Introduction à la vision artificielle III

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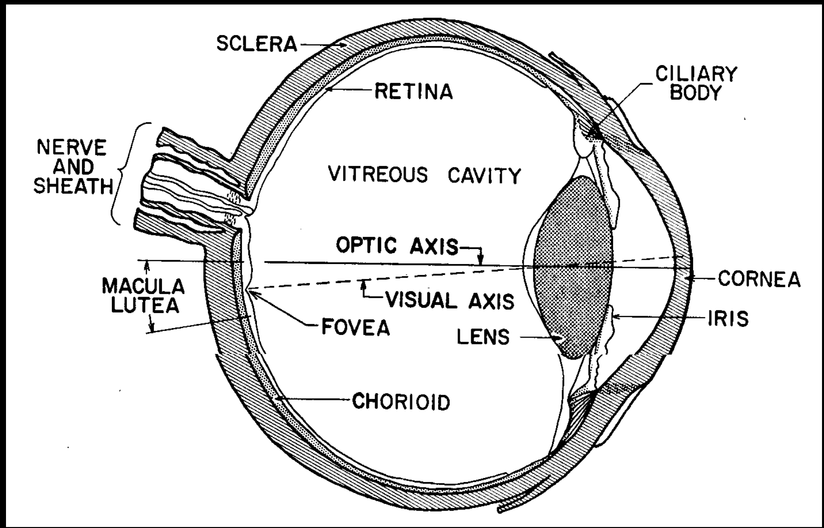
http://www.di.ens.fr/~ponce/introvis/lect3.pptx
http://www.di.ens.fr/~ponce/introvis/lect3.pdf
http://www.di.ens.fr/~ponce/introvis/sbook.pdf

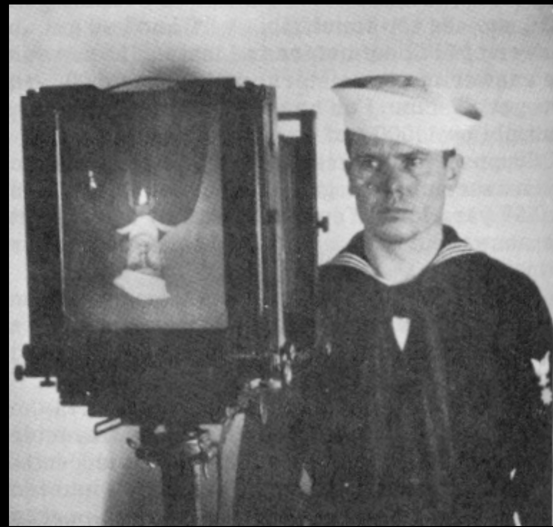
- Le premier exo est du le 6 octobre
  http://www.di.ens.fr/willow/teaching/introvis14/assignment1/

Camera geometry and calibration I

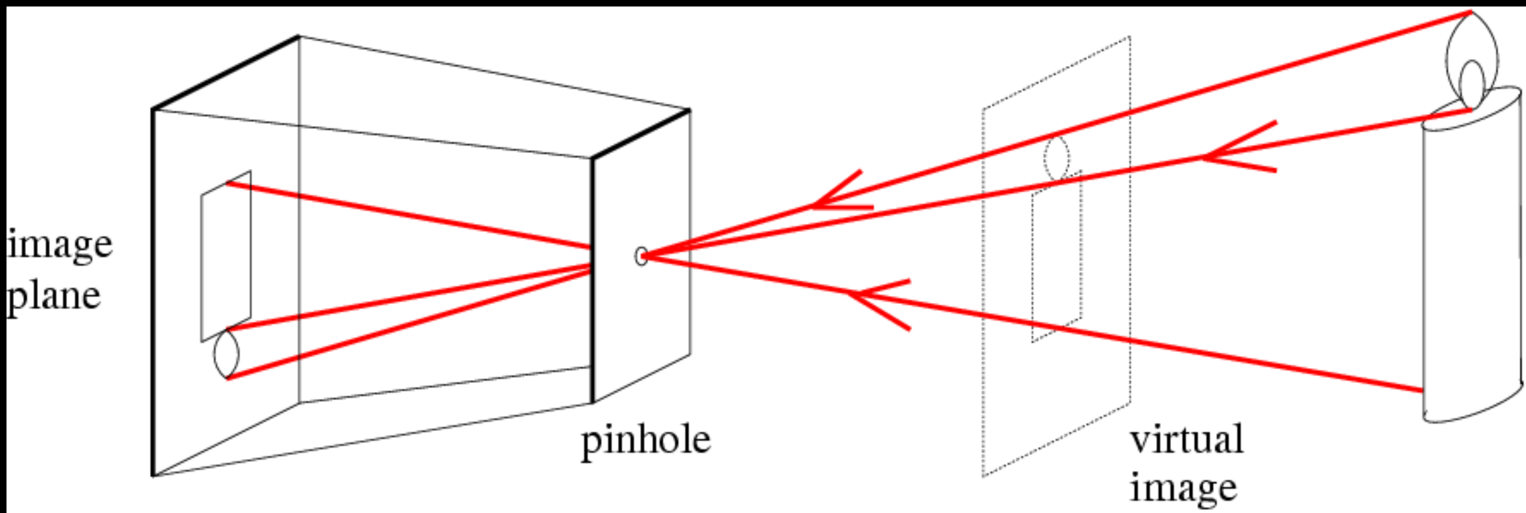
- Pinhole perspective projection
- Orthographic and weak-perspective models
- Non-standard models
- A detour through sensing country
- Intrinsic and extrinsic parameters



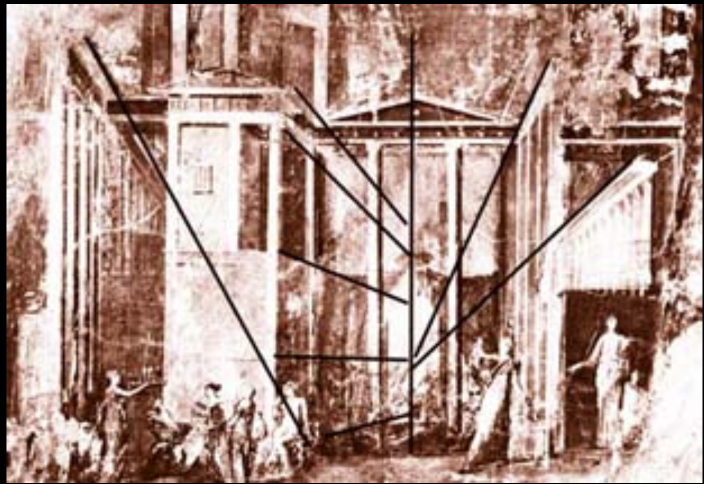
Animal eye: a looonnng time ago.



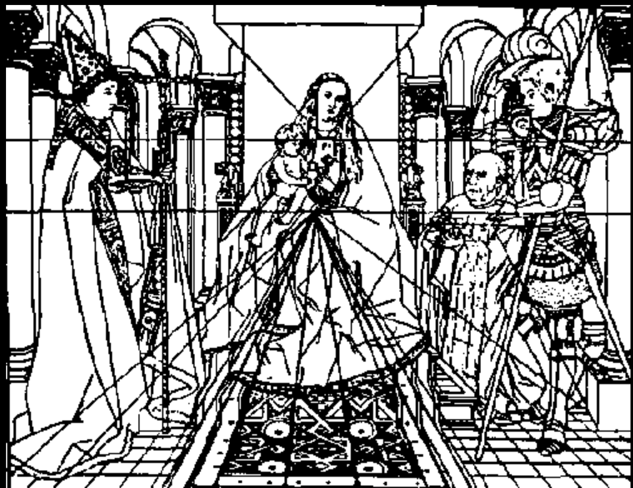
Photographic camera: Niepce, 1816.



Pinhole perspective projection: Brunelleschi, XVth Century.
Camera obscura: XVIth Century.

