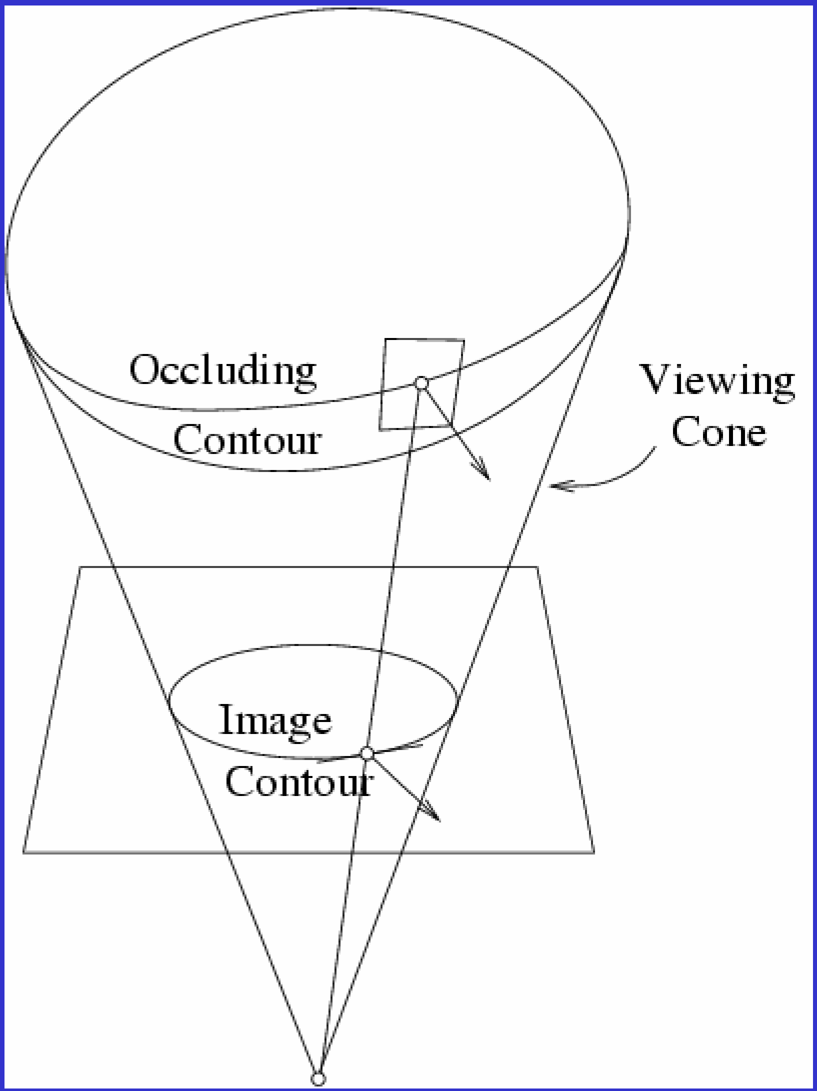
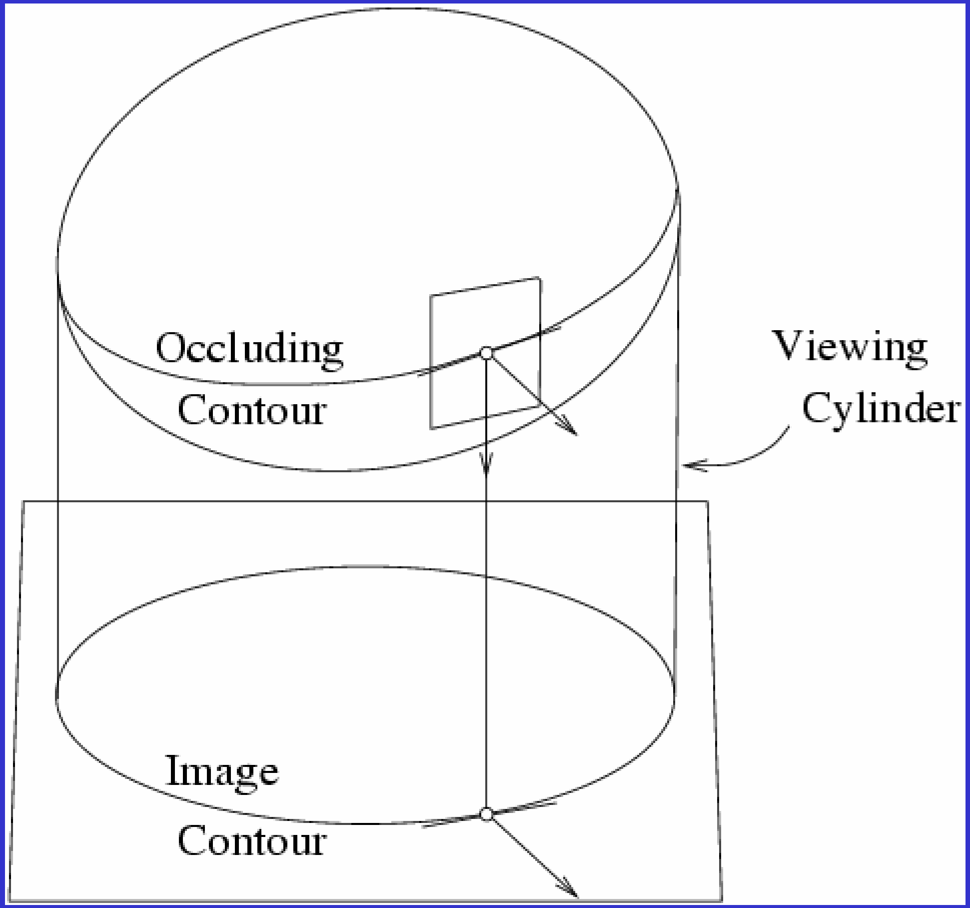
SMOOTH SURFACES AND THEIR OUTLINES II

- The second fundamental form
- Koenderink's Theorem
- Aspect graphs
- More differential geometry
- A catalogue of visual events
- Computing the aspect graph

- http://www.di.ens.fr/~ponce/geomvis/lect8.ppt
- http://www.di.ens.fr/~ponce/geomvis/lect8.pdf

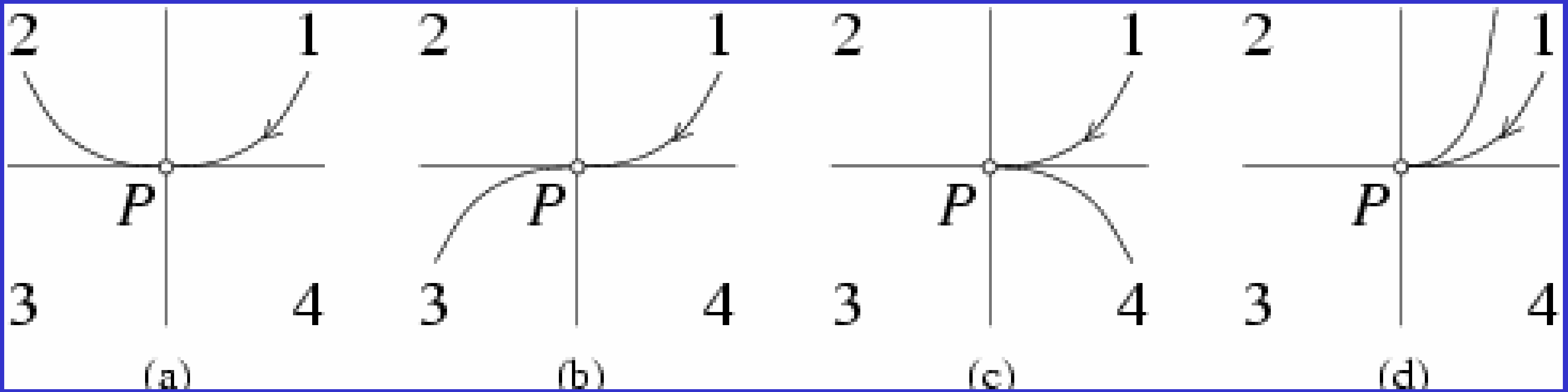
Smooth Shapes and their Outlines





Can we say anything about a 3D shape from the shape of its contour?

What can happen to a curve in the vicinity of a point?



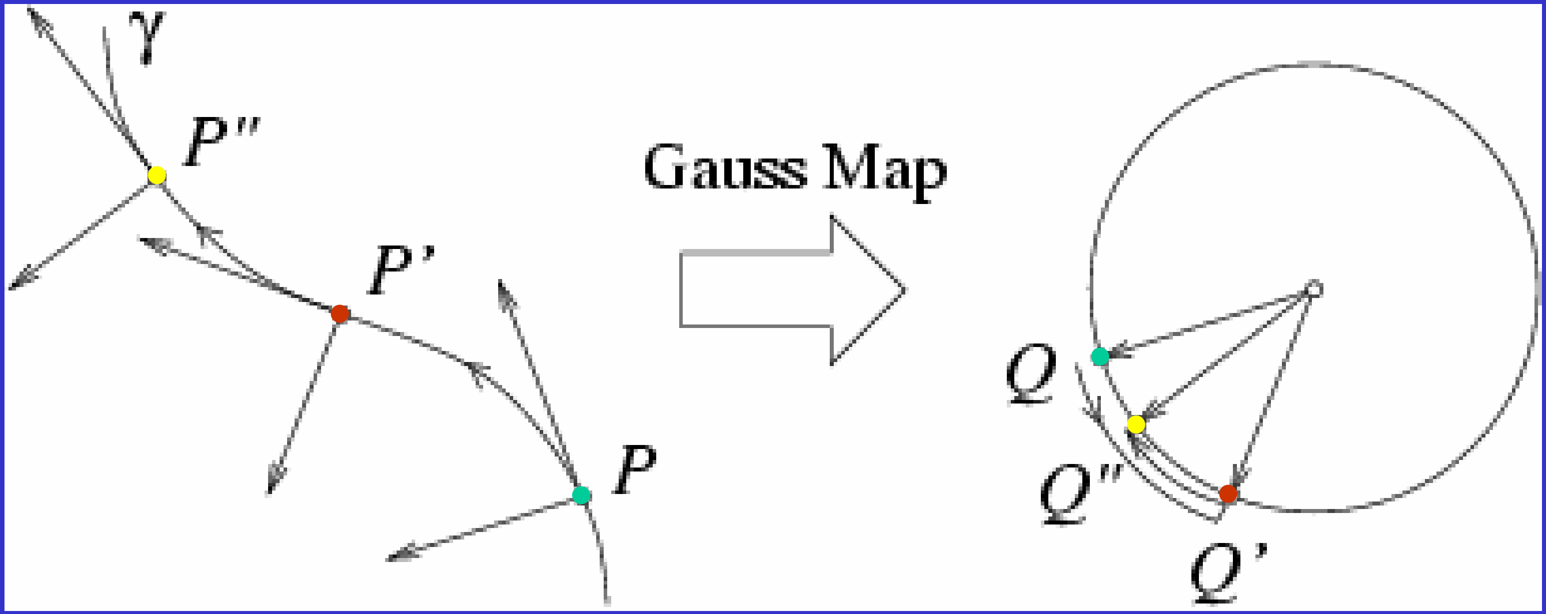
(a) Regular point;

(b) inflection;

(c) cusp of the first kind;

(d) cusp of the second kind.

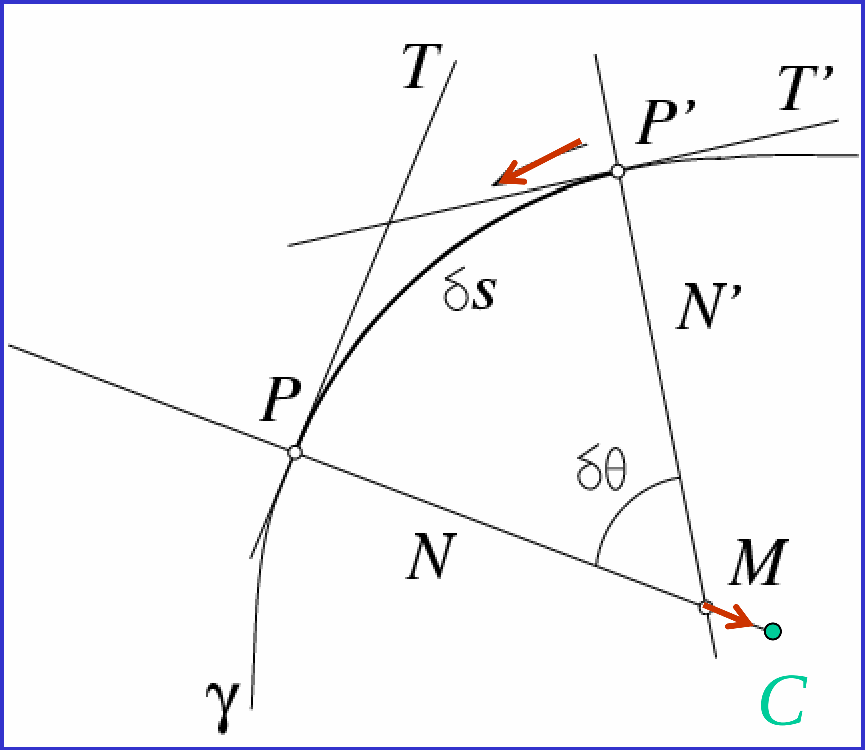
The Gauss Map



- It maps points on a curve onto points on the unit circle.

- The direction of traversal of the Gaussian image reverts at inflections: it folds there.

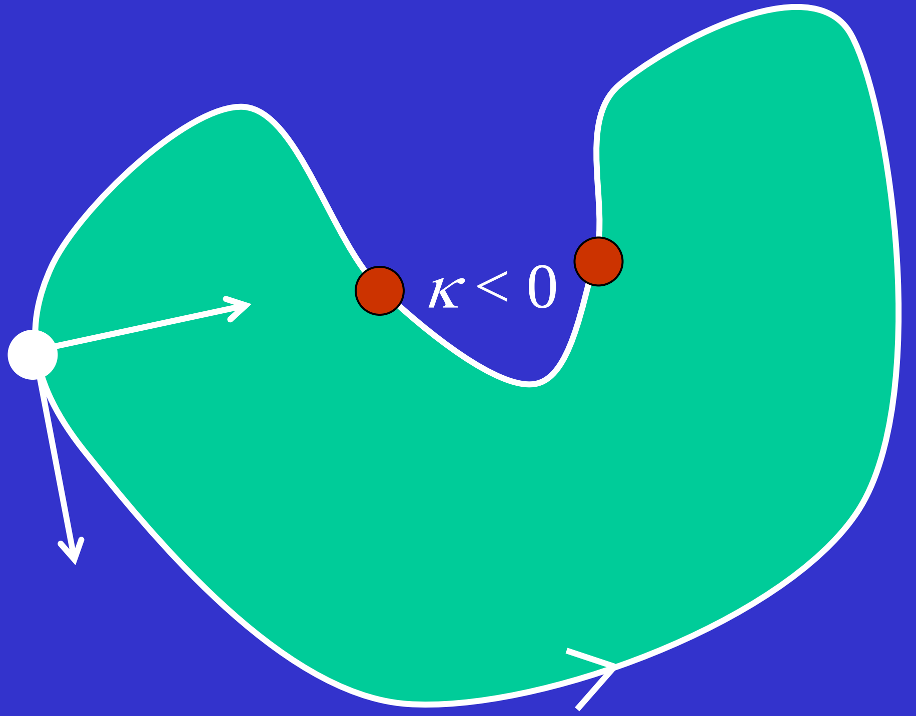
The curvature



- C is the center of curvature;
- $R = CP$ is the radius of curvature;
- $\kappa = \lim \delta\theta/\delta s = 1/R$ is the curvature.

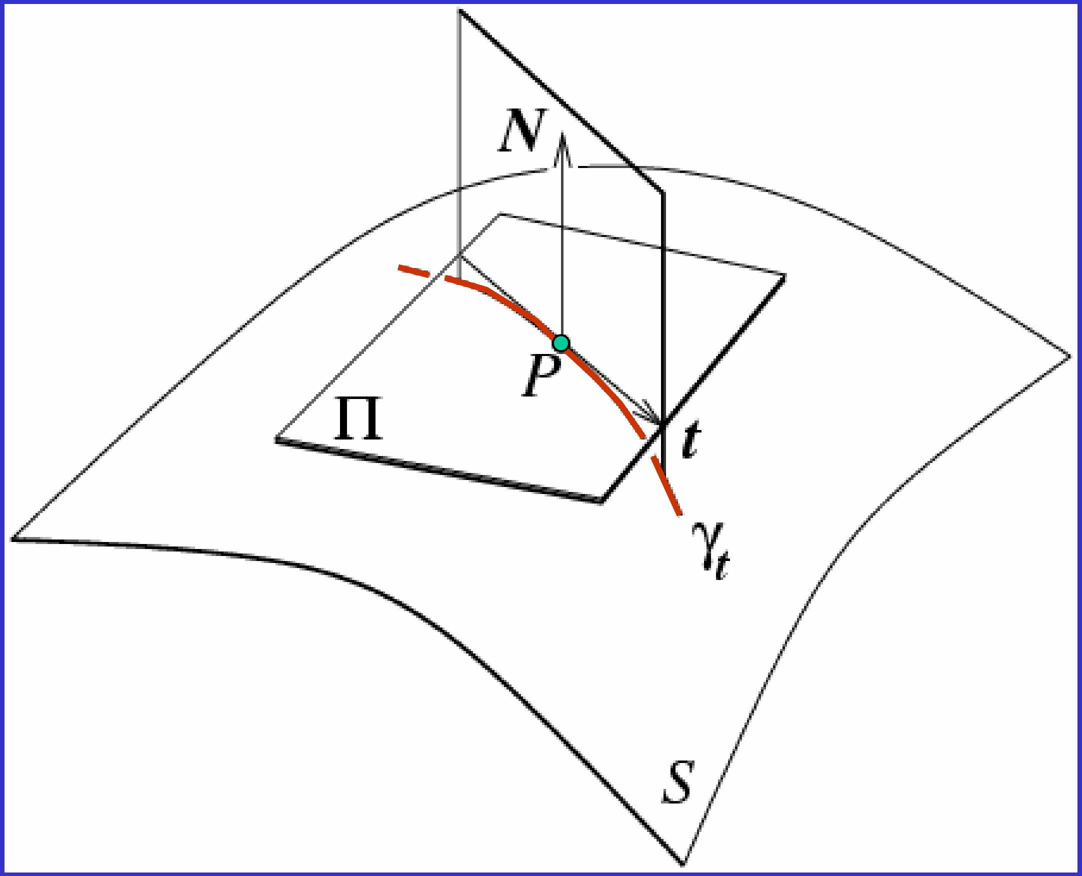
$$d\mathbf{t}/ds = \kappa\,\mathbf{n}$$

Closed curves admit a canonical orientation..



