xss yss`

$\text{stream}(T_i) \rightarrow \text{stream}(T_o)$

i	F	T	F	F	F	F	F	T	F	⋯
n	3	3	3	3	3	3	3	3	3	⋯
edge	F	T	F	F	F	F	F	T	F	⋯
v	0	3	2	1	0	0	0	3	2	⋯
ck	F	T	T	T	T	F	F	T	T	⋯
n when ck		3	3	3	3			3	3	⋯
dcount(n when ck)		3	2	1	0			3	2	⋯
o	F	T	T	T	F	F	F	T	T	⋯

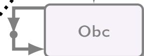
Untyped Lustre → Lustre → NLustre → Stc → Obc → Clight → Assembly

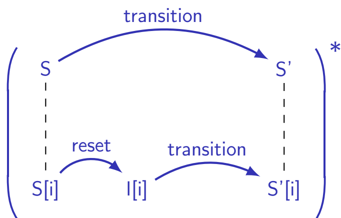
Correctness of NLustre to Stc translation

`sem_node G f xss yss`

$\text{stream}(T_i) \rightarrow \text{stream}(T_o)$

i	F	T	F	F	F	F	F	T	F	···
n	3	3	3	3	3	3	3	3	3	···
edge	F	T	F	F	F	F	F	T	F	···
v	0	3	2	1	0	0	0	3	2	···
ck	F	T	T	T	T	F	F	T	T	···
n when ck		3	3	3	3			3	3	···
dcount(n when ck)		3	2	1	0			3	2	···
o	F	T	T	T	F	F	F	T	T	···





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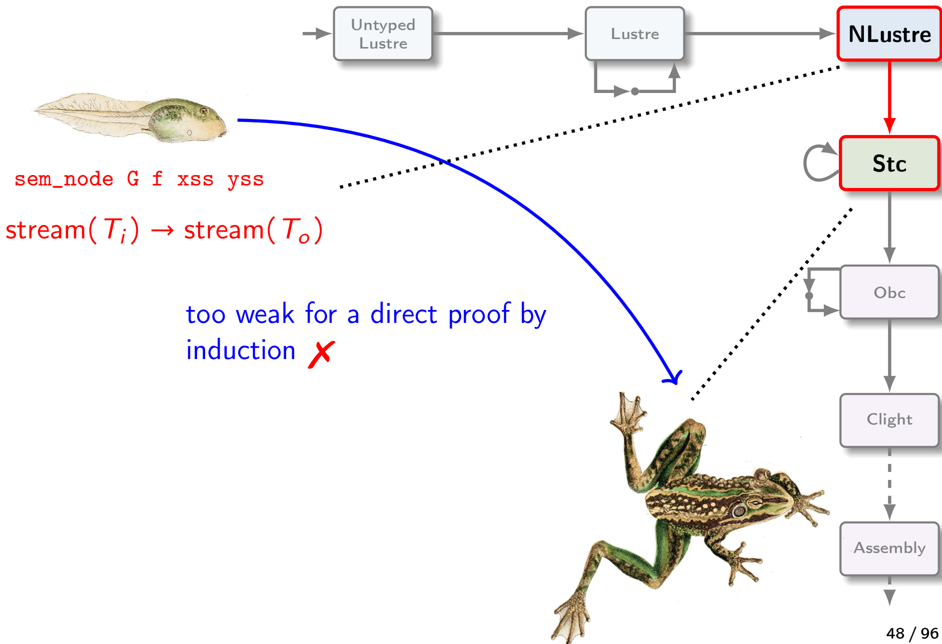

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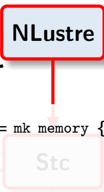
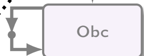


sem_node G f xss yss

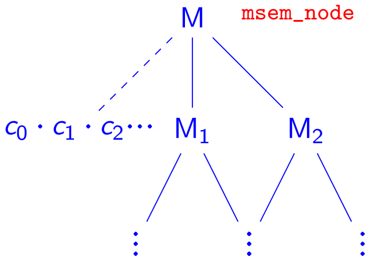
$\text{stream}(T_i) \rightarrow \text{stream}(T_o)$

```
Inductive memory (A: Type): Type := mk memory {
  mm_values : PM.t A;
  mm_instances : PM.t (memory A)
}.

Definition memory := memory (stream const).
```



msem_node G f xss M yss



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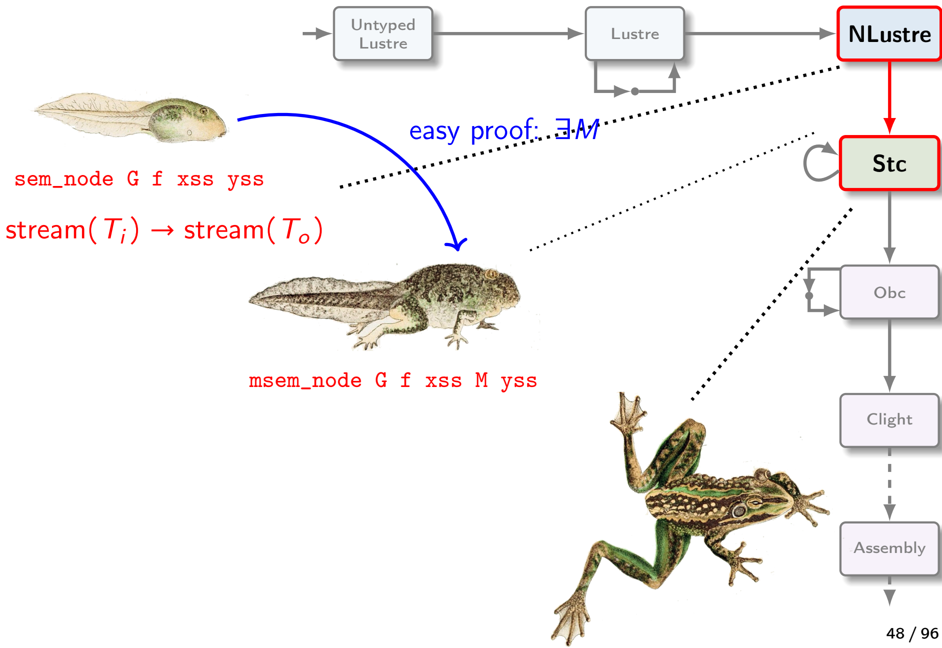

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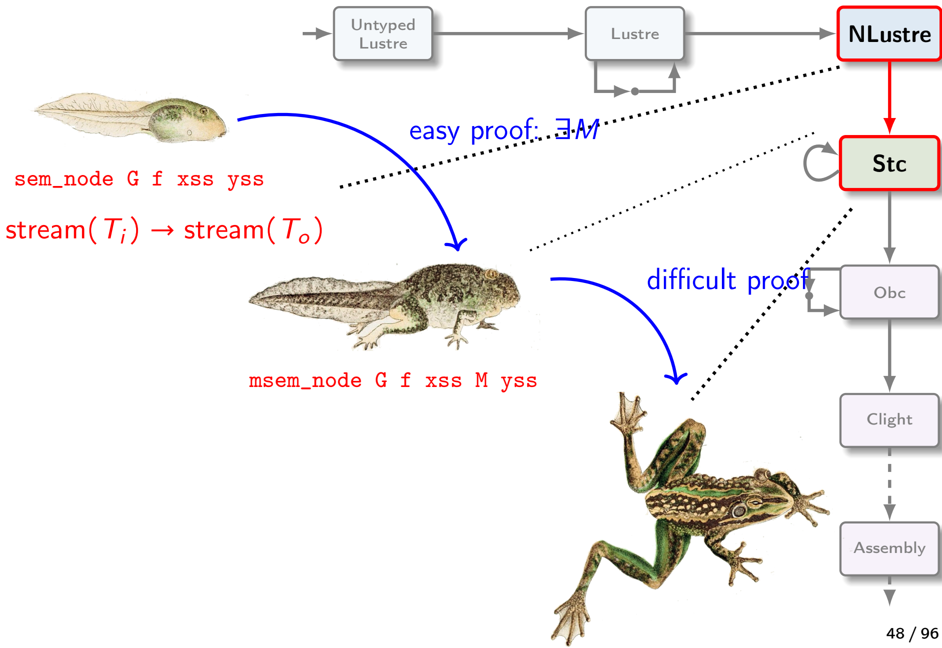

```
Inductive sem_equation (G: global)
  : history → equation → Prop :=

| (SEqDef: )
    sem_var H x xs →
    sem_caexp H cae xs →
   ─────────────────────────────────
    sem_equation G H (EqDef x cae)
```

$$x = (ce)^{ck}$$

```
...

| (SEqFby: )
    sem_laexp H lae ls →
    sem_var H x xs →
    xs = fby v0 ls →
   ─────────────────────────────────────
    sem_equation G H (EqFby x v0 lae)
```

$$x = (v0 \text{ fby } le)^{ck}$$

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```

$x = (v0 \text{ fby } le)^{ck}$

```
Inductive msem_equation G
  : history → memory → equation
  → Prop :=
| SEqDef:
    sem_var H x xs →
    sem_caexp H cae xs →
   ─────────────────────────────────────
    msem_equation G H M (EqDef x cae)
...
| SEqFby:
    mfind_mem x M = Some ms →
    ms 0 = v0 →
    sem_laexp H lae ls →
    sem_var H x xS →
    (∀ n,
      match ls n with
      | absent    ⇒ ms (S n) = ms n
                    ∧ xs n = absent
      | present v ⇒ ms (S n) = v
                    ∧ xs n = present (ms n)
      end) →
   ─────────────────────────────────────
    msem_equation G H M (EqFby x v0 lae)
```

n	0	1	2	3	4	5	6	···	base
ck	F	F	T	F	T	T	F	···	base
x = n when (ck = T)			3		5	6		···	base on (ck = T)
y = 0 fby x			0		3	5		···	base on (ck = T)
y_M = 0 mby x	0	0	0	3	3	5	6	···	base

```
...

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    sem_laexp H lae ls →
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ck	F	F	T	F	T	T	F	···	base
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y = 0 fby x			0		3	5		···	base on (ck = T)
y_M = 0 mby x	0	0	0	3	3	5	6	···	base

```
...

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y_M = 0 mby x	0→	0→	0	3	3	5	6	···	base

```
...

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```
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```
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```
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```

	0	1	2	3	4	5	6	...	
n	0	1	2	3	4	5	6	...	base
ck	F	F	T	F	T	T	F	...	base
x = n when (ck = T)			3		5	6		...	base on (ck = T)
y = 0 fby x			0		3	5		...	base on (ck = T)
y_M = 0 mby x	0	0	0	3	3	5	6	...	base

```
...

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    sem_laexp H lae ls →
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...

```
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    sem_var H x xs →
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    ─────────────────────────────
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```

...

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    ─────────────────────────────
    msem_equation G H M (EqFby x v0 lae)
```

	0	1	2	3	4	5	6	⋯	
n	0	1	2	3	4	5	6	⋯	base
ck	F	F	T	F	T	T	F	⋯	base
x = n when (ck = T)			3		5	6		⋯	base on (ck = T)
y = 0 fby x			0		3	5		⋯	base on (ck = T)
y_M = 0 mby x	0	0	0	3	3	5	6	⋯	base

...

```
| SEqFby:
    sem_laexp H lae ls →
    sem_var H x xs →
    xs = fby v0 ls →
    ─────────────────────────────
    sem_equation G H (EqFby x v0 lae)
```

...

```
| SEqFby:
    mfind_mem x M = Some ms →
    ms 0 = v0 →
    sem_laexp H lae ls →
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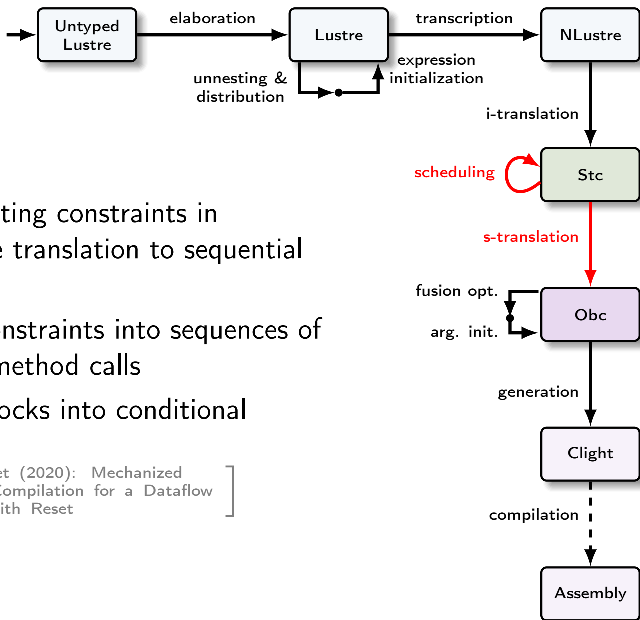
```

s-translation: Stc to Obc



- Schedule the resulting constraints in anticipation of the translation to sequential code
- Convert lists of constraints into sequences of assignments and method calls
- Translate static clocks into conditional statements
- [Bourke, Brun, and Pouzet (2020): Mechanized Semantics and Verified Compilation for a Dataflow Synchronous Language with Reset]

Simple Imperative Language: Obc

c ::=	x^{ty}	variable
|	$\text{state}(x)^{ty}$	memory
|	k	constant
|	$\diamond^{ty}\ e$	unary operator
|	$e \oplus^{ty} e$	binary operator

s ::=	$x := c$	variable assignment
|	$\text{state}(x) := c$	memory assignment
|	$\text{if}\ c\ \{s\}\ \text{else}\ \{s\}$	conditional branching
|	$s;\ s$	sequential composition
|	$x, \ldots, x := cl.\ m\ o\ (e, \ldots, e)$	class method call
|	skip	nop

Big-step semantics: $me, ve \vdash_{prog} s \Downarrow me', ve'$

$me : \cdots \qquad ve : ident \to option\ val$

Obc: methods and classes

```
Record method : Type :=
  mk_method {
      m_name : ident;
      m_in   : list (ident * type);
      m_vars : list (ident * type);
      m_out  : list (ident * type);
      m_body : stmt;

      m_nodupvars : NoDupMembers (m_in ++ m_vars ++ m_out);
      m_good      : Forall NotReserved (m_in ++ m_vars ++ m_out)   }.

Record class : Type :=
  mk_class {
      c_name    : ident;
      c_mems    : list (ident * type);
      c_objs    : list (ident * ident);   (* (instance, class) *)
      c_methods : list method;

      c_nodups  : NoDup (map fst c_mems ++ map fst c_objs)
      c_nodupm  : NoDup (map m_name c_methods)
      c_good    : Forall (fun xt ⇒ valid (fst xt)) c_objs ∧ valid c_name  }.

Definition program : Type := list class.
```

- Structural invariants included in dependent records (as in CompCert).
- Typing predicate not shown:
  - Memories used in methods are declared in class.
  - Variables used in methods are declared in methods.
  - Classes and methods used in methods are declared in programs.

Obc: semantic model

```
Inductive stmt_eval :
  program → heap → stack → stmt → heap * stack → Prop :=

| Iassign:
    exp_eval menv env e v →
    PM.add x v env = env' →
   ------------------------------------------------
    stmt_eval prog menv env (Assign x e) (menv, env')
```

x := e

```
| Iassignst:
    exp_eval menv env e v →
    madd_mem x v menv = menv' →
   ------------------------------------------------
    stmt_eval prog menv env (AssignSt x e) (menv', env)
```

state(x) := e

```
| Istep:
    omenv = match mfind_inst o menv with None ⇒ hempty | Some om ⇒ om end →
    Forall2 (exp_eval menv env) es vos →
    stmt_call_eval prog omenv cls m vos omenv' rvos →
    madd_obj o omenv' menv = menv' →
    updates ys rvos env = env' →
   ------------------------------------------------
    stmt_eval prog menv env (Call ys cls o m es) (menv', env')
⋮
```

x,...,x := cls.m o (e,...,e)

Obc: semantic model

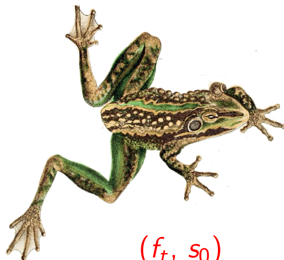
```
Inductive stmt_eval :
  program → heap → stack → stmt → heap * stack → Prop :=

| Iassign:
    exp_eval menv env e v →
    PM.add x v env = env' →
    ────────────────────────────────────────────
    stmt_eval prog menv env (Assign x e) (menv, env')
```

x := e

```
| Iassignst:
    exp_eval menv env e v →
    madd_mem x v menv = menv' →
    ────────────────────────────────────────────
    stmt_eval prog menv env (AssignSt x e) (menv', env)
```

state(x) := e



(f_t, s_0)

$S \times T \to T' \times S$ $\quad S$

```
| Istep:
    omenv = match mfind_inst o menv with None ⇒ hempty | Some om ⇒ om end →
    Forall2 (exp_eval menv env) es vos →
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    ────────────────────────────────────────────
    stmt_eval prog menv env (Call ys cls o m es) (menv', env')
  ⋮
```

x,...,x := cls.m o (e,...,e)

Translation to simple imperative code (Obc)

```
system rising_edge_retrigger {
  init norm1$1 = 0,
       norm1$2 = 0;
  sub <elab$3> : dcount;

  transition(i : bool; n : int32)
  returns (o : bool)
  var edge, ck : bool; v : int32;
      elab$3 : int32 when ck;
  {
    next norm1$2 = not i;
    edge = i and norm1$2;
    next norm1$1 = o;
    ck = edge or norm1$1;
    elab$3 = dcount<elab$3>(n when ck);
    reset(dcount<elab$3>) every (. on edge);
    v = merge ck
          (true -> elab$3)
          (false -> 0 when not ck);
    o = v > 0;
  }
}
```

Translation to simple imperative code (Obc)

```
system rising_edge_retrigger {
  init norm1$1 = 0,
       norm1$2 = 0;
  sub <elab$3> : dcount;

  transition(i : bool; n : int32)
  returns (o : bool)
  var edge, ck : bool; v : int32;
     elab$3 : int32 when ck;
  {
    next norm1$2 = not i;
    edge = i and norm1$2;
    next norm1$1 = o;
    ck = edge or norm1$1;
    elab$3 = dcount<elab$3>(n when ck);
    reset(dcount<elab$3>) every (. on edge);
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Schedule Stc constraints

- Normal variables: write before reads
- Next variables: reads before write
- Semantics is invariant under constraint reordering

- Scheduling in OCaml using a heuristic to group constraints with related guards
- Use result to sort constraints in Coq
- Correctness proof is trivial

Translation to simple imperative code (Obc)

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system rising_edge_retrigger {
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s-translation

```
class rising_edge_retrigger {
  state norm1$1 : bool;
  state norm1$2 : bool;
  instance elab$3 : dcount;

  step(i : bool; n : int32)
  returns (o : bool)
  var edge, ck : bool; v, elab$3 : int32
  {
    edge := i and state(norm1$2);
    state(norm1$2) := not i;
    ck := edge or state(norm1$1);
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    if ck {
      elab$3 := dcount(elab$3).step([n])
    };
    if ck {
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    };
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```
Variable mems : PS.t.
Definition tovar (x: ident) : exp := if PS.mem x mems then State x else Var x.