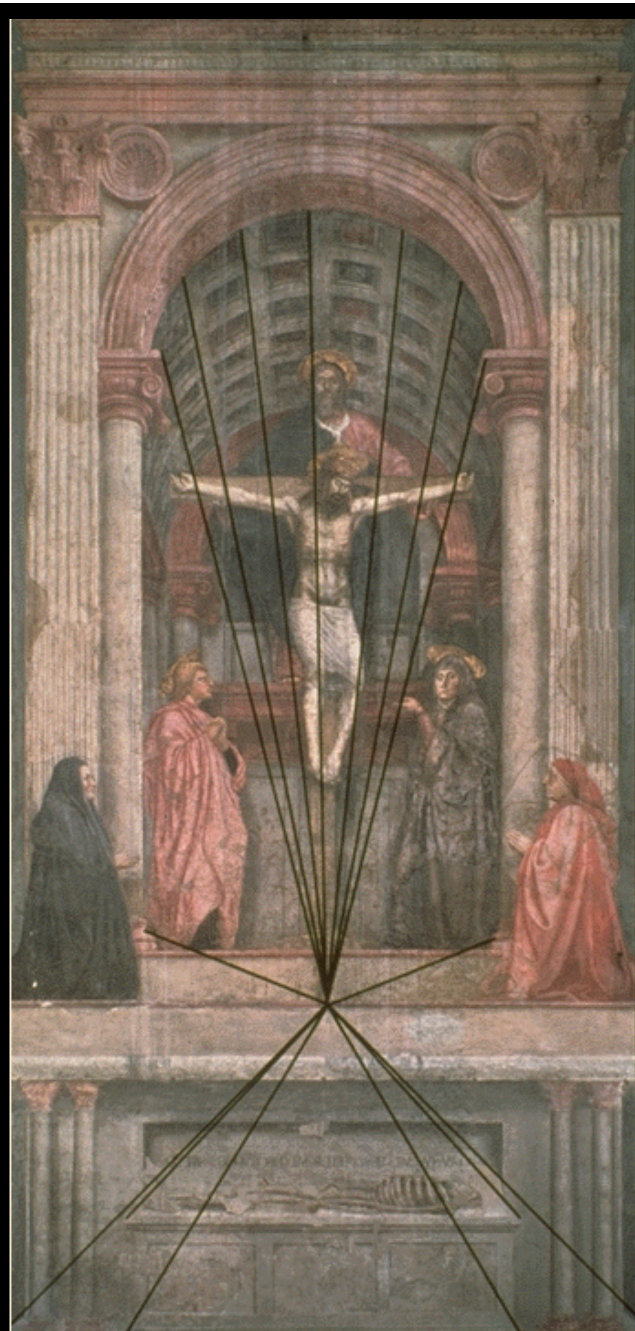


Pompei painting, 2000 years ago

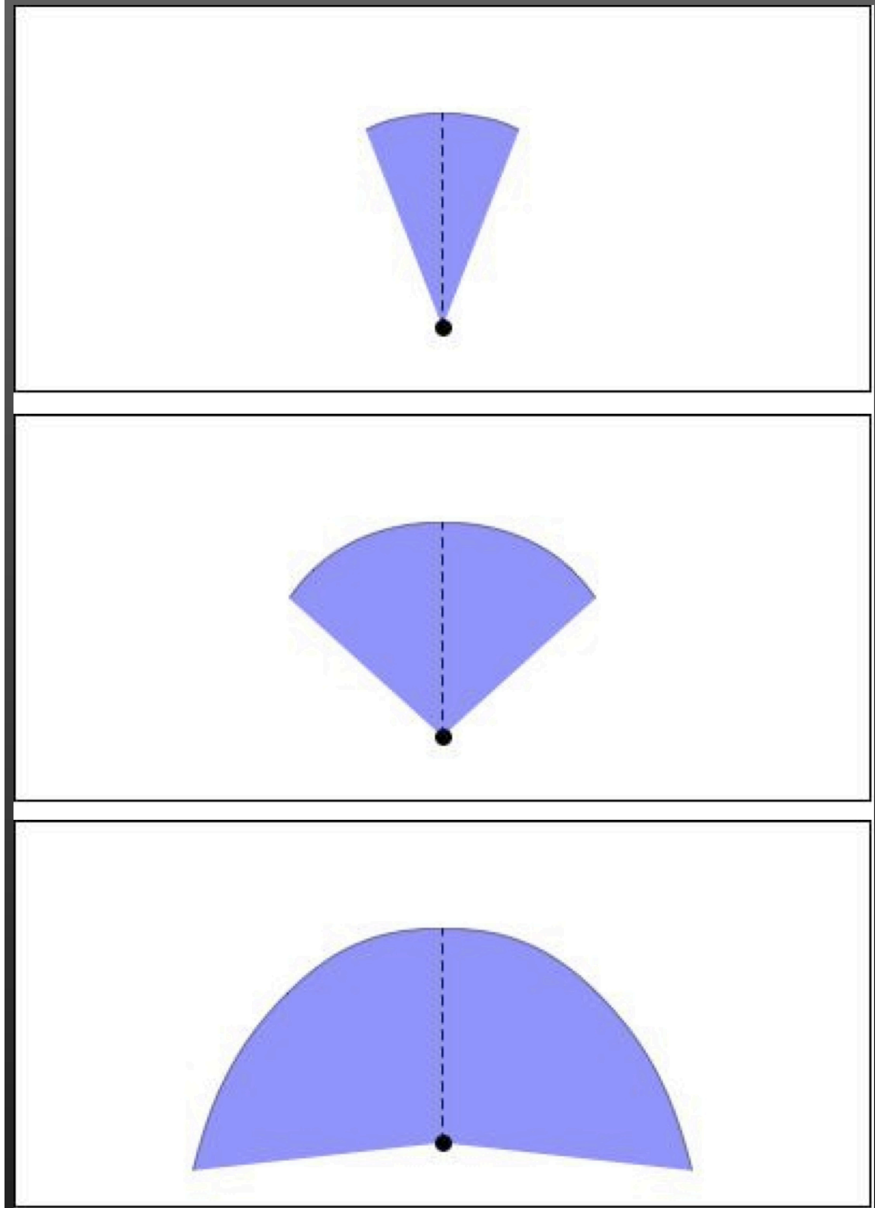


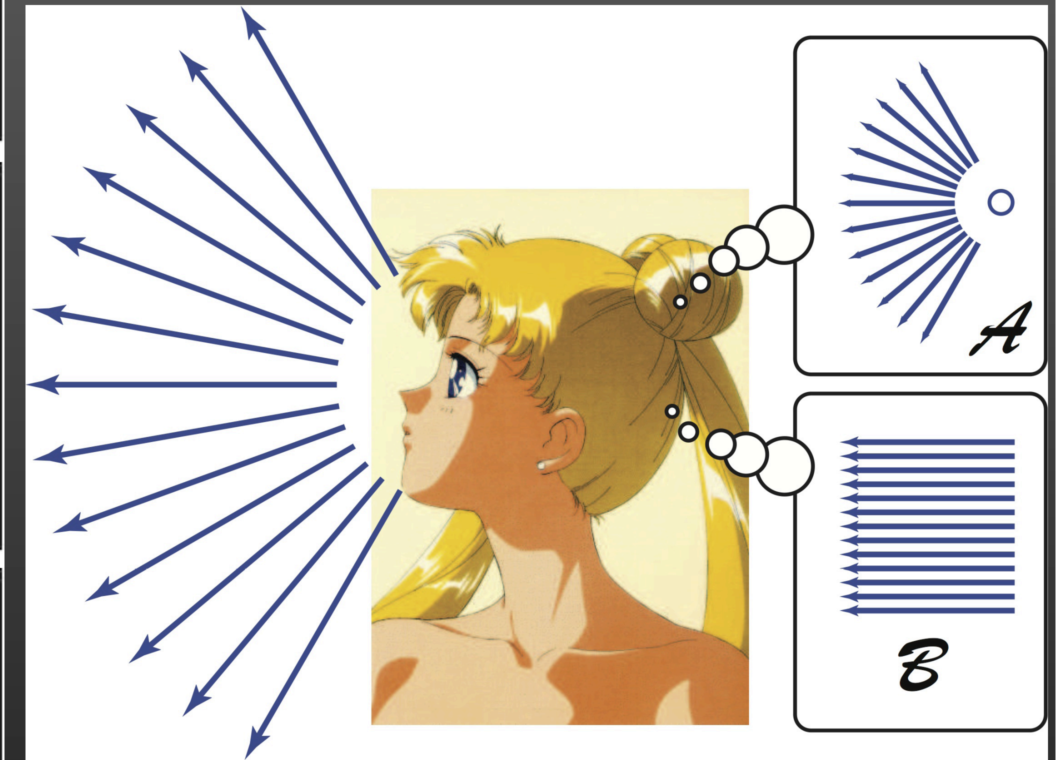
Van Eyk, XIVth Century

Brunelleschi, 1415

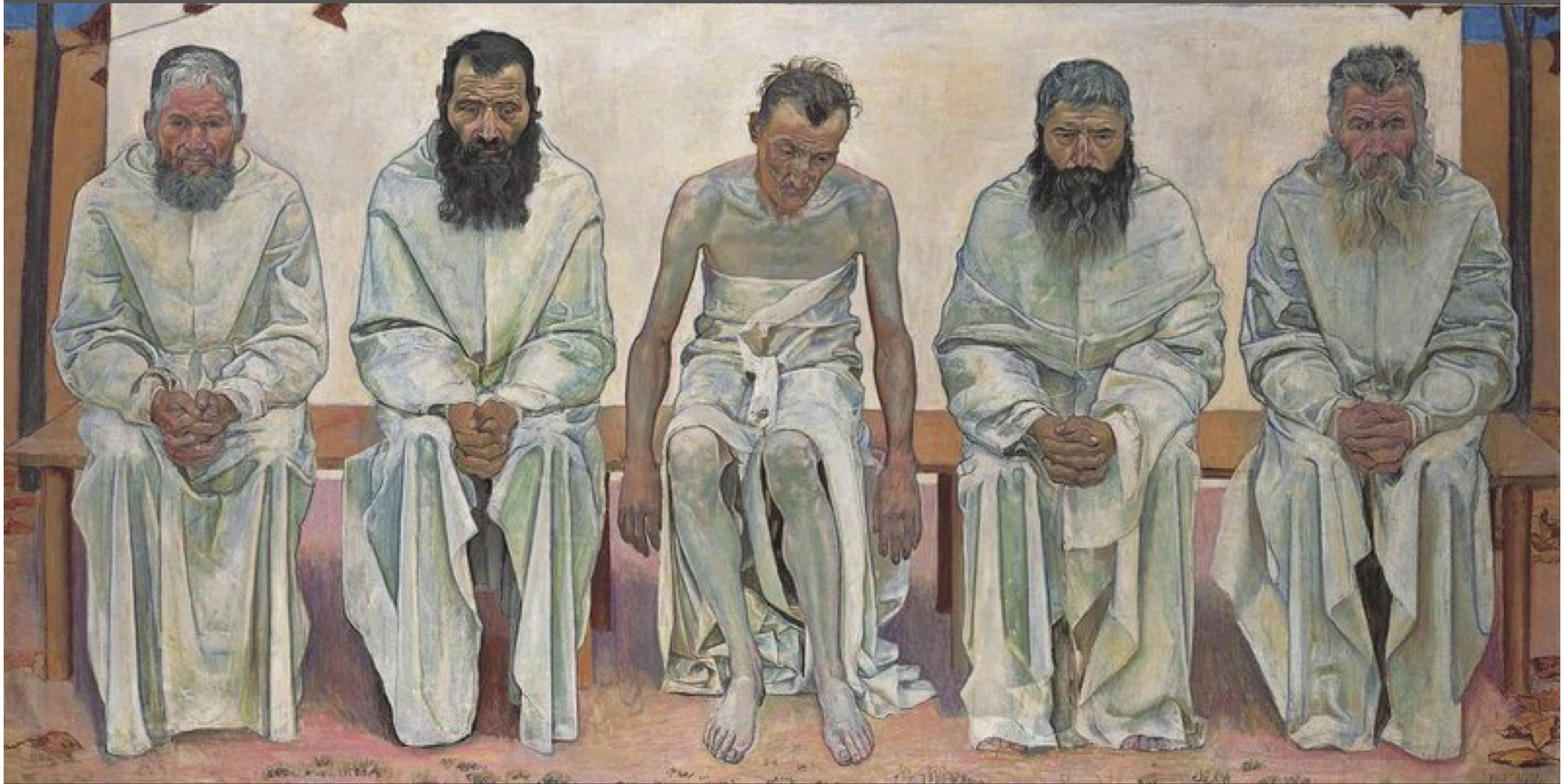


Massaccio's Trinity, 1425

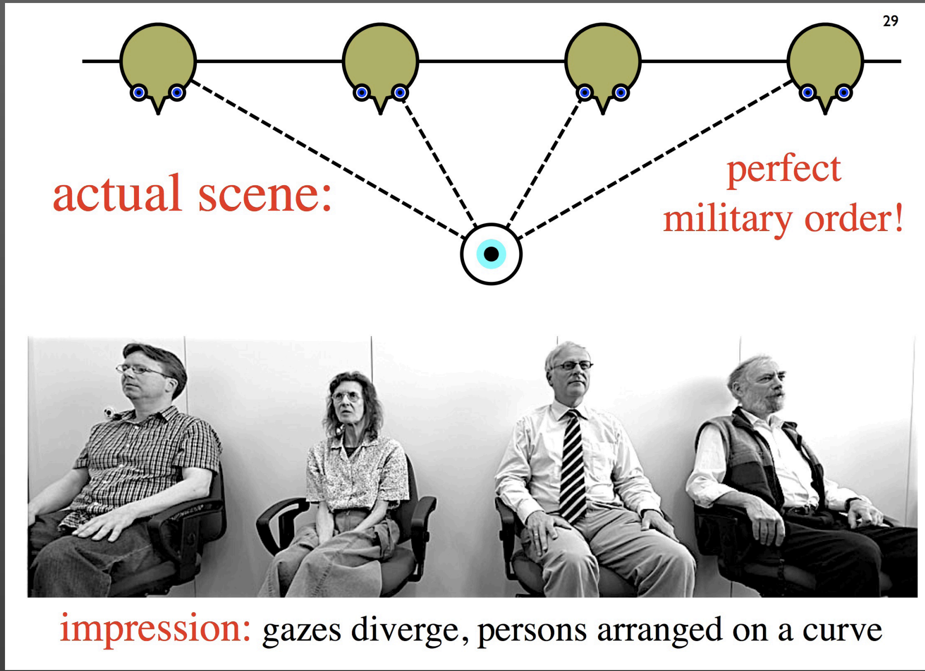


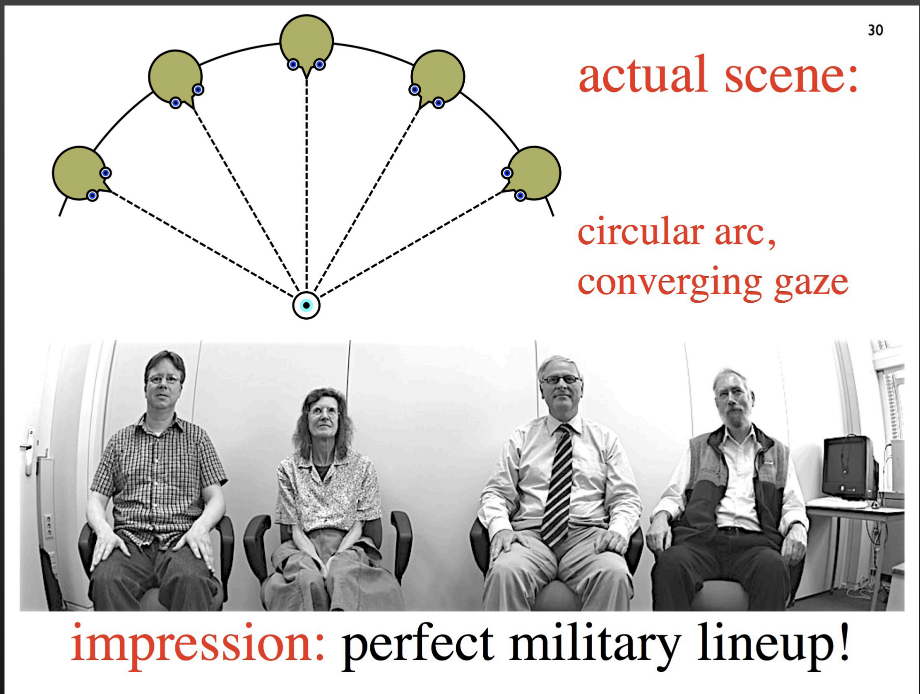


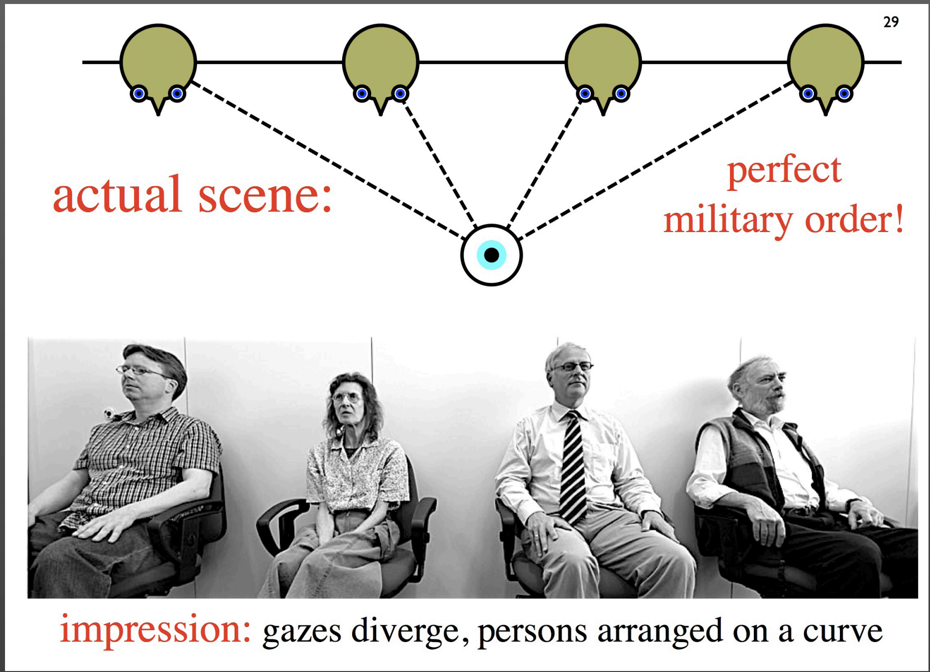
Most people don’t experience the divergence of visual rays in a veridical manner. This is fine. [Koenderink]