$\kappa = d\theta / ds$ $\leftarrow$ derivative of the Gauss map!

Normal sections and normal curvatures



Principal curvatures:

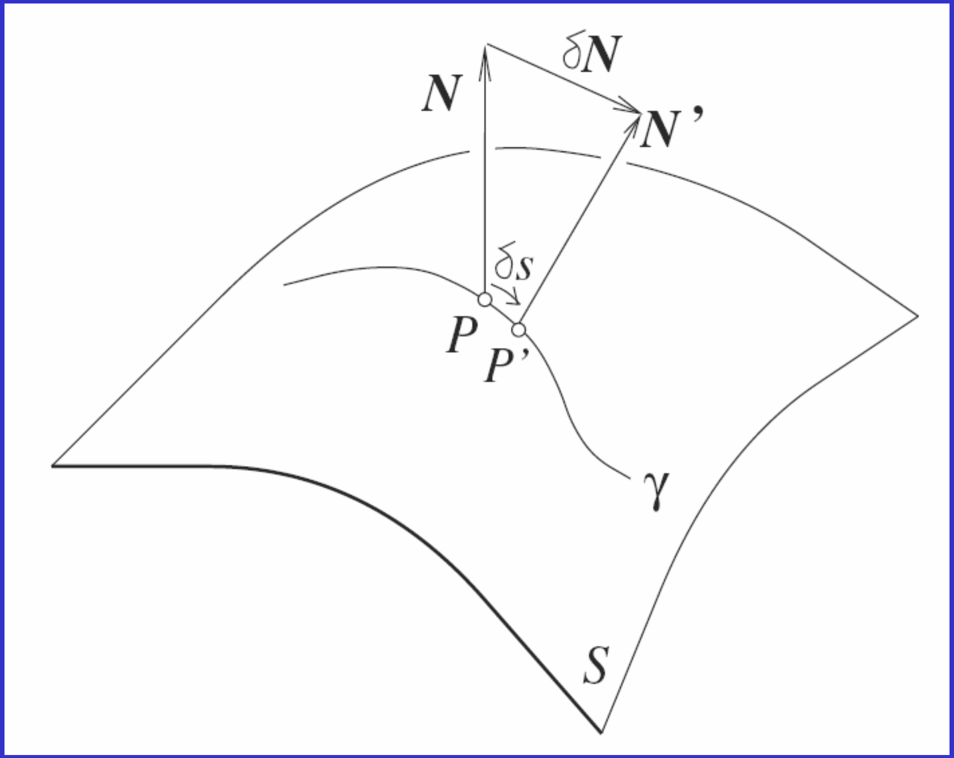
minimum value κ_1

maximum value κ_2

Gaussian curvature:

$$K = \kappa_1 \kappa_2$$

The differential of the Gauss map



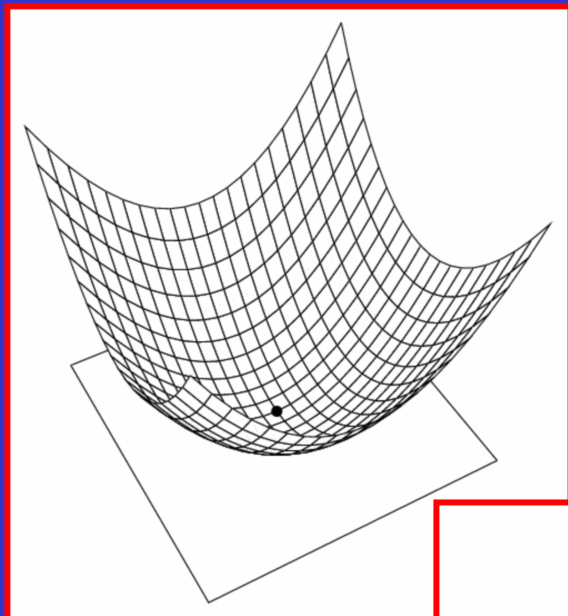
$$dN(\mathbf{t}) = \lim_{\delta s \to 0} \frac{1}{\delta s} \delta \mathbf{N}$$

Second fundamental form:
$\mathrm{II}(\mathbf{u}, \mathbf{v}) = \mathbf{u}^T dN(\mathbf{v})$

(II is symmetric.)

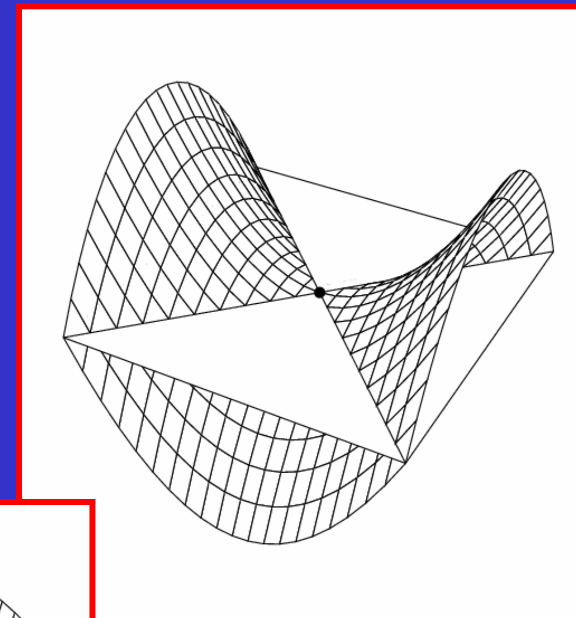
- The normal curvature is $\kappa_t = \mathrm{II}(\mathbf{t}, \mathbf{t})$.
- Two directions are said to be conjugated when $\mathrm{II}(\mathbf{u}, \mathbf{v}) = 0$.

The local shape of a smooth surface



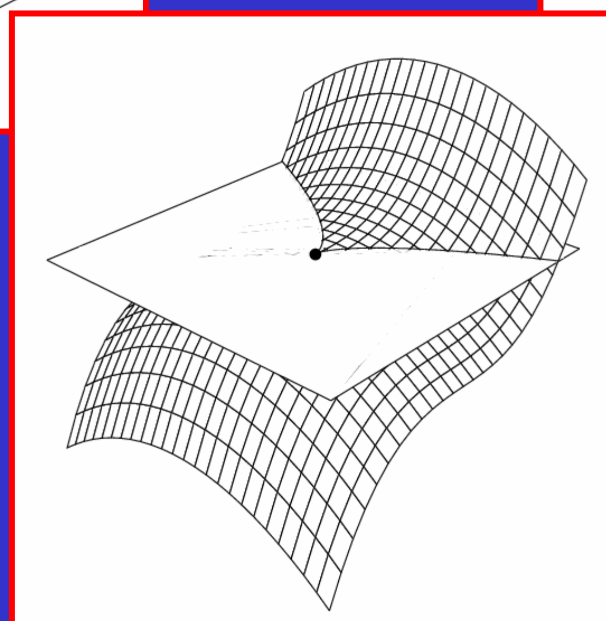
Elliptic point

$K > 0$



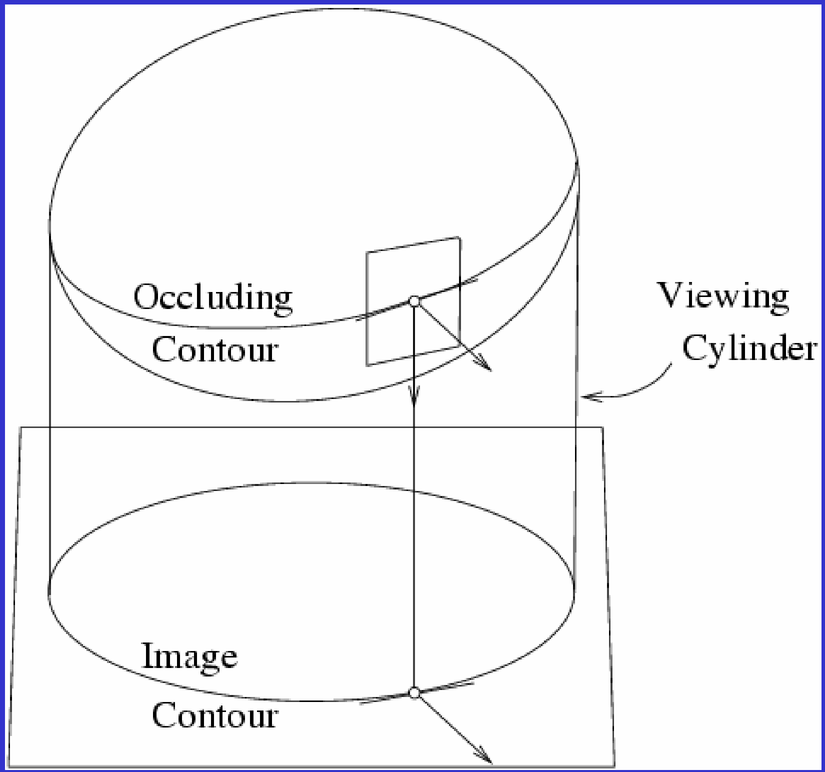
Hyperbolic point

$K < 0$



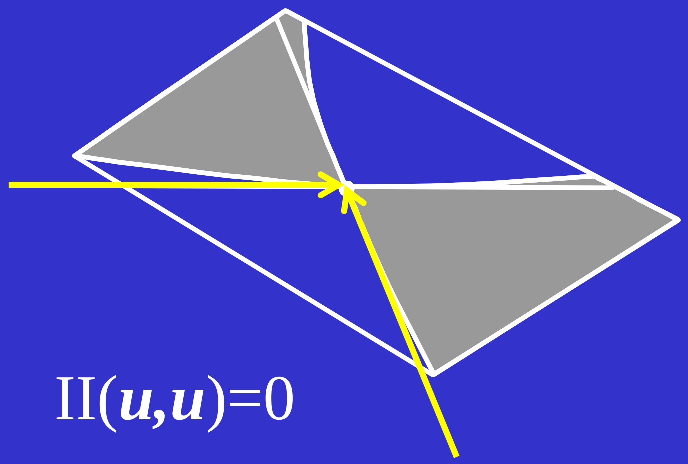
Parabolic point $K = 0$

The parabolic lines marked on the Apollo Belvedere by Felix Klein



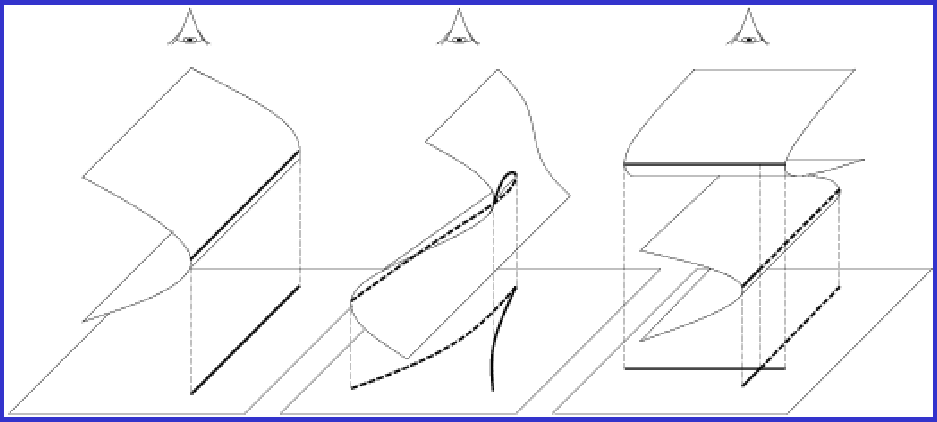
$$N \,.\, v = 0 \Rightarrow \mathrm{II}(\, t \,,\, v \,)=0$$

Asymptotic directions:

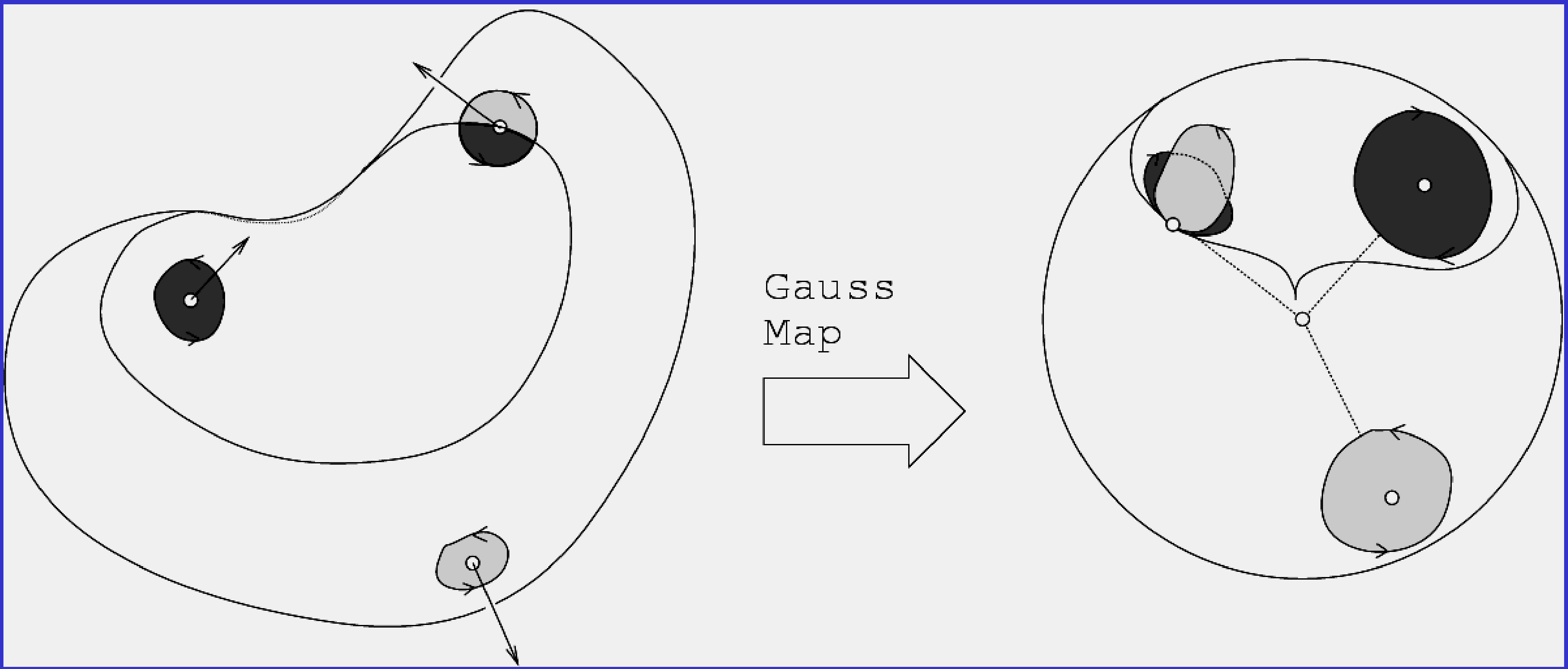


$\mathrm{II}(\boldsymbol{u},\boldsymbol{u})=0$

The contour cusps when when a viewing ray grazes the surface along an asymptotic direction.