Fixpoint Control (ck: clock) (s: stmt) : stmt :=
  match ck with
  | Cbase ⇒ s
  | Con ck x true  ⇒ Control ck (Ifte (tovar x) s Skip)
  | Con ck x false ⇒ Control ck (Ifte (tovar x) Skip s)
  end.

Fixpoint translate_cexp (x: ident) (e : cexp) {struct e} : stmt :=
  match e with
  | Emerge y t f ⇒ Ifte (tovar y) (translate_cexp x t) (translate_cexp x f)
  | Eexp l ⇒ Assign x (translate_exp l)
  end.

Definition translate_tc (tc: trconstr) : stmt :=
  match tc with
  | TcDef x ck ce ⇒ Control ck (translate_cexp x ce)
  | TcNext x ck le ⇒ Control ck (AssignSt x (translate_exp le))
  | TcCall s xs ck rst f es ⇒
      Control ck (Call xs f s step (map (translate_arg ck) es))
  | TcReset s ck f ⇒ Control ck (Call [] f s reset [])
  end.
```

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Fixpoint translate_cexp (x: ident)(e : cexp) {struct e} : stmt := ...

Definition translate_tc (tc: trconstr) : stmt :=
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  | TcNext x ck le ⇒ Control ck (AssignSt x (translate_exp le))
  | TcCall s xs ck rst f es ⇒
      Control ck (Call xs f s step (map (translate_arg ck) es))
  | TcReset s ck f ⇒ Control ck (Call [] f s reset [])
  end.

Definition translate_tcs (tcs: list trconstr) : stmt :=
  fold_left (fun i tc ⇒ Comp (translate_tc tc) i) tcs Skip.
```

Implementation of translation

- Translation pass: small set of functions on abstract syntax.
- Challenge: going from one semantic model to another.

```
Definition tovar (x: ident) : exp :=
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Fixpoint Control (ck: clock) (s: stmt) : stmt :=
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  end.

Fixpoint translate_exp (e : lexp) : exp :=
  match e with
  | Econst c ⇒ Const c
  | Evar x ⇒ tovar x
  | Ewhen e c x ⇒ translate_exp e
  | Eop op es ⇒ Op op (map translate_exp es)
  end.

Fixpoint translate_cexp (x: ident) (e: cexp) : stmt :=
  match e with
  | Emerge y t f ⇒ Ifte (tovar y) (translate_cexp x t)
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  end.

Definition translate_tc (tc: trconstr) : stmt :=
  match tc with
  | TcDef x ck ce ⇒ Control ck (translate_cexp x ce)
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  end.

Definition translate_tcs (tcs: list trconstr) : stmt :=
  fold_left (fun i tc ⇒ Comp (translate_tc tc) i) tcs Skip.

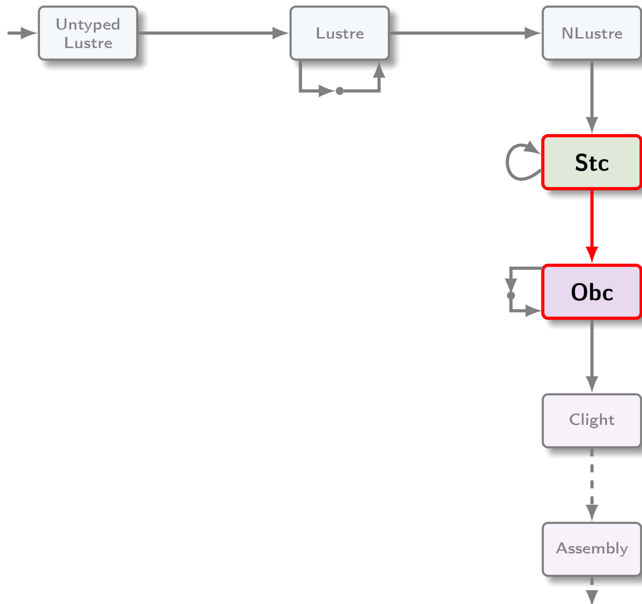
Definition ps_from_list (l: list ident) : PS.t :=
  fold_left (fun s i⇒PS.add i s) l PS.empty.

Program Definition step_method (s: system) : method :=
  let memids := map fst s.(s_lasts) in
  let mems := ps_from_list memids in
  let clkvars := Env.adds_with snd s.(s_out)
                  (Env.adds_with snd s.(s_vars)
                    (Env.from_list_with snd s.(s_in)))
  in
  {| m_name := step;
     m_in   := idty s.(s_in);
     m_vars := idty s.(s_vars);
     m_out  := idty s.(s_out);
     m_body := translate_tcs mems clkvars s.(s_tcs)
  |}.

Program Definition translate_system (b: system) : class :=
  {| c_name    := b.(s_name);
     c_mems    :=
      map (fun xc ⇒ (fst xc, type_const (fst (snd xc)))) b.(s_lasts);
     c_objs    := b.(s_subs);
     c_methods := [ step_method b; reset_method b ]
  |}.

Definition translate (P: SynStc.program) : program :=
  map translate_system P.
```

Correctness of Stc to Obc translation



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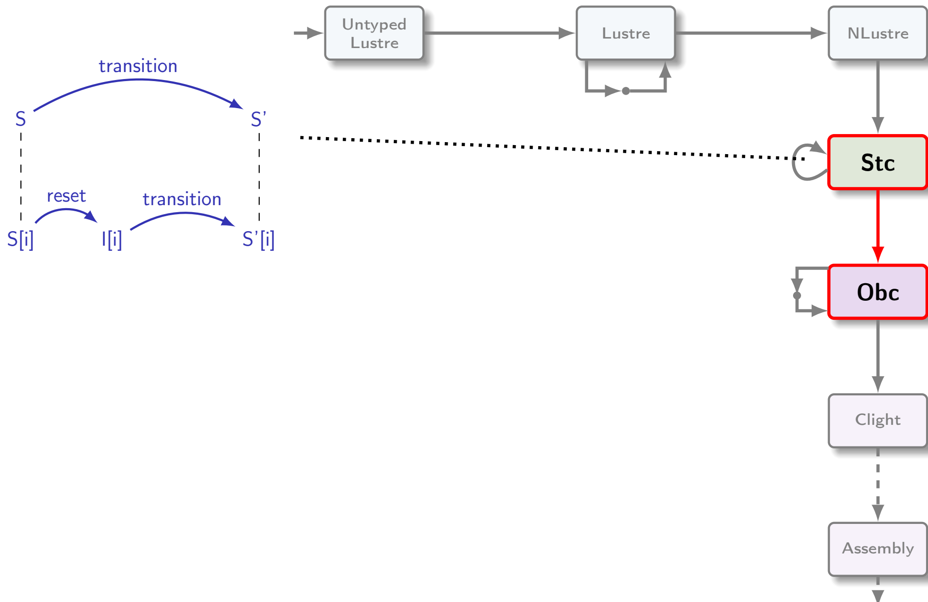

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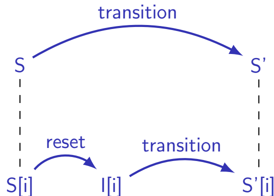

Untyped Lustre → Lustre → NLustre → **Stc** → **Obc** → Clight → Assembly

$$\frac{p, me, ve \vdash s_1 \Downarrow (me_1, ve_1) \qquad p, me_1, ve_1 \vdash s_2 \Downarrow (me_2, ve_2)}{p, me, ve \vdash s_1 \ ; \ s_2 \Downarrow (me_2, ve_2)}$$

$$\frac{me, ve \vdash e \Downarrow \lfloor \mathsf{T} \rfloor \qquad p, me, ve \vdash s_t \Downarrow (me', ve')}{p, me, ve \vdash \texttt{if}\ e\ \{\ s_t\ \}\ \texttt{else}\ \{\ s_f\ \} \Downarrow (me', ve')}$$

$\vdots$

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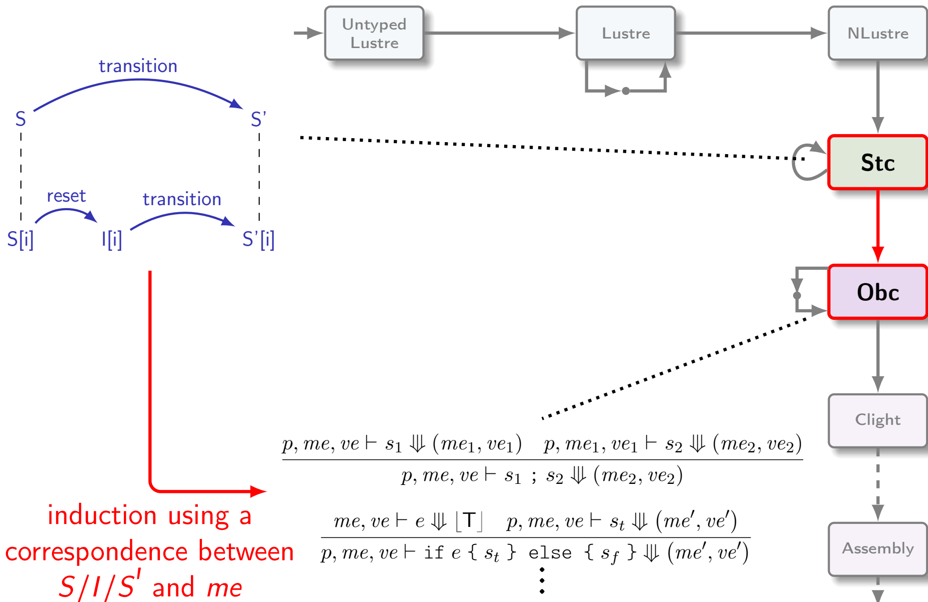

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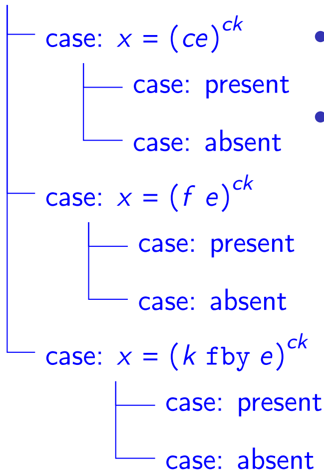


- Tricky proof, many technicalities.
- ≈100 lemmas
- Several iterations to find the right definitions.
- The intermediate states are central.

Correctness of translation to Obc

induction n

- induction G
  - induction eqs
    - case: $x = (ce)^{ck}$
      - case: present
      - case: absent
    - case: $x = (f\ e)^{ck}$
      - case: present
      - case: absent
    - case: $x = (k \texttt{ fby } e)^{ck}$
      - case: present
      - case: absent

- Tricky proof, many technicalities.
- ≈100 lemmas
- Several iterations to find the right definitions.
- The intermediate states are central.

alleqs

oeqs — eq — eqs

$[\cdots; w = v_0 \texttt{ fby } e\ ; \cdots] \; +\!+ \; (x = e :: [\cdots; y = e; \cdots])$ input

Stc to Obc: invariant

```
Definition state := memory val.
Definition menv := memory val.

Definition value_corres (x: ident) (S: state) (me: menv) : Prop :=
  find_val x S = find_val x me.

Definition state_corres (s: ident) (S: state) (me: menv) : Prop :=
  find_inst s S ⌈≡⌉ find_inst s me.

Definition Memory_Corres (tcs: list trconstr) (S I S': state) (me: menv) : Prop
  :=
    (∀ x, (Is_last_in x tcs → value_corres x S' me)
          ∧ (¬ Is_last_in x tcs → value_corres x S me))

   ∧ (∀ s, (¬ Step_in s tcs ∧ ¬ Reset_in s tcs → state_corres s S me)
          ∧ (¬ Step_in s tcs ∧ Reset_in s tcs → state_corres s I me)
          ∧ (Step_in s tcs → state_corres s S' me)).
```

Overview

Lustre: syntax and semantics

Lustre: normalization

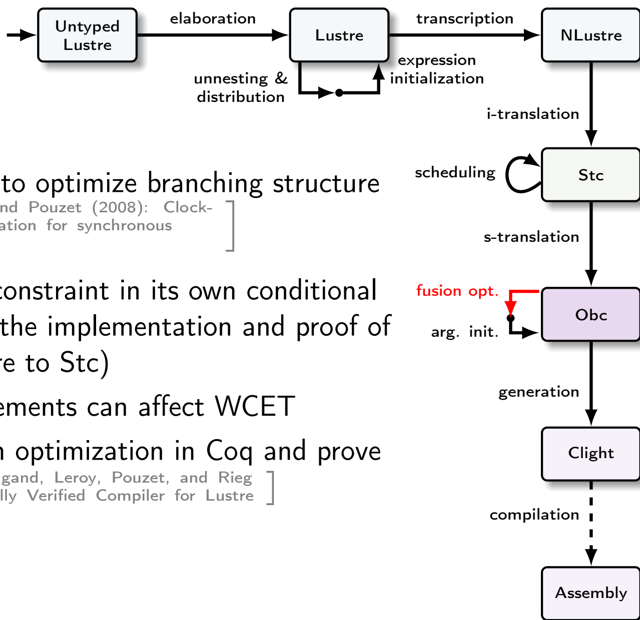
Translation: from dataflow programs to imperative code

Optimization: control structure fusion

Generation: from Obc to Clight

Conclusion

Fusion optimization: Obc to Obc



- Classic optimization to optimize branching structure [Biernacki, Colaço, Hamon, and Pouzet (2008): Clock-directed modular code generation for synchronous data-flow languages]
- Wrapping each Stc constraint in its own conditional statement simplifies the implementation and proof of s-translation (NLustre to Stc)
- But conditional statements can affect WCET
- So, implement fusion optimization in Coq and prove correct [Bourke, Brun, Dagand, Leroy, Pouzet, and Rieg (2017): A Formally Verified Compiler for Lustre]

Control structure fusion [Biernacki, Colaço, Hamon, and Pouzet (2008): Clock-directed modular code generation for synchronous data-flow languages]

```
step(delta: int, sec: bool)
   returns (v: int) {
  var r, t : int;

  r := count.step o1 (0, delta, false);
  if sec then {
    t := count.step o2 (1, 1, false)
  };
  if sec then {
    v := r / t
  } else {
    v := state(w)
  };
  state(w) := v
}
```

```
step(delta: int, sec: bool)
   returns (v: int) {
  var r, t : int;

  r := count.step o1 (0, delta, false);
  if sec then {
    t := count.step o2 (1, 1, false);
    v := r / t
  } else {
    v := state(w)
  };

  state(w) := v
}
```

- Generate control for each equation; splits proof obligation in two.
- Fuse afterward: scheduler places similarly clocked equations together.
- Use whole framework to justify required invariant.
- Easier to reason in intermediate language than in Clight.