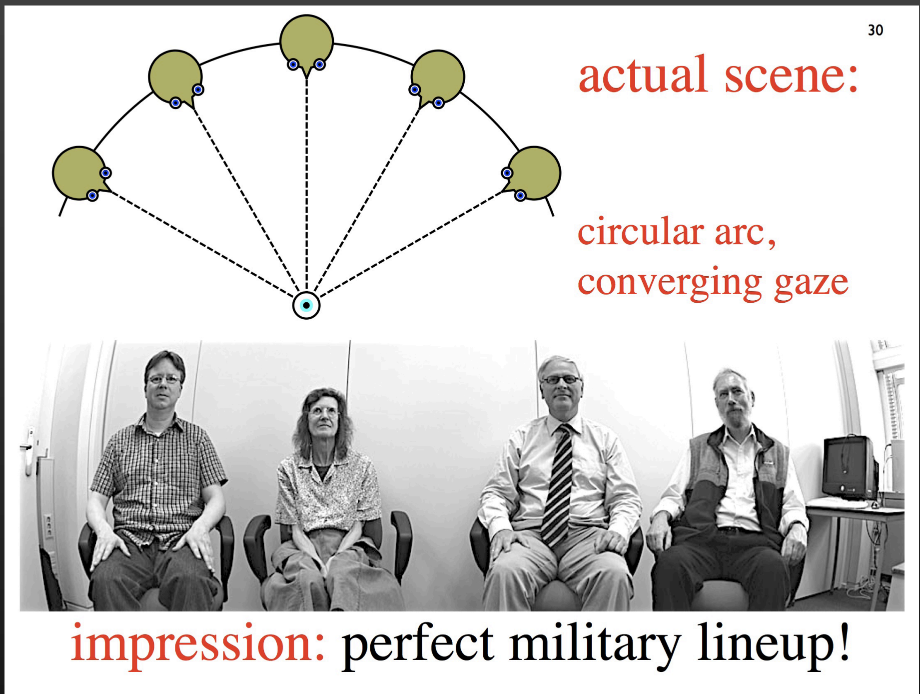


Ferdinand Hodler

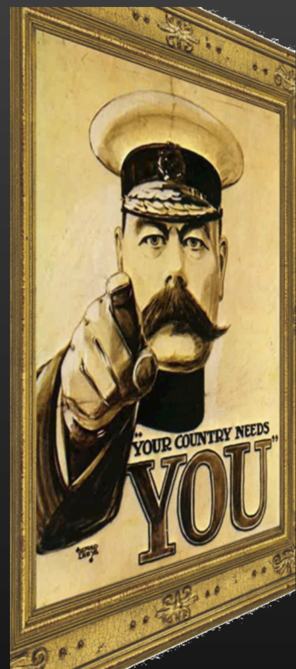

29
actual scene:
perfect
military order!
impression: gazes diverge, persons arranged on a curve

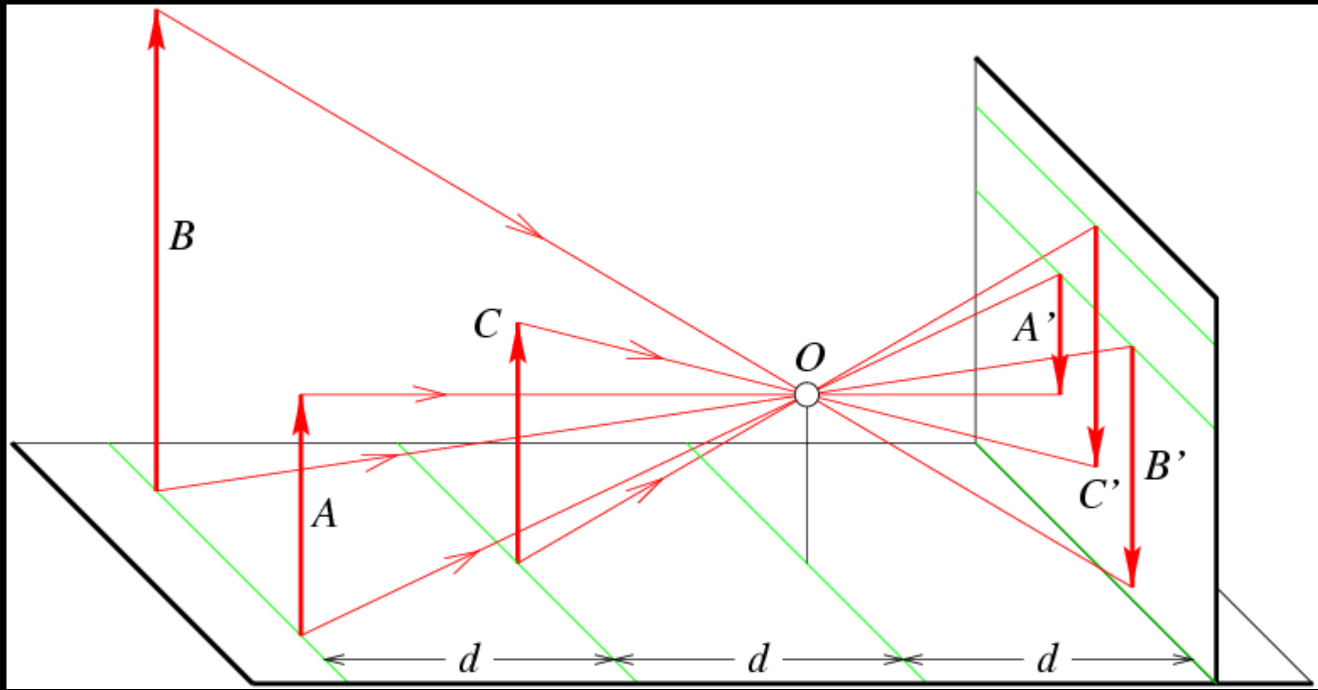

30
actual scene:
circular arc,
converging gaze
impression: perfect military lineup!


29
actual scene:
perfect
military order!
impression: gazes diverge, persons arranged on a curve


30
actual scene:
circular arc,
converging gaze
impression: perfect military lineup!


"YOUR COUNTRY NEEDS
YOU"


"YOUR COUNTRY NEEDS
YOU"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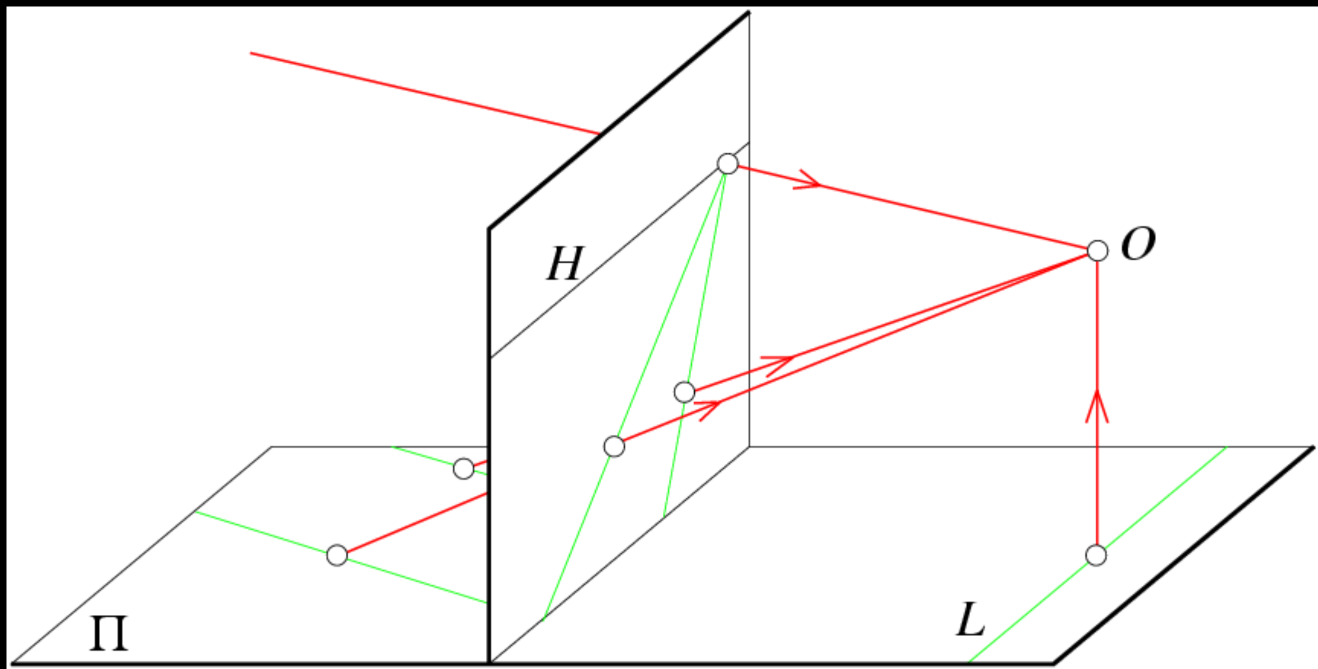
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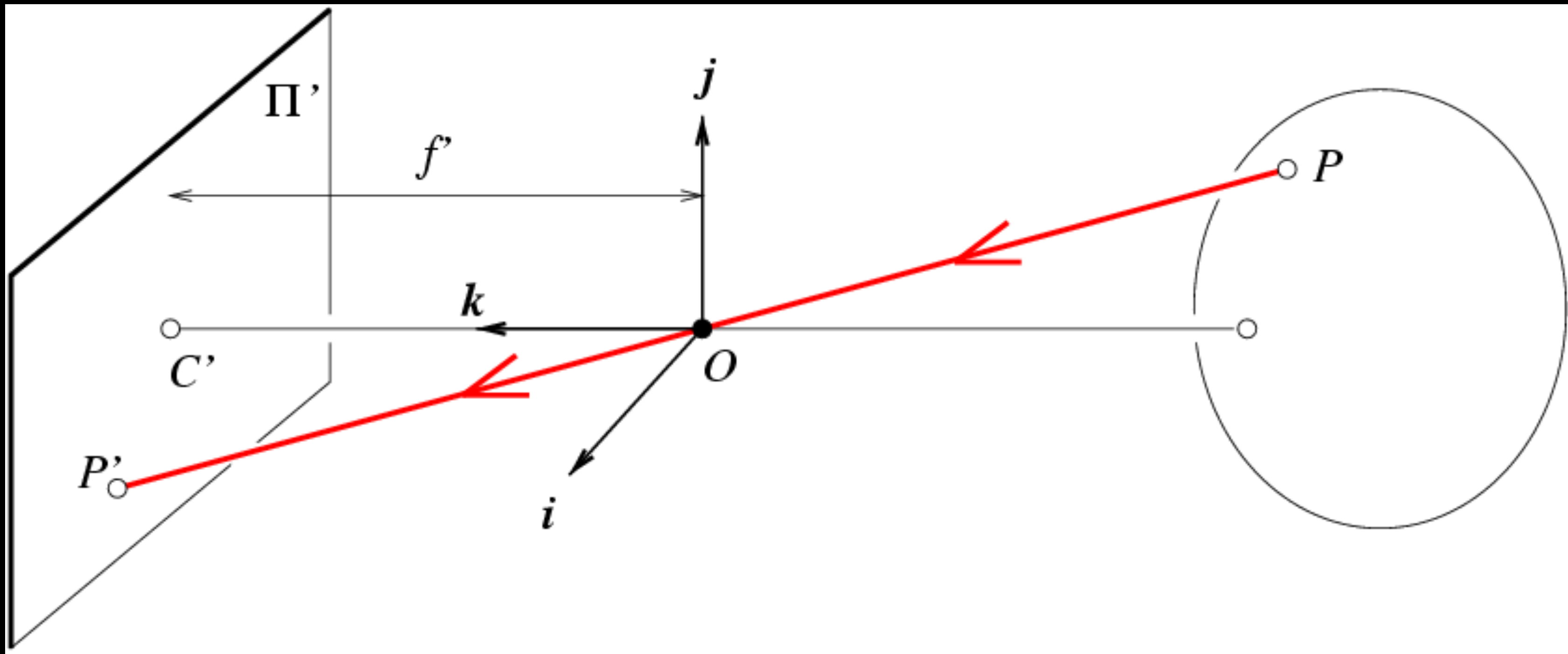
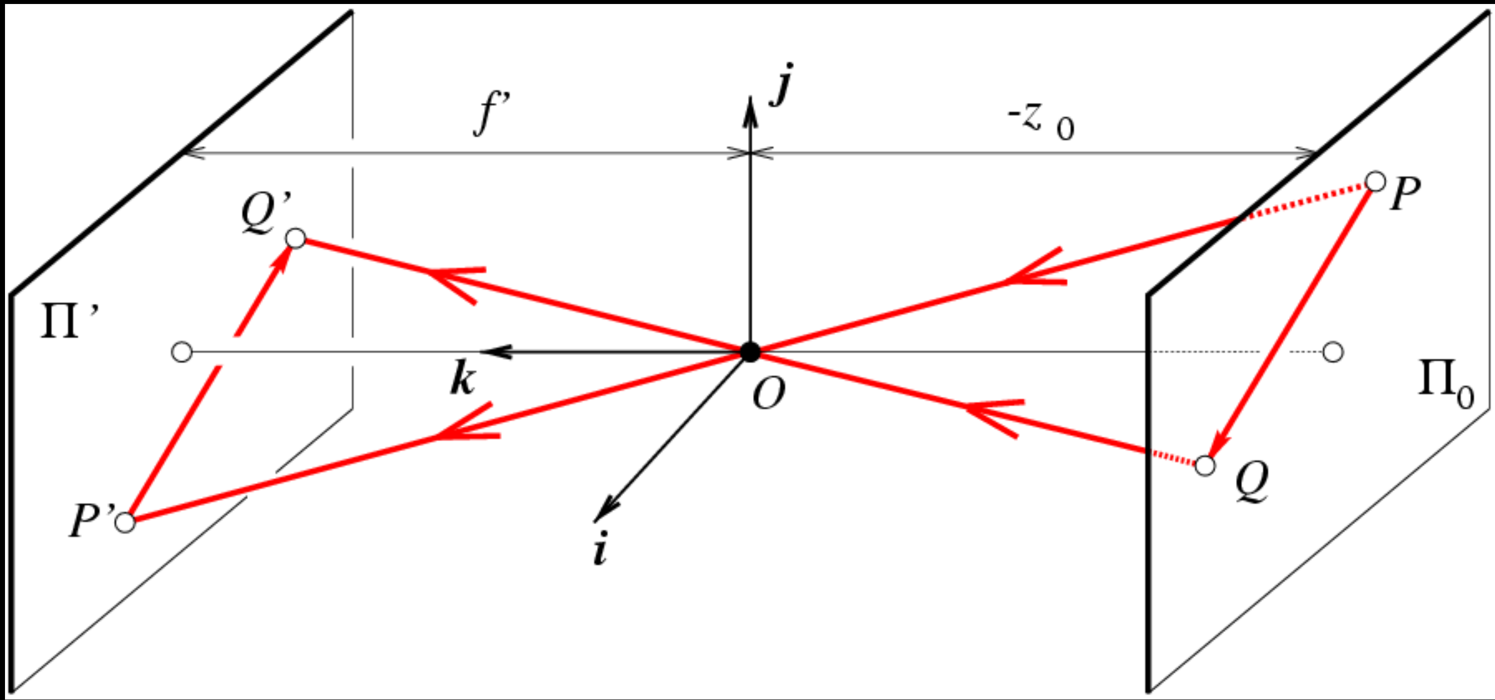
Pinhole Perspective Equation