The Gauss map

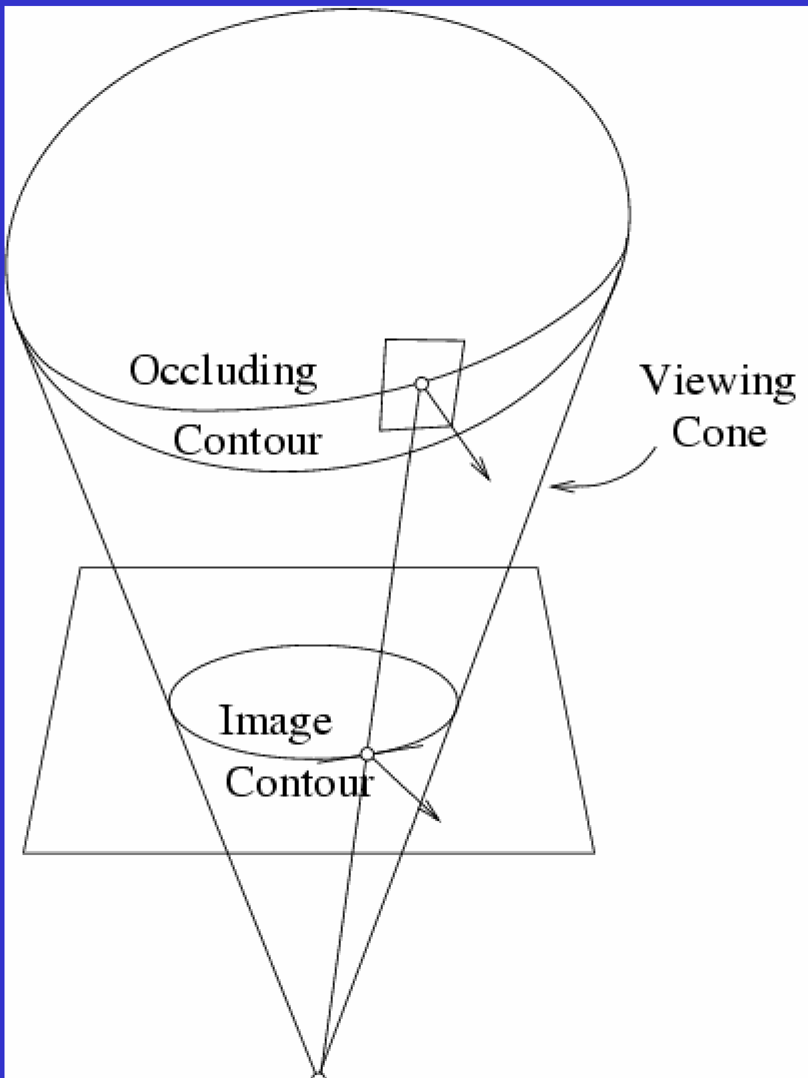


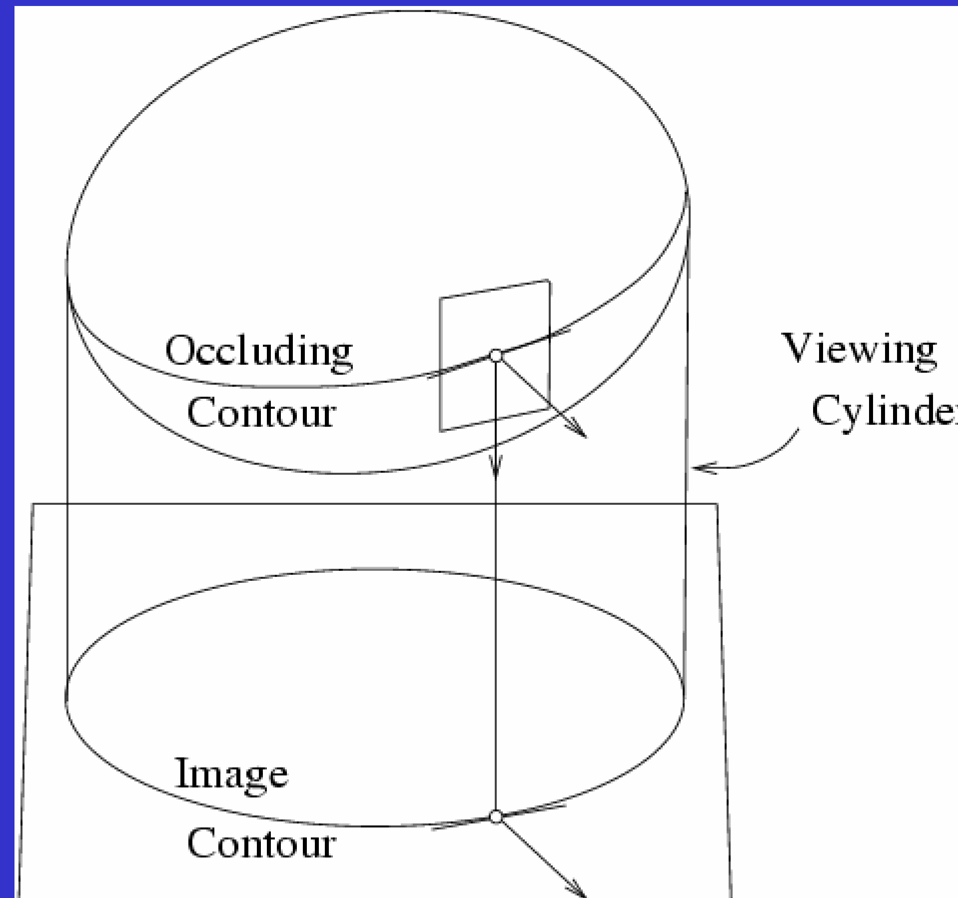
The Gauss map folds at parabolic points.

$$K = dA'/dA$$

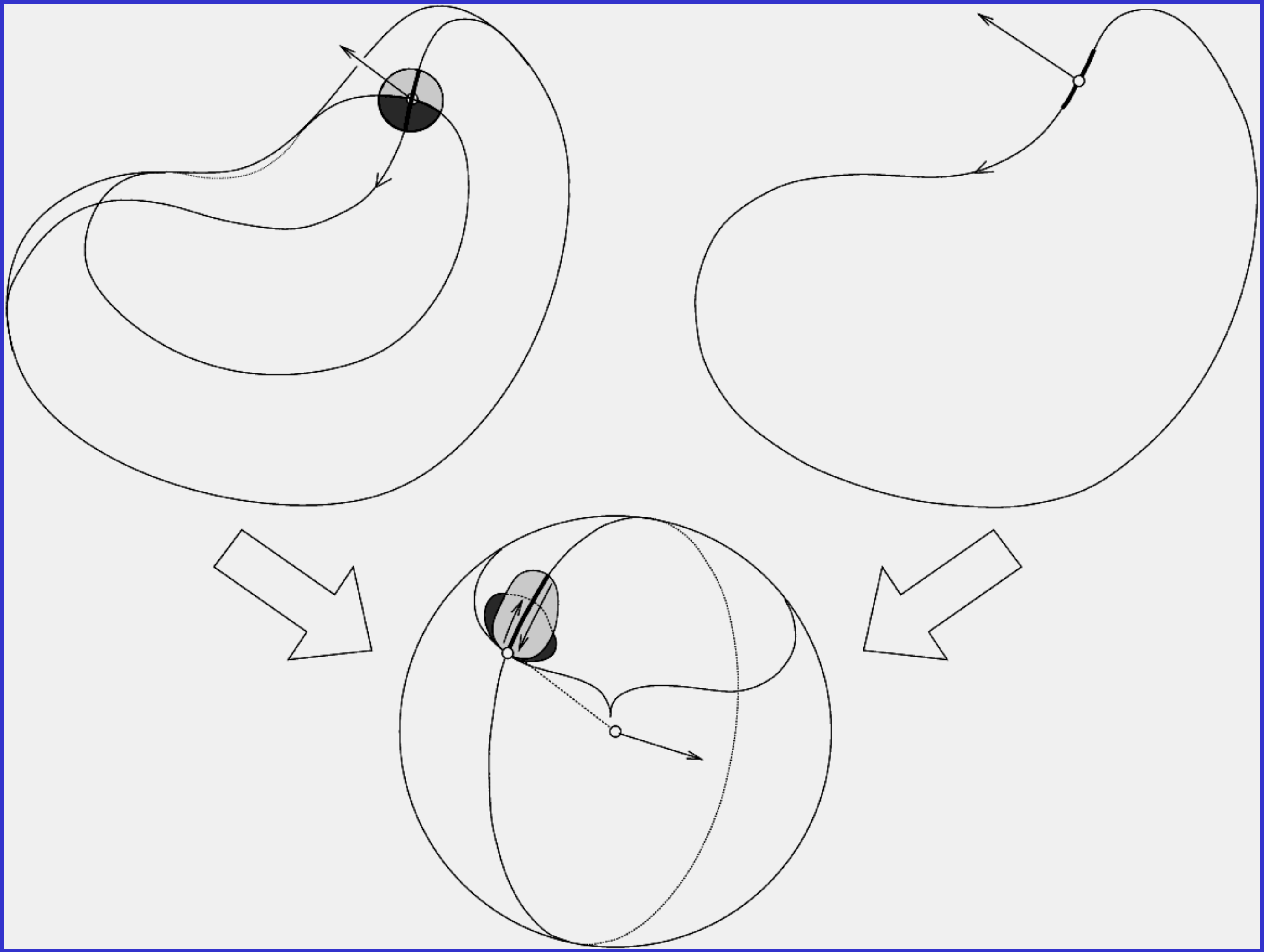
Reprinted from "On Computing Structural Changes in Evolving Surfaces and their Appearance," By S. Pae and J. Ponce, the International Journal of Computer Vision, 43(2):113-131 (2001).

Smooth Shapes and their Outlines





Can we say anything about a 3D shape from the shape of its contour?



Theorem [Koenderink, 1984]: the inflections of the silhouette are the projections of parabolic points.

Koenderink's Theorem (1984)

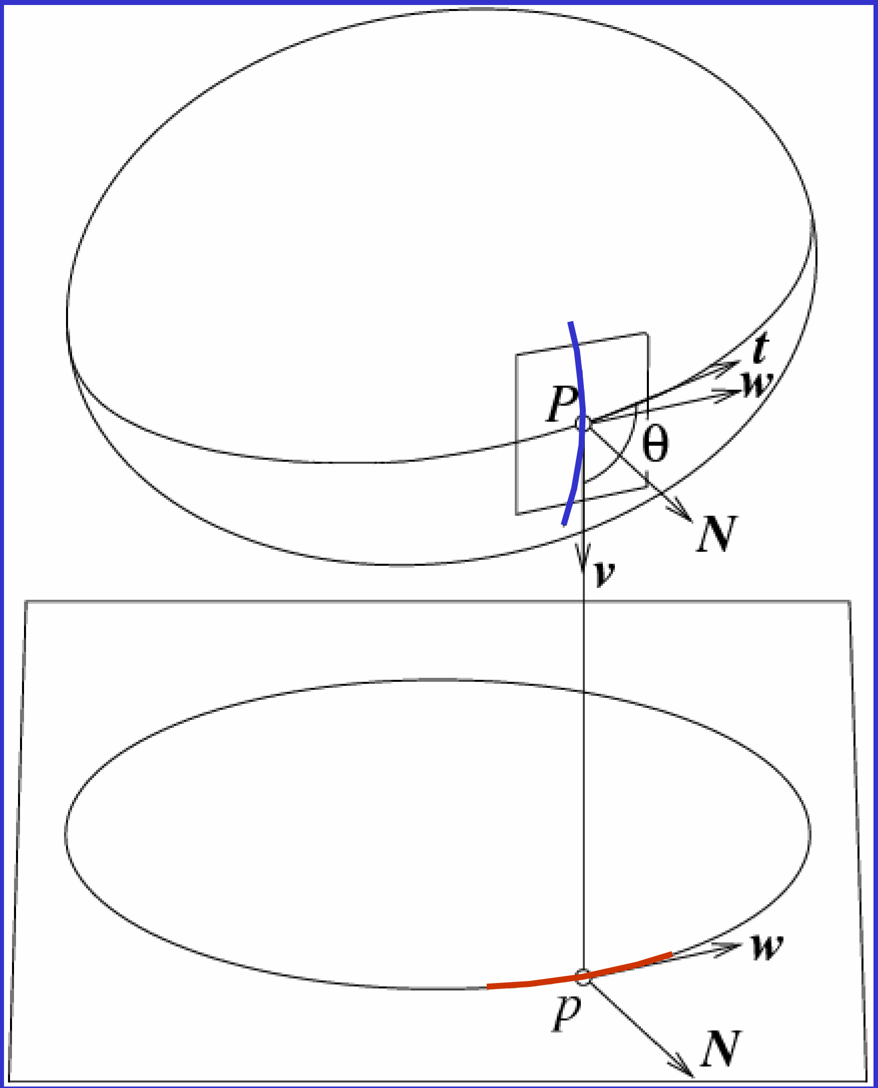
$$K = \kappa_r \kappa_c$$

Note: $\kappa_r > 0$.

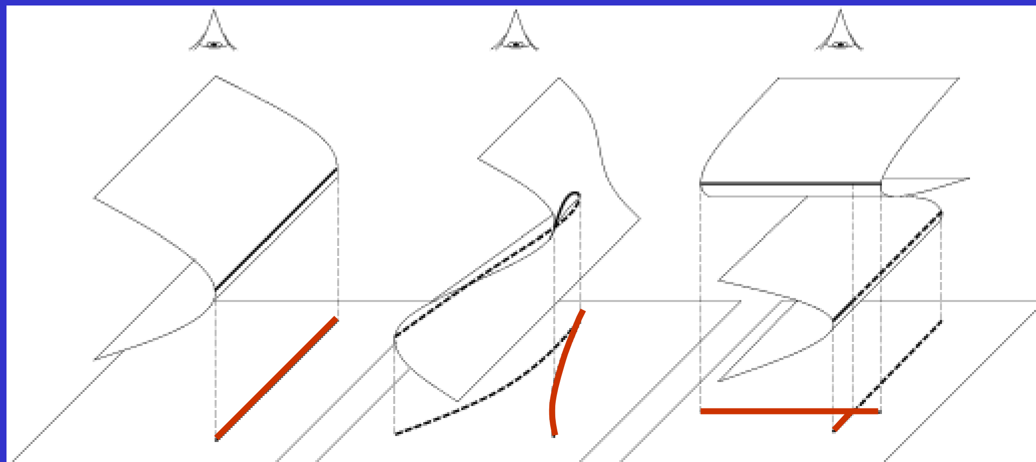
Corollary: K and κ_c have the same sign!

Proof: Based on the idea that, given two conjugated directions,

$$K \sin^2\theta = \kappa_u \kappa_v$$

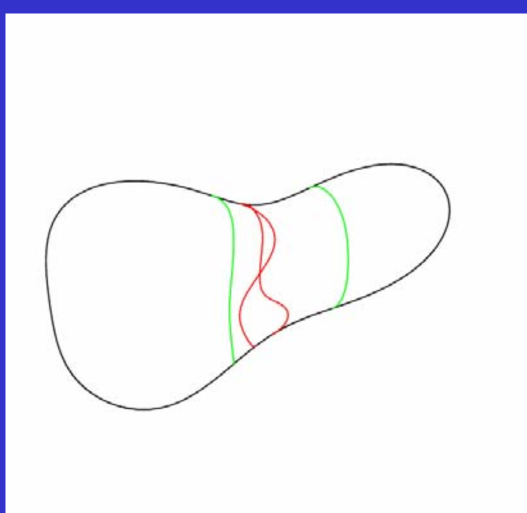


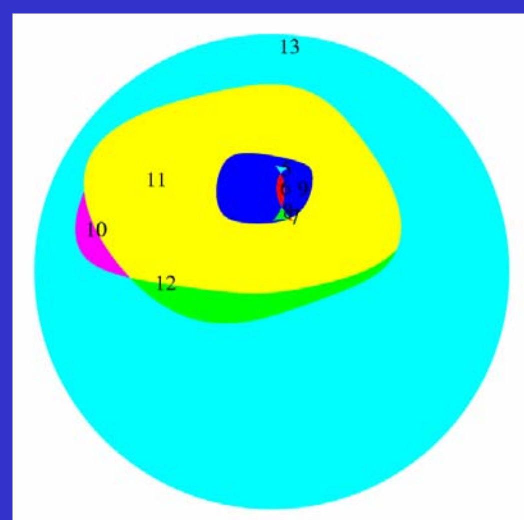
What are the contour stable features??

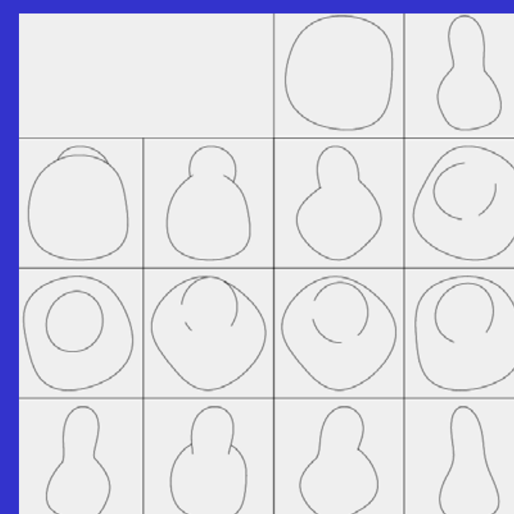


folds cusps T-junctions

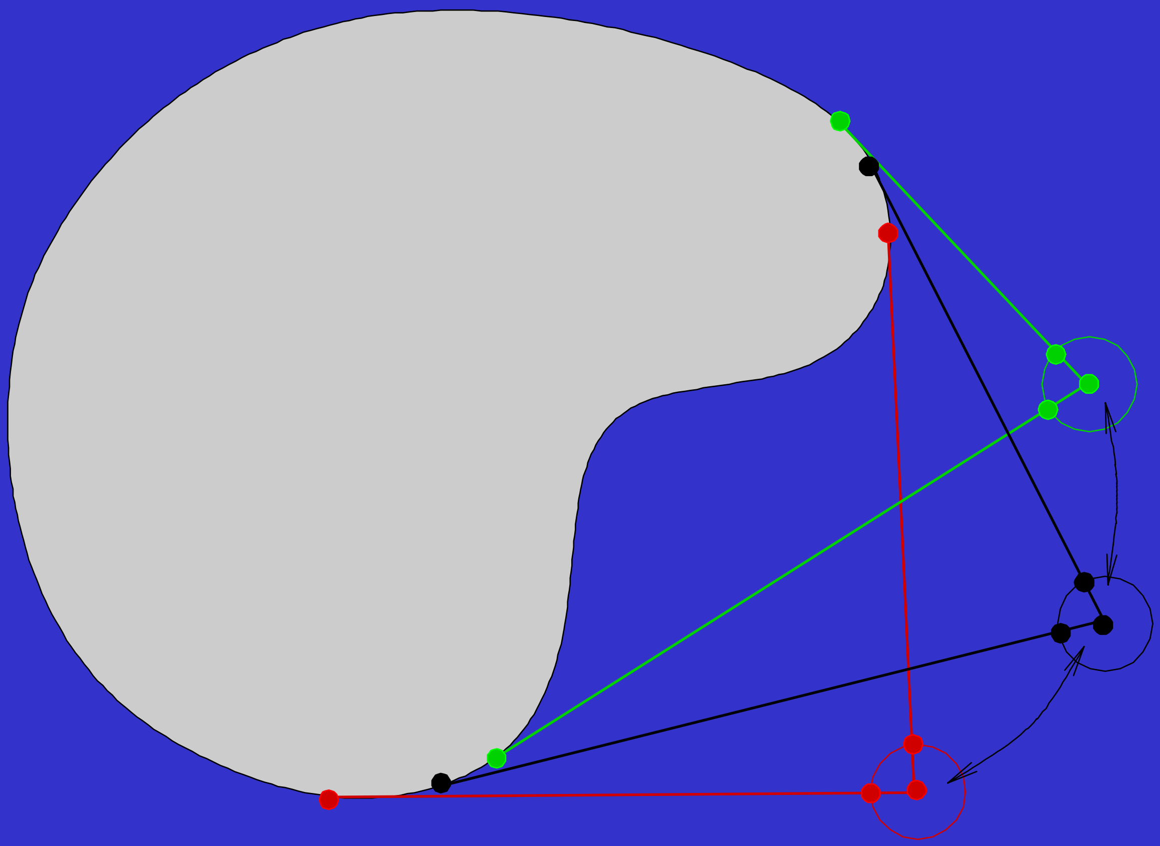
How does the appearance of an object change with viewpoint?