We also define the function $Join(.,.)$ which merges two control structures gathered by the same guards:

$$
\begin{aligned}
& Join(\texttt{ case } (x)\ \{C_1 : S_1; ...; C_n : S_n\}, \\
& \qquad \texttt{case } (x)\ \{C_1 : S'_1; ...; C_n : S'_n\}) \\
& \quad = \texttt{case } (x)\ \{C_1 : Join(S_1, S'_1); ...; C_n : Join(S_n, S'_n)\} \\
& Join(S_1, S_2) = S_1; S_2
\end{aligned}
$$

$$
\begin{aligned}
& JoinList(S) = S \\
& JoinList(S_1, ..., S_n) = Join(S_1, JoinList(S_2, ..., S_n))
\end{aligned}
$$

[Biernacki, Colaço, Hamon, and Pouzet (2008): Clock-directed modular code generation for synchronous data-flow languages]

We also define the function $Join(.,.)$ which merges two control structures gathered by the same guards:

$$\begin{aligned} &Join(\texttt{case}\ (x)\ \{C_1 : S_1; ...; C_n : S_n\}, \\ &\qquad \texttt{case}\ (x)\ \{C_1 : S'_1; ...; C_n : S'_n\}) \\ &\quad = \texttt{case}\ (x)\ \{C_1 : Join(S_1, S'_1); ...; C_n : Join(S_n, S'_n)\} \\ &Join(S_1, S_2) = S_1; S_2 \end{aligned}$$

$$\begin{aligned} &JoinList(S) = S \\ &JoinList(S_1, ..., S_n) = Join(S_1, JoinList(S_2, ..., S_n)) \end{aligned}$$

```
Fixpoint zip s1 s2 : stmt :=
  match s1, s2 with
  | Ifte e1 t1 f1, Ifte e2 t2 f2 ⇒
    if equiv_decb e1 e2
    then Ifte e1 (zip t1 t2) (zip f1 f2)
    else Comp s1 s2
  | Skip, s ⇒ s
  | s,    Skip ⇒ s
  | Comp s1' s2', _ ⇒ Comp s1' (zip s2' s2)
  | s1,   s2 ⇒ Comp s1 s2
  end.

Fixpoint fuse' s1 s2 : stmt :=
  match s1, s2 with
  | s1, Comp s2 s3 ⇒ fuse' (zip s1 s2) s3
  | s1, s2 ⇒ zip s1 s2
  end.

Definition fuse s : stmt :=
  match s with
  | Comp s1 s2 ⇒ fuse' s1 s2
  | _ ⇒ s
  end.
```

```
Fixpoint zip s1 s2 : stmt :=
  match s1, s2 with
  | Ifte e1 t1 f1, Ifte e2 t2 f2 ⇒
    if equiv_decb e1 e2
    then Ifte e1 (zip t1 t2) (zip f1 f2)
    else Comp s1 s2
  | Skip, s ⇒ s
  | s,    Skip ⇒ s
  | Comp s1' s2', _ ⇒ Comp s1' (zip s2' s2)
  | s1,   s2 ⇒ Comp s1 s2
  end.

Fixpoint fuse' s1 s2 : stmt :=
  match s1, s2 with
  | s1, Comp s2 s3 ⇒ fuse' (zip s1 s2) s3
  | s1, s2 ⇒ zip s1 s2
  end.

Definition fuse s : stmt :=
  match s with
  | Comp s1 s2 ⇒ fuse' s1 s2
  | _ ⇒ s
  end.
```

```
Require Import Coq.FSets.FMapPositive.
Require Import PArith.
Require Import Velus.Common.
Require Import Velus.Operators.
Require Import Velus.Obc.Syntax.
Require Import Velus.Obc.Semantics.
Require Import Velus.Obc.Typing.
Require Import Velus.Obc.Equiv.

Require Import Velus.NLustre.Syntax.
Require Import Velus.RMemory.

Require Import List.
Import List.ListNotations.
Open Scope list_scope.

Ltac inv H := inversion H; subst; clear H.

Module Type FUSION
       (Import Ids   : IDS)
       (Import Op    : OPERATORS)
       (Import OpAux : OPERATORS_AUX Op)
       (Import SynObc: Velus.Obc.Syntax.SYNTAX Ids Op OpAux)
       (Import SemObc: Velus.Obc.Semantics.SEMANTICS Ids Op OpAux SynObc)
       (Import TypObc: Velus.Obc.Typing.TYPING Ids Op OpAux SynObc SemObc)
       (Import Equ   : Velus.Obc.Equiv.EQUIV Ids Op OpAux SynObc SemObc).

  (** ** Determine whether an Obc command can modify a variable. *)

  Inductive Can_write_in : ident → stmt → Prop :=
  | CWIAssign: ∀ x e,
      Can_write_in x (Assign x e)
  | CWIAssignSt: ∀ x e,
      Can_write_in x (AssignSt x e)
  | CWIIfteTrue: ∀ x e s1 s2,
      Can_write_in x s1 →
      Can_write_in x (Ifte e s1 s2)
  | CWIIfteFalse: ∀ x e s1 s2,
      Can_write_in x s2 →
      Can_write_in x (Ifte e s1 s2)
  | CWICall_ap: ∀ x xs cls i f es,
      In x xs →
      Can_write_in x (Call xs cls i f es)
  | CWIComp1: ∀ x s1 s2,
      Can_write_in x s1 →
      Can_write_in x (Comp s1 s2)
  | CWIComp2: ∀ x s1 s2,
      Can_write_in x s2 →
      Can_write_in x (Comp s1 s2).

  Lemma cannot_write_in_Ifte:
    ∀ x e s1 s2,
      ¬ Can_write_in x (Ifte e s1 s2)
      ↔
      ¬ Can_write_in x s1 ∧ ¬ Can_write_in x s2.
  Proof.
    Hint Constructors Can_write_in.
    intros; split; intro; try (intro HH; inversion_clear HH); intuition.
  Qed.

  ...

  (** ** Fusion functions *)

  Fixpoint zip s1 s2 : stmt :=
    match s1, s2 with
    | Ifte e1 t1 f1, Ifte e2 t2 f2 ⇒
      if equiv_decb e1 e2
      then Ifte e1 (zip t1 t2) (zip f1 f2)
      else Comp s1 s2
    | Skip, s ⇒ s
    | s,    Skip ⇒ s
    | Comp s1' s2', _ ⇒ Comp s1' (zip s2' s2)
    | s1,   s2 ⇒ Comp s1 s2
    end.

  Fixpoint fuse' s1 s2 : stmt :=
    match s1, s2 with
    | s1, Comp s2 s3 ⇒ fuse' (zip s1 s2) s3
    | s1, s2 ⇒ zip s1 s2
    end.

  (*
     NB: almost got rid of zip, but this function decreases on the first
     argument, whereas the remaining instance of zip requires a decrease
     on the second argument...

     Fixpoint fuse' s1 s2 : stmt :=
       match s1, s2 with
       ...
         then Ifte e1 (fuse' t1 t2) (fuse' f1 f2)
         else Comp s1 s2
       | s1, Comp s2 s3 => fuse' (fuse' s1 s2) s3
       | Skip, s => s
       | s,    Skip => s
       | Comp s1' s2', Ifte _ _ _ => Comp s1' (zip s2' s2)
       | s1,   s2 => Comp s1 s2
       end.
   *)

  Definition fuse s : stmt :=
    match s with
    | Comp s1 s2 ⇒ fuse' s1 s2
    | _ ⇒ s
    end.

  Definition fuse_method (m: method): method :=
    match m with
      mk_method name ins vars out body nodup good ⇒
      mk_method name ins vars out (fuse body) nodup good
    end.

  Lemma map_m_name_fuse_methods:
    ∀ methods,
      map m_name (map fuse_method methods) = map m_name methods.
  Proof.
    intro ms; induction ms as [|m ms]; auto.
    simpl. rewrite IHms.
    now destruct m.
  Qed.

  Program Definition fuse_class (c: class): class :=
    match c with
      mk_class name mems objs methods nodup nodupm ⇒
      mk_class name mems objs (map fuse_method methods) nodup _
    end.
  Next Obligation.
    now rewrite map_m_name_fuse_methods.
  Qed.

  (** ** Basic lemmas around [fuse_class] and [fuse_method]. *)

  Lemma fuse_class_name:
    ∀ c, (fuse_class c).(c_name) = c.(c_name).
  Proof. destruct c; auto. Qed.

  Lemma fuse_method_name:
  ...

  (** ** Invariant sufficient to justify semantic preservation of fusion. *)

  (*
   The property "Fusible (translate_eqns mems eqs)" is obtained above
   using scheduling assumptions for EqDef, since they are needed to treat
   the translation of merge expressions, and clocking assumptions for EqApp and
   EqFby. While the EqApp case can also be obtained from the scheduling
   assumptions alone, the EqFby case cannot. Consider the equations:
      y = (true when x) :: Con Cbase x true
      x = true fby y    :: Con Cbase x true

   These equations and their clocks are incorrect, since "Con Cbase x"
   requires that the clock of x be "Cbase", whereas the second equation
   requires that it be "Con Cbase x true". Still, we could try compiling
   them program:
      if x { y = true };
      if x { mem(x) = y }

   This program is well scheduled, but it does not satisfy the invariant
   Fusible. We could imagine a weaker invariant that allows the
   right-most write under an expression to change free variables in the
   expression, which suffices to justify the optimization, but the
   preservation of this invariant under the optimization likely becomes much
   trickier to prove. And, in any case, such programs are fundamentally
   incorrect.

   What about trying to reject such programs using sem_node rather than
   introducing clocking assumptions? The problem is that certain circular
   programs (like the one above) still have a semantics (since x and y
   are effectively always present).
  *)

  Inductive Fusible : stmt → Prop :=
  | IFAssign: ∀ x e,
      Fusible (Assign x e)
  | IFAssignSt: ∀ x e,
      Fusible (AssignSt x e)
  | IFIfte: ∀ e s1 s2,
      Fusible s1 →
      Fusible s2 →
      (∀ x, Is_free_in_exp x e → ¬Can_write_in x s1 ∧ ¬Can_write_in x s2)
      →
      Fusible (Ifte e s1 s2)
  | IFStep_ap: ∀ xs cls i f es,
      Fusible (Call xs cls i f es)
  | IFComp: ∀ s1 s2,
      Fusible s1 →
      Fusible s2 →
      Fusible (Comp s1 s2)
  | IFSkip:
      Fusible Skip.

  Definition ClassFusible (c: class) : Prop :=
    Forall (fun m⇒ Fusible m.(m_body)) c.(c_methods).

  Lemma lift_Fusible:
    ∀ e s1 s2 t1 t2,
      Fusible (Comp (Ifte e s1 s2) (Ifte e t1 t2))
      ↔
      Fusible (Ifte e (Comp s1 t1) (Comp s2 t2)).
  Proof.
    Hint Constructors Fusible.
    intros e s1 s2 t1 t2.
    split; intro Hifw.
  ...

    + intros i' Hin' Hdef.
      apply Hin, Exists_∃.
      ∃ i'. intuition.
    - apply translate_reset_eqns_Fusible.
    - inv WdG. apply IHG; auto.
  Qed.

End FUSIBLE.

Module FusibleFun
       (Import Ids   : IDS)
       (Import Op    : OPERATORS)
       (Import OpAux : OPERATORS_AUX Op)
       (Import NLus  : Velus.NLustre.NLUSTRE Ids Op OpAux)
       (Import Obc   : Velus.Obc.OBC Ids Op OpAux)
       (Import Trans : Velus.NLustreToObc.Translation.TRANSLATION
                         Ids Op OpAux NLus.Syn Obc.Syn NLus.Mem)
  <: FUSIBLE Ids Op OpAux NLus Obc Trans.

  Include FUSIBLE Ids Op OpAux NLus Obc Trans.

End FusibleFun.
```

Fusion of control structures: implementation

$$\mathrm{fuse}\left(\begin{array}{c} ; \\ / \ \backslash \\ s \quad t \end{array}\right) = \mathrm{fuse}'(s, t)$$

$$\mathrm{fuse}(s) = s$$

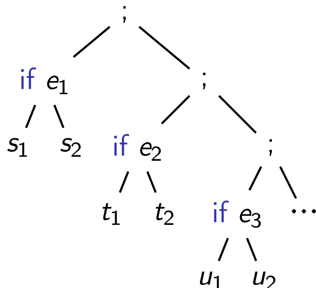
$$\mathrm{fuse}'\left(s, \begin{array}{c} ; \\ / \ \backslash \\ t_1 \quad t_2 \end{array}\right) = \mathrm{fuse}'(\mathrm{zip}(s, t_1), t_2)$$

$$\mathrm{fuse}'(s, t) = \mathrm{zip}(s, t)$$

$$\mathrm{zip}\left(\begin{array}{c} \mathrm{if}\ e \\ / \ \backslash \\ s_1 \quad s_2 \end{array}, \begin{array}{c} \mathrm{if}\ e \\ / \ \backslash \\ t_1 \quad t_2 \end{array}\right) = \begin{array}{c} \mathrm{if}\ e \\ / \qquad \backslash \\ \mathrm{zip}(s_1, t_1) \quad \mathrm{zip}(s_2, t_2) \end{array}$$

$$\mathrm{zip}\left(\begin{array}{c} ; \\ / \ \backslash \\ s_1 \quad s_2 \end{array}, t\right) = \begin{array}{c} ; \\ / \quad \backslash \\ s_1 \quad \mathrm{zip}(s_2, t) \end{array}$$

$$\mathrm{zip}(s, t) = \begin{array}{c} ; \\ / \ \backslash \\ s \quad t \end{array}$$



Fusion of control structures: requires invariant

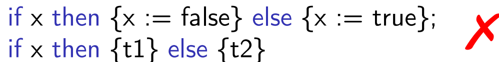
```
if e then {s1} else {s2};
if e then {t1} else {t2}
```

➡

```
if e then {s1; t1} else {s2; t2};
```

Fusion of control structures: requires invariant

Fusion of control structures: requires invariant

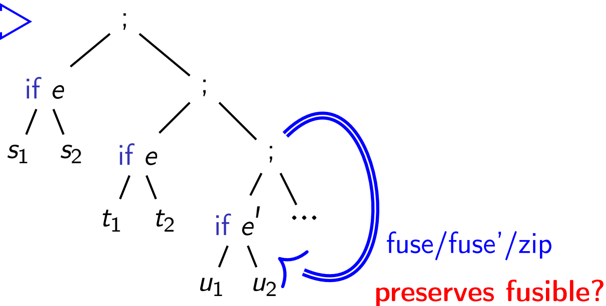


$$\frac{\text{fusionnable}(s_1) \qquad \text{fusionnable}(s_2) \qquad \forall x \in \text{libres}(e), \neg\text{peut-écrire}\ x\ s_1 \wedge \neg\text{peut-écrire}\ x\ s_2}{\text{fusionnable}(\text{if}\ e\ \{s_1\}\ \text{else}\ \{s_2\})}$$

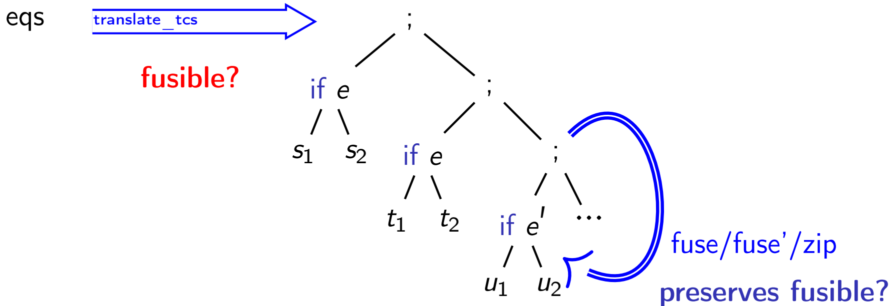
$$\frac{\text{fusionnable}(s_1) \quad \text{fusionnable}(s_2)}{\text{fusionnable}(s_1; s_2)} \qquad \ldots$$

Fusion of control structures: correctness

eqs



Fusion of control structures: correctness

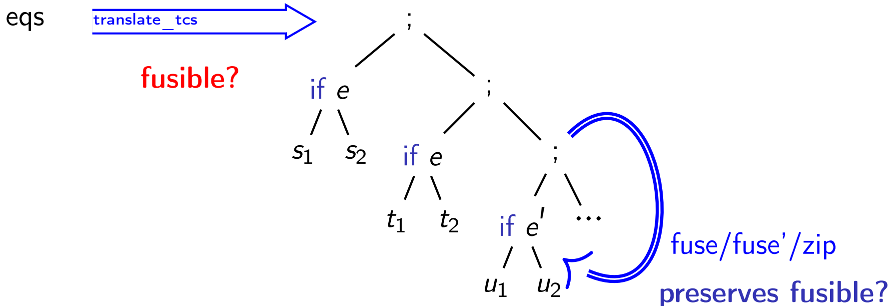


$x = (\text{merge } b\ e1\ e2)^{\text{base on ck}}$

```
if ck {
  if b {
    x := e1
  } else {
    x := e2
  }
}
```

- In a well scheduled dataflow program it is not possible to read x before writing it.
- Compiling $x = (ce)^{ck}$ and $x = (f\ le)^{ck}$ gives fusible imperative code.

Fusion of control structures: correctness



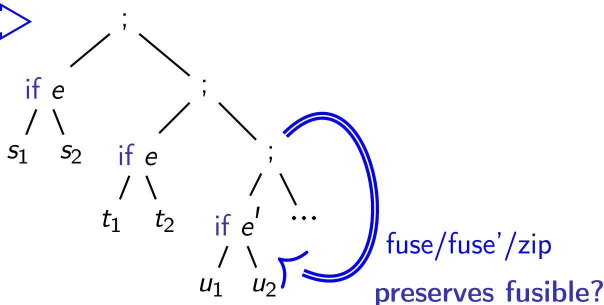
$x = (0 \text{ fby } (x + 1))^{\text{base on ck}}$

```
if ck {
  mem(x) := mem(x) + 1
}
```

- But for fby equations, we must read x before writing it.
- A different invariant?
  Once we write x, we never read it again.
  Trickier to express. Trickier to work with.