$$\begin{cases} x' = f'\dfrac{x}{z} \\ y' = f'\dfrac{y}{z} \end{cases}$$

NOTE: *z* is always negative..

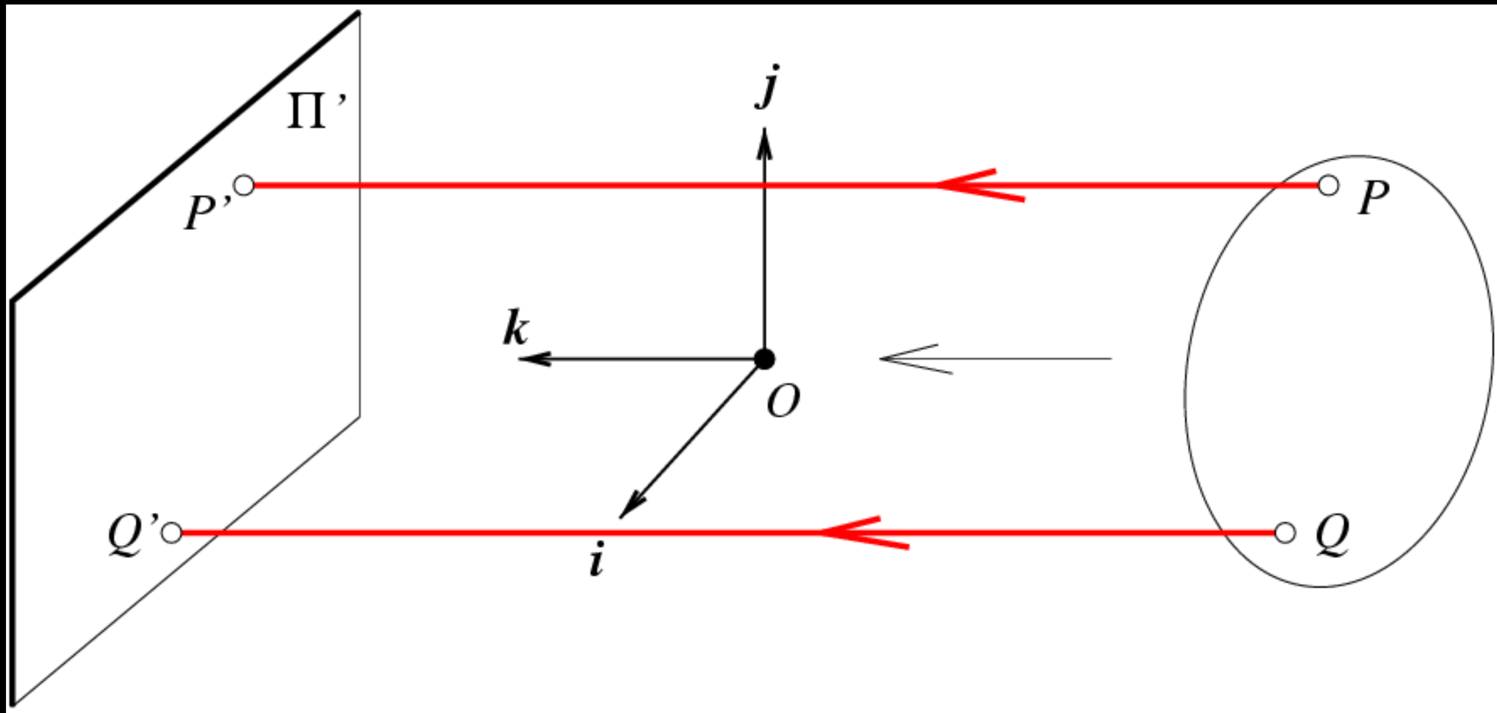
Affine projection models: Weak perspective projection



$$\begin{cases} x' = -mx \\ y' = -my \end{cases} \quad \text{where} \quad m = -\frac{f'}{z_0}$$ is the magnification.

When the scene relief is small compared its distance from the Camera, *m* can be taken constant: weak perspective projection.

Affine projection models: Orthographic projection

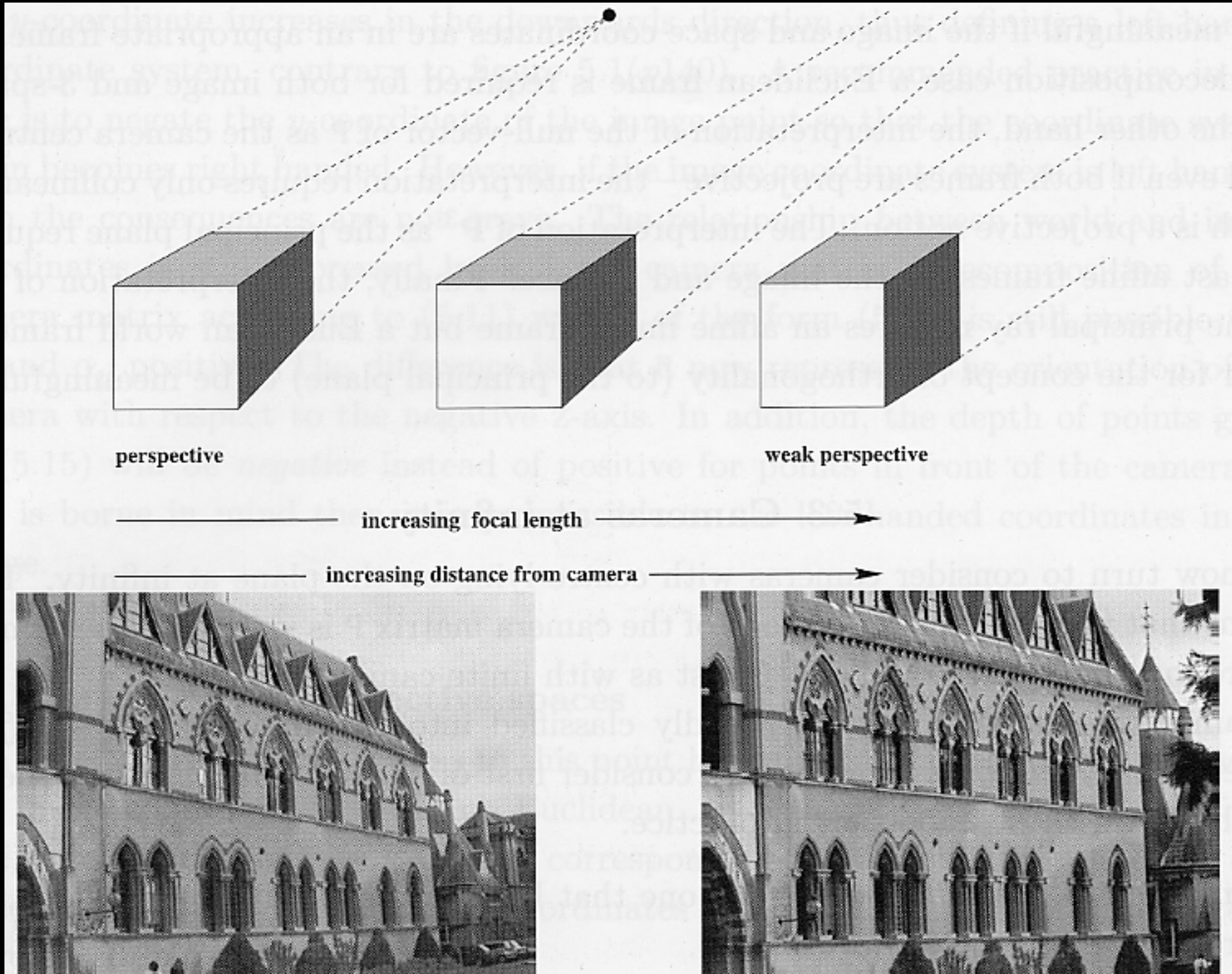


$$\begin{cases} x' = x \\ y' = y \end{cases}$$

When the camera is at a (roughly constant) distance from the scene, take m=1.