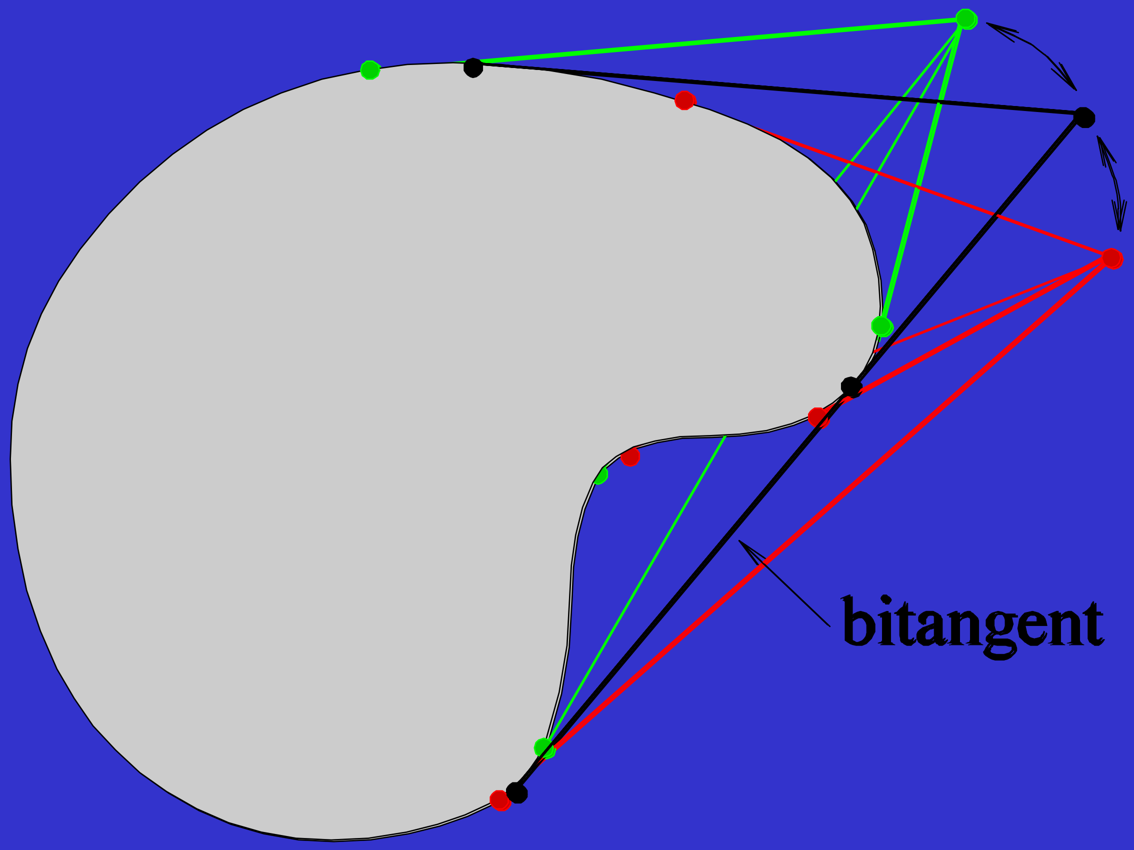




Imaging in Flatland: Stable Views

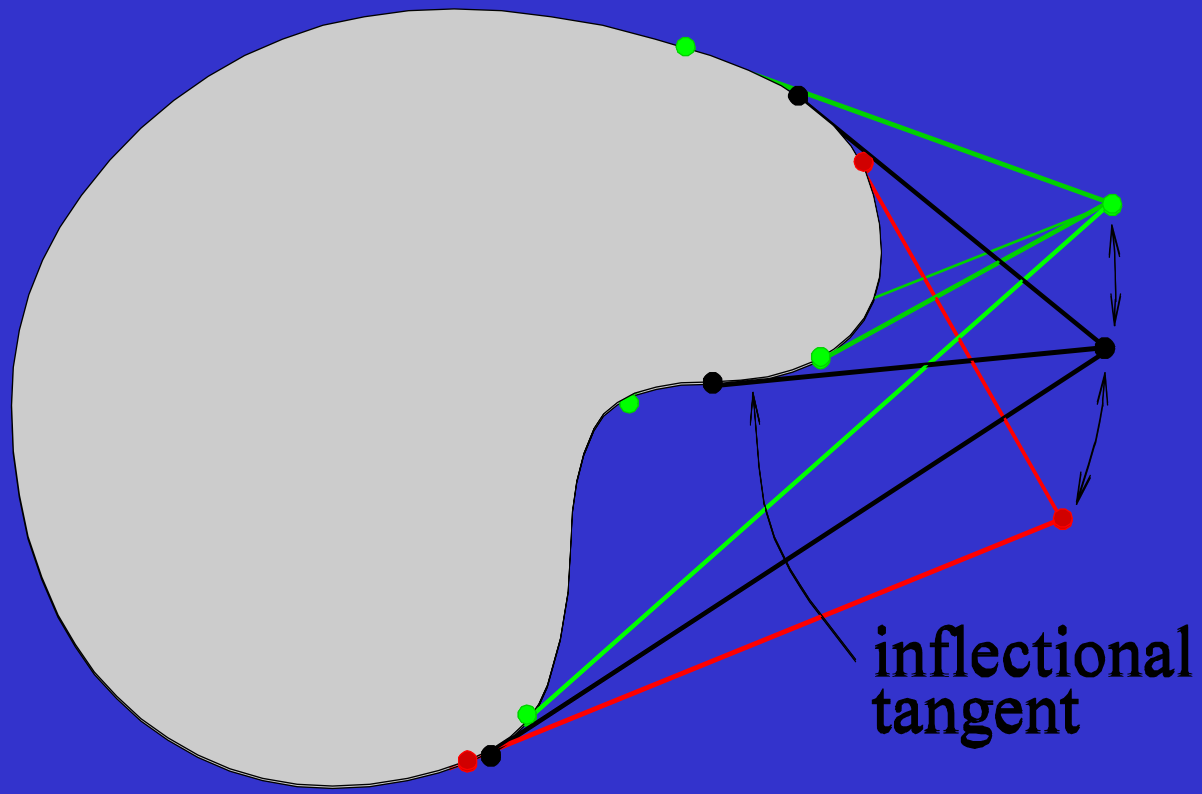


Visual Event: Change in Ordering of Contour Points



Transparent Object
Opaque Object

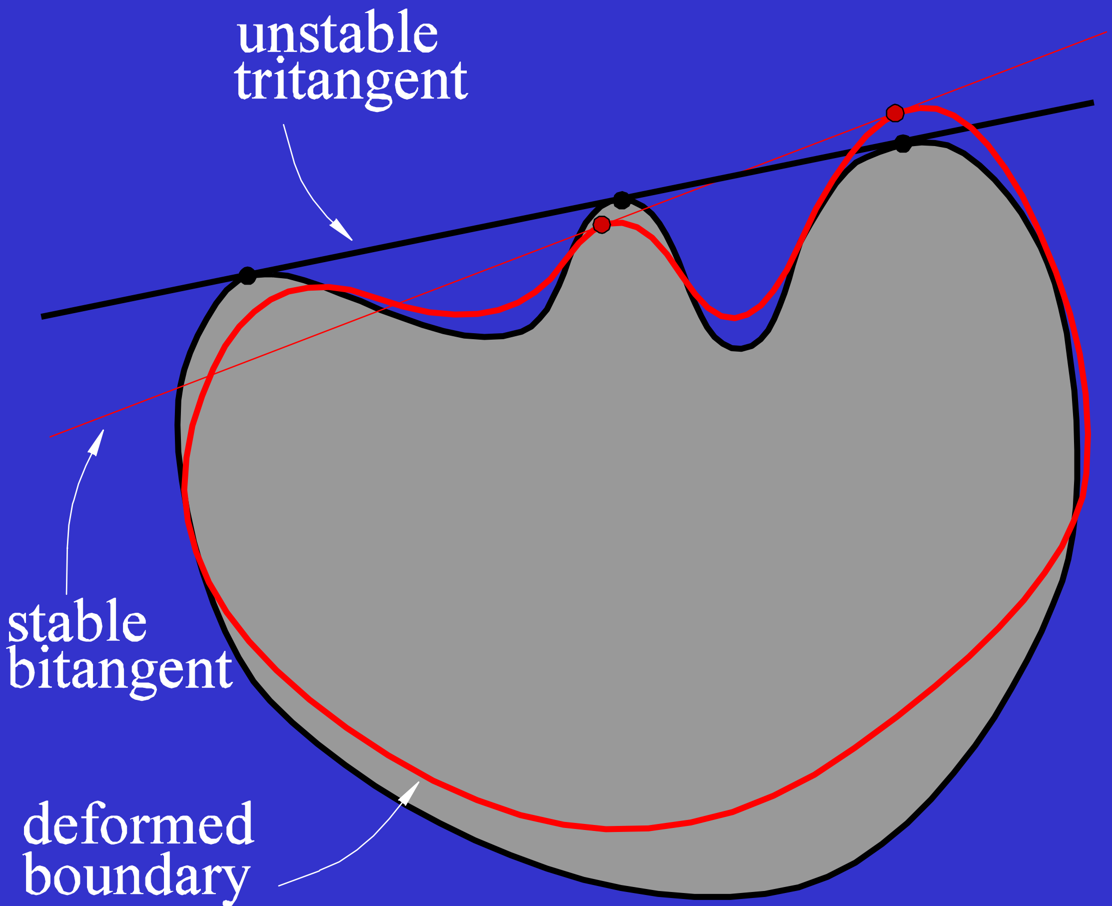
Visual Event: Change in Number of Contour Points