Fusion of control structures: correctness

eqs



$$y = (\text{true when } x)^{\text{base on } x}$$

$$x = (\text{true fby } y)^{\text{base on } x}$$

```
if mem(x) {
  y := true
}
if mem(x) {
  mem(x) := y
}
```

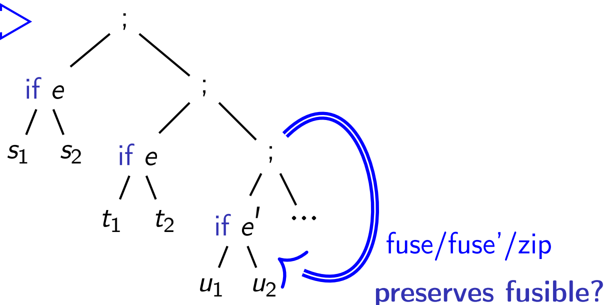
- Happily, such programs are not well clocked.

$$\frac{C \vdash \mathit{true} :: \mathit{base} \qquad C \vdash x :: \mathit{base}}{C \vdash \mathit{true}\ \texttt{when}\ x :: \mathit{base}\ \text{on}\ (x = T)}$$

$$C \vdash x :: \mathit{base}\ \text{on}\ (x = T)$$

Fusion of control structures: correctness

eqs



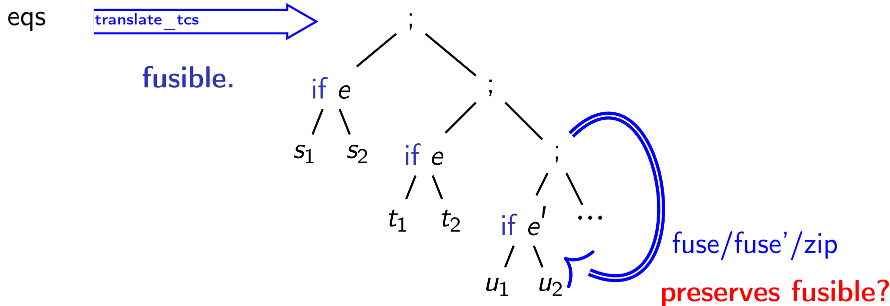
y = (true when x)$^{\text{base on x}}$

x = (true fby y)$^{\text{base on x}}$

```
if mem(x) {
  y := true
}
if mem(x) {
  mem(x) := y
}
```

- Happily, such programs are not well clocked.
- Show that a variable x is never free in its own clock in a well clocked program:
  $C \not\vdash x :: base \text{ on } \cdots \text{ on } x \text{ on } \cdots$
- Compiling x = (v0 fby le)^{ck} also gives fusible imperative code.

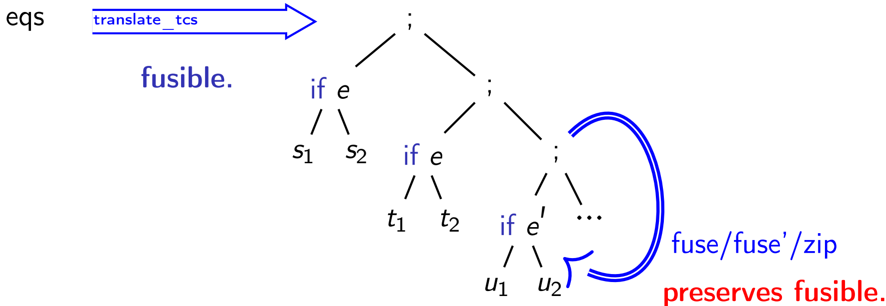
Fusion of control structures: correctness



- Define $s_1 \approx_{eval} s_2$

```
Definition stmt_eval_eq s1 s2: Prop :=
  ∀ prog menv env menv' env',
    stmt_eval prog menv env s1 (menv', env')
    ↔
    stmt_eval prog menv env s2 (menv', env').
```

Fusion of control structures: correctness



- Define $s_1 \approx_{eval} s_2$
- Define $s_1 \approx_{fuse} s_2$ as
  $s_1 \approx_{eval} s_2 \wedge \text{fusionnable}(s_1) \wedge \text{fusionnable}(s_2)$
- Show congruence ('Proper' instances) for ;/fuse/fuse'/zip.
- Rewrite until $\dfrac{\text{fusionnable}(s)}{\text{fuse}\,(s) \approx_{eval} s}$

Overview

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Lustre: normalization

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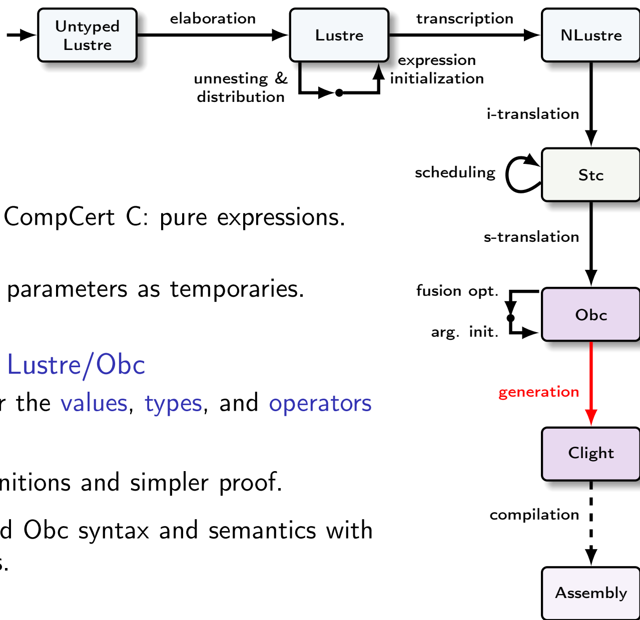
Conclusion

Generation: Obc to Clight



- Clight
  - Simplified version of CompCert C: pure expressions.
  - 4 semantic variants:
    we use big-step with parameters as temporaries.
- Integrate Clight into Lustre/Obc
  - Abstract interface for the values, types, and operators of Lustre and Obc.
  - Result: modular definitions and simpler proof.
  - Instantiate Lustre and Obc syntax and semantics with CompCert definitions.

- Introduce an abstract interface for values, types, and operators.
  - Define N-Lustre and Obc syntax and semantics against this interface.
  - Likewise for the N-Lustre to Obc translation and proof.
- Instantiate with definitions for the Obc to Clight translation and proof.

```
Module Type OPERATORS.

 Parameter val   : Type.
 Parameter type  : Type.
 Parameter const : Type.
```

- Introduce an abstract interface for values, types, and operators.
  - Define N-Lustre and Obc syntax and semantics against this interface.
  - Likewise for the N-Lustre to Obc translation and proof.

- Instantiate with definitions for the Obc to Clight translation and proof.

```
Module Type OPERATORS.

 Parameter val   : Type.
 Parameter type  : Type.
 Parameter const : Type.

 (* Boolean values *)
 Parameter bool_type : type.

 Parameter true_val  : val.
 Parameter false_val : val.
 Axiom true_not_false_val :
   true_val <> false_val.
```

- Introduce an abstract interface for values, types, and operators.
  - Define N-Lustre and Obc syntax and semantics against this interface.
  - Likewise for the N-Lustre to Obc translation and proof.
- Instantiate with definitions for the Obc to Clight translation and proof.

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 (* Boolean values *)
 Parameter bool_type : type.

 Parameter true_val  : val.
 Parameter false_val : val.
 Axiom true_not_false_val :
   true_val <> false_val.

 (* Constants *)
 Parameter type_const : const → type.
 Parameter sem_const  : const → val.
```

- Introduce an abstract interface for values, types, and operators.
  - Define N-Lustre and Obc syntax and semantics against this interface.
  - Likewise for the N-Lustre to Obc translation and proof.
- Instantiate with definitions for the Obc to Clight translation and proof.

```
Module Type OPERATORS.

 Parameter val   : Type.
 Parameter type  : Type.
 Parameter const : Type.

 (* Boolean values *)
 Parameter bool_type : type.

 Parameter true_val  : val.
 Parameter false_val : val.
 Axiom true_not_false_val :
   true_val <> false_val.

 (* Constants *)
 Parameter type_const : const → type.
 Parameter sem_const  : const → val.

 (* Operators *)
 Parameter unop  : Type.
 Parameter binop : Type.

 Parameter sem_unop  :
   unop → val → type → option val.

 Parameter sem_binop :
   binop → val → type → val → type
      → option val.

 Parameter type_unop  :
   unop → type → option type.

 Parameter type_binop :
   binop → type → type → option type.

 (* ... *)
End OPERATORS.
```

- Introduce an abstract interface for values, types, and operators.
  - Define N-Lustre and Obc syntax and semantics against this interface.
  - Likewise for the N-Lustre to Obc translation and proof.
- Instantiate with definitions for the Obc to Clight translation and proof.

```
Module Type OPERATORS.

 Parameter val   : Type.
 Parameter type  : Type.
 Parameter const : Type.

 (* Boolean values *)
 Parameter bool_type : type.

 Parameter true_val  : val.
 Parameter false_val : val.
 Axiom true_not_false_val :
   true_val <> false_val.

 (* Constants *)
 Parameter type_const : const → type.
 Parameter sem_const  : const → val.

 (* Operators *)
 Parameter unop  : Type.
 Parameter binop : Type.

 Parameter sem_unop  :
   unop → val → type → option val.

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 (* ... *)
End OPERATORS.
```

```
Module Export Op <: OPERATORS.

 Definition val: Type := Values.val.

            Inductive val: Type :=
              | Vundef  : val
              | Vint    : int → val
              | Vlong   : int64 → val
              | Vfloat  : float → val
              | Vsingle : float32 → val
              | Vptr    : block → int → val.
```

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 Parameter bool_type : type.

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 (* ... *)
End OPERATORS.
```

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 Definition val: Type := Values.val.

 Inductive type : Type :=
 | Tint   : intsize → signedness → type
 | Tlong  : signedness → type
 | Tfloat : floatsize → type.

 Inductive signedness : Type :=
   | Signed   : signedness
   | Unsigned : signedness.

 Inductive intsize : Type :=
   | I8    : intsize    (* char  *)
   | I16   : intsize    (* short *)
   | I32   : intsize    (* int   *)
   | IBool : intsize.   (* bool  *)

 Inductive floatsize : Type :=
   | F32 : floatsize    (* float  *)
   | F64 : floatsize.   (* double *)
```

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 | Cint    : int → intsize → signedness → const
 | Clong   : int64 → signedness → const
 | Cfloat  : float → const
 | Csingle : float32 → const.
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 Definition true_val  := Vtrue.  (* Vint Int.one  *)
 Definition false_val := Vfalse. (* Vint Int.zero *)

 Lemma true_not_false_val: true_val <> false_val.
 Proof. discriminate. Qed.

 Definition bool_type : type := Tint IBool Signed.
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 Definition bool_type : type := Tint IBool Signed.

 Inductive unop : Type :=
 | UnaryOp: Cop.unary_operation → unop
 | CastOp:  type → unop.

 Definition binop := Cop.binary_operation.

 Definition sem_unop (uop: unop) (v: val) (ty: type) : option val
 := match uop with
    | UnaryOp op ⇒ sem_unary_operation op v (cltype ty) Mem.empty
    | CastOp ty' ⇒ sem_cast v (cltype ty) (cltype ty') Mem.empty
    end.

 (* ... *)
End Op.
```

Language design: Operator types

- $(+) : \mathsf{int} \to \mathsf{int} \to \mathsf{int}$

Language design: Operator types

- $(+) : \mathsf{int} \to \mathsf{int} \to \mathsf{int}$
- $(+) : \mathsf{double} \to \mathsf{double} \to \mathsf{double}$

Language design: Operator types

- $(+)$: int → int → int
- $(+)$: double → double → double
- $(+)$: unsigned char → unsigned char → ?

Language design: Operator types

- $(+) : \text{int} \rightarrow \text{int} \rightarrow \text{int}$
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Language design: Operator types

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Obc

```
var x : uint8,
    y : int;

x := y
```

Clight

```
unsigned char x;
int y;

x = y;
```

Language design: Operator types

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implicit cast: x = (**unsigned char**) y

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Obc

```
var x : uint8,
    y : int;

x := (y : uint8)
```

Clight

```
unsigned char x;
int y;

x = y;
```

implicit cast: x = (**unsigned char**) y

- Choice: no implicit casting in Obc.
- Explicit casts simplify substitution (referential transparency).
- Alternative: extend `OPERATORS` with an explicit operation for assignment

  `store` : `type` → `type` → `val` → `val`

Language design: operator types (unfortunate consequences)

```
node pita(x: signed char; y: signed char)
  returns (z: signed char)
  var w : signed char;
let
  w = x + y;
  z = (w / 2) + x;
tel
```

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let
  w = (x + y : signed char);
  z = ((w / (2 : signed char)) + x : signed char);
tel
```

Language design: operator types (unfortunate consequences)

```
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  returns (z: signed char)
  var w : signed char;
let
  w = x + y;
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tel
node pita(x: signed char; y: signed char)
  returns (z: signed char)
  var w : signed char;
let
  w = (x + y : signed char);
  z = ((w / (2 : signed char)) + x : signed char);
tel
```

```
node pita(x: signed char; y: signed char)
  returns (z: signed char)
  var w : signed char, t1 : int;
let
  w = (x + y : signed char);
  t1 = w / (2 : signed char);
  z = (t1 + x : signed char);
tel
```

Operator types: bool

- _Bool: a special kind of integer that is normally 0 or 1.
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Obc	Clight
var x, y : bool;	_Bool x, y
x := y	x = y;
explicit cast not mandated	implicit cast: x = (_Bool) y

Operator types: bool

- _Bool: a special kind of integer that is normally 0 or 1.
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var x, y : bool;	_Bool x, y
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explicit cast not mandated	implicit cast: x = (_Bool) y

- The Clight type system is not strong enough for our purposes:
  - There is no typing invariant on the memory.
  - A _Bool is stored in 8-bits.
  - E.g., no way to know that y does not contain 7 (without explicitly adding such an invariant).
  - Problematic for the correctness proof.

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  - There is no typing invariant on the memory.
  - A _Bool is stored in 8-bits.
  - E.g., no way to know that y does not contain 7 (without explicitly adding such an invariant).
  - Problematic for the correctness proof.
- We refine the types of operators and use a typing invariant.

$$(<) : \text{int} \to \text{int} \to \text{int} \quad \Longrightarrow \quad (<) : \text{int} \to \text{int} \to \text{bool}$$

$$(\&) : \text{bool} \to \text{bool} \to \text{int} \quad \Longrightarrow \quad (\&) : \text{bool} \to \text{bool} \to \text{bool}$$

Operator domains

- Partial operators:
  - integer division/modulo x/y, $x \% y$: $y = 0 \lor (x = \text{INT_MIN} \land y = -1)$
  - shifts $x \ll y$, $x \gg y$: $y < 0 \lor y \geq 32$

- 'Dynamic' precondition in the existence proof?
  ($\forall i$, `sem_binop_i <> None`)
- Explicitly model the possibility of a failure?

- Alternative `OPERATORS` implementation and translation:
  x / y becomes **if** x != INT_MIN && y != 0 then x / y **else** 0

Unused types

```
Inductive type : Type :=
  | Tvoid: type                                       (**r the [void] type *)
  | Tint: intsize → signedness → attr → type          (**r integer types *)
  | Tlong: signedness → attr → type                   (**r 64-bit integer types *)
  | Tfloat: floatsize → attr → type                   (**r floating-point types *)
  | Tpointer: type → attr → type                      (**r pointer types ([*ty]) *)
  | Tarray: type → Z → attr → type                    (**r array types ([ty[len]]) *)
  | Tfunction: typelist → type → calling_convention → type    (**r function types *)
  | Tstruct: ident → attr → type                      (**r struct types *)
  | Tunion: ident → attr → type                       (**r union types *)
```

- We only use a very limited subset of Clight (Obc is much simpler).
- How to treat variant types in our compiler?
- How to treat arrays in our compiler?

Unknown values

- In Clight (`Ctyping`): ∀ `ty`, `wt_val Vundef ty`
- In Obc: `Vundef` is never well typed, `None` represents uninitialized values.
- Assuming the semantics of an Obc program precludes ever reading `None`.

```
class count { ... }

class avgvelocity {
  memory w : int;
  class count o1, o2;

  reset() {
    count.reset o1;
    count.reset o2;
    state(w) := 0
  }

  step(delta: int, sec: bool) returns (r, v: int)
  {
    var t : int;

    r := count.step o1 (0, delta, false);
    if sec
      then (t := count.step o2 (1, 1, false);
            v := r / t)
      else v := state(w);
    state(w) := v
  }
}
```

- Standard technique for encapsulating state.
- Each detail entails complications in the proof.