Strong perspective:

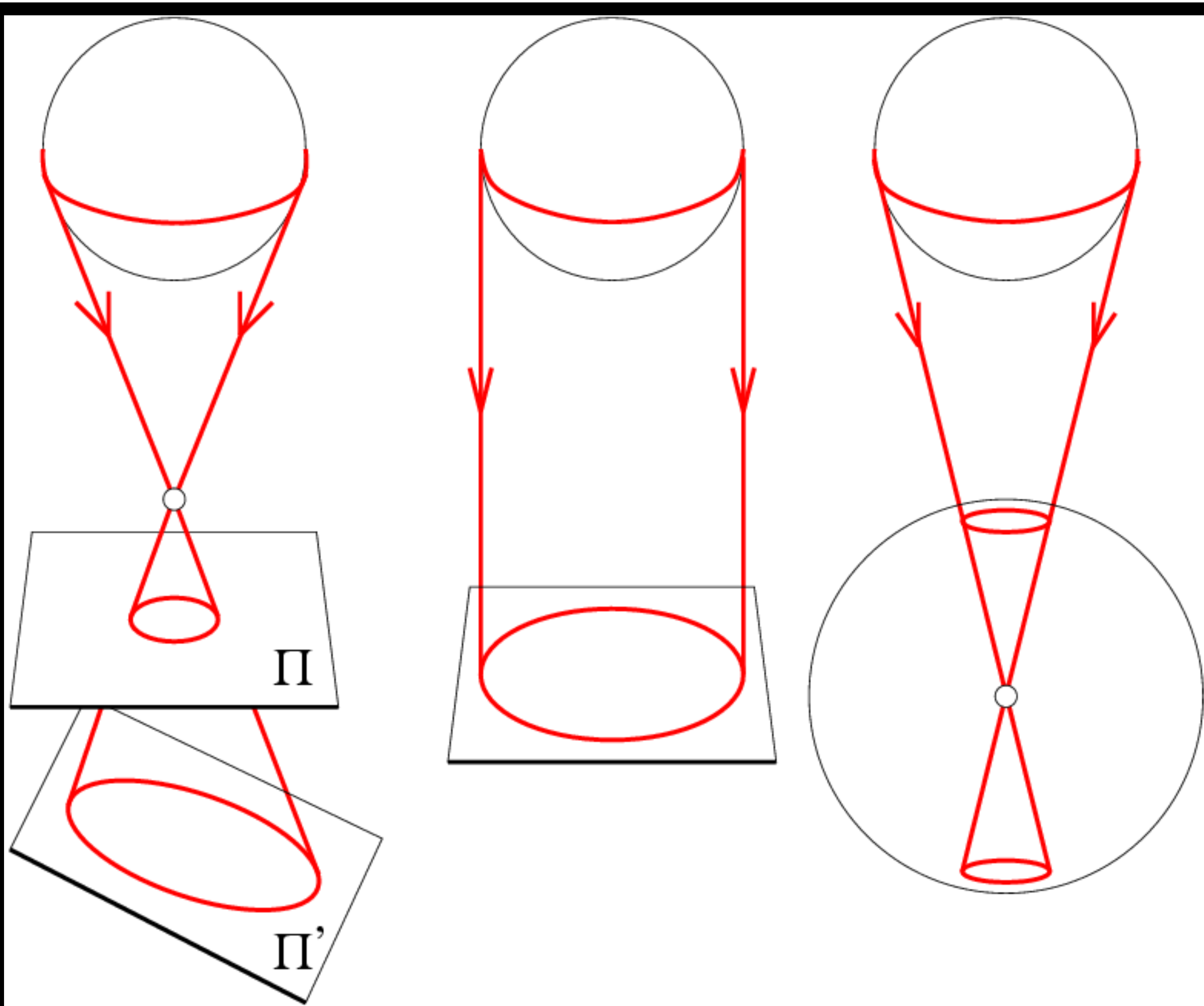
- Angles are not preserved
- The projections of parallel lines intersect at one point



From Zisserman & Hartley

Strong perspective:
Angles are not preserved
The projections of parallel lines intersect at one point

Weak perspective:
Angles are better preserved
The projections of parallel lines are (almost) parallel



Planar pinhole perspective

Orthographic projection

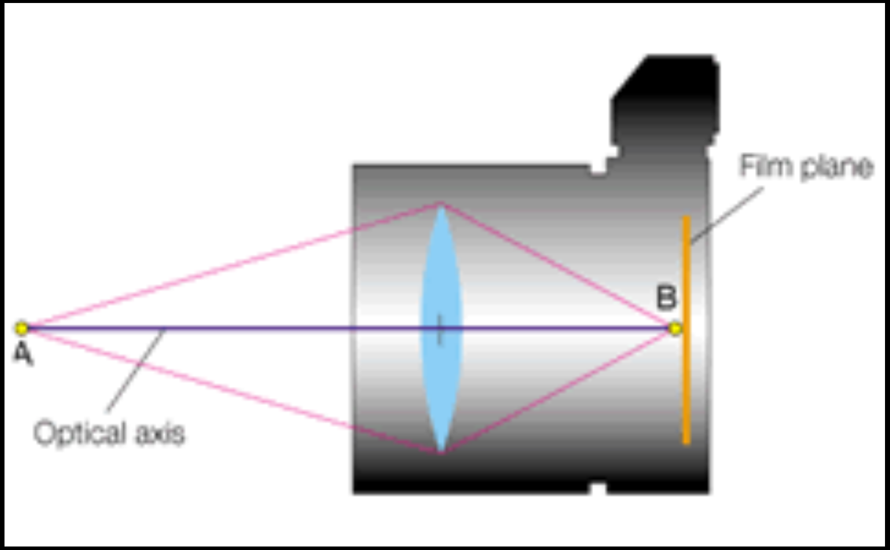
Spherical pinhole perspective

Diffraction effects in pinhole cameras.

Shrinking pinhole size

Use a lens!

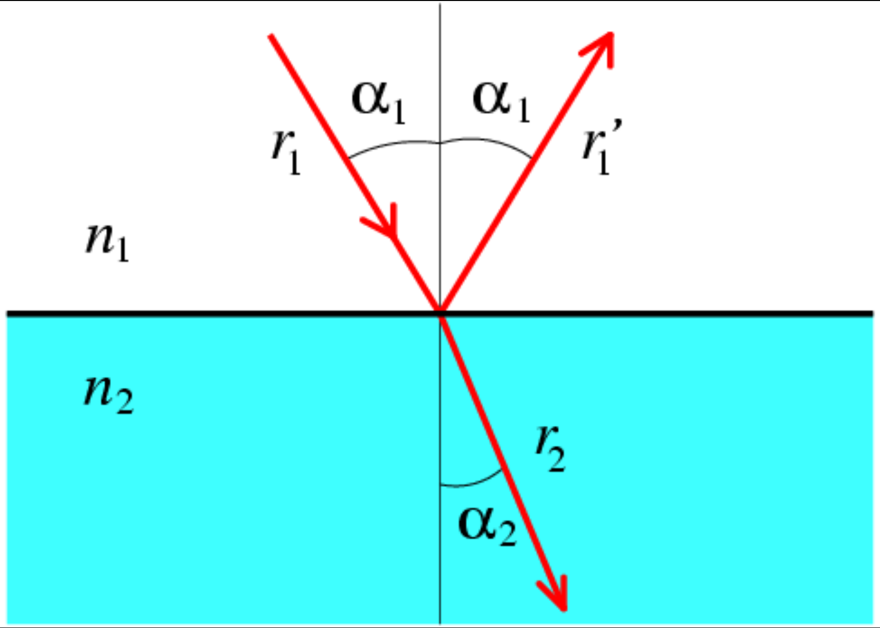
Lenses



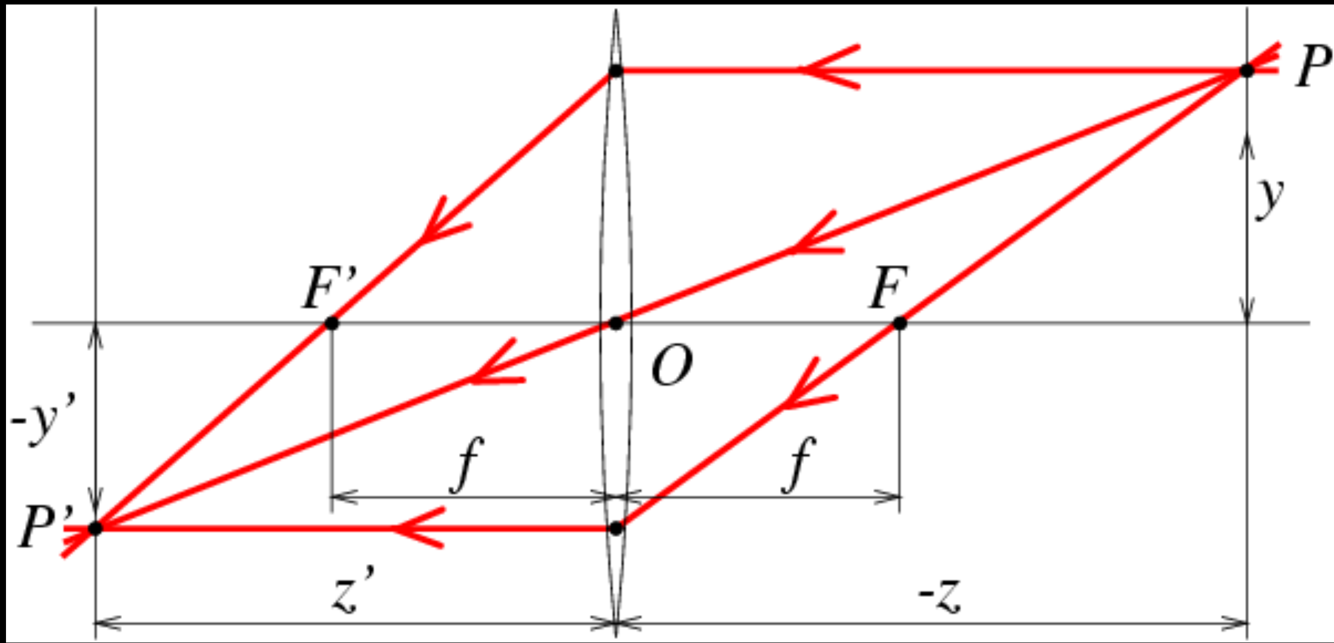
Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

Descartes' law

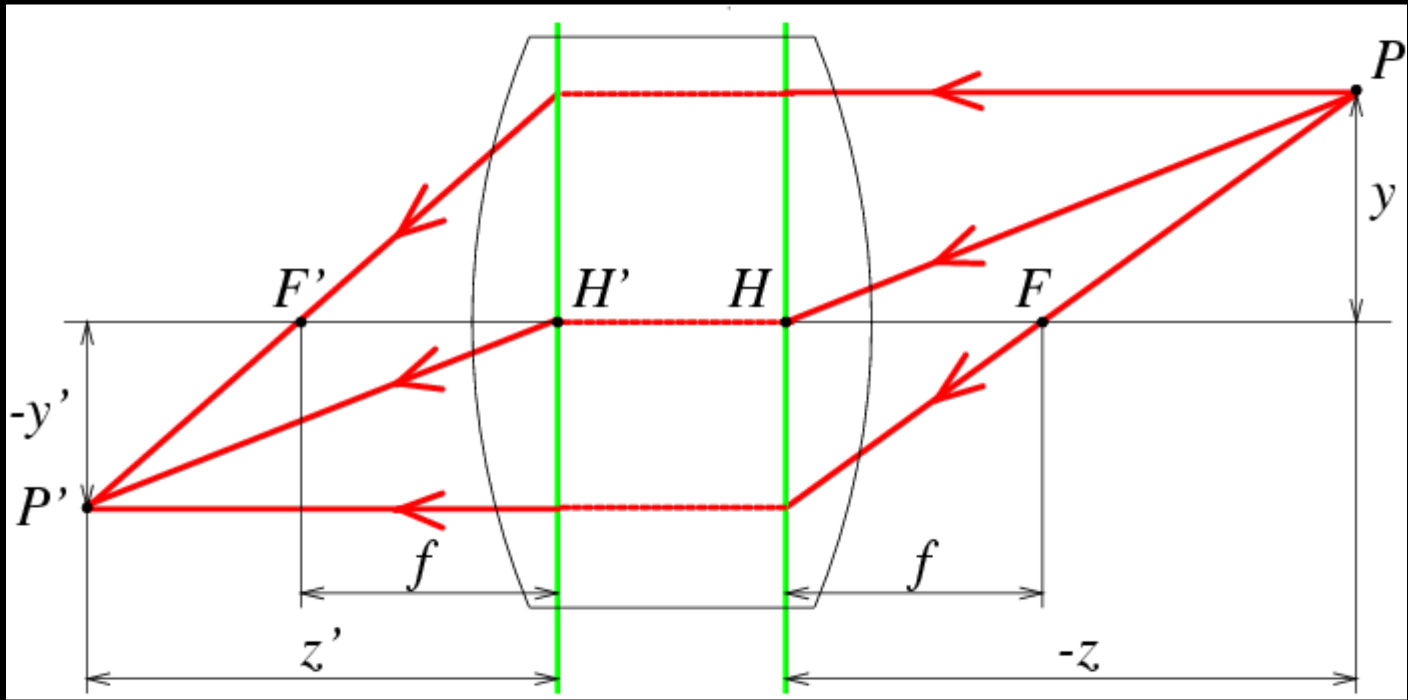


Thin Lenses

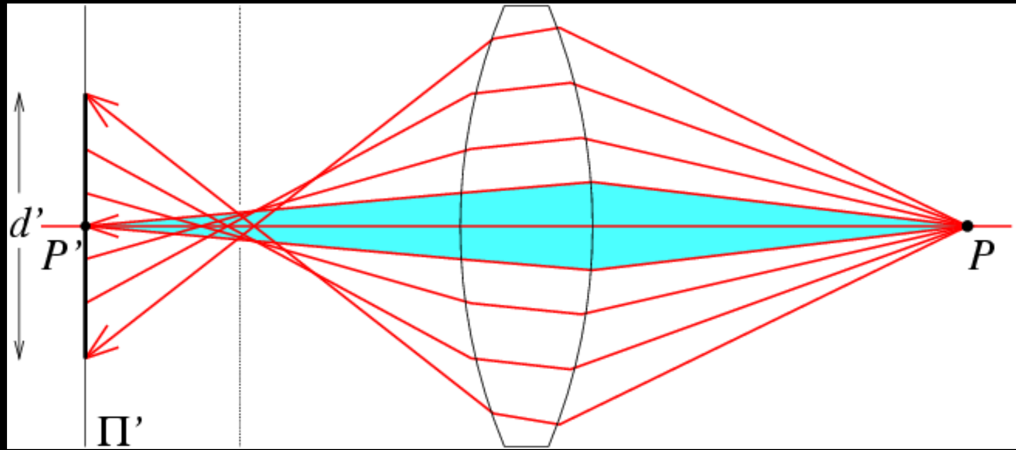


$$\begin{cases} x' = z'\dfrac{x}{z} \\ y' = z'\dfrac{y}{z} \end{cases} \quad \text{where} \quad \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \quad \text{and} \quad f = \frac{R}{2(n-1)}$$