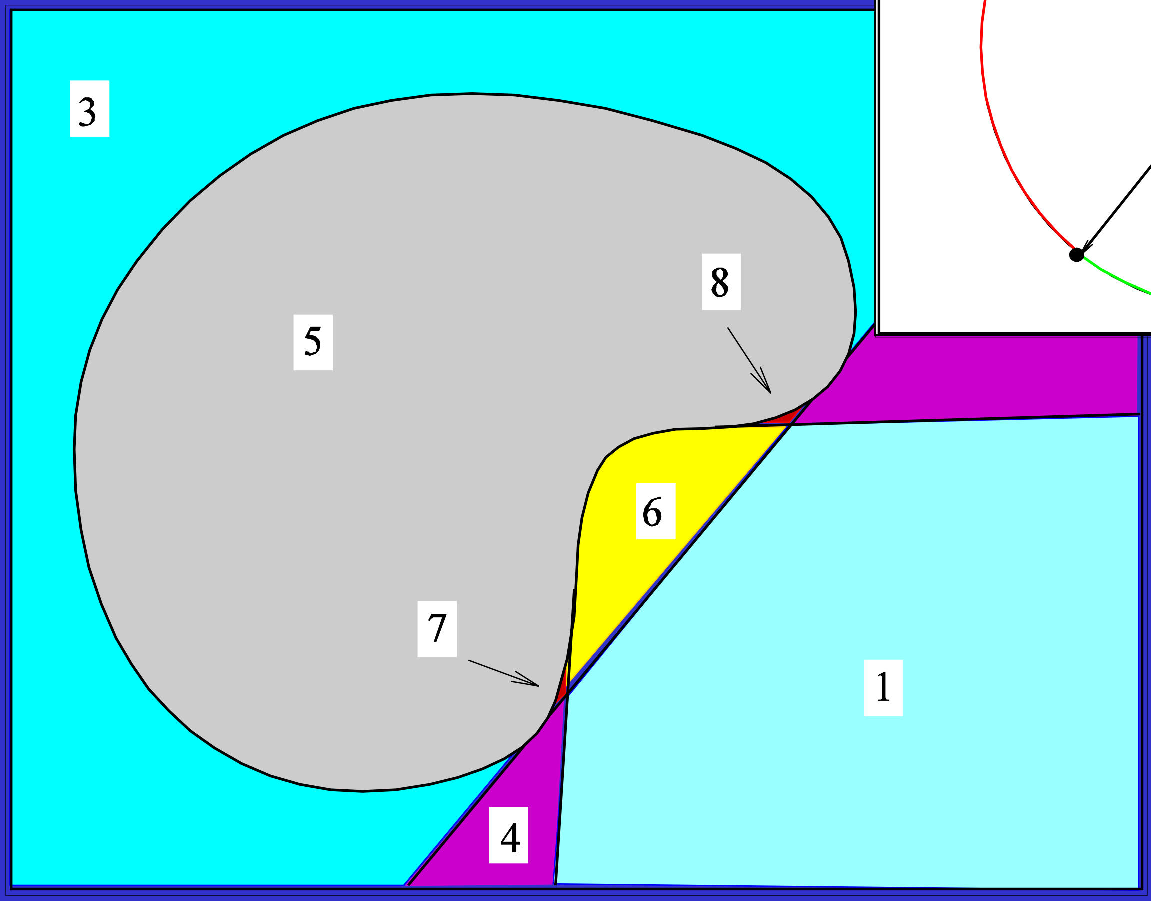
Transparent Object

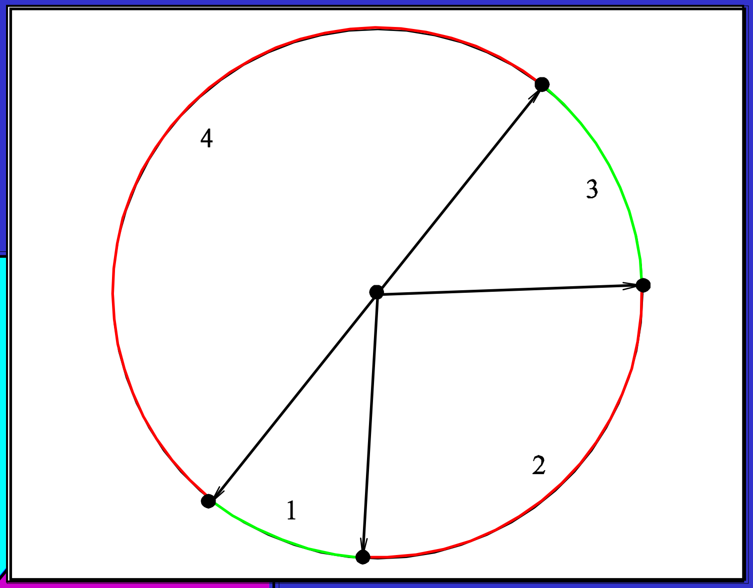
Opaque Object

Exceptional and Generic Curves

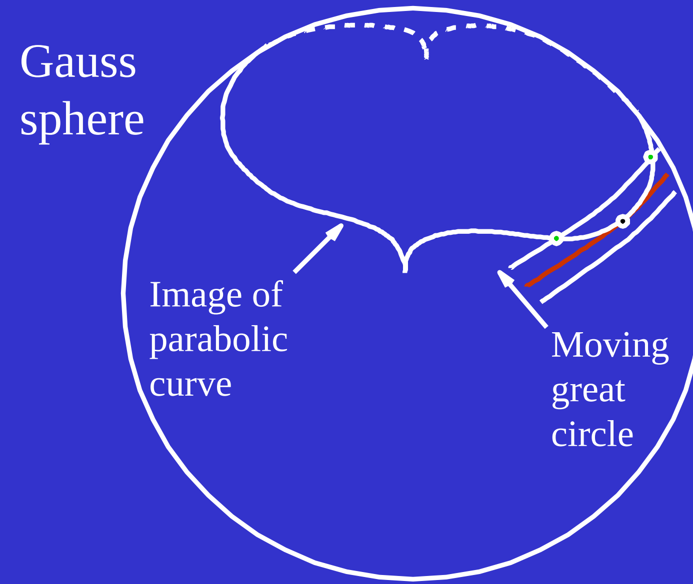


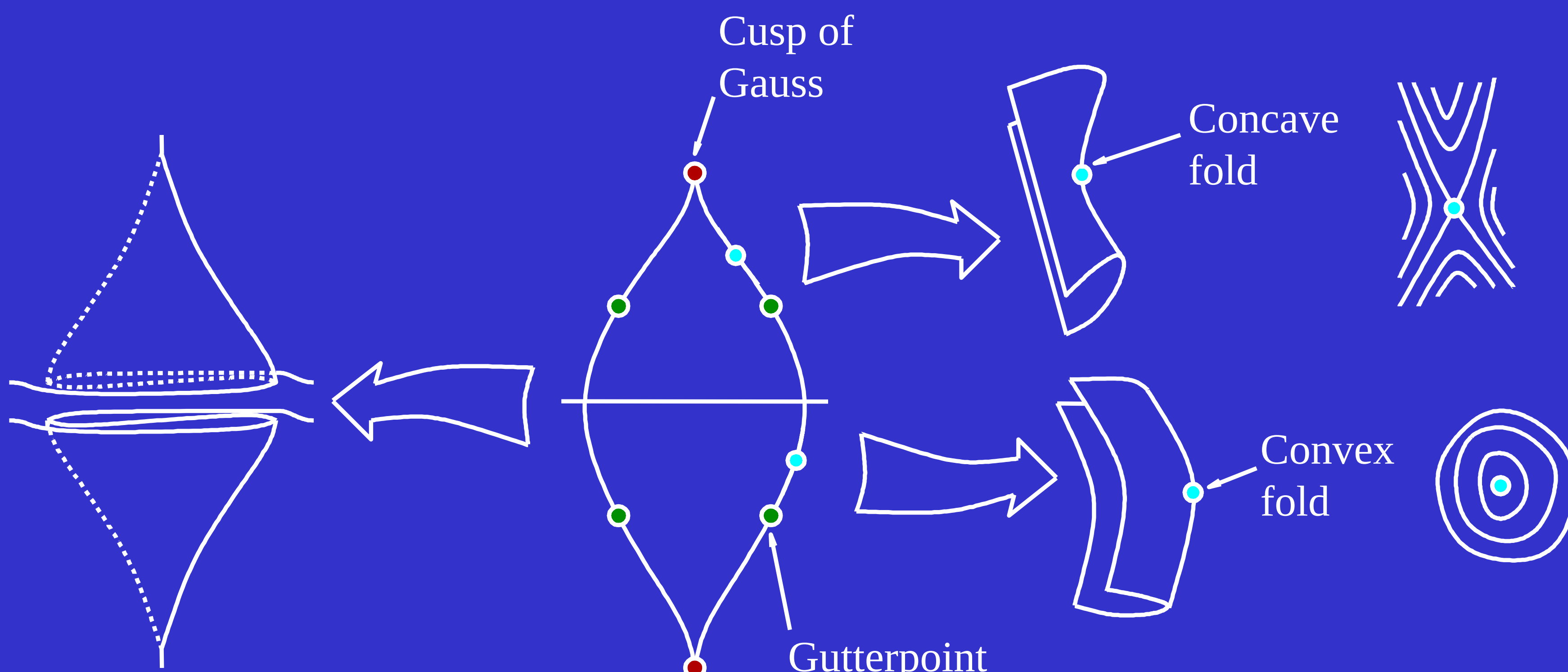
The Aspect Graph
In Flatland



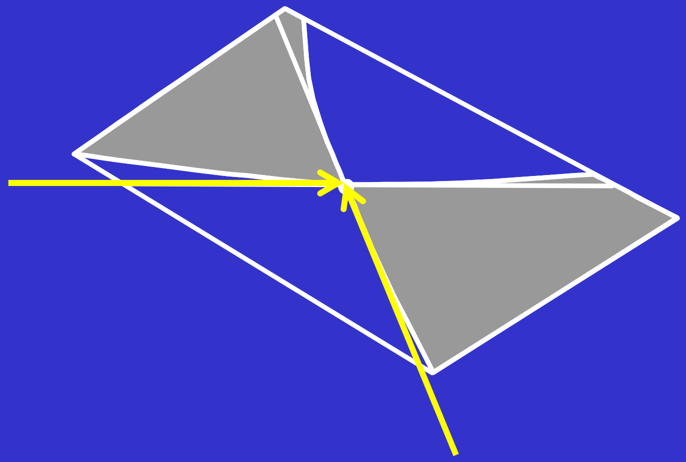


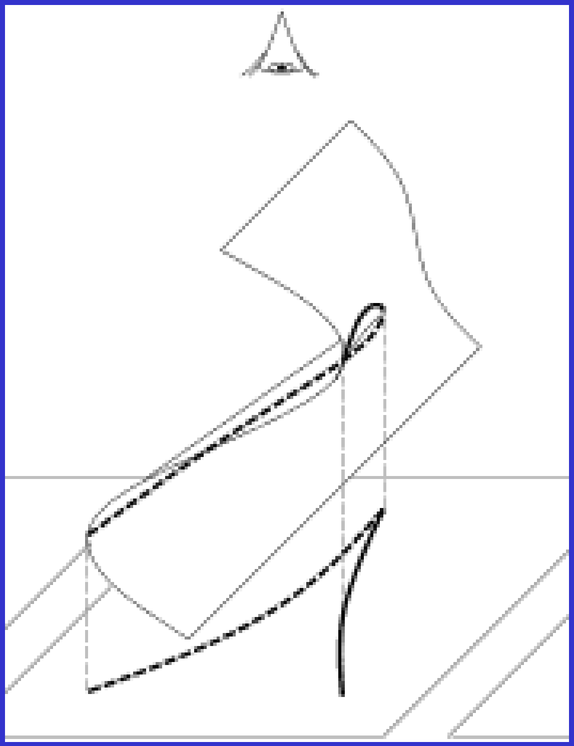
The Geometry of the Gauss Map

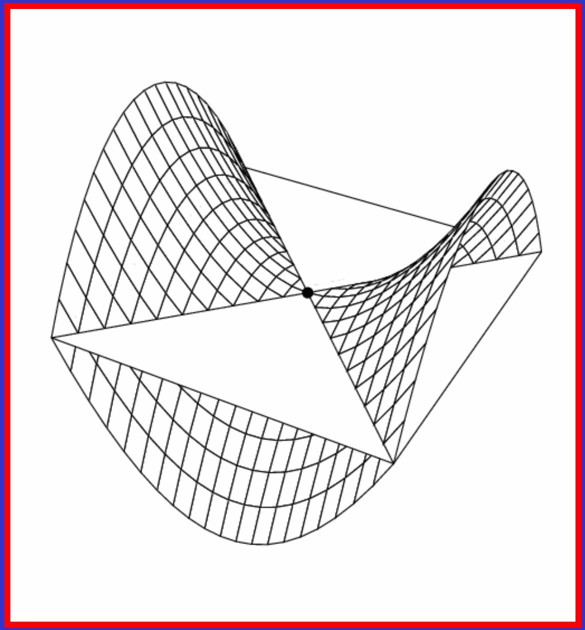




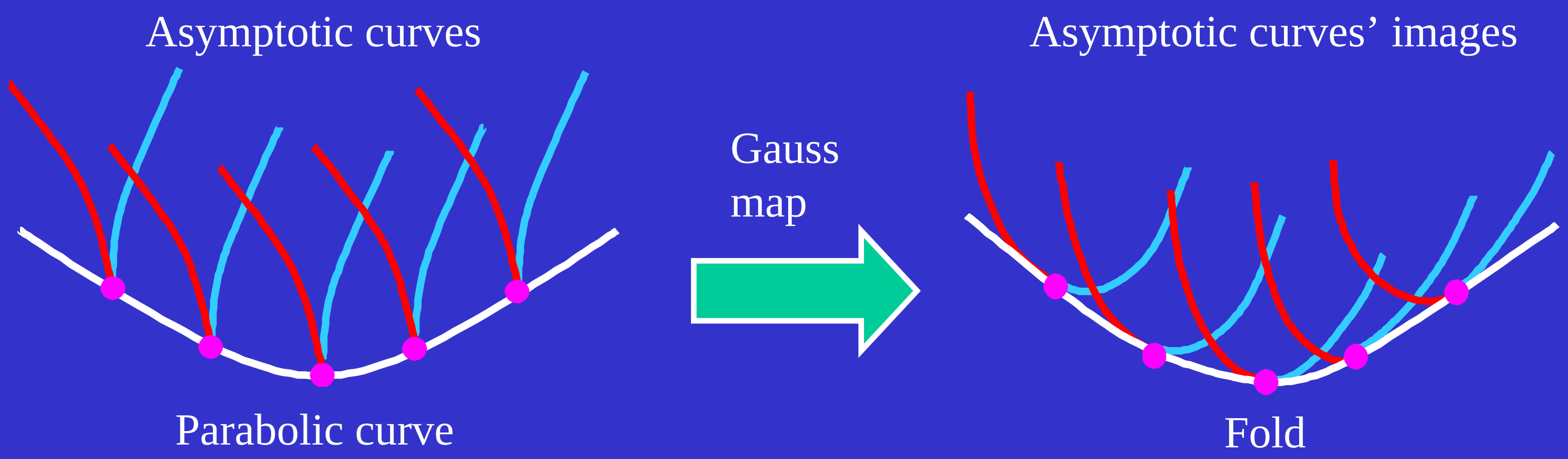
Asymptotic directions at ordinary hyperbolic points







The integral curves of the asymptotic directions form two families of asymptotic curves (red and blue)



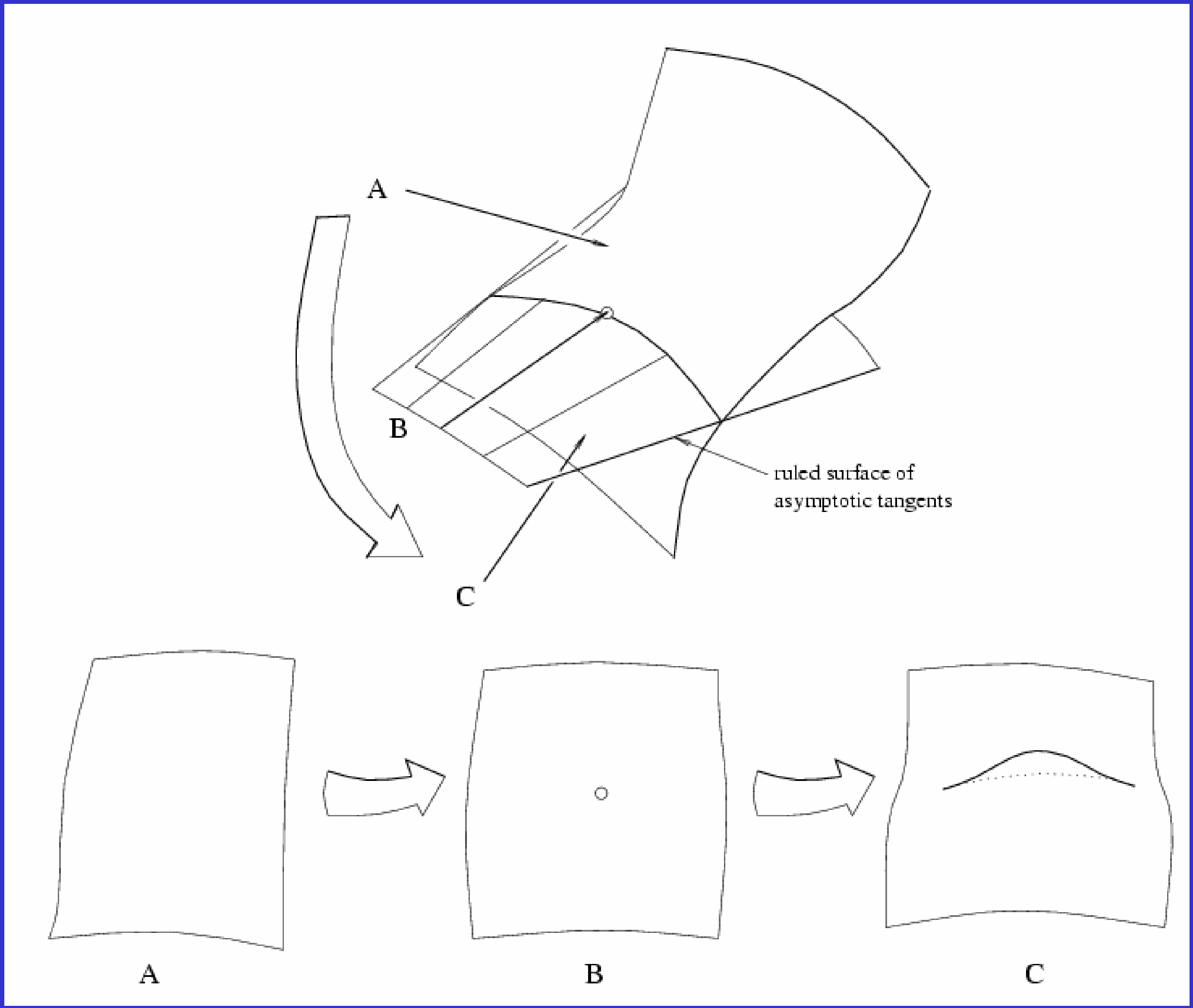
- Asymptotic directions are self conjugate: $\boldsymbol{a} \cdot dN(\boldsymbol{a}) = 0$

- At a parabolic point $dN(\boldsymbol{a}) = 0$, so for any curve
  $\boldsymbol{t} \cdot \mathrm{d}N(\boldsymbol{a}) = \boldsymbol{a} \cdot dN(\boldsymbol{t}) = 0$

- In particular, if t is the tangent to the parabolic curve itself
  $dN(\boldsymbol{a}) \approx dN(\boldsymbol{t})$

The Lip Event

$$\mathbf{v} \,.\, dN\,(\boldsymbol{a}) = 0 \Rightarrow \mathbf{v} \approx \boldsymbol{a}$$



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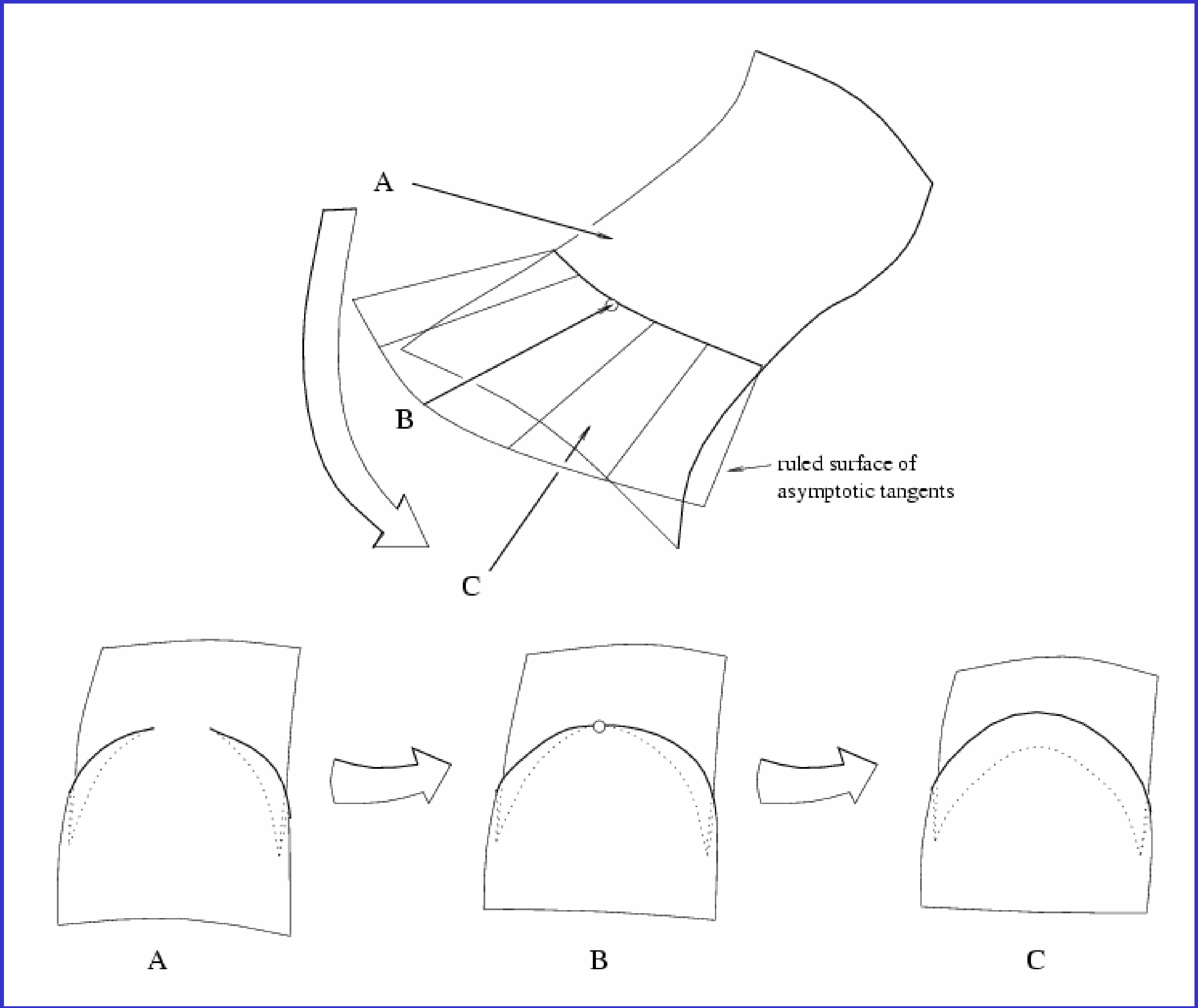
The Beak-to-Beak Event

$$\mathbf{v} \, . \, dN\,(\mathbf{a}) = 0 \Rightarrow \mathbf{v} \approx \mathbf{a}$$