```
struct count { _Bool f; int c; };
void count$reset(struct count *self) { ... }
int count$step(struct count *self, int ini, int inc, _Bool res) { ... }

struct avgvelocity {
  int w;
  struct count o1;
  struct count o2;
};

struct avgvelocity$step {
  int r;
  int v;
};

void avgvelocity$reset(struct avgvelocity *self)
{
  count$reset(&(self→o1));
  count$reset(&(self→o2));
  self→w = 0;
}

void avgvelocity$step(struct avgvelocity *self,
                      struct avgvelocity$step *out, int delta, _Bool sec)
{
  register int t, step$n;

  step$n = count$step(&(self→o1), 0, delta, 0);
  out→r = step$n;
  if (sec) {
    step$n = count$step(&(self→o2), 1, 1, 0);
    t = step$n;
    out→v = out→r / t;
  } else {
    out→v = self→w;
  }
  self→w = out→v;
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class count { ... }
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struct count { _Bool f; int c; };
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  out→r = step$n;
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    step$n = count$step(&(self→o2), 1, 1, 0);
    t = step$n;
    out→v = out→r / t;
  } else {
    out→v = self→w;
  }
  self→w = out→v;
}
```

```
class count { ... }

class avgvelocity {
  memory w : int;
  class count o1, o2;

  reset() {
    count.reset o1;
    count.reset o2;
    state(w) := 0
  }

  step(delta: int, sec: bool) returns (r, v: int)
  {
    var t : int;

    r := count.step o1 (0, delta, false);
    if sec
      then (t := count.step o2 (1, 1, false);
            v := r / t)
      else v := state(w);
    state(w) := v
  }
}
```

- Standard technique for encapsulating state.
- Each detail entails complications in the proof.

```
struct count { _Bool f; int c; };
void count$reset(struct count *self) { ... }
int count$step(struct count *self, int ini, int inc, _Bool res) { ... }

struct avgvelocity {
  int w;
  struct count o1;
  struct count o2;
};

struct avgvelocity$step {
  int r;
  int v;
};

void avgvelocity$reset(struct avgvelocity *self)
{
  count$reset(&(self→o1));
  count$reset(&(self→o2));
  self→w = 0;
}

void avgvelocity$step(struct avgvelocity *self,
                      struct avgvelocity$step *out, int delta, _Bool sec)
{
  register int t, step$n;

  step$n = count$step(&(self→o1), 0, delta, 0);
  out→r = step$n;
  if (sec) {
    step$n = count$step(&(self→o2), 1, 1, 0);
    t = step$n;
    out→v = out→r / t;
  } else {
    out→v = self→w;
  }
  self→w = out→v;
}
```

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class count { ... }

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```

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  out→r = step$n;
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    step$n = count$step(&(self→o2), 1, 1, 0);
    t = step$n;
    out→v = out→r / t;
  } else {
    out→v = self→w;
  }
  self→w = out→v;
}
```

CompCert's Clight frontend

Formal model

context:	genv	ident → {type, function, global}
state:	m	block → offset → memval × permission
	e	ident → block × type
	le	ident → val

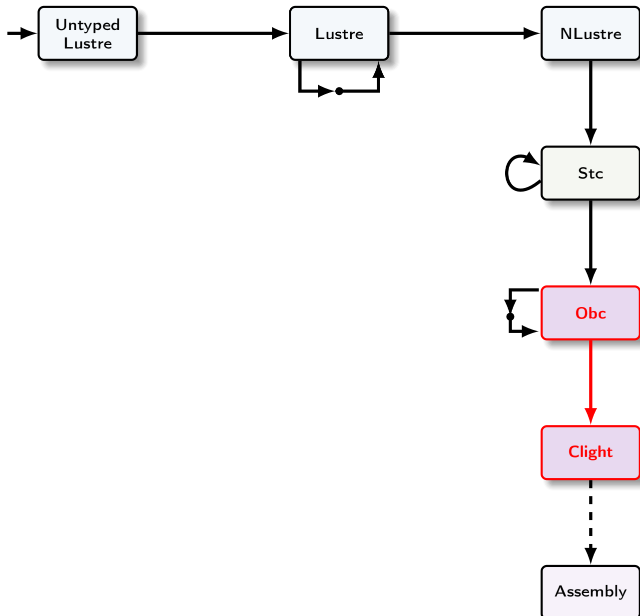
Four semantic models:

	func. params in m	*func. params in le*
$(m, e, le, s) \rightarrow (m', e', le', s')$	clight1	clight2
$genv, e \vdash m, le, s \Downarrow m', le'$	bigstep clight1	bigstep clight2

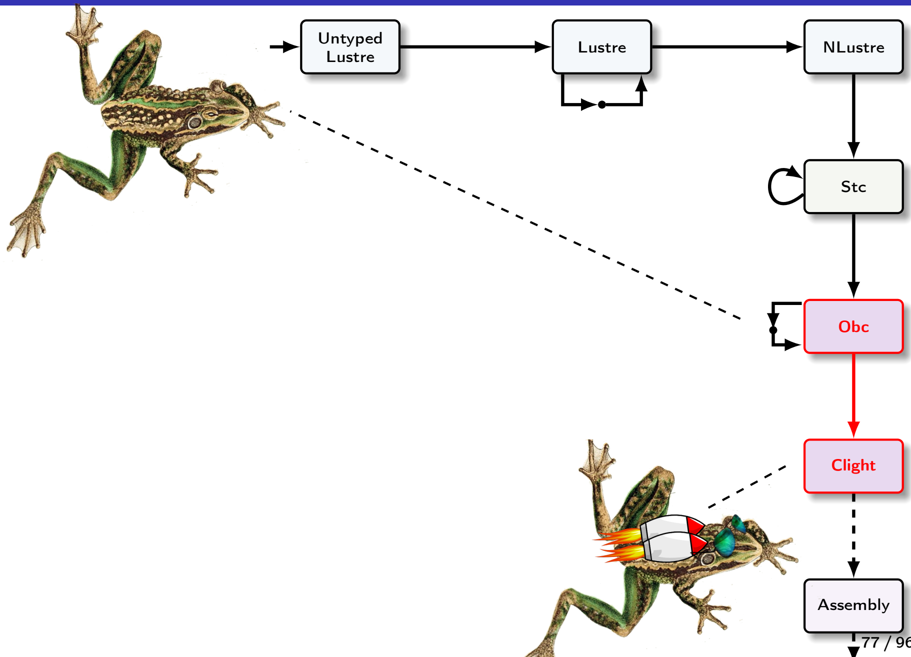
bigstep clight2 $\Longrightarrow$ clight2 $\Longrightarrow$ clight1 and bigstep clight1 $\Longrightarrow$ clight1

- Model of the C 'abstract machine' (standard + implementation).
- Data representations, types, and operators.
- Byte-level memory model: load(), store(), alloc(), and free().

Correctness of Clight generation



Correctness of Clight generation



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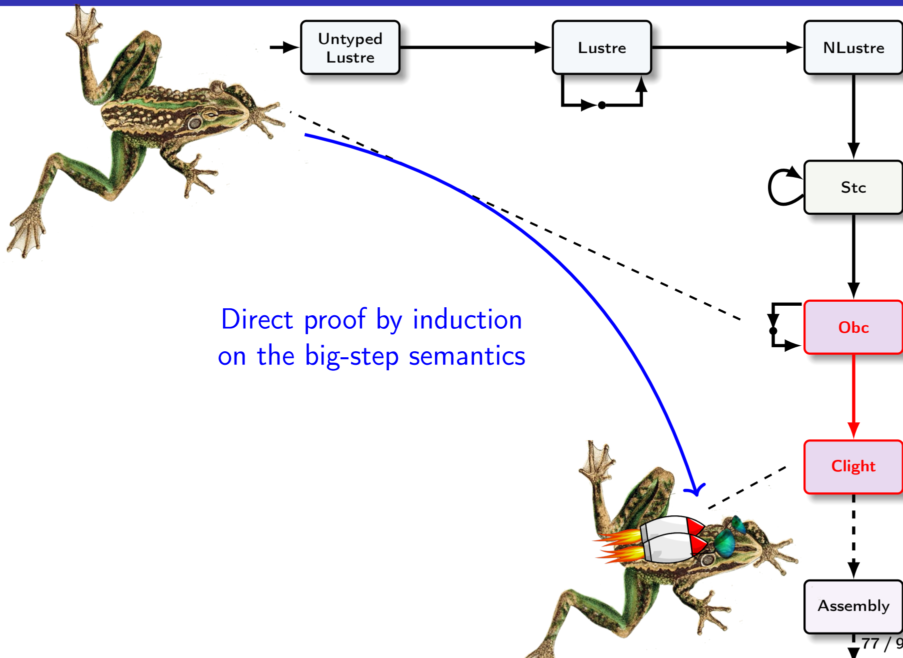

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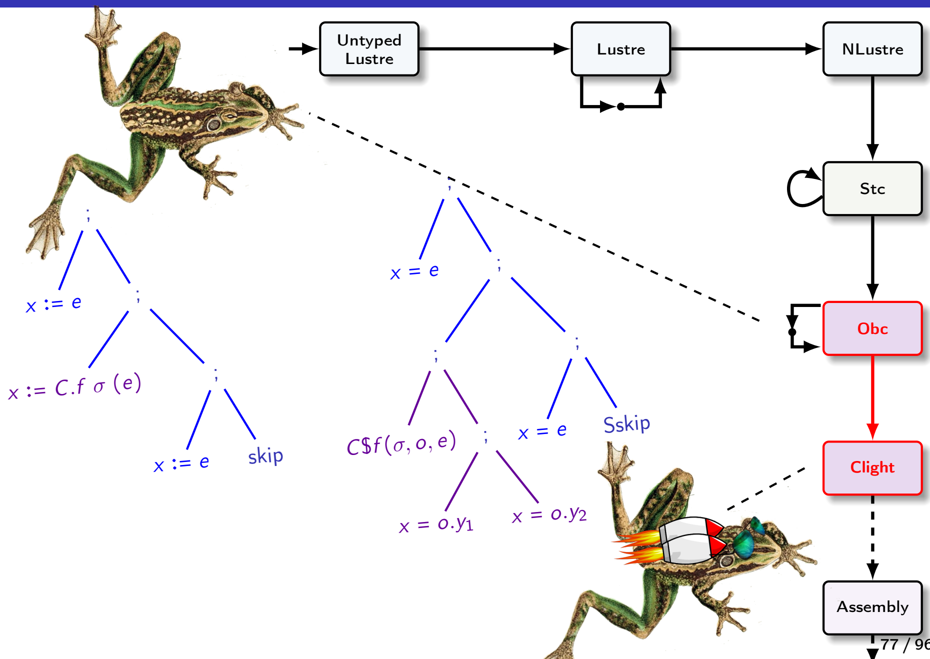

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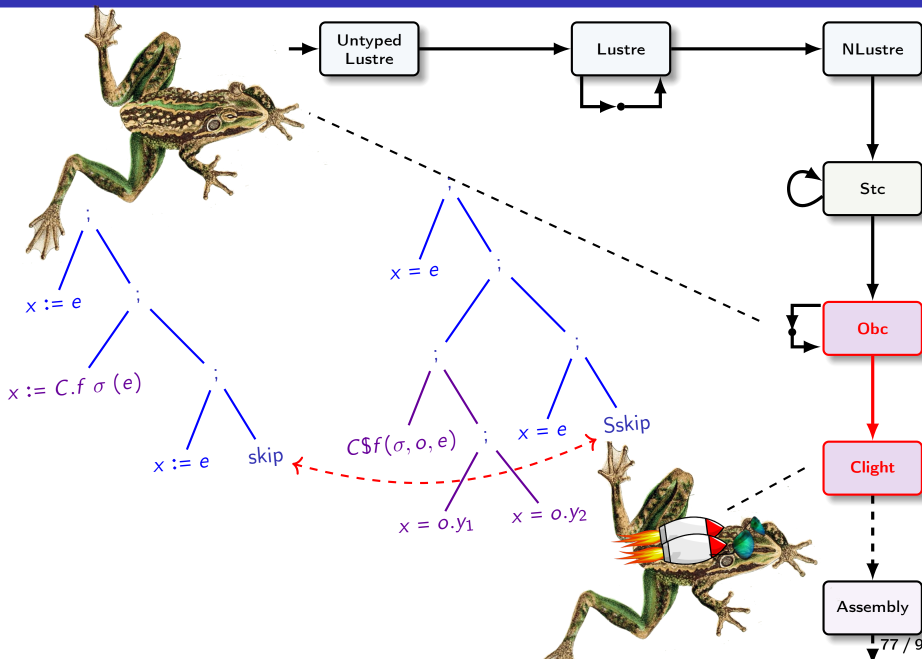

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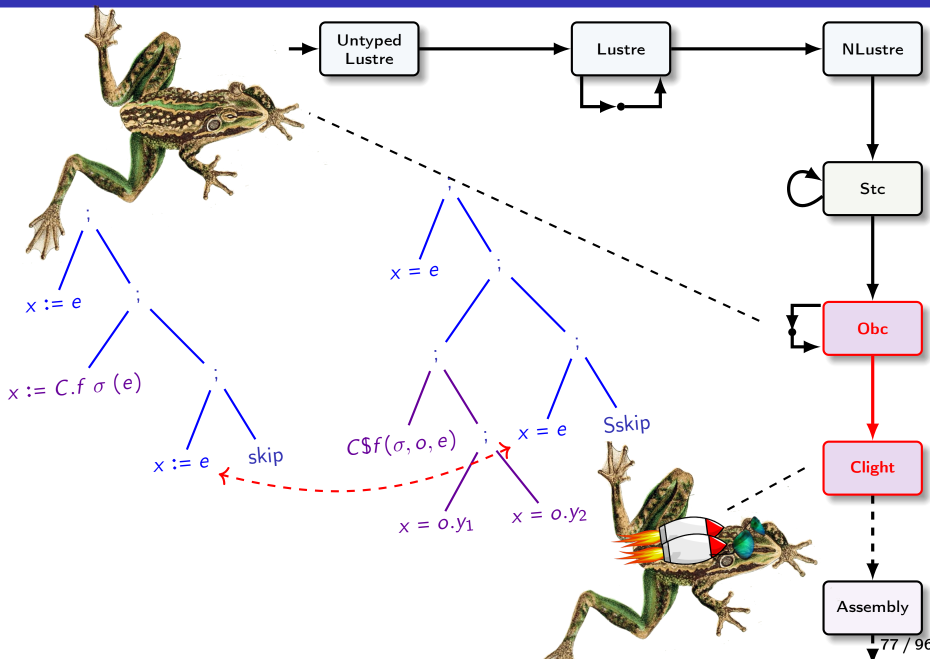

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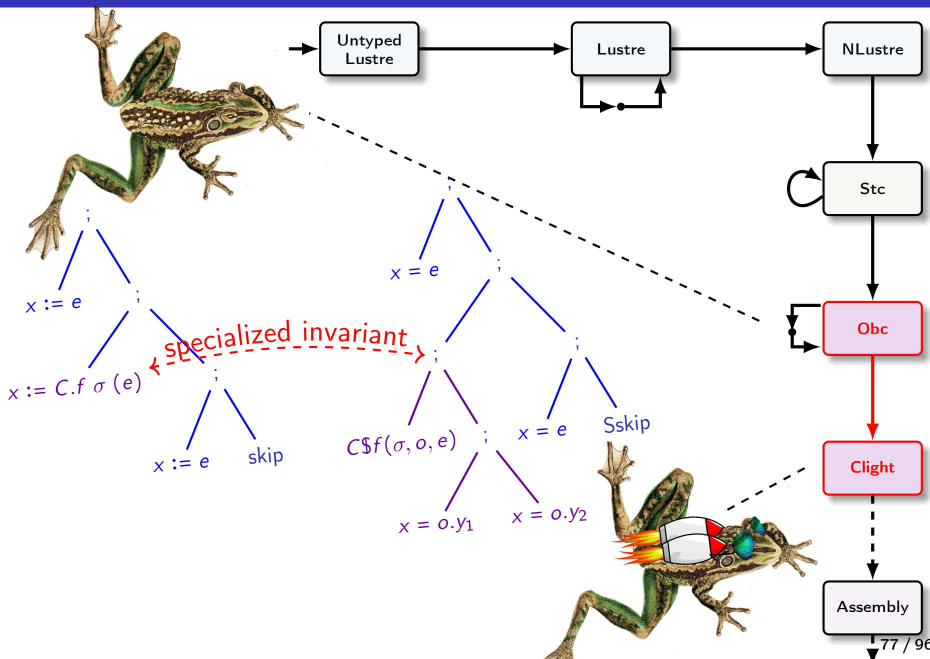

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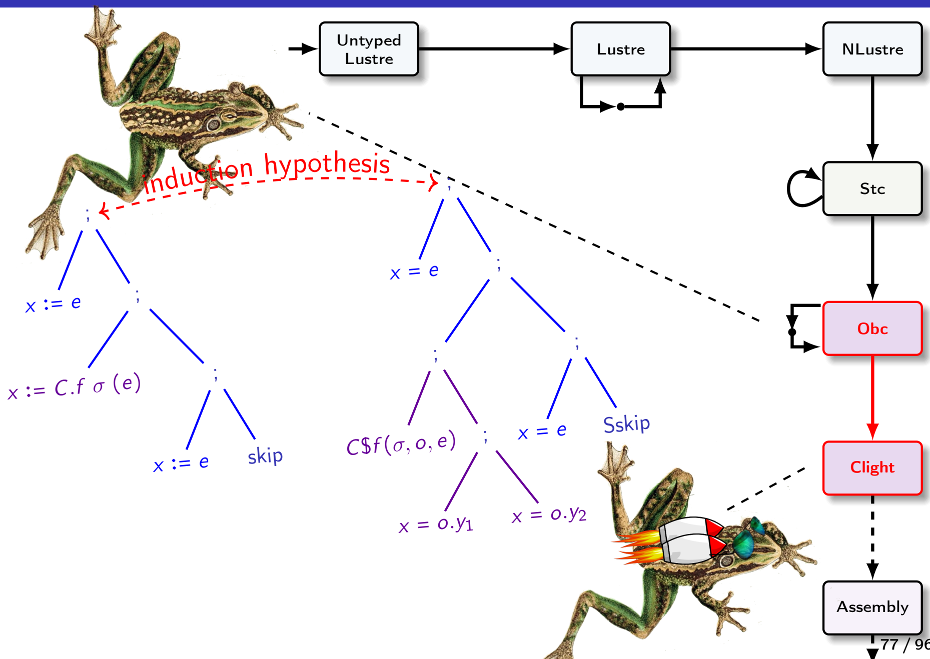