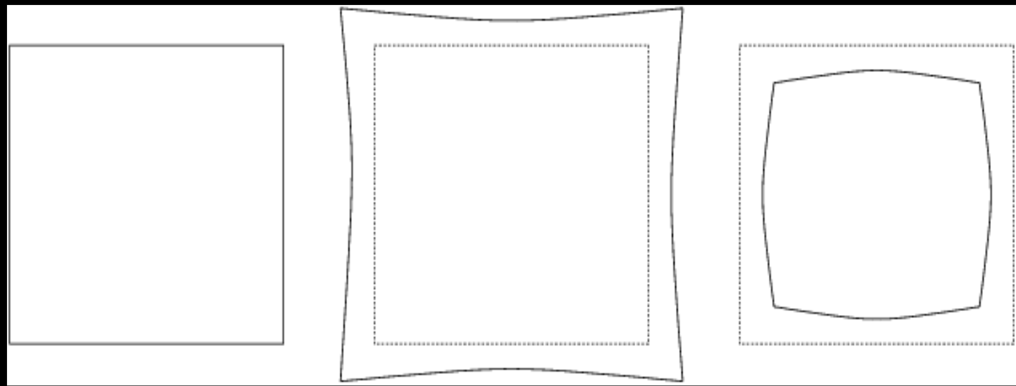
Thick Lenses



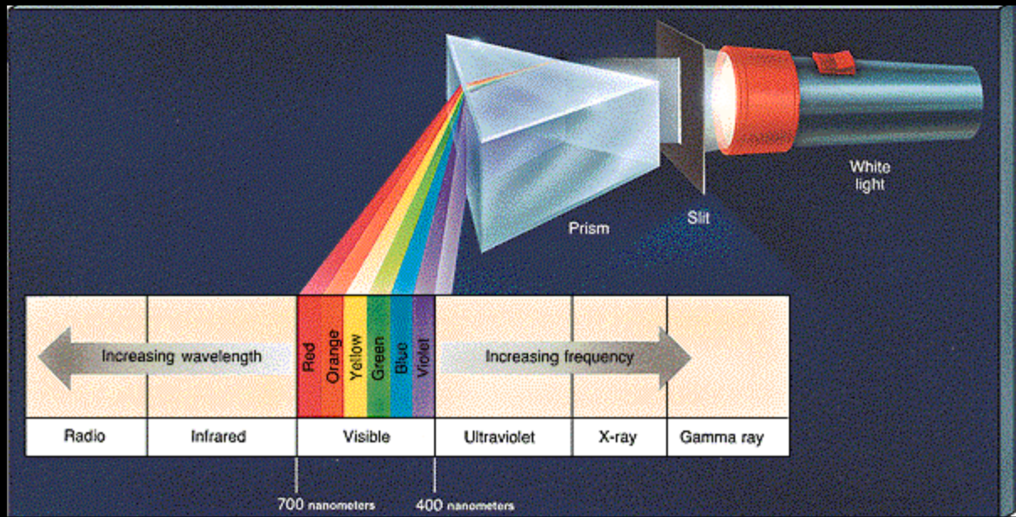
Spherical Aberration



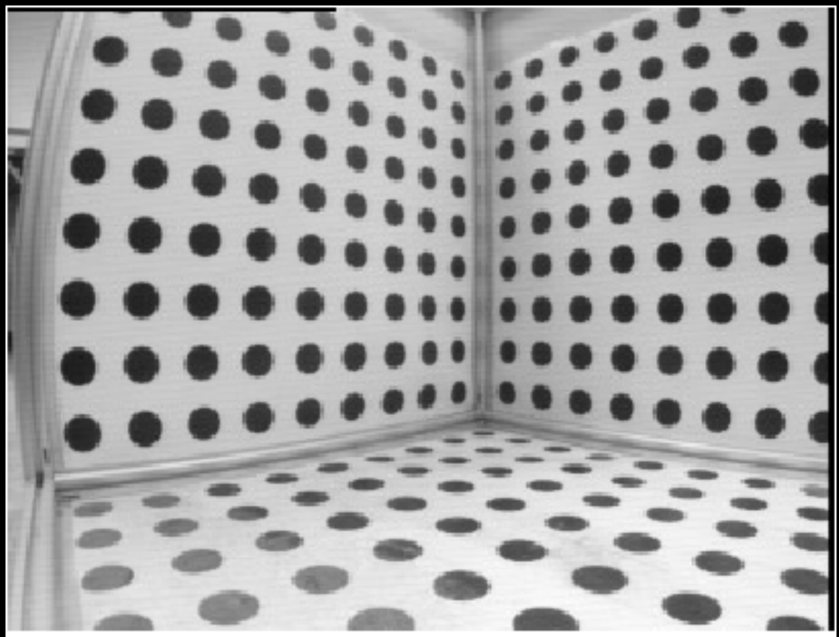
Distortion

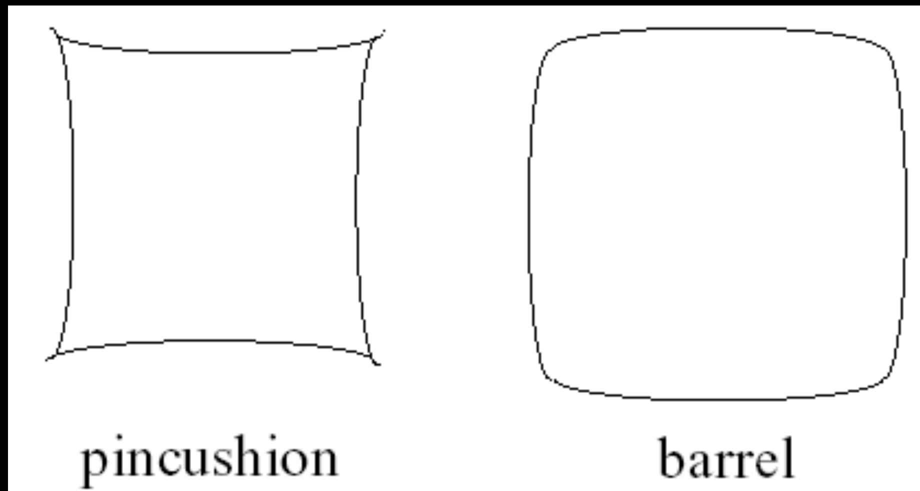


Chromatic Aberration



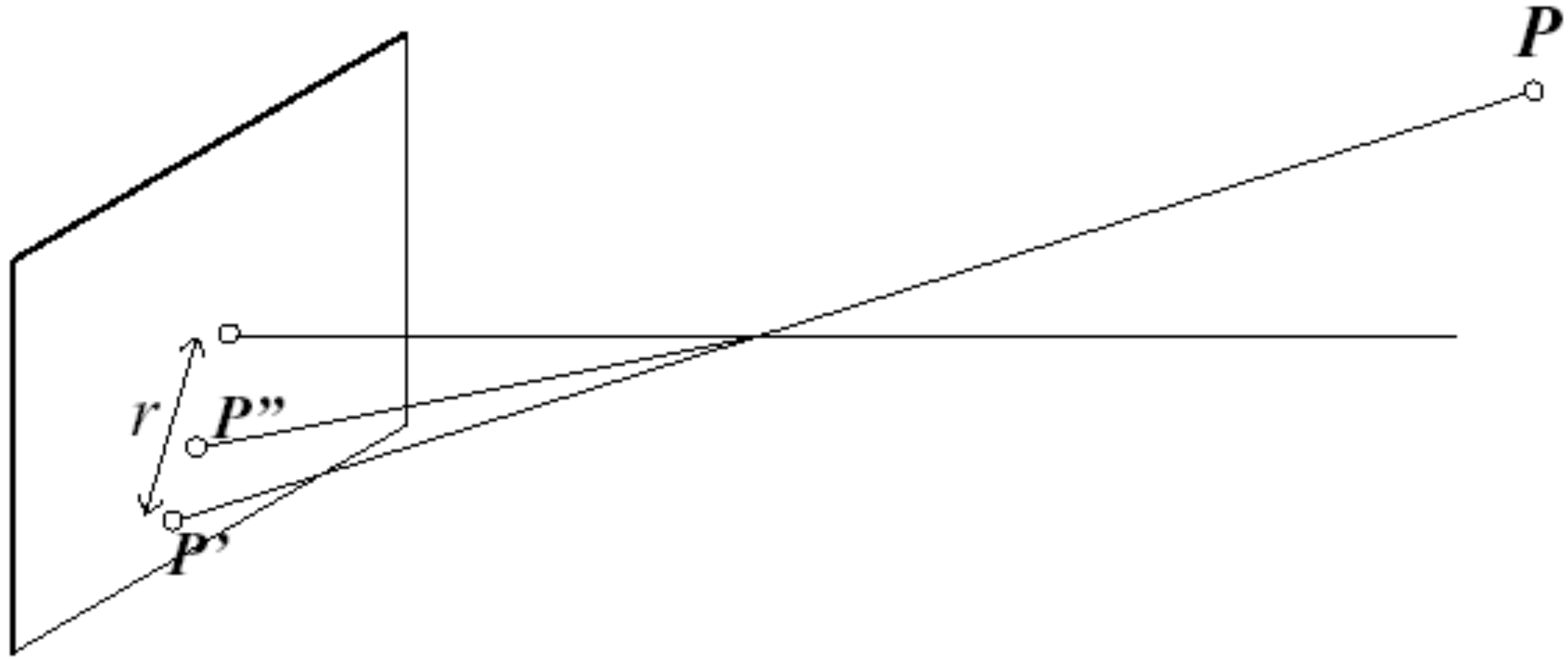
Geometric Distortion





Rectification

Radial Distortion Model



Ideal:

$$x' = f\frac{x}{z}$$

$$y' = f\frac{y}{z}$$

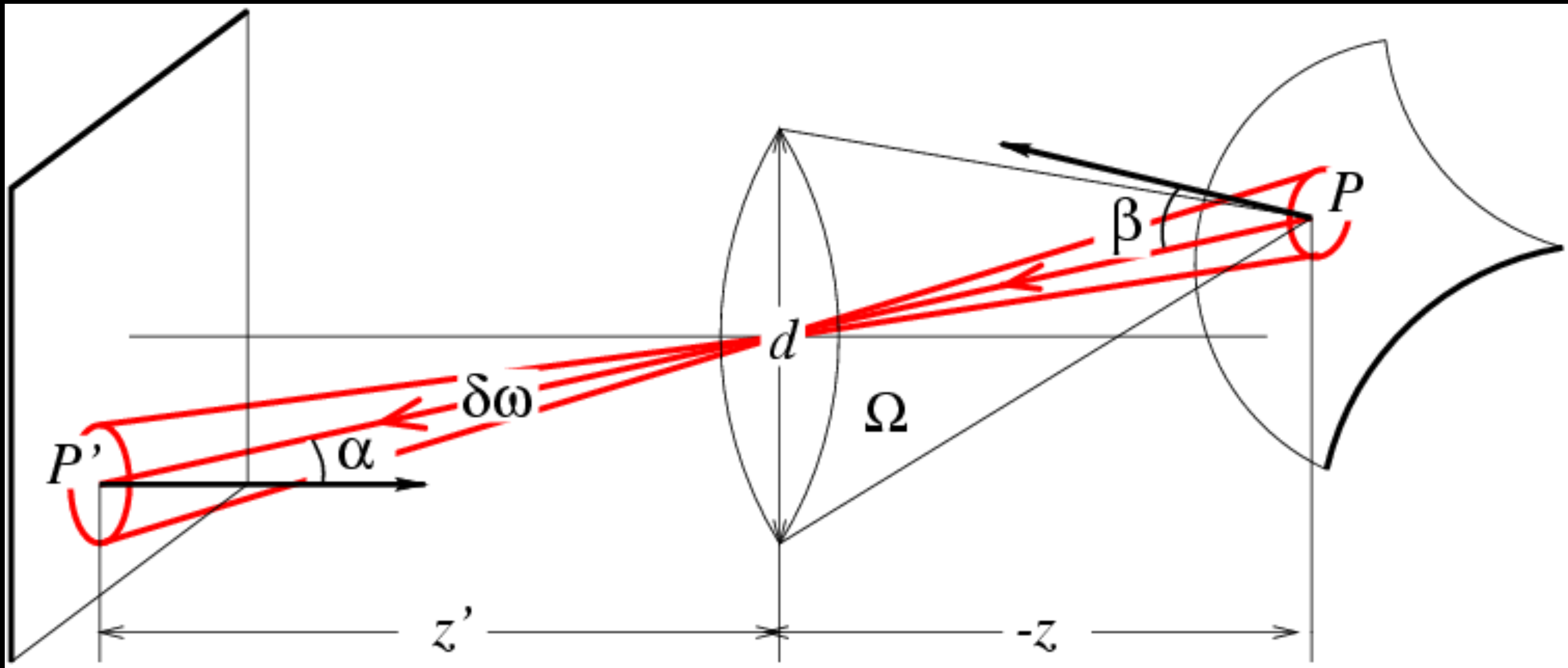
Distorted:

$$x'' = \frac{1}{\lambda}x'$$

$$y'' = \frac{1}{\lambda}y'$$

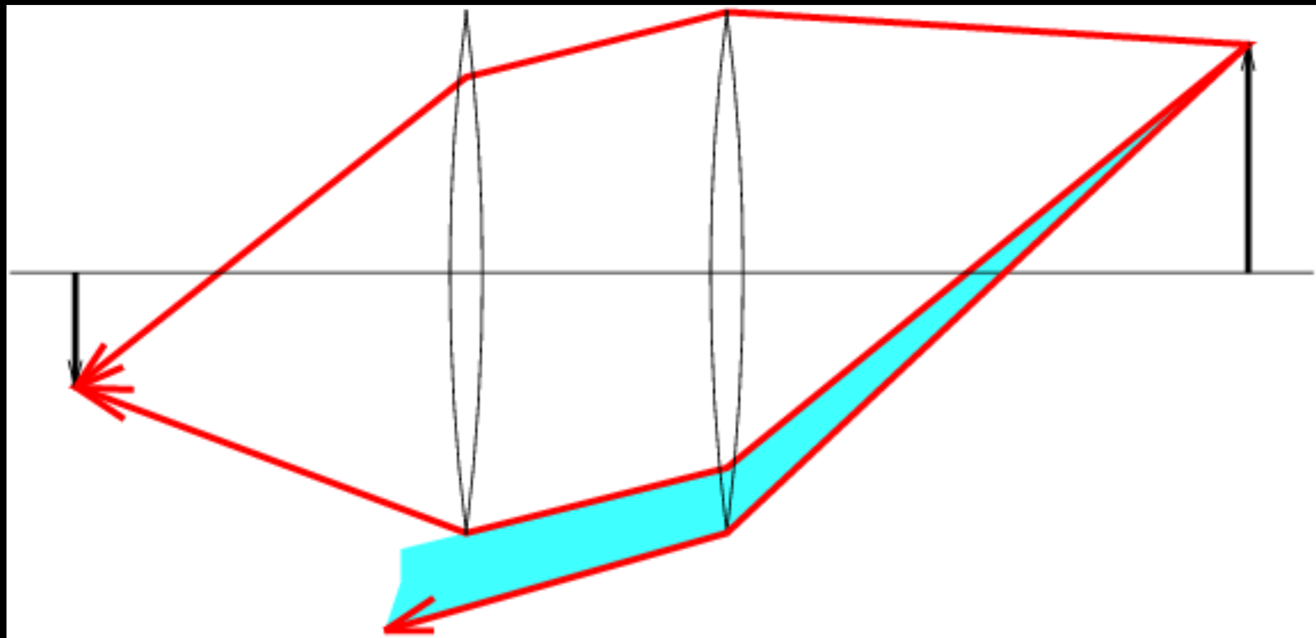
$$\lambda = 1 + k_1 r^2 + k_2 r^4 + \cdots$$

A compound lens



$$E = (\Pi/4)\left[\,(d/z')^2 \cos^4\alpha\,\right] L$$

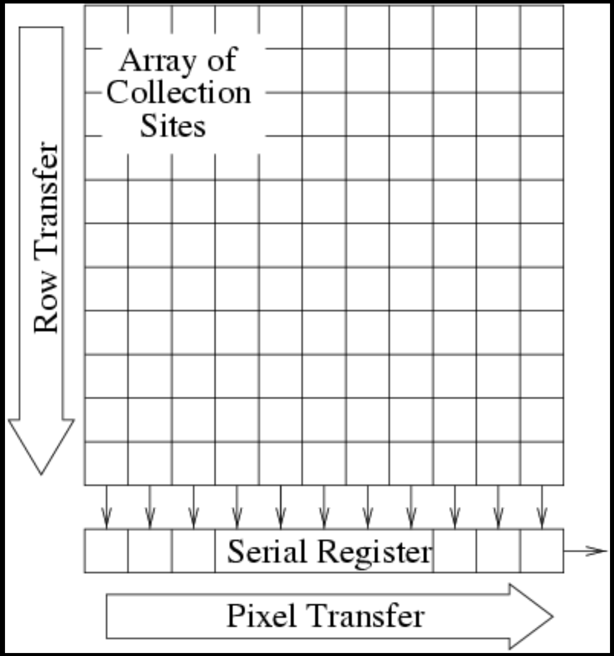
Vignetting



Challenge: Illumination – What is wrong with these pictures?

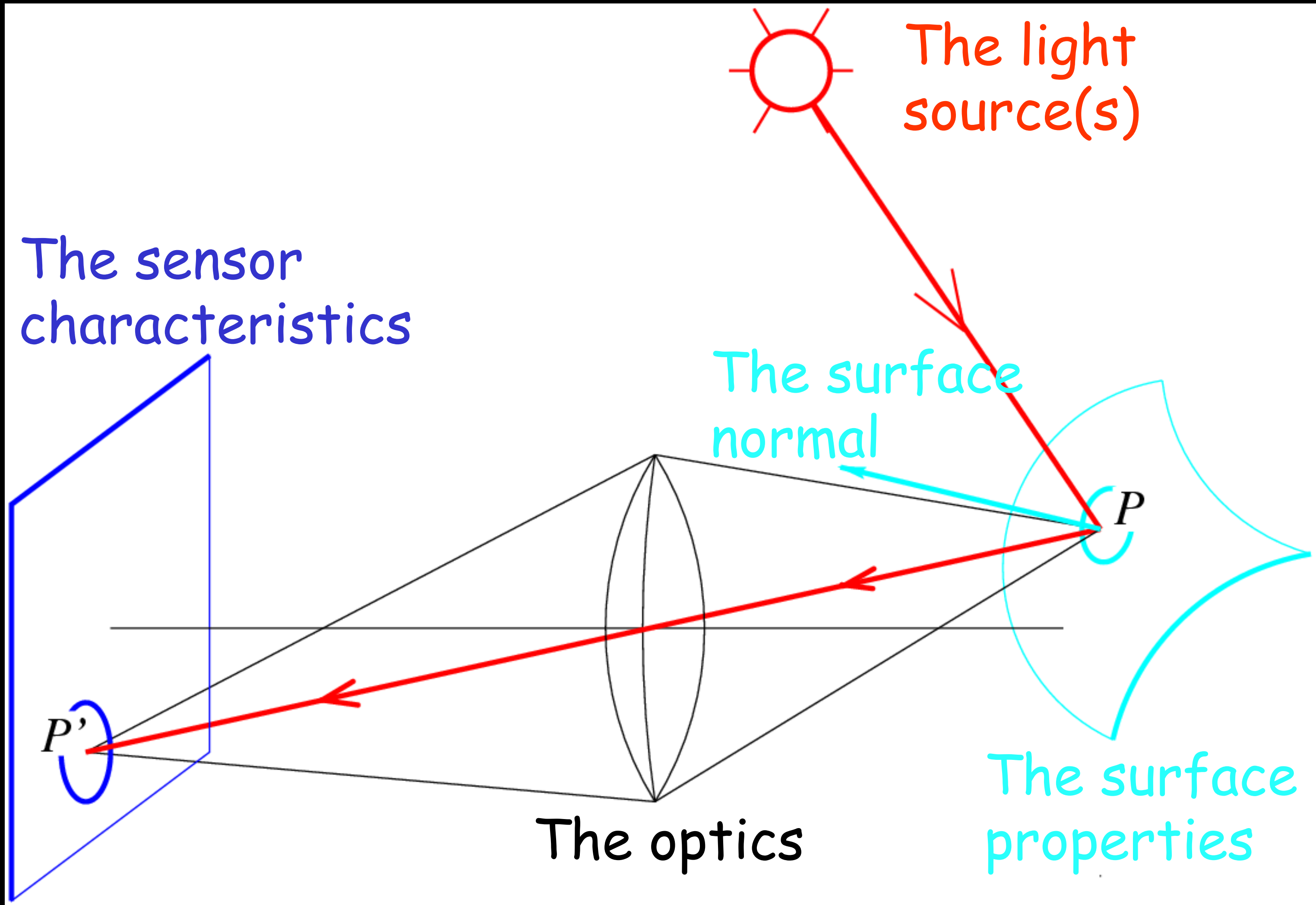
Photography
(Niepce, “La Table Servie,” 1822)

Milestones:
• Daguerréotypes (1839)
• Photographic Film (Eastman, 1889)
• Cinema (Lumière brothers, 1895)
• Color Photography (Lumière brothers, again, 1908)
• Television (Baird, Farnsworth, Zworykin, 1920s)



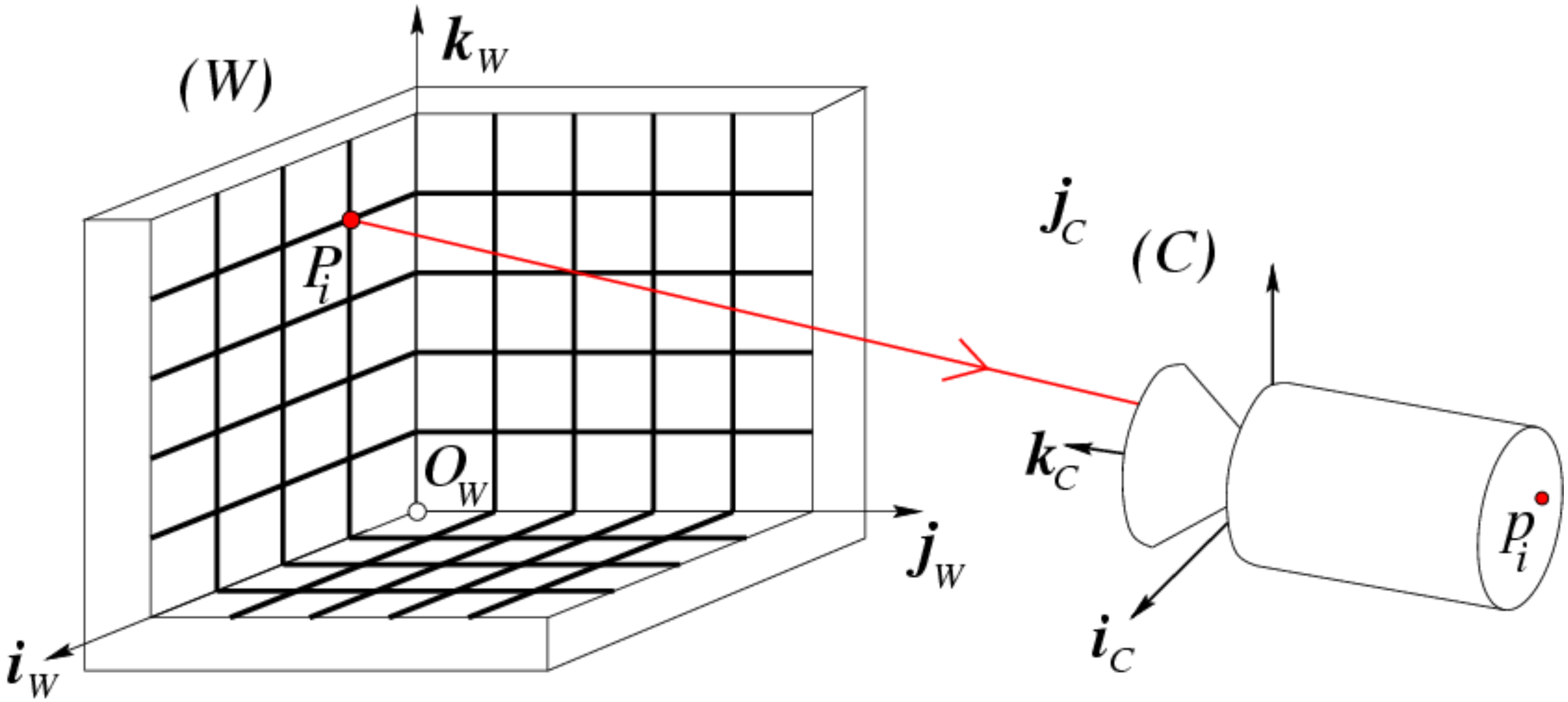
CCD Devices (1970), etc.