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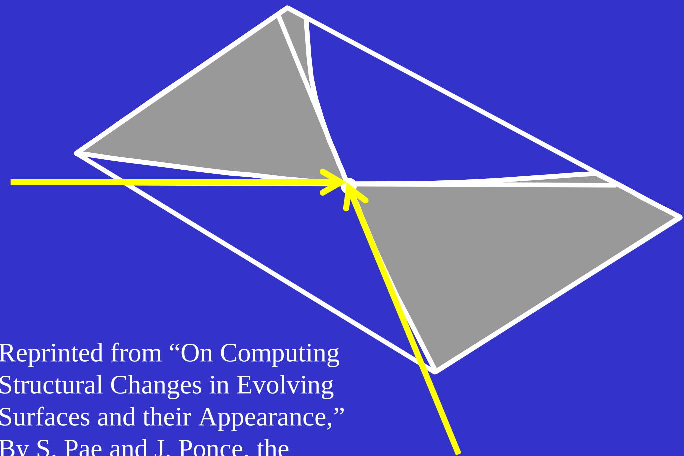
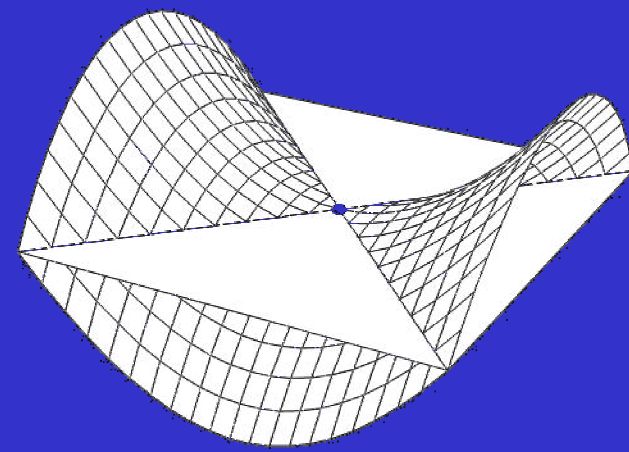
Ordinary Hyperbolic Point





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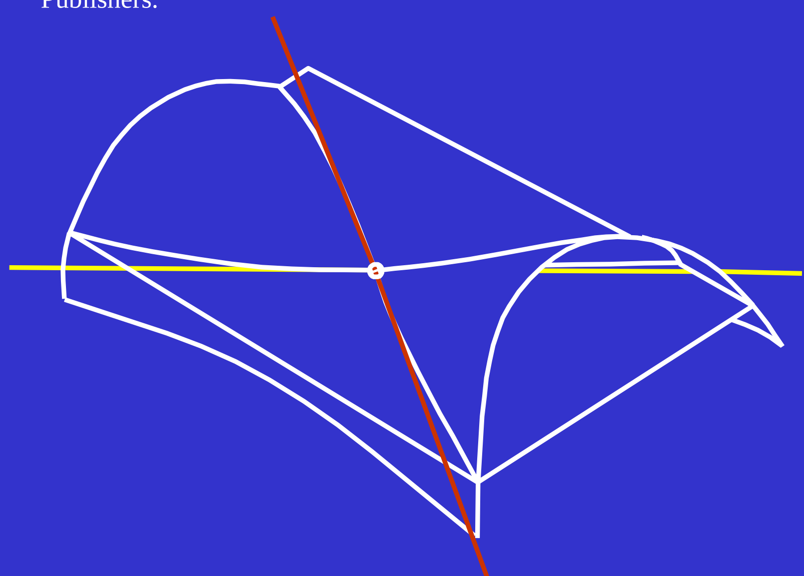
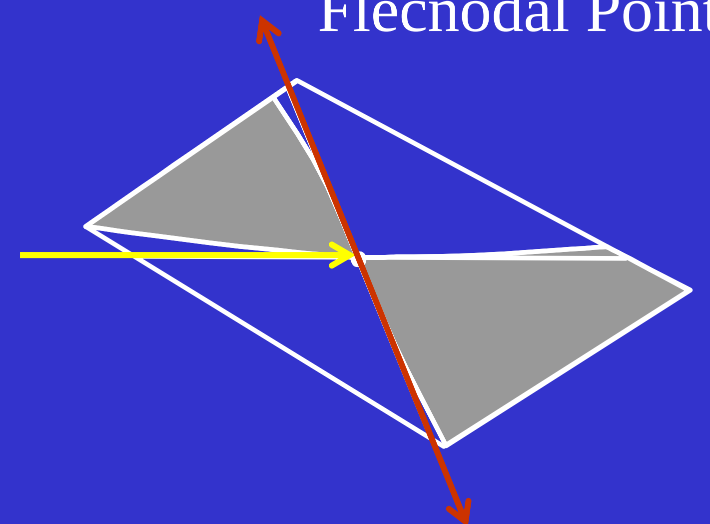
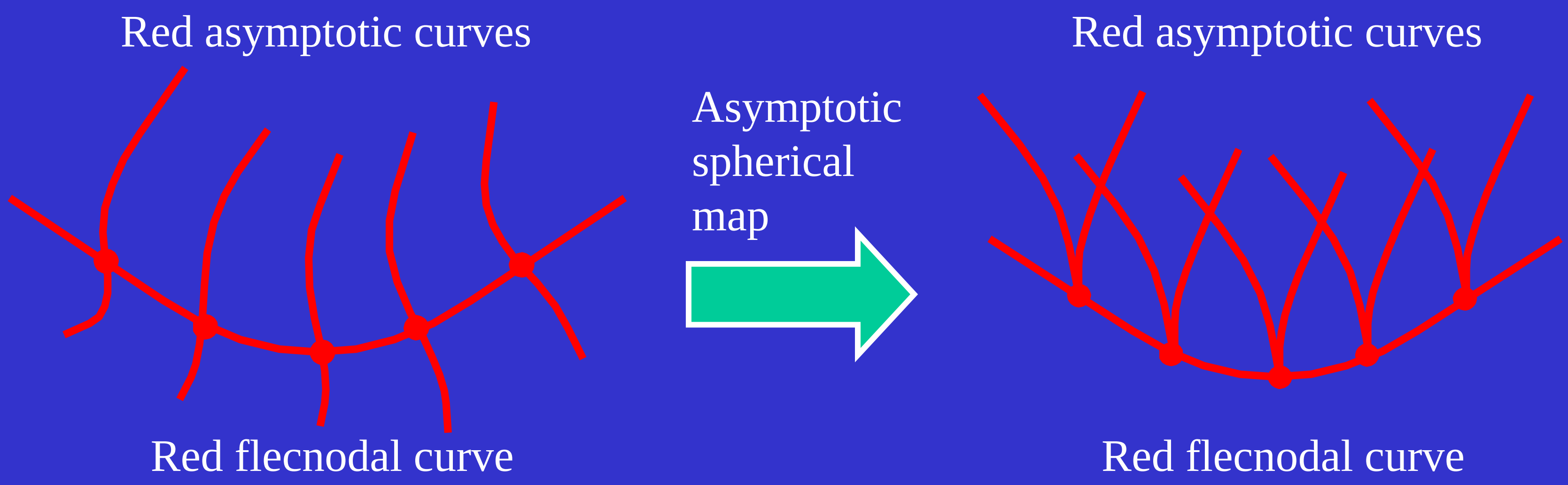
Flecnodal Point

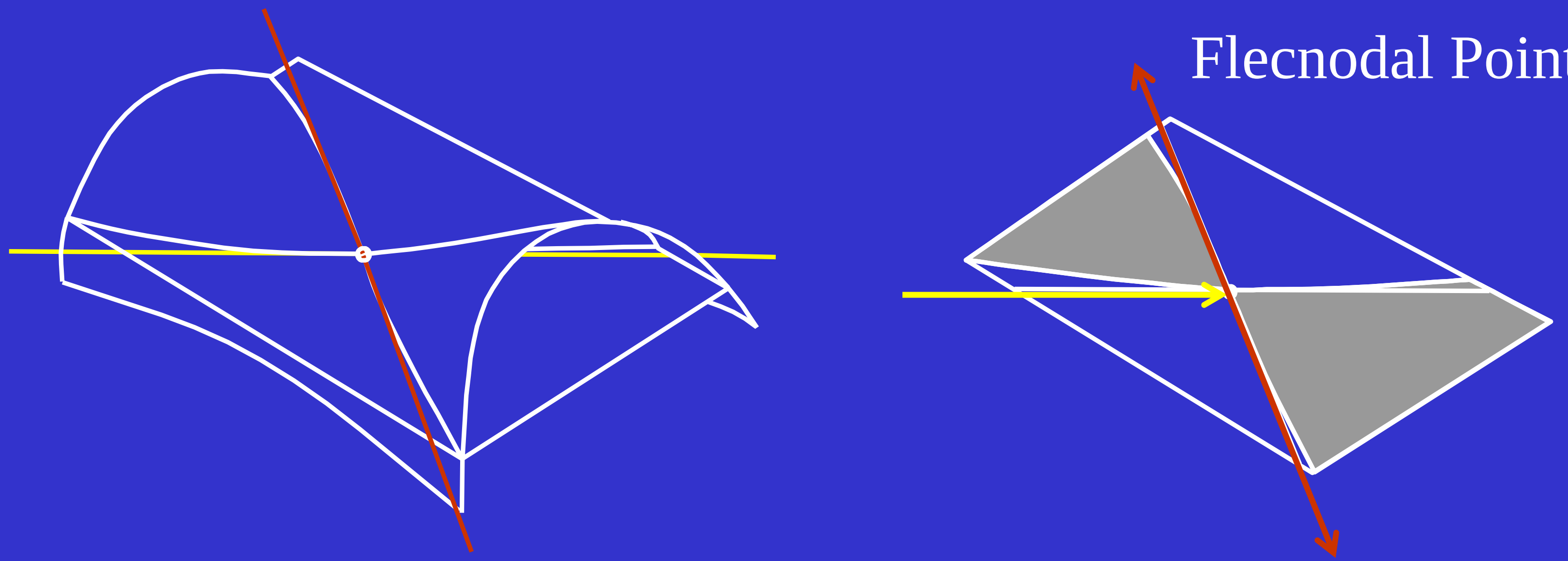


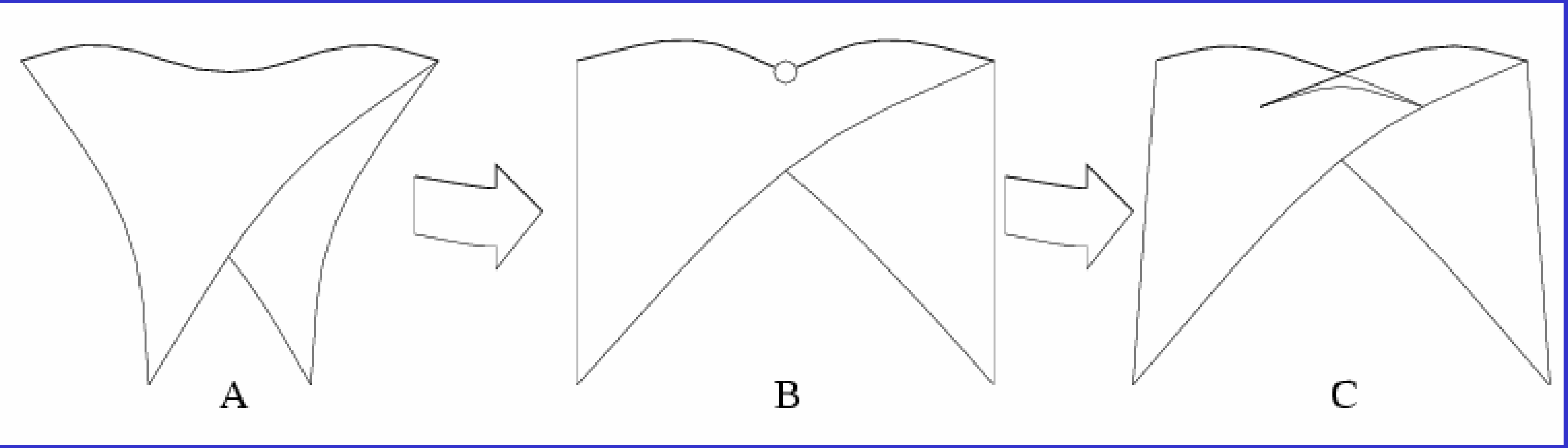




Cusp pairs appear or disappear as one crosses the fold of the asymptotic spherical map.
This happens at asymptotic directions along parabolic curves, and asymptotic directions along flecnodal curves.

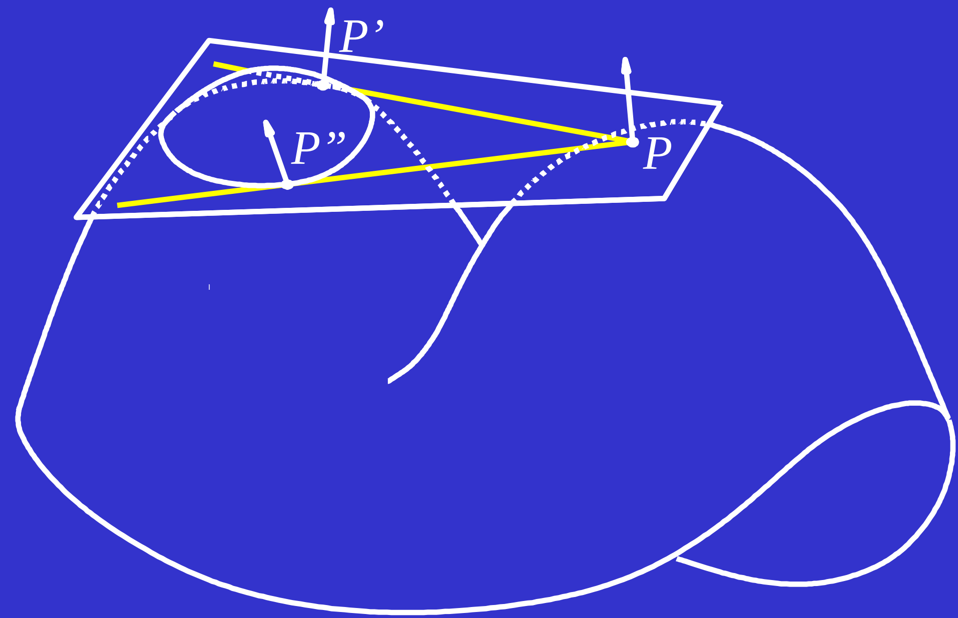
The Swallowtail Event

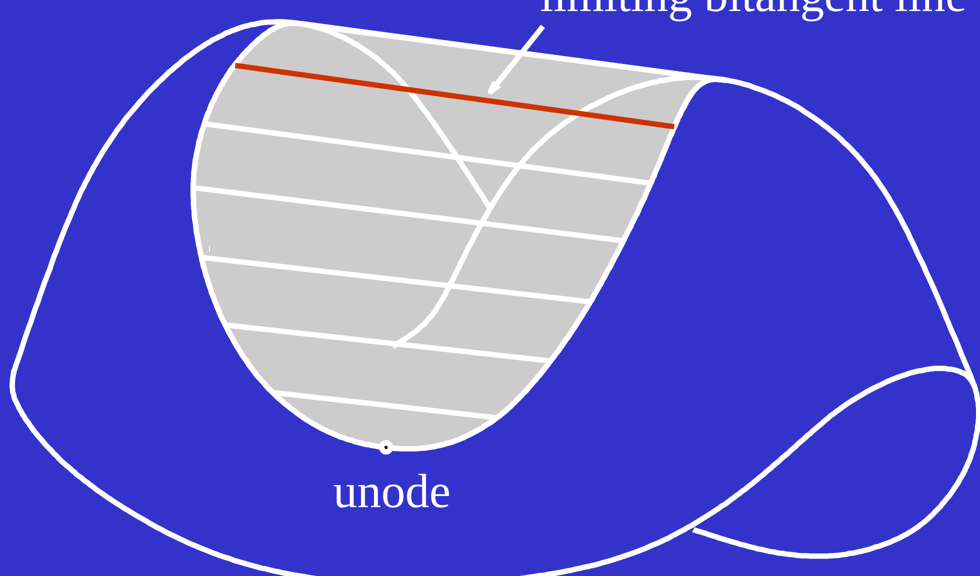




The Bitangent Ray Manifold:

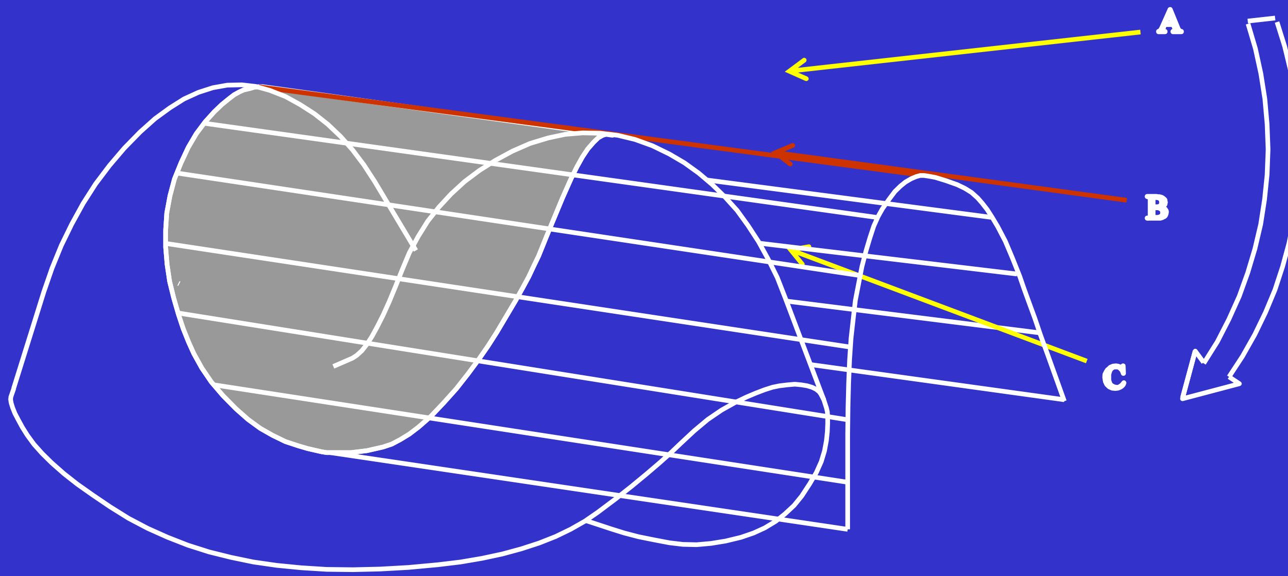
Ordinary bitangents..