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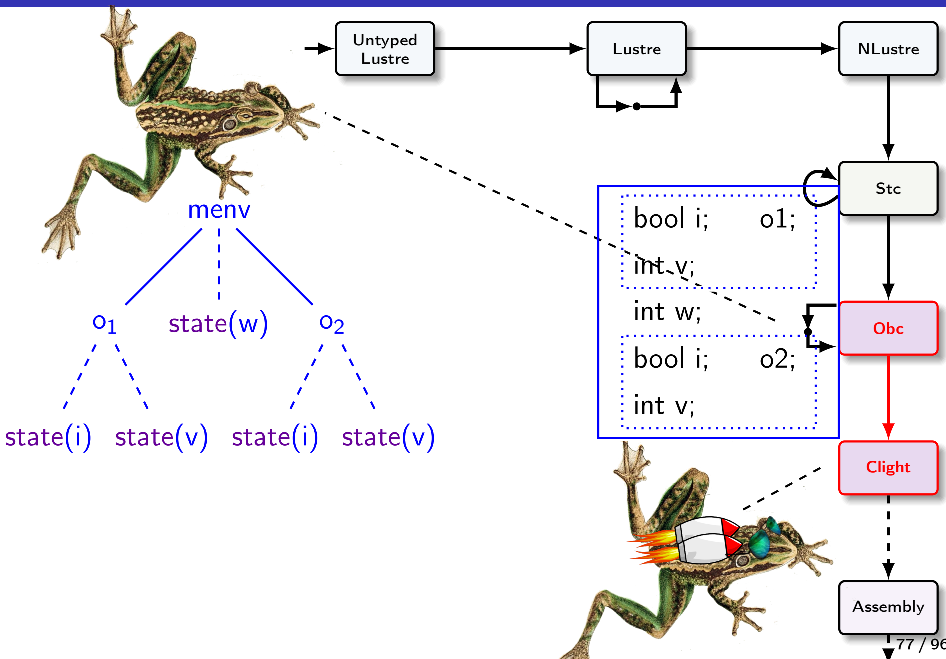

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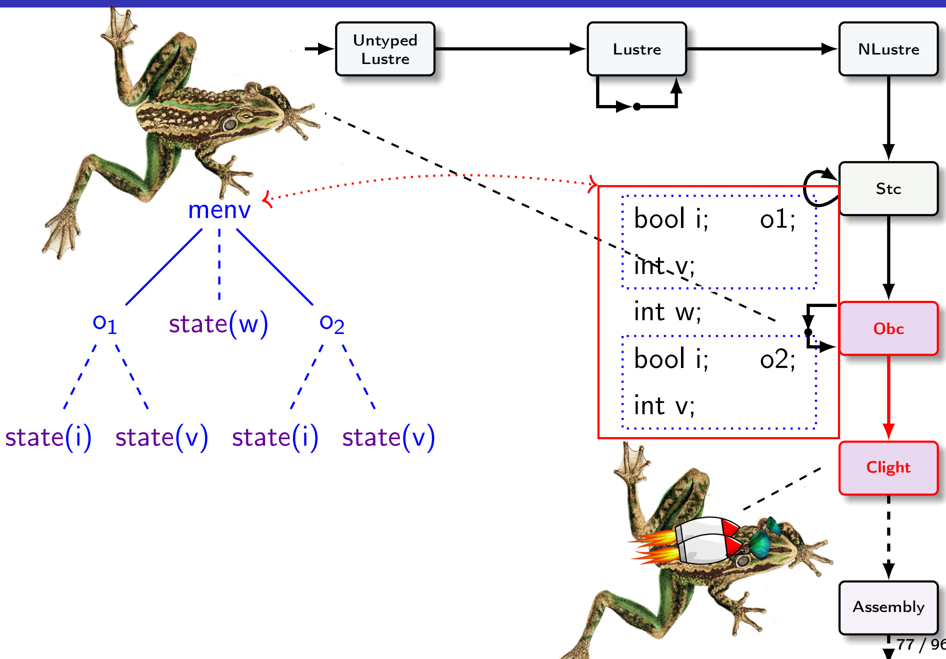

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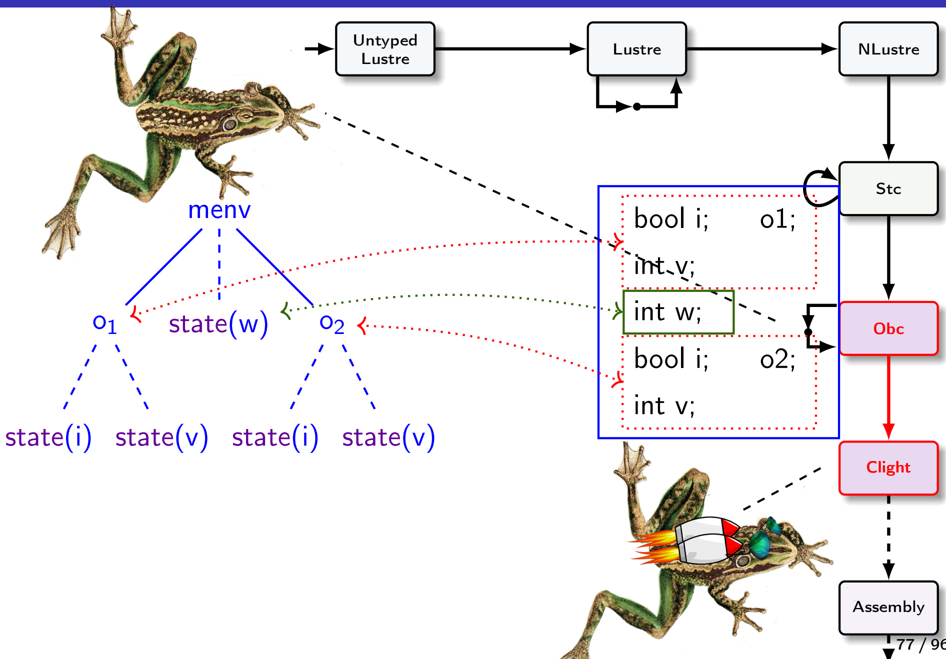

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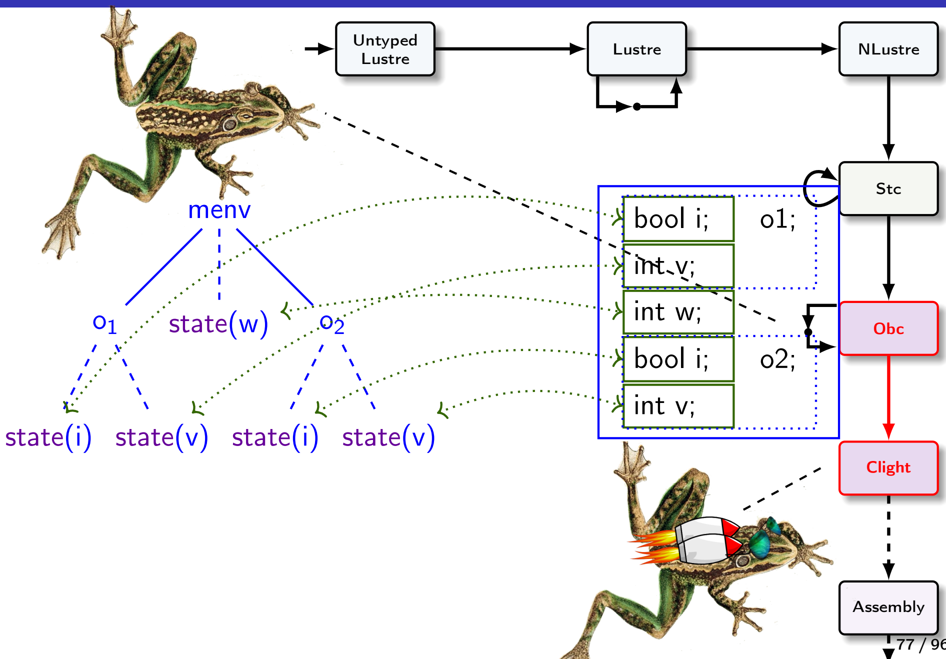


Semantics preservation

Obc : (me, ve) ; Clight : (e, le, m)

$$me_1, ve_1 \qquad \vdash s \Downarrow \qquad (me_2, ve_2)$$

$$\text{match_states} \wr\!\wr$$

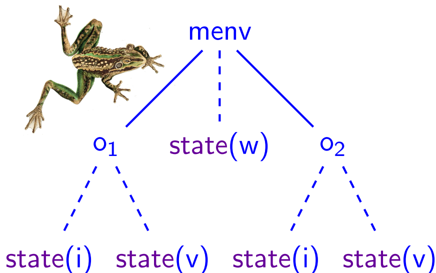
$$e_1, le_1, m_1$$

Semantics preservation

Obc : (me, ve) ; Clight : (e, le, m)

$$
\begin{array}{ccc}
me_1, ve_1 & \vdash s \Downarrow & (me_2, ve_2) \\
\text{match_states} \wr & & \wr\ \text{match_states} \\
e_1, le_1, m_1 & \vdash_{\text{Clight}} \text{generate}(s) \Downarrow & (e_1, le_2, m_2)
\end{array}
$$

Obc to Clight: memory correspondence




```
bool i;    o1;
int v;
int w;
bool i;    o2;
int v;
```

- This time the semantic models are similar (Clight: very detailed)
- The real challenge is to relate the memory models.
  - Obc: tree structure, variable separation is manifest.
  - Clight: block-based, must treat aliasing, alignment, and sizes.

- Extend CompCert's lightweight library of separating assertions: https://github.com/AbsInt/CompCert/common/Separation.v.
- Encode simplicity of source model in richer memory model.
- General (and very useful) technique for interfacing with CompCert.

Separation logic in CompCert

predicate

$$massert \triangleq \left\{ \begin{array}{l} \text{pred} : \mathit{memory} \to \mathbb{P} \\ \text{foot} : \mathit{block} \to \mathit{int} \to \mathbb{P} \\ \text{invar} : \forall m\, m', \text{pred}\ m \to \\ \qquad \text{unchanged_on foot}\ m\ m' \to \\ \qquad \text{pred}\ m' \end{array} \right\}$$

notation: $m \vDash P \triangleq P.\text{pred}\ m$

conjonction

$$P * Q \triangleq \left\{ \begin{array}{l} \text{pred} = \lambda\, m.\ (m \vDash P) \wedge (m \vDash Q) \\ \qquad \wedge\ \text{disjoint}\ P.\text{foot}\ Q.\text{foot} \\ \text{foot} = \lambda\, b\, \mathit{ofs}.\ P.\text{foot}\, b\, \mathit{ofs} \vee Q.\text{foot}\, b\, \mathit{ofs} \end{array} \right\}$$

pure formula $m \vDash \text{pure}\,(P) * Q \leftrightarrow P \wedge m \vDash Q$

```
(* Xavier's Separation.v *)

Record massert : Type := { m_pred : mem → Prop;
                           m_footprint : block → Z → Prop; ...  }.

Notation "m |= p" := (m_pred p m) : sep_scope.

Definition disjoint_footprint (P Q: massert) : Prop :=
  ∀ b ofs, m_footprint P b ofs → m_footprint Q b ofs → False.

Definition sepconj (P Q: massert) : massert := {|
  m_pred := fun m ⇒ m_pred P m ∧ m_pred Q m ∧ disjoint_footprint P Q;
  m_footprint := fun b ofs ⇒ m_footprint P b ofs ∨ m_footprint Q b ofs |}.

Infix "**" := sepconj : sep_scope.

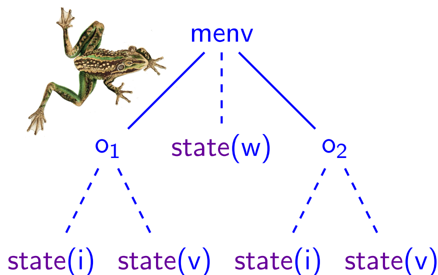
(* Blockrep *)

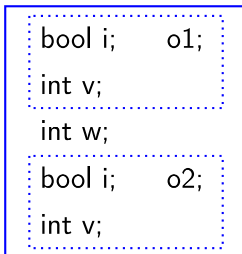
Fixpoint sepall (p: A → massert) (xs: list A) : massert := match xs with
                                                           | nil ⇒ sepemp
                                                           | x:: xs ⇒ p x ** sepall p xs
                                                           end.

Definition match_value (e: PM.t val) (x: ident) (v': val) : Prop := match PM.find x e with
                                                                    | None ⇒ True
                                                                    | Some v ⇒ v' = v
                                                                    end.

Definition blockrep (ve: venv) (flds: members) (b: block) : massert :=
  sepall (fun (x, ty) : ident * type ⇒
          match field_offset ge x flds, access_mode ty with
          | OK d, By_value chunk ⇒ contains chunk b d (match_value ve x)
          | _, _ ⇒ sepfalse
          end) flds.
```

Obc to Clight: memory correspondence





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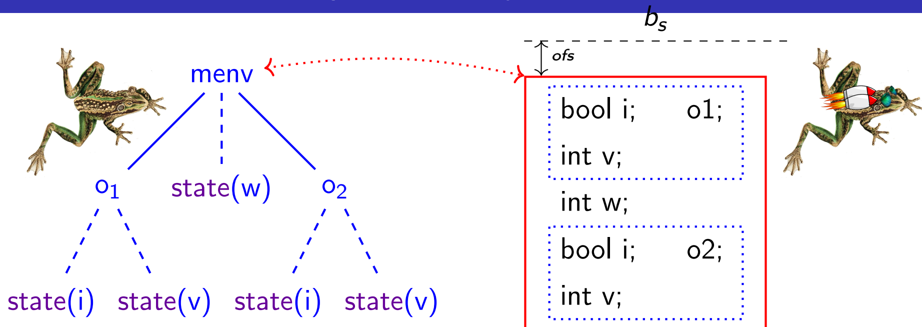
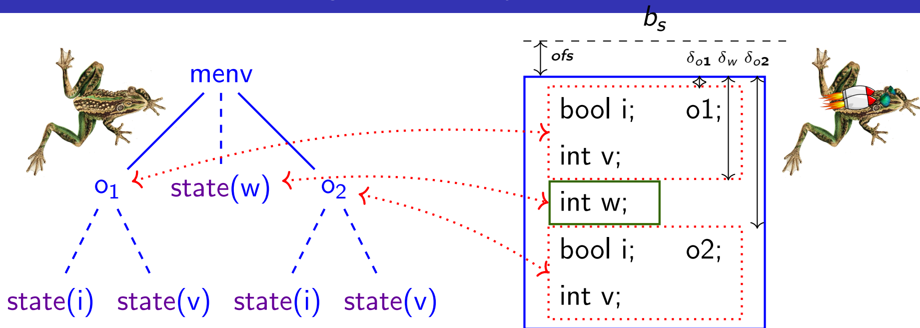


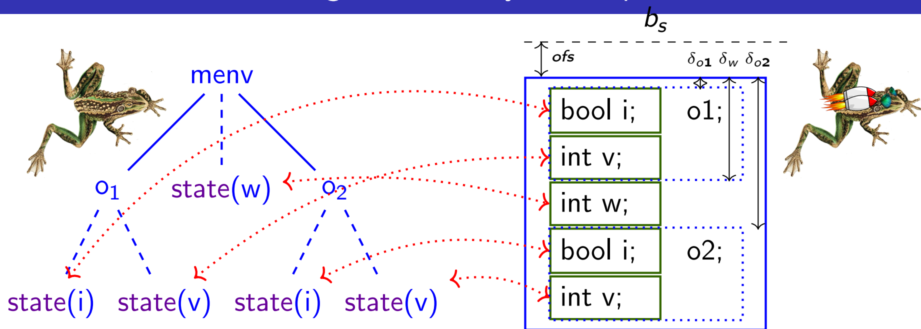
$m \models$ staterep avgvelocity me (b_s, ofs)

Obc to Clight: memory correspondence



$$
\begin{aligned}
m \vDash \quad & \text{staterep count } me(o1)\ (b_s, ofs + \delta_{o1}) \\
* & \text{ contains } tyint32s\ (b_s, ofs + \delta_w)\ \lceil me.\text{state}(w) \rceil \\
* & \text{ staterep count } me(o2)\ (b_s, ofs + \delta_{o2})
\end{aligned}
$$