Image Formation: Radiometry



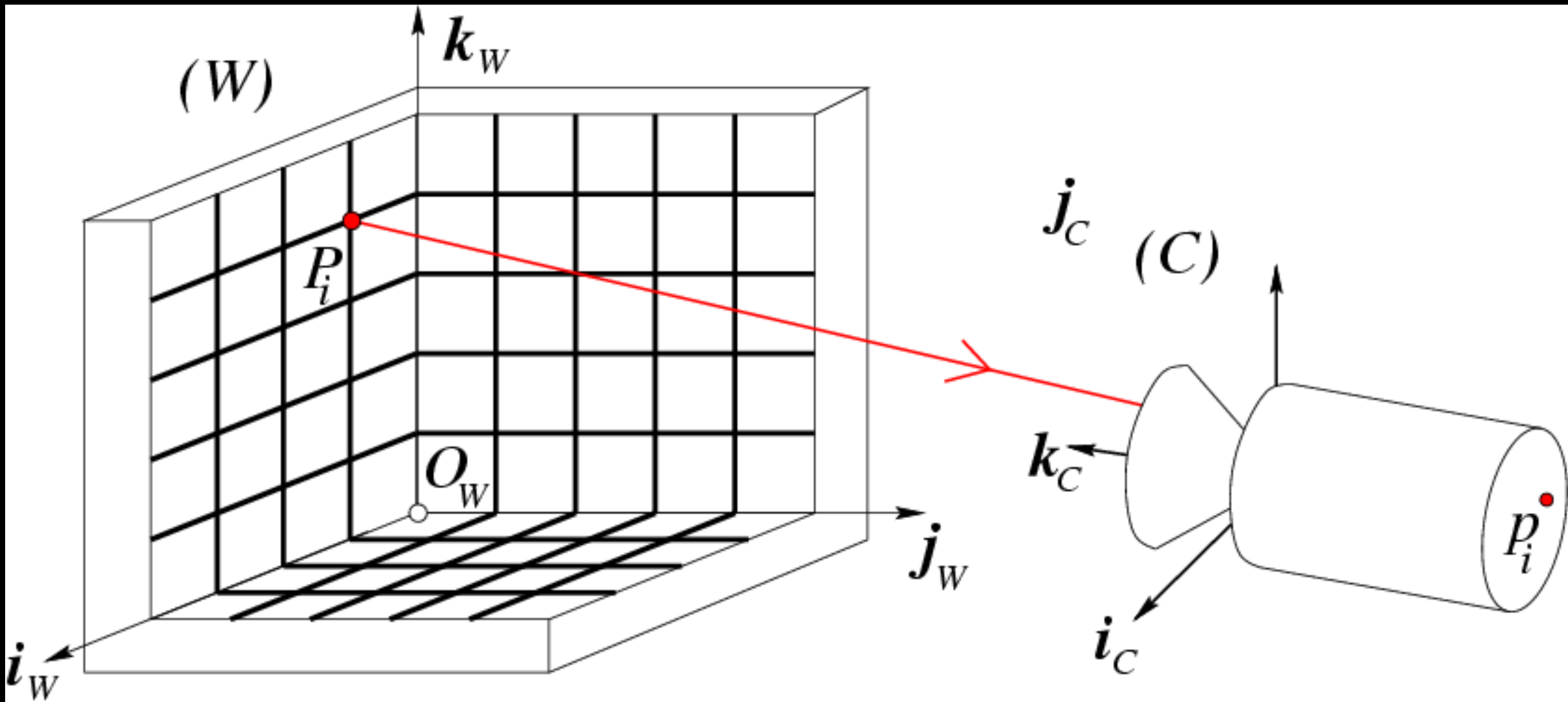
What determines the brightness of an image pixel?

Perspective Projection	$x' = f\frac{x}{z}$ $y' = f\frac{y}{z}$	*x,y*: World coordinates *x',y'*: Image coordinates *f*: pinhole-to-retina distance
Weak-Perspective Projection (Affine)	$x' \approx -mx$ $y' \approx -my$ $\quad m = -\frac{f}{\bar{z}}$	*x,y*: World coordinates *x',y'*: Image coordinates *m*: magnification
Orthographic Projection (Affine)	$x' \approx x$ $y' \approx y$	*x,y*: World coordinates *x',y'*: Image coordinates
Common distortion model	$x'' = \frac{1}{\lambda}x'$ $y'' = \frac{1}{\lambda}y'$ $\lambda = 1 + k_1 r^2 + k_2 r^4 + \cdots$	*x',y'*: Ideal image coordinates *x'',y''*: Actual image coordinates



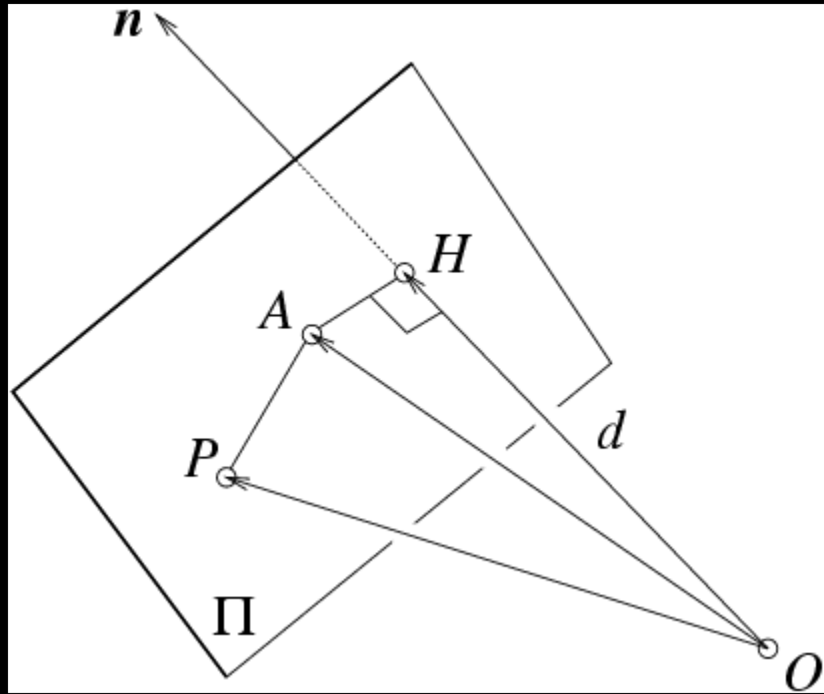
(W)
k_W
P_i
O_W
j_W
i_W
j_C
(C)
k_C
i_C
p_i

Quantitative Measurements and Calibration



Euclidean Geometry

Planes and homogeneous coordinates



$$\overrightarrow{AP}.\mathbf{n} = 0 \Leftrightarrow ax + by + cz - d = 0 \Leftrightarrow \mathbf{\Pi}.\mathbf{P} = 0$$

where $\mathbf{\Pi} = \begin{bmatrix} a \\ b \\ c \\ -d \end{bmatrix}$ and $\mathbf{P} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

The Extrinsic Parameters of a Camera

- When the camera frame (C) is different from the world frame (W),

$$\begin{pmatrix} {}^C P \\ 1 \end{pmatrix} = \begin{pmatrix} {}^C_W \mathcal{R} & {}^C O_W \\ \mathbf{0}^T & 1 \end{pmatrix} \begin{pmatrix} {}^W P \\ 1 \end{pmatrix}.$$

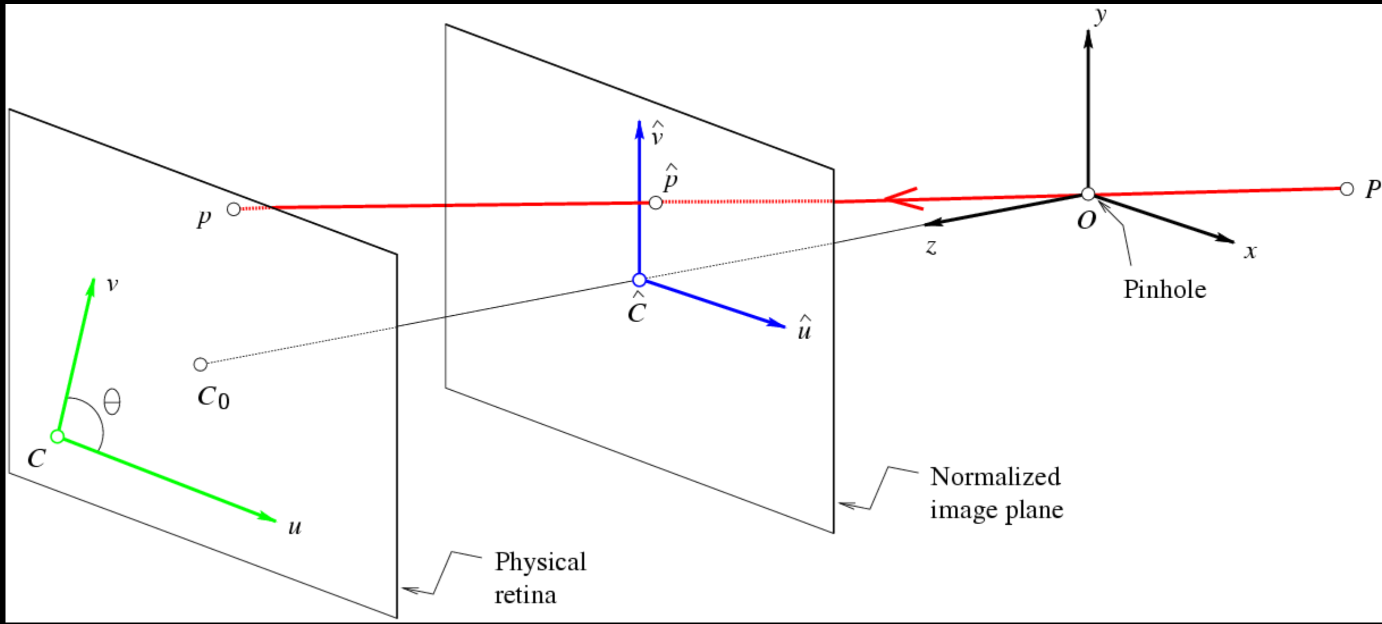
- Thus,

$$\boxed{\boldsymbol{p} = \frac{1}{z} \mathcal{M} \boldsymbol{P},} \quad \text{where} \quad \begin{cases} \mathcal{M} = \mathcal{K} (\mathcal{R} \quad \boldsymbol{t}), \\ \mathcal{R} = {}^C_W \mathcal{R}, \\ \boldsymbol{t} = {}^C O_W, \\ \boldsymbol{P} = \begin{pmatrix} {}^W P \\ 1 \end{pmatrix}. \end{cases}$$

- Note: z is *not* independent of $\mathcal{M}$ and $\boldsymbol{P}$:

$$\mathcal{M} = \begin{pmatrix} \boldsymbol{m}_1^T \\ \boldsymbol{m}_2^T \\ \boldsymbol{m}_3^T \end{pmatrix} \Longrightarrow z = \boldsymbol{m}_3 \cdot \boldsymbol{P}, \quad \text{or} \quad \begin{cases} u = \dfrac{\boldsymbol{m}_1 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}}, \\ v = \dfrac{\boldsymbol{m}_2 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}}. \end{cases}$$

The Intrinsic Parameters of a Camera



The calibration Matrix

$$\boldsymbol{p} = \mathcal{K}\hat{\boldsymbol{p}}, \quad \text{where} \quad \boldsymbol{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \quad \text{and} \quad \mathcal{K} \stackrel{\text{def}}{=} \begin{pmatrix} \alpha & -\alpha \cot\theta & u_0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

The Perspective Projection Equation

$$\boldsymbol{p} = \frac{1}{z}\mathcal{M}\boldsymbol{P}, \quad \text{where} \quad \mathcal{M} \stackrel{\text{def}}{=} (\mathcal{K} \quad \mathbf{0})$$