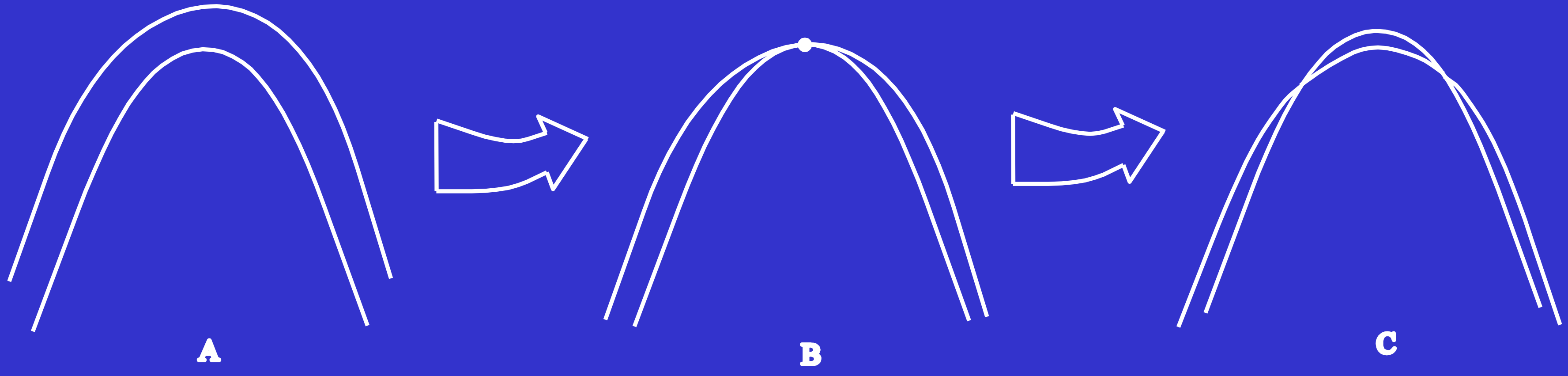




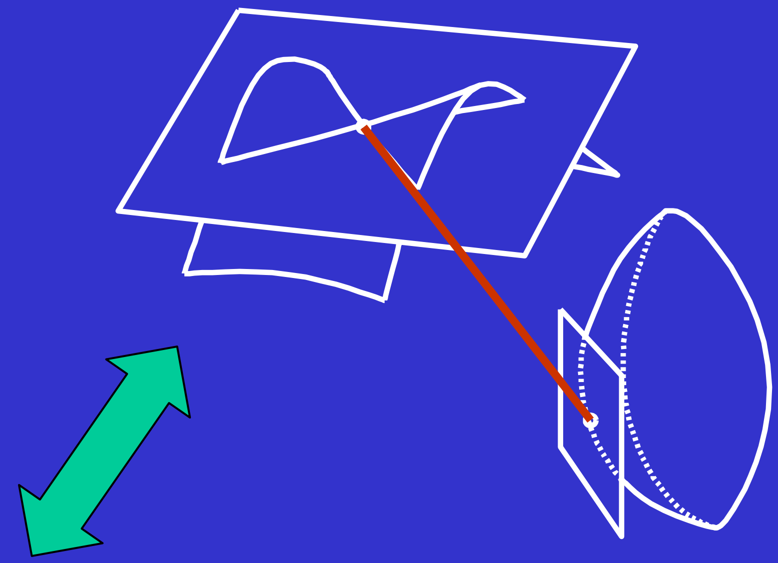
..and exceptional (limiting) ones.

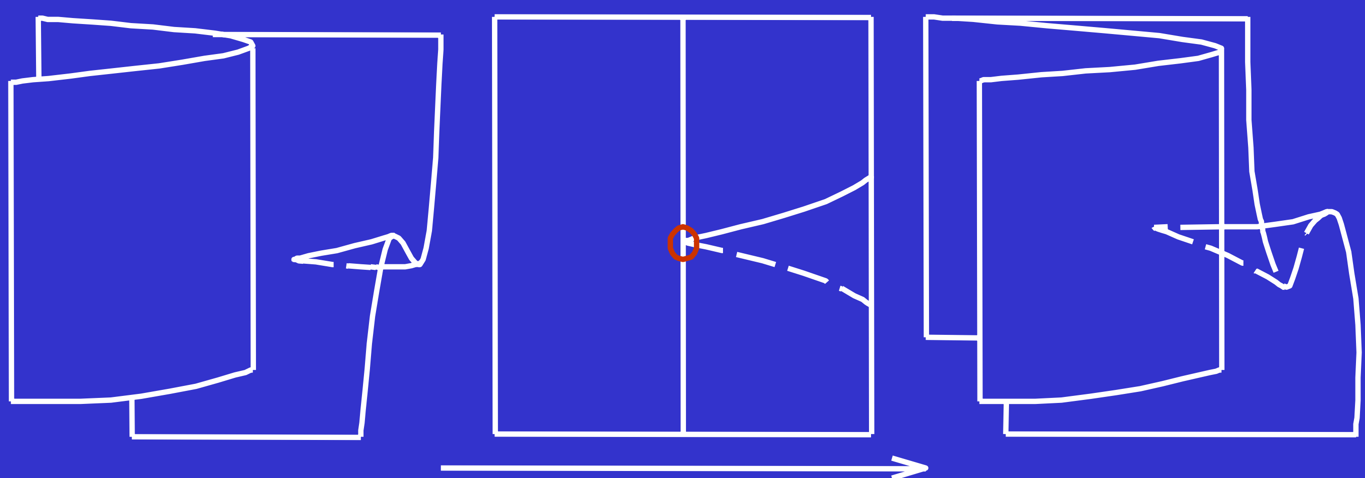
The Tangent Crossing Event





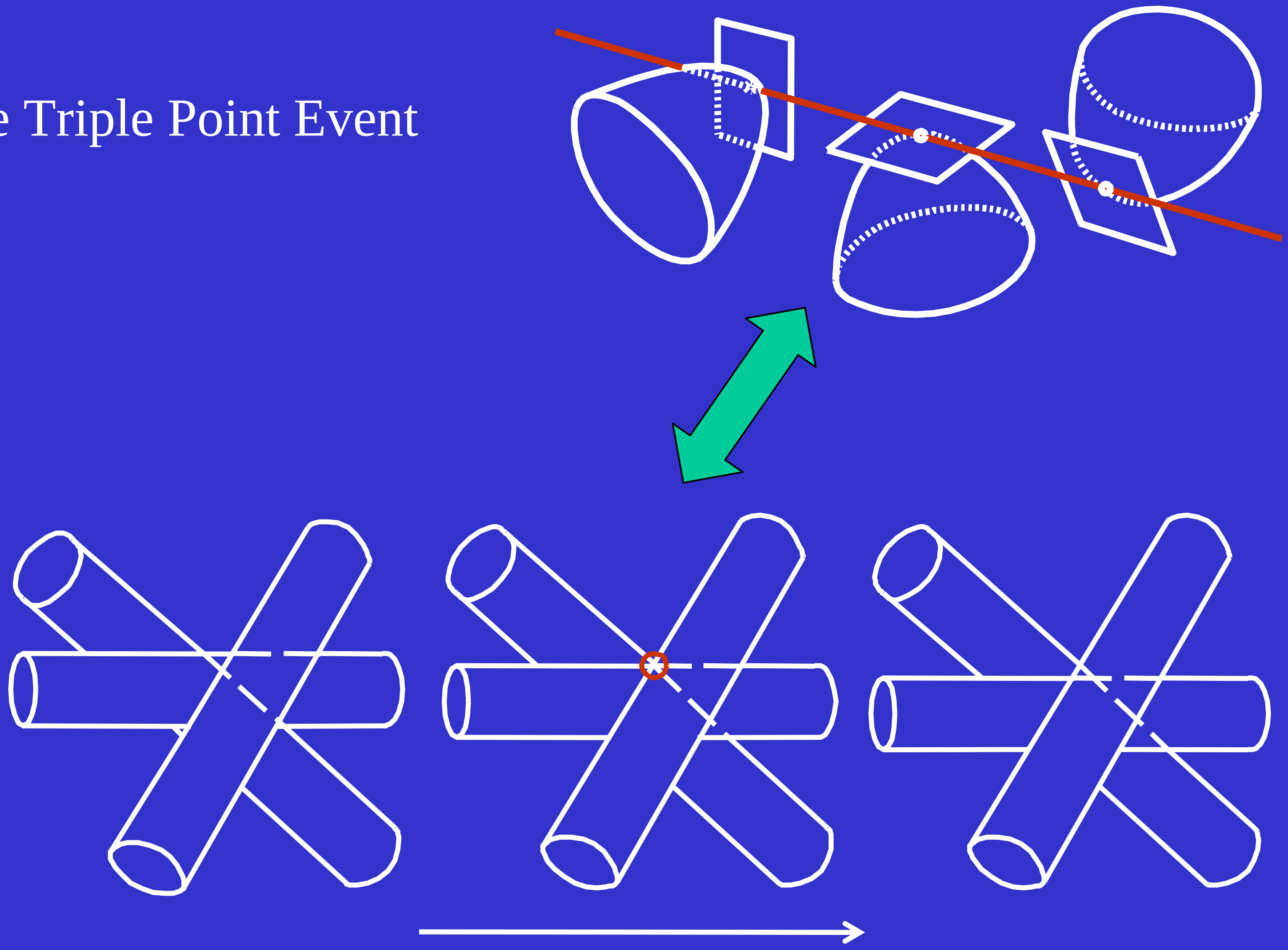
The Cusp Crossing Event





After “Computing Exact Aspect Graphs of Curved Objects: Algebraic Surfaces,” by S. Petitjean, J. Ponce, and D.J. Kriegman, the International Journal of Computer Vision, 9(3):231-255 (1992). © 1992 Kluwer Academic Publishers.

The Triple Point Event



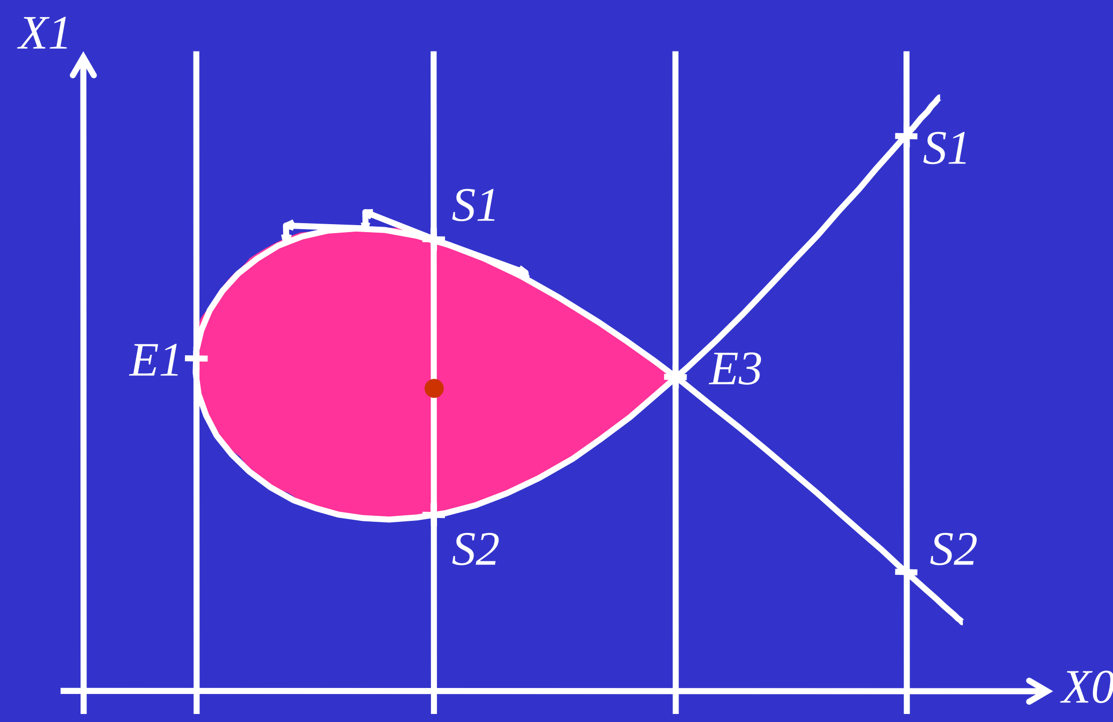
After "Computing Exact Aspect Graphs of Curved Objects: Algebraic Surfaces," by S. Petitjean, J. Ponce, and D.J. Kriegman, the International Journal of Computer Vision, 9(3):231-255 (1992).

Tracing Visual Events

Computing the Aspect Graph

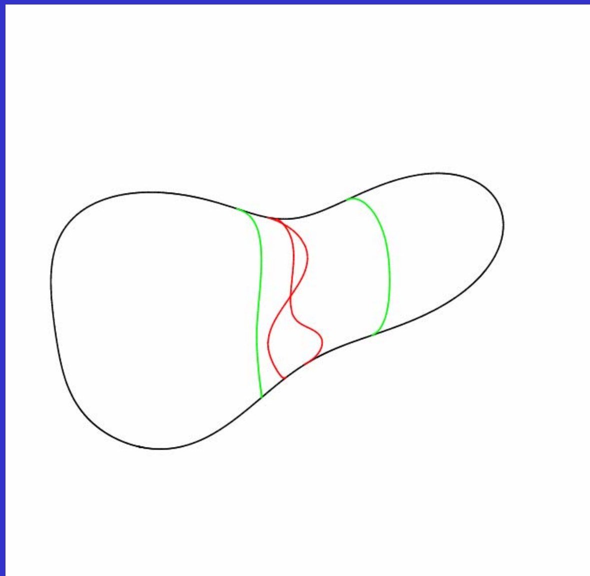
$$F(x,y,z)=0$$

$$P_1(x_1,\ldots,x_n)=0$$
$$\ldots$$
$$P_n(x_1,\ldots,x_n)=0$$

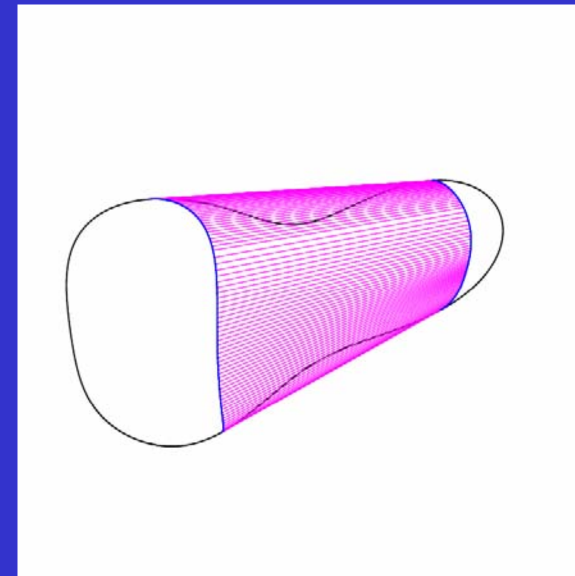


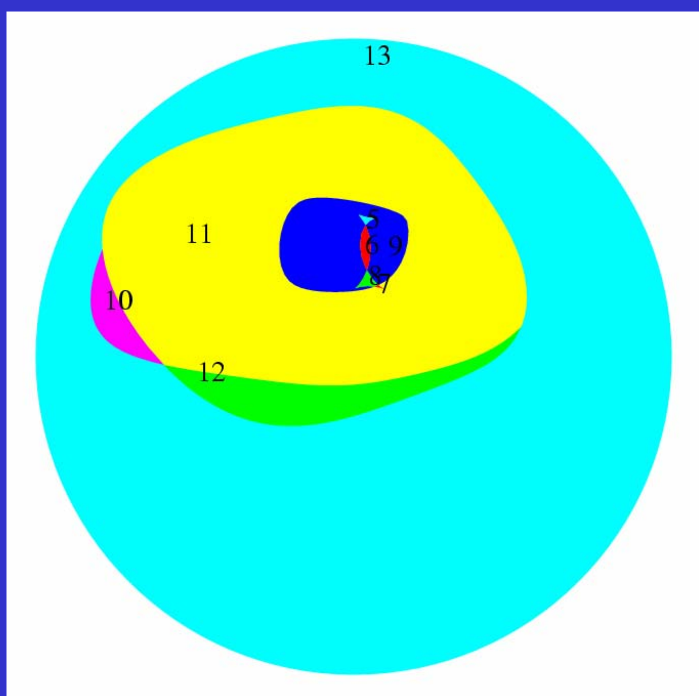
After "Computing Exact Aspect Graphs of Curved Objects: Algebraic Surfaces," by S. Petitjean, J. Ponce, and D.J. Kriegman, the International Journal of Computer Vision, 9(3):231-255 (1992). © 1992 Kluwer Academic Publishers.