Obc to Clight: memory correspondence



$$
\begin{aligned}
m \vDash \quad & \text{contains} \;\; \mathit{tybool} \;\; (b_s, \mathit{ofs} + \delta_{o1} + \delta_i) \;\; \lceil \mathit{me}.o_1.\text{state}(i) \rceil \\
* \; & \text{contains} \;\; \mathit{tyint32s} \;\; (b_s, \mathit{ofs} + \delta_{o1} + \delta_v) \;\; \lceil \mathit{me}.o_1.\text{state}(v) \rceil \\
* \; & \text{contains} \;\; \mathit{tyint32s} \;\; (b_s, \mathit{ofs} + \delta_w) \;\; \lceil \mathit{me}.\text{state}(w) \rceil \\
* \; & \text{contains} \;\; \mathit{tybool} \;\; (b_s, \mathit{ofs} + \delta_{o2} + \delta_i) \;\; \lceil \mathit{me}.o_2.\text{state}(i) \rceil \\
* \; & \text{contains} \;\; \mathit{tyint32s} \;\; (b_s, \mathit{ofs} + \delta_{o2} + \delta_v) \;\; \lceil \mathit{me}.o_2.\text{state}(v) \rceil
\end{aligned}
$$

Invariant: staterep

```
Inductive memory (V: Type): Type := mk_memory {
  mm_values : PM.t V;
  mm_instances : PM.t (memory V) }.

Definition staterep_mems (cls: class) (me: menv) (b: block) (ofs: Z) ((x, ty) : ident * typ) :=
  match field_offset ge x (make_members cls) with
  | OK d ⇒ contains (chunk_of_type ty) b (ofs + d) (match_value me.( mm_values) x)
  | Error _ ⇒ sepfalse
  end.

Fixpoint staterep (p: program) (clsnm: ident) (me: menv) (b: block) (ofs: Z): massert :=
  match p with
  | nil ⇒ sepfalse
  | cls :: p' ⇒ if ident_eqb clsnm cls.( c_name) then
      sepall (staterep_mems cls me b ofs) cls.( c_mems)
      ** sepall (fun ((i, c) : ident * ident) ⇒ match field_offset ge i (make_members cls) with
                                                  | OK d ⇒ staterep p' c (instance_match me i) b (ofs + d)
                                                  | Error _ ⇒ sepfalse
                                                  end) cls.( c_objs)
    else staterep p' clsnm me b ofs
  end.
```

Freeing memory/struct padding

- Keep permissions on local memory to 'free' it before return.
- Permissions are 'in' the `contains` predicate.
- But, the padding is not included. How can we free it?

$$\begin{aligned} \text{match_states} = & \forall c\ f\ \mathit{me}\ \mathit{ve}\ e\ \mathit{le}\ \mathit{sb}\ \mathit{sofs}\ \mathit{outb}\ \mathit{outco}, \\ & \quad \ldots \\ & \quad * \text{subrep}\ f\ e \\ & \quad * \text{subrep}\ f\ e \mathrel{-\!\!*} \text{subrep_range}\ e \end{aligned}$$

Separating wand

- Key property:

$$P * (P \mathrel{-\!\!*} Q) \longrightarrow\!\!\!\triangleright\ Q$$

- Works together with the frame condition on other lemmas.

C99: non-defined arguments

- In the C99 standard, the effect of passing an undefined value in a function call is undefined:
  - *in preparing for the call to a function, the arguments are evaluated, and each parameter is assigned the value of the corresponding argument*,
    (C99 §6.5.2.2)
  - *if an lvalue does not designate an object when it is evaluated, the behavior is undefined*.
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  - » *if an lvalue does not designate an object when it is evaluated, the behavior is undefined*. (C99 §6.3.2.1)

- In the Clight semantics:
  - » The arguments of a function call are evaluated from left to right, (`cfrontend/Clight.v: eval_exprlist`)
  - » The type conversion applied to each element must return `Some` value, while a conversion of `Vundef` to any type other than `void` gives `None`. (`cfrontend/Cop.v: sem_cast`)

C99: non-defined arguments

- In the Clight semantics:
  - The arguments of a function call are evaluated from left to right, (`cfrontend/Clight.v: eval_exprlist`)

```
Inductive eval_exprlist: list expr → typelist → list val → Prop :=
  | eval_Enil:
      eval_exprlist nil Tnil nil
  | eval_Econs:  ∀ a bl ty tyl v1 v2 vl,
      eval_expr a v1 →
      sem_cast v1 (typeof a) ty m = Some v2 →
      eval_exprlist bl tyl vl →
      eval_exprlist (a :: bl) (Tcons ty tyl) (v2 :: vl).
```

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```
Definition sem_cast (v: val) (t1 t2: type) (m: mem): option val :=
  match classify_cast t1 t2 with
  ...
  | cast_case_i2i sz2 si2 =>
      match v with
      | Vint i => Some (Vint (cast_int_int sz2 si2 i))
      | _ => None
      end
  ...
  | cast_case_void =>
      Some v
  ...
  end
```

Undefined arguments: possible solutions

- Forbid using subclocks in node interfaces.
- Change the semantics of Clight?
- Compile to Cminor?
- Use inline expansion for nodes with clocked arguments (Scade)?
- Pass a pointer to a struct containing the arguments?
- Put the arguments in the node state?
- Initialize all the variables?
- Initialize all the variables that are passed to a node?

```
node n1(c : bool;
        x : bool when c)
returns(v : int;
        o : int when c);

node n2(c1       : bool;
        c2, w, y : bool when c1;
        x        : bool when c2;
        z        : bool when not c1)
returns(o : int);

node f(m : bool;
       h  : bool when m;
       x  : bool)
returns(o1 : int;
        o2 : int when m);
var e0, e3 : bool when h;
    w      : int  when m;
    y      : int  when h;
    r1, e1 : bool when m;
    r2, e2 : bool when not m;
let
  e1 = r1 and (x when m);
  e2 = r2 and (x when not m);
  r2 = true fby not r2;
  e0 = (r1 when h) and ((not (x when m)) when h);
  e3 = e0 or (r1 when h) or ((x when m) when h);
  r1 = false fby not r1;
  w, y = n1(h, e3);
  o2 = merge h y (0 when not h);
  o1 = n2(m, h, (not x) when m, e1, e0, e2);
tel
```

```
method step(m, h, x : bool) returns (o1, o2 : int32)
var e0, e1, e2, e3 : bool; w, y : int32
{
    if (m) {
      e1 := state(r1) & x
    };
    if (m) { skip } else {
      e2 := state(r2) & x
    };
    if (m) { skip } else {
      state(r2) := ! state(r2)
    };
    if (m) {
      if (h) {
        e0 := state(r1) & ! x
      }
    };
    if (m) {
      if (h) {
        e3 := e0 | state(r1) | x
      }
    };
    if (m) {
      state(r1) := ! state(r1)
    };
    if (m) {
      w, y := n1(w).step(⟨h⟩, e3)
    };
    if (m) {
      if (h) {
        o2 := y
      } else {
        o2 := 0
      }
    };
    o1 := n2(o1).step(⟨m⟩, h, ! x, e1, e0, e2)
}
```

```
node n1(c : bool;
        x : bool when c)
returns(v : int;
       o : int when c);

node n2(c1        : bool;
        c2, w, y : bool when c1;
        x         : bool when c2;
        z         : bool when not c1)
returns(o : int);

node f(m : bool;
       h  : bool when m;
       x  : bool)
returns(o1 : int;
        o2 : int when m);
var e0, e3 : bool when h;
    w      : int  when m;
    y      : int  when h;
    r1, e1 : bool when m;
    r2, e2 : bool when not m;
let
  e1 = r1 and (x when m);
  e2 = r2 and (x when not m);
  r2 = true fby not r2;
  e0 = (r1 when h) and ((not (x when m)) when h);
  e3 = e0 or (r1 when h) or ((x when m) when h);
  r1 = false fby not r1;
  w, y = n1(h, e3);
  o2 = merge h y (0 when not h);
  o1 = n2(m, h, (not x) when m, e1, e0, e2);
tel
```

```
method step(m, h, x : bool)
returns (o1, o2 : int32)
var e0, e1, e2, e3 : bool; w, y : int32
{

  if (m) {

    e1 := state(r1) & x;
    state(r1) := ! state(r1);
    if (h) {
      e0 := ! x;
      e3 := e0 | x
    } else {

      skip
    };
    y := n1(i1).step(⟨h⟩, e3);
    if (h) {
      o2 := y
    } else {
      o2 := 0
    }
  } else {


    e2 := state(r2) & x;
    state(r2) := ! state(r2)
  };
  o1 := n2(i2).step(⟨m⟩, h, ! x, e1, e0, e2)
}
```

```
node n1(c : bool;
        x : bool when c)
returns(v : int;
        o : int when c);

node n2(c1          : bool;
        c2, w, y : bool when c1;
        x           : bool when c2;
        z           : bool when not c1)
returns(o : int);

node f(m : bool;
       h  : bool when m;
       x  : bool)
returns(o1 : int;
        o2 : int when m);
var e0, e3 : bool when h;
    w      : int  when m;
    y      : int  when h;
    r1, e1 : bool when m;
    r2, e2 : bool when not m;
let
  e1 = r1 and (x when m);
  e2 = r2 and (x when not m);
  r2 = true fby not r2;
  e0 = (r1 when h) and ((not (x when m)) when h);
  e3 = e0 or (r1 when h) or ((x when m) when h);
  r1 = false fby not r1;
  w, y = n1(h, e3);
  o2 = merge h y (0 when not h);
  o1 = n2(m, h, (not x) when m, e1, e0, e2);
tel
```

```
method step(m, h, x : bool)
returns (o1, o2 : int32)
var e0, e1, e2, e3 : bool; w, y : int32
{

  if (m) {

    e1 := state(r1) & x;
    state(r1) := ! state(r1);
    if (h) {
      e0 := ! x;
      e3 := e0 | x
    } else {

      skip
    };
    y := n1(i1).step(⟨h⟩, e3);
    if (h) {
      o2 := y
    } else {
      o2 := 0
    }
  } else {


    e2 := state(r2) & x;
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  };
  o1 := n2(i2).step(⟨m⟩, ⟨h⟩, ! x, e1, e0, e2)
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```

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        o : int when c);

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returns(o1 : int;
        o2 : int when m);
var e0, e3 : bool when h;
    w      : int  when m;
    y      : int  when h;
    r1, e1 : bool when m;
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let
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      e0 := ! x;
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    e2 := 0;
    e1 := state(r1) & x;
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    if (h) {
      e0 := ! x;
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    } else {

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    o2 := 0;
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  if (m) {
    e2 := 0;
    e1 := state(r1) & x;
    state(r1) := ! state(r1);
    if (h) {
      e0 := ! x;
      e3 := e0 | x
    } else {
      e3 := 0;
      skip
    };
    y := n1(i1).step(⟨h⟩, ⟨e3⟩);
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    o2 := 0;
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method step(m, h, x : bool)
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  e0 := 0;
  if (m) {
    e2 := 0;
    e1 := state(r1) & x;
    state(r1) := ! state(r1);
    if (h) {
      e0 := ! x;
      e3 := e0 | x
    } else {
      e3 := 0;
      skip
    };
    y := n1(i1).step(⟨h⟩, ⟨e3⟩);
    if (h) {
      o2 := y
    } else {
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    o2 := 0;
    e1 := 0;
    e2 := state(r2) & x;
    state(r2) := ! state(r2)
  };
  o1 := n2(i2).step(⟨m⟩, ⟨h⟩, ! x, ⟨e1⟩, ⟨e0⟩, ⟨e2⟩)
}
```

Extra normalization condition on node arguments

$$\frac{\text{NoOps base } e \qquad \text{NoOps } ck\ c \qquad \text{NoOps } ck\ x \qquad \text{NoOps } ck\ e}{\text{NoOps } (ck \text{ on } xb)\ (e \texttt{ when } (\texttt{not})\ x)}$$

- A static condition on a clock in a node interface together with an argument in a node instance

- Expressions on the base clock are allowed.
- Constants are allowed.
- Variables are allowed.
- `when`s are removed.

- Sampled expressions are forbidden.

Extra normalization condition on node arguments

$$\frac{\text{NoOps base } e \qquad \text{NoOps } ck\ c \qquad \text{NoOps } ck\ x}{\text{NoOps } ck\ e}$$

$$\overline{\text{NoOps } (ck \text{ on } xb)\ (e\ \texttt{when}\ (\texttt{not})\ x)}$$

- Allowed: NoOps (base on cktrue) I((!x) when m)
- Forbidden: $\neg$NoOps (base on cktrue) (!(x when m))

```
Definition add_write x s :=
  match type_of_var x with None ⇒ s |  Some ty ⇒ (x := (init_type ty)) ; s end.

Definition add_valid (e : exp) (esreq : list exp * PS.t) :=
  match e, esreq with
  | Var x ty, (es, req) ⇒ (Valid e ::  es, req ∪ {x})
  | _,        (es, req) ⇒ (      e ::  es, req)
  end.

/*                                                     (s', req', sometimes, always)    */
Fixpoint add_defaults_stmt (req: PS.t) (s: stmt) : stmt * PS.t * PS.t * PS.t :=
  match s with
  | skip ⇒ (s, req, ∅, ∅)
  | x := e ⇒ (s, req − {x}, ∅, {x})
  | st(x) := e ⇒ (s, req, ∅, ∅)

  | xs := f(o).m(es) ⇒
    let (es', req') := fold_right add_valid ([], ps_removes xs req) es
    in (xs := f(o).m(es'), req', ∅, of_list xs)

  | s1 ; s2 ⇒
    let (t2, req2, st2, al2) := add_defaults_stmt req s2 in
    let (t1, req1, st1, al1) := add_defaults_stmt req2 s1 in
    (t1 ; t2, req1, (st1 − al2) ∪ (st2 − al1), al1 ∪ al2)

  | if e { s1 } else { s2 } ⇒
    let (t1, req1, st1, al1) := add_defaults_stmt ∅  s1 in
    let (t2, req2, st2, al2) := add_defaults_stmt ∅  s2 in
    let (al1_req, al2_req) := (al1 ∩ req, al2 ∩ req) in
    let (w1, w2) := (al2_req − al1_req, al1_req − al2_req) in
    let w := ((st1 ∩ req) − w1) ∪ ((st2 ∩ req) − w2) in
    let (al1', al2') := (al1 ∪ w1, al2 ∪ w2) in
    let (st1', st2') := (st1 − w1, st2 − w2) in
    (add_writes w (if e { add_writes w1 t1 } else { add_writes w2 t2 }),
     (((req − al1_req) − al2_req) ∪ req1 ∪ req2) − w,
     (st1' ∪ st2' ∪ (al1' − al2') ∪ (al2' − al1')) − w,
     (al1' ∩ al2') ∪ w)
  end.
```

Correctness of variable initalization

- A refinement relation between environments:

$$ve_2 \sqsubseteq ve_1 \equiv \forall x\ v,\ ve_2\ x = \text{Some}\ v \implies ve_1\ x = \text{Some}\ v$$

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- Extend to statements and add a precondition P:

$$\begin{aligned} s_2 \sqsubseteq_P^{p_2,p_1} s_1 \equiv\ & \forall me\ me'\ ve_1\ ve_2\ ve_2', \\ & P\ ve_1\ env_2 \implies \\ & ve_2 \sqsubseteq ve_1 \implies \\ & p_2, me, ve_2 \vdash s_2 \Downarrow (me', ve_2') \implies \\ & \exists ve_1',\ p_1, me, ve_1 \vdash s_1 \Downarrow (me', ve_1') \land ve_2' \sqsubseteq ve_1' \end{aligned}$$

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- Extend to programs:

$$\begin{aligned} p_2 \sqsubseteq_P p_1 \equiv \forall n\ c_2\ p_2', \text{find-class}\ n\ p_2 = \text{Some}\ (c_2, p_2') \implies \\ \exists c_1\ p_1', \text{find-class}\ n\ p_1 = \text{Some}\ (c_1, p_1') \\ \land\ c_1 \sqsubseteq_{(P\ n)}^{p_1,p_2} c_2 \\ \land\ p_2' \sqsubseteq_P p_1' \end{aligned}$$

Initializing variables: correctness

- The core of the correctness lemma:

```
add_defaults_stmt tyenv req s = (t, req', st, al) →
```

$$s \sqsubseteq^{p,p'}_{(\text{in1_notin2}\ req'\ (st \cup al))}\ t$$

Initializing variables: correctness

- The core of the correctness lemma:

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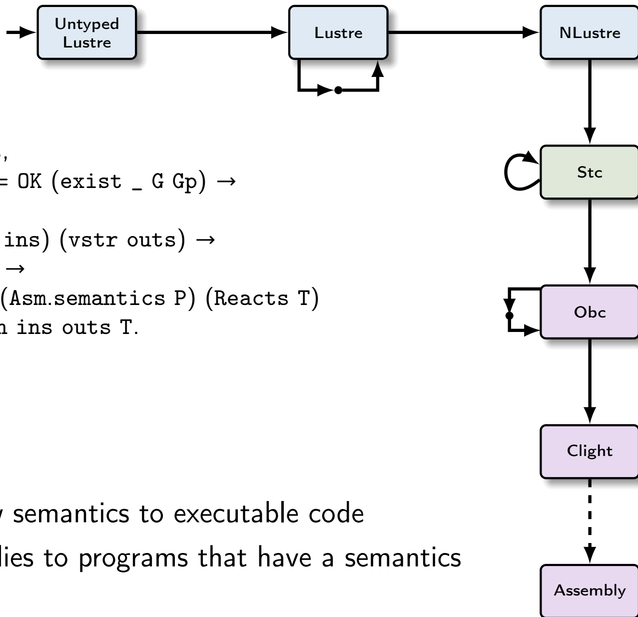
- The input Obc program must be in SSA form.
  (SSA = Static Single Assignment form)
- The output Obc program is no longer in SSA form.

$$\frac{(\forall x, \text{CanWrite}\ x\ s_1 \implies \neg\text{CanWrite}\ x\ s_2) \quad (\forall x, \text{CanWrite}\ x\ s_2 \implies \neg\text{CanWrite}\ x\ s_1) \quad \text{NoOverwrites}\ s_1 \quad \text{NoOverwrites}\ s_2}{\text{NoOverwrites}\ (s_1\ ;\ s_2)}$$

$$\frac{\text{Forall}\ (\lambda e,\ e \neq x)\ es}{\text{NoNakedVars}\ (xs := cls(i).f(es))}$$

Main theorem