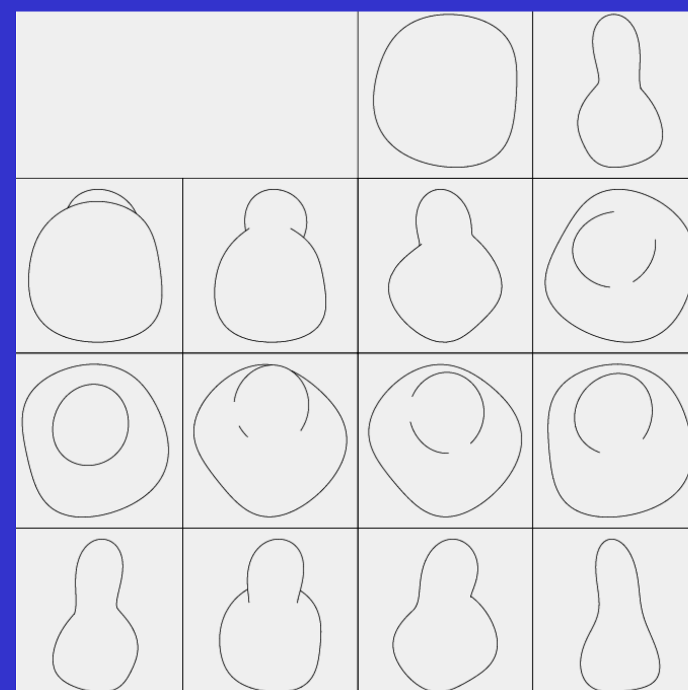
- Curve Tracing
- Cell Decomposition



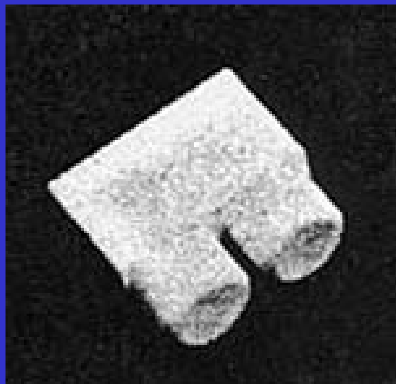
An Example

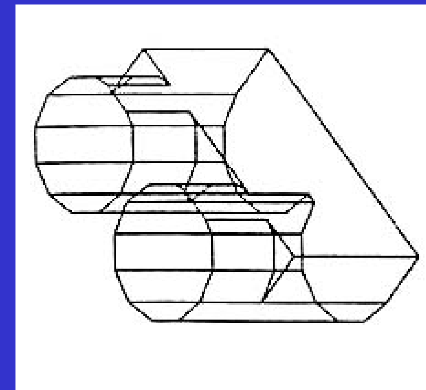


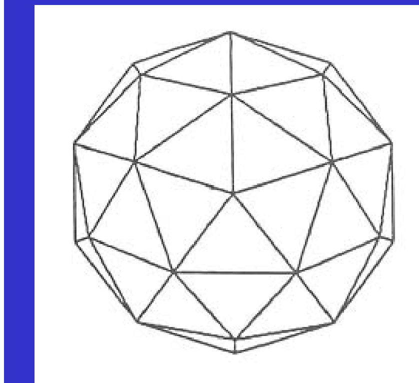


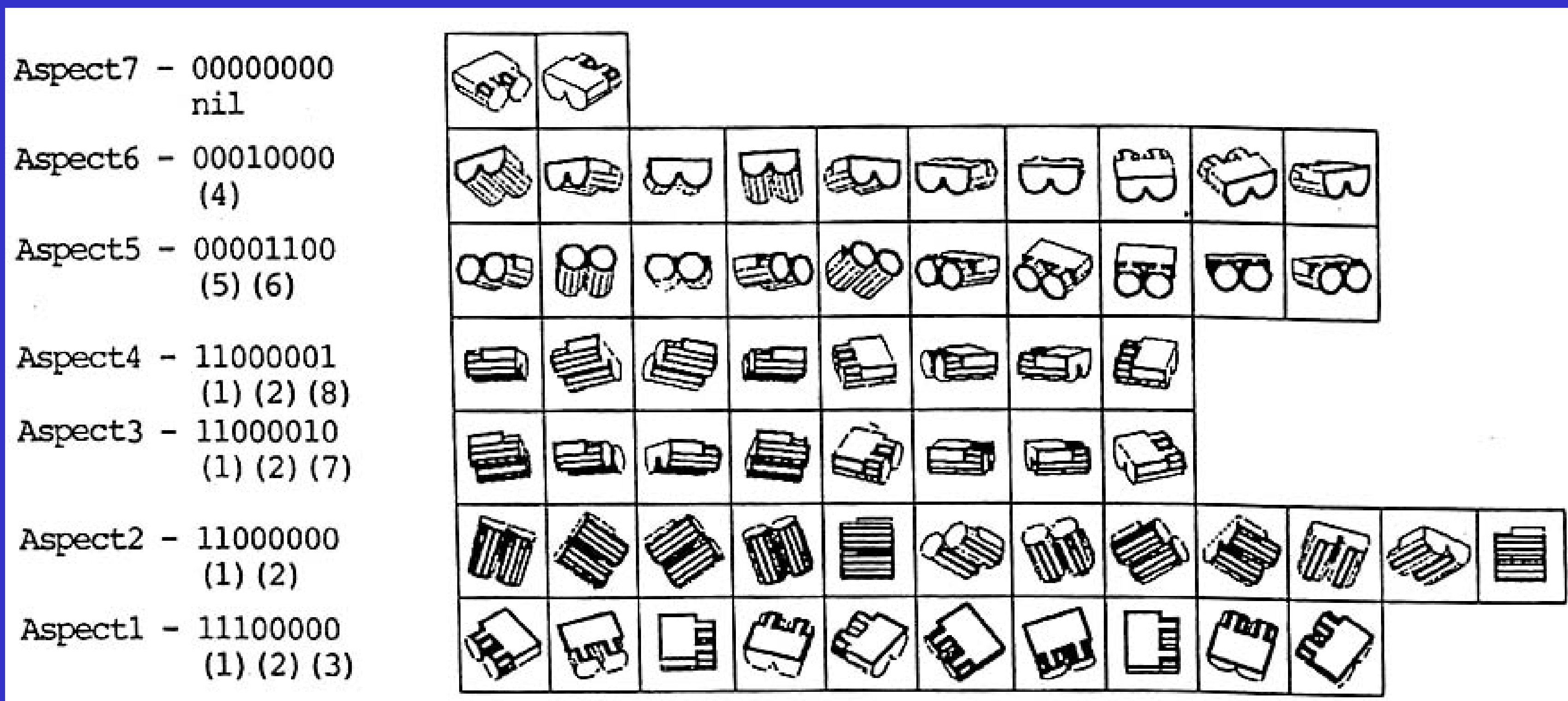


Approximate Aspect Graphs (Ikeuchi & Kanade, 1987)



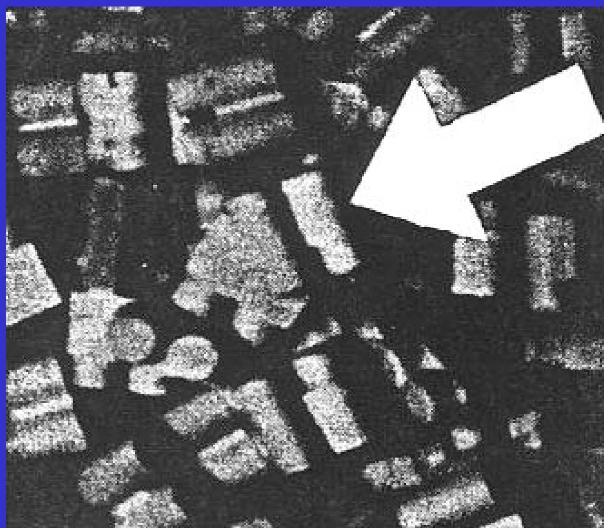


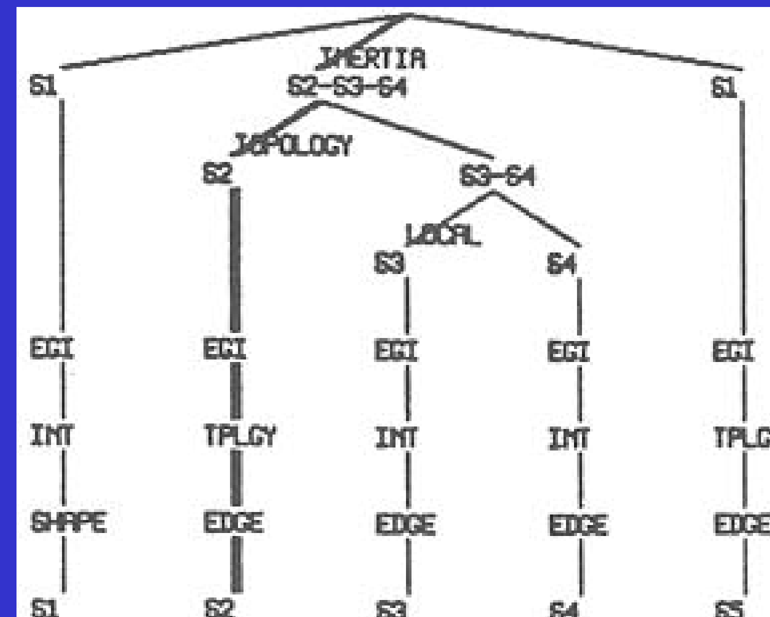


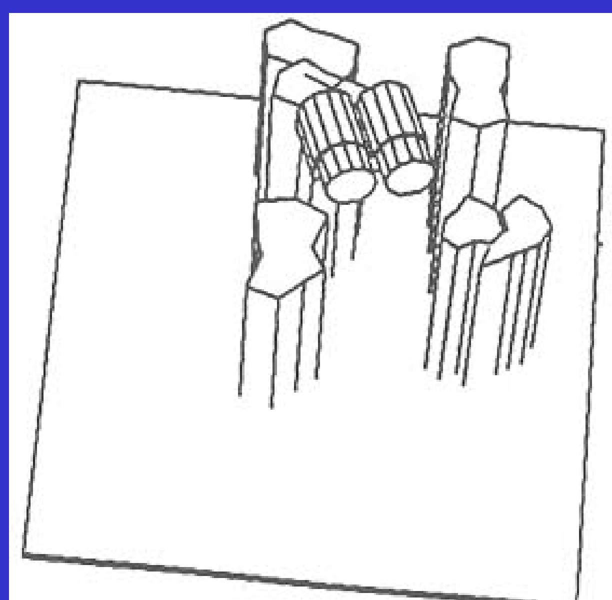


Reprinted from "Automatic Generation of Object Recognition Programs," by K. Ikeuchi and T. Kanade, Proc. of the IEEE, 76(8):1016-1035 (1988).

Approximate Aspect Graphs II: Object Localization (Ikeuchi & Kanade, 1987)







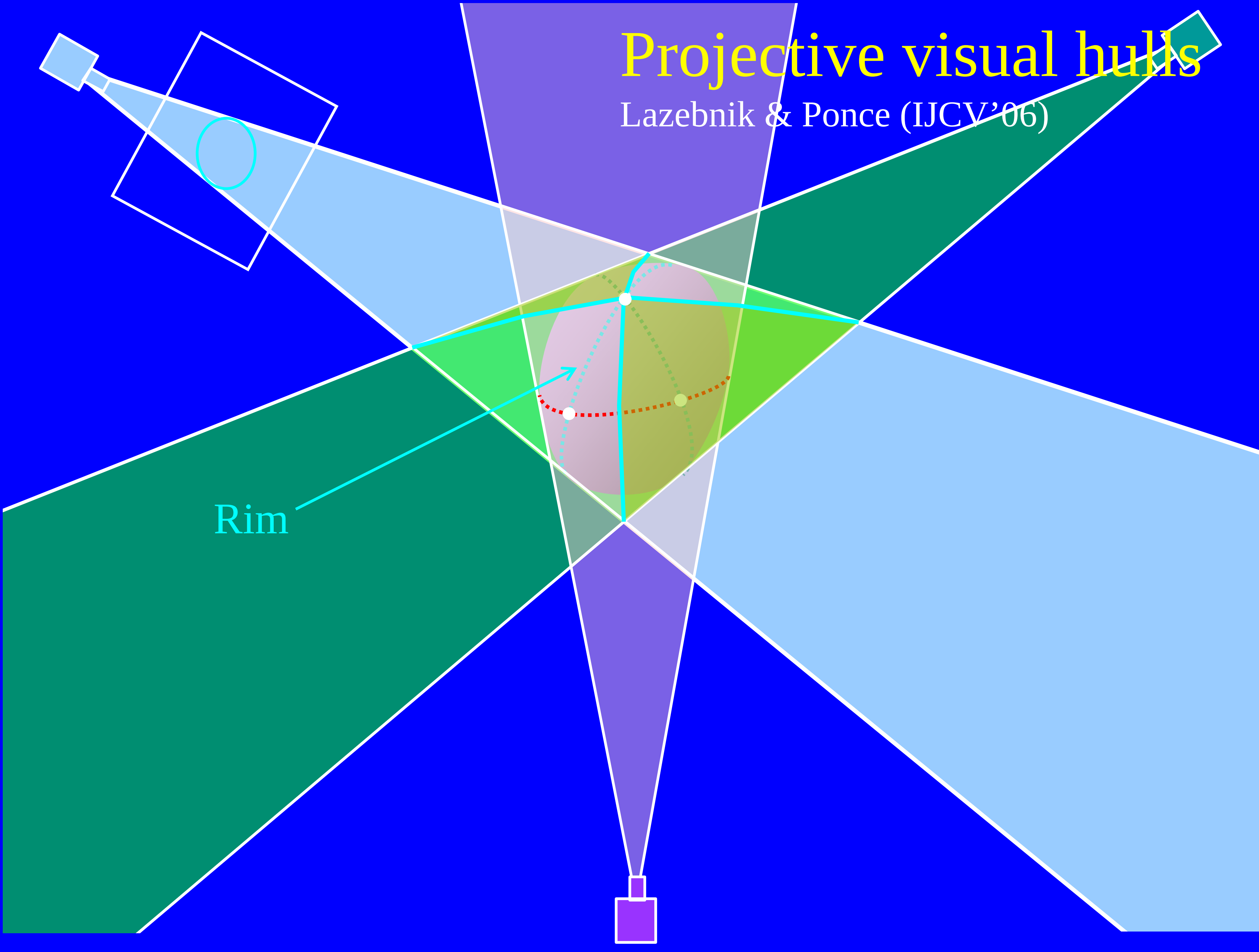
Reprinted from "Precompiling a Geometrical Model into an Interpretation Tree for Object Recognition in Bin-Picking Tasks," by K. Ikeuchi, Proc. DARPA Image Understanding Workshop, 1987.

VISUAL HULLS

- Visual hulls
- Differential projective geometry
- Oriented differential projective geometry
- Image-based computation of projective visual hulls

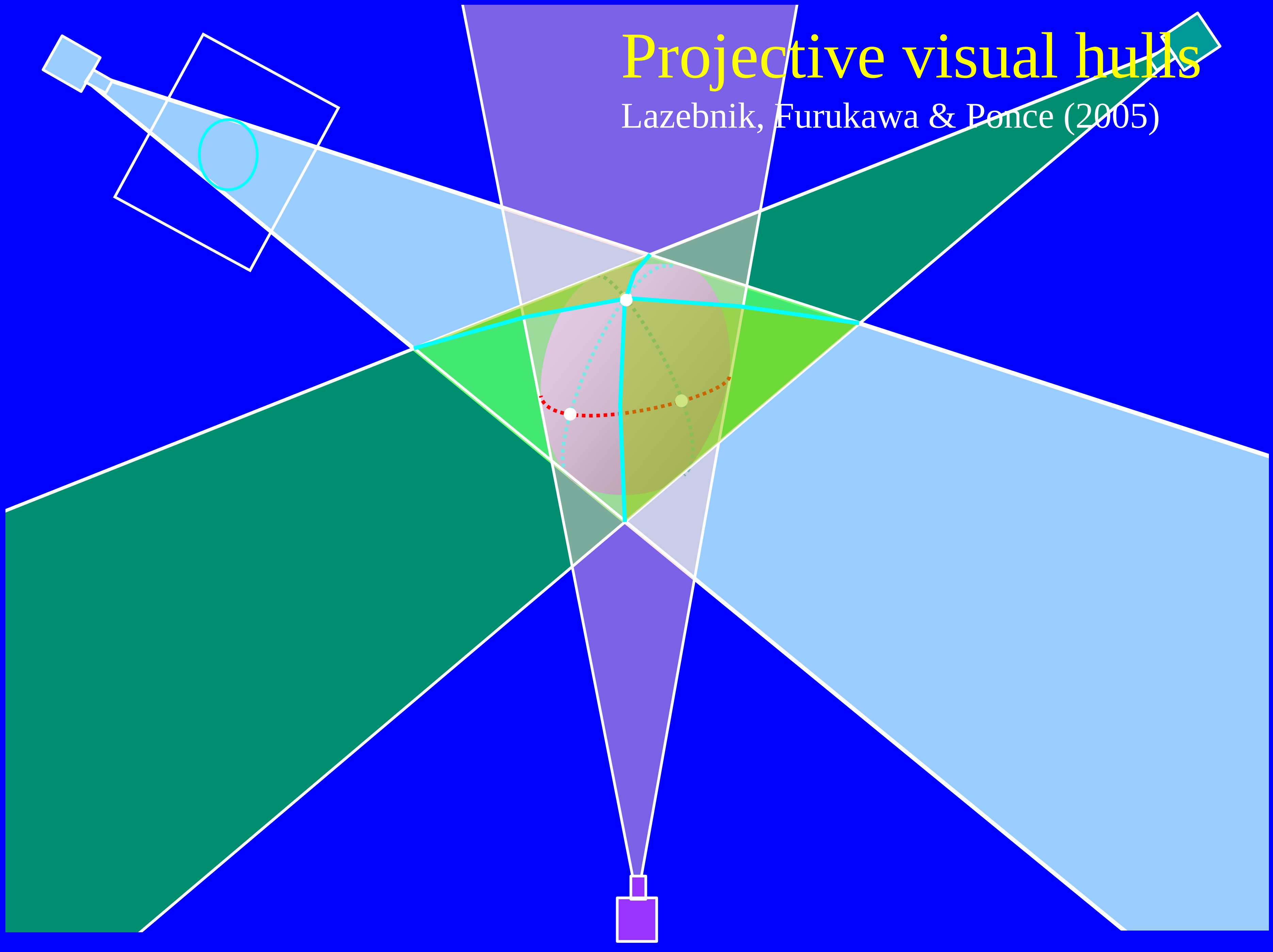
Projective visual hulls