```
Theorem behavior_asm:
  ∀ D G Gp P main ins outs,
    elab_declarations D = OK (exist _ G Gp) →
    wt_ins G main ins →
    sem_node G main (vstr ins) (vstr outs) →
    compile D main = OK P →
    ∃ T, program_behaves (Asm.semantics P) (Reacts T)
          ∧ bisim_io G main ins outs T.
```

- Good: links dataflow semantics to executable code
- Less good: only applies to programs that have a semantics

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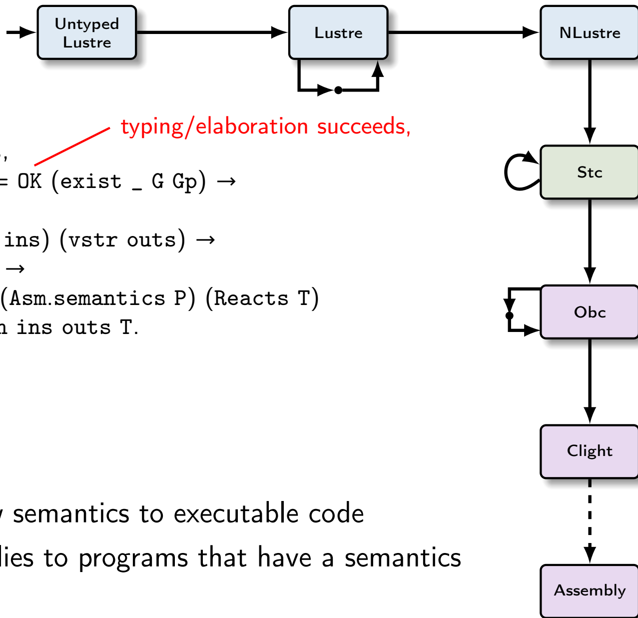


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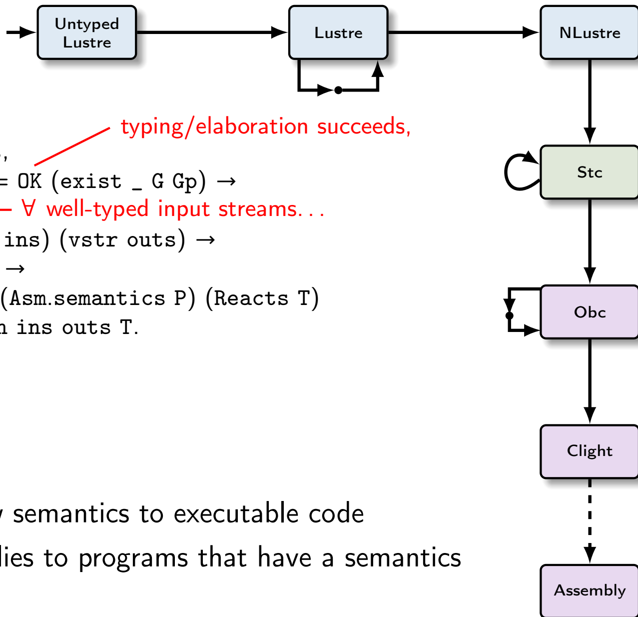


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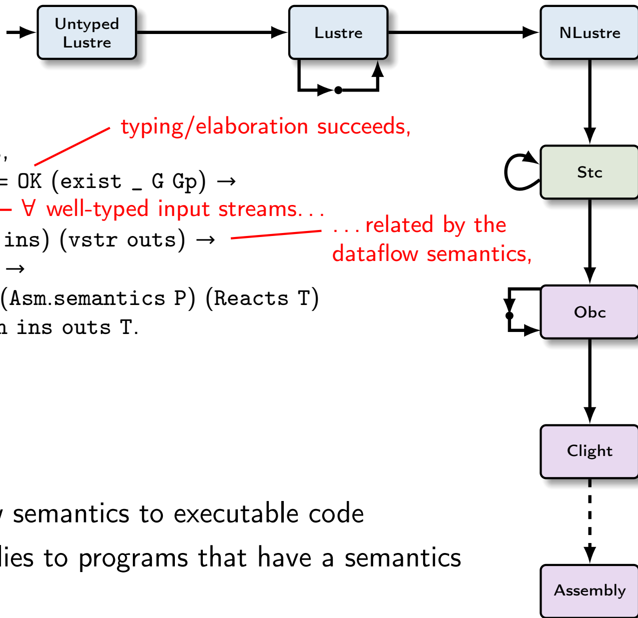


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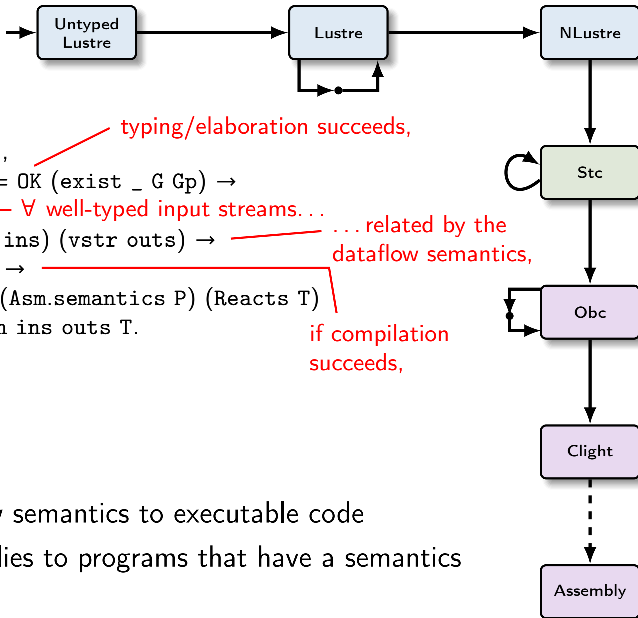


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Theorem behavior_asm:
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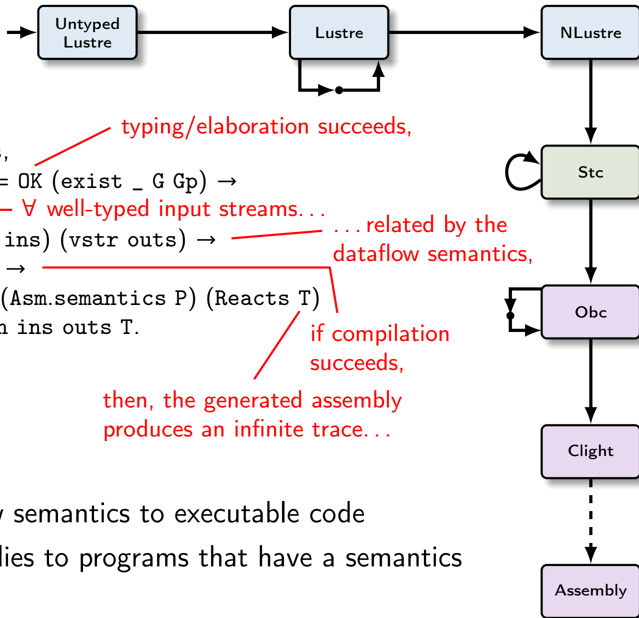
    elab_declarations D = OK (exist _ G Gp) →
    wt_ins G main ins →
    sem_node G main (vstr ins) (vstr outs) →
    compile D main = OK P →
    ∃ T, program_behaves (Asm.semantics P) (Reacts T)
         ∧ bisim_io G main ins outs T.
```

- Good: links dataflow semantics to executable code
- Less good: only applies to programs that have a semantics

Main theorem

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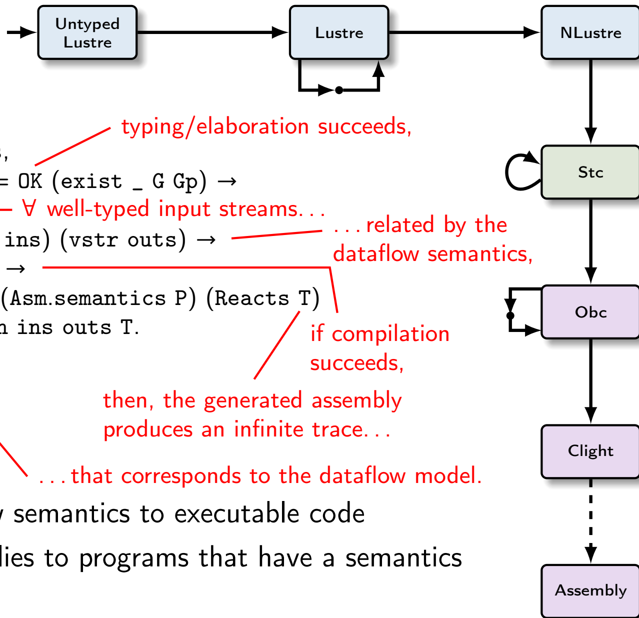


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Experimental results

Industrial application

- ≈6 000 nodes
- ≈162 000 equations
- ≈12 MB source file (minus comments)
- Modifications:
  - Remove constant lookup tables.
  - Replace calls to assembly code.
- Vélus compilation: ≈1 min 40 s

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	Vélus	*Hept+CC*	*Hept+gcc*	*Hept+gcci*	*Lus6+CC*	*Lus6+gcc*	*Lus6+gcci*
avgvelocity	315	385 (22%)	265 (-15%)	70 (-77%)	1 150 (265%)	625 (98%)	350 (11%)
count	55	55 (0%)	25 (-54%)	25 (-54%)	300 (445%)	160 (190%)	50 (-9%)
tracker	680	790 (16%)	530 (-22%)	500 (-26%)	2 610 (283%)	1 515 (122%)	735 (8%)
pip_ex	4 415	4 065 (-7%)	2 565 (-41%)	2 040 (-53%)	10 845 (145%)	6 245 (41%)	2 905 (-34%)
mp_longitudinal [16]	5 525	6 465 (17%)	3 465 (-37%)	2 835 (-48%)	11 675 (111%)	6 785 (22%)	3 135 (-43%)
cruise [54]	1 760	1 875 (6%)	1 230 (-30%)	1 230 (-30%)	5 855 (232%)	3 595 (104%)	1 965 (11%)
risingedgeretrigger [19]	285	300 (5%)	190 (-33%)	190 (-33%)	1 440 (405%)	820 (187%)	335 (17%)
chrono [20]	410	425 (3%)	305 (-25%)	305 (-25%)	2 490 (507%)	1 500 (265%)	670 (63%)
watchdog3 [26]	610	575 (-5%)	355 (-41%)	310 (-49%)	2 015 (230%)	1 135 (86%)	530 (-13%)
functionalchain [17]	11 550	13 535 (17%)	8 545 (-26%)	7 525 (-34%)	23 085 (99%)	14 280 (23%)	8 240 (-28%)
landing_gear [11]	9 660	8 475 (-12%)	5 880 (-39%)	5 810 (-39%)	25 470 (163%)	15 055 (55%)	8 025 (-16%)
minus [57]	890	900 (1%)	580 (-34%)	580 (-34%)	2 825 (217%)	1 620 (82%)	800 (-10%)
prodcell [32]	1 020	990 (-2%)	620 (-39%)	410 (-59%)	3 615 (254%)	2 050 (100%)	1 070 (4%)
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Figure 12. WCET estimates in cycles [4] for step functions compiled for an armv7-a/vfpv3-d16 target with CompCert 2.6 (CC) and GCC 4.4.8 `-O1` without inlining (gcc) and with inlining (gcci). Percentages indicate the difference relative to the first column.

It performs loads and stores of volatile variables to model, respectively, input consumption and output production. The coinductive predicate presented in Section 1 is introduced to relate the trace of these events to input and output streams.

Finally, we exploit an existing CompCert lemma to transfer our results from the big-step model to the small-step one, from whence they can be extended to the generated assembly code to give the property stated at the beginning of the paper. The transfer lemma requires showing that a program does not diverge. This is possible because the body of the main loop always produces observable events.

5. Experimental Results

Our prototype compiler, Vélus, generates code for the platforms supported by CompCert (PowerPC, ARM, and x86). The code can be executed in a 'test mode' that `scanf`s inputs and `printf`s outputs using an alternative (unverified) entry point. The *verified* integration of generated code into a complete system where it would be triggered by interrupts and interact with hardware is the subject of ongoing work.

As there is no standard benchmark suite for Lustre, we adapted examples from the literature and the Lustre v4 distribution [57]. The resulting test suite comprises 14 programs, totaling about 160 nodes and 960 equations. We compared the code generated by Vélus with that produced by the Heptagon 1.03 [23] and Lustre v6 [35, 57] academic compilers. For the example with the deepest nesting of clocks (3 levels), both Heptagon and our prototype found the same optimal schedule. Otherwise, we follow the approach of [23, §6.2] and estimate the Worst-Case Execution Time (WCET) of the generated code using the open-source OTAWA v5 framework [4] with the 'trivial' script and default parameters.[10] For the targeted domain, an over-approximation to the WCET is usually more valuable than raw performance numbers. We compiled with CompCert 2.6 and GCC 4.8.4 (`-O1`) for the `arm-none-eabi` target (armv7-a) with a hardware floating-point unit (vfpv3-d16).

The results of our experiments are presented in Figure 12. The first column shows the worst-case estimates in cycles for the step functions produced by Vélus. These estimates compare favorably with those for generation with either Heptagon or Lustre v6 and then compilation with CompCert. Both Heptagon and Lustre (automatically) re-normalize the code to have one operator per equation, which can be costly for nested conditional statements, whereas our prototype simply maintains the (manually) normalized form. This re-normalization is unsurprising: both compilers must treat a richer input language, including arrays and automata, and both expect the generated code to be post-optimized by a C compiler. Compiling the generated code with GCC but still without any inlining greatly reduces the estimated WCETs, and the Heptagon code then outperforms the Vélus code. GCC applies 'if-conversions' to exploit predicated ARM instructions which avoids branching and thereby improves WCET estimates. The estimated WCETs for the Lustre v6 generated code only become competitive when inlining is enabled because Lustre v6 implements operators, like **pre** and −>, using separate functions. CompCert can perform inlining, but the default heuristic has not yet been adapted for this particular case. We note also that we use the modular compilation scheme of Lustre v6, while the code generator also provides more aggressive schemes like clock enumeration and automaton minimization [29, 56].

Finally, we tested our prototype on a large industrial application (≈6 000 nodes, ≈162 000 equations, ≈12 MB source file without comments). The source code was already normalized since it was generated with a graphical interface,

[10] This configuration is quite pessimistic but suffices for the present analysis.

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- Vélus compilation: ≈1 min 40 s

- Compare WCET of generated code with two academic compilers on smaller examples.

[Ballabriga, Cassé, Rochange, and Sainrat (2010): OTAWA: An Open Toolbox for Adaptive WCET Analysis]

- Results depend on C compiler:
  - CompCert: Vélus code same/better
  - gcc -O1 no-inlining: Vélus code slower
  - gcc -O1: Vélus code much slower
- [TODO]: adjust CompCert inlining heuristic.

12

Overview

Lustre: syntax and semantics

Lustre: normalization

Translation: from dataflow programs to imperative code

Optimization: control structure fusion

Generation: from Obc to Clight

Conclusion

Conclusion

Working compiler from Lustre to assembler in Coq.

[Bourke, Dagand, Pouzet, and Rieg (2017): Vérification de la génération modulaire du code impératif pour Lustre] [Bourke, Brun, Dagand, Leroy, Pouzet, and Rieg (2017): A Formally Verified Compiler for Lustre]

- Relate dataflow model to imperative code with `if`-fusion
- Generate Clight for CompCert; change to richer memory model
- Intermediate language and separation predicates were decisive

Support for modular reset of function instances.

[Bourke, Brun, and Pouzet (2020): Mechanized Semantics and Verified Compilation for a Dataflow Synchronous Language with Reset] [Brun (2020): Mechanized Semantics and Verified Compilation for a Dataflow Synchronous Language with Reset]

- Semantics of reset on function instances
- Stc intermediate language and optimized scheduling
- Enumerated types

Conclusion

Normalization and Activation Blocks

[Bourke, Jeanmaire, Pesin, and Pouzet (2021): Verified Lustre Normalization with Node Subsampling] [Bourke, Pesin, and Pouzet (2023): Verified Compilation of Synchronous Dataflow with State Machines]

- Rewriting to normalize programs and compile state machines
- PhD Defense of B. Pesin this Friday!

Ongoing work

- Denotational model: existence of semantics + fixpoint induction
- Aim: Interactive verification of Lustre programs

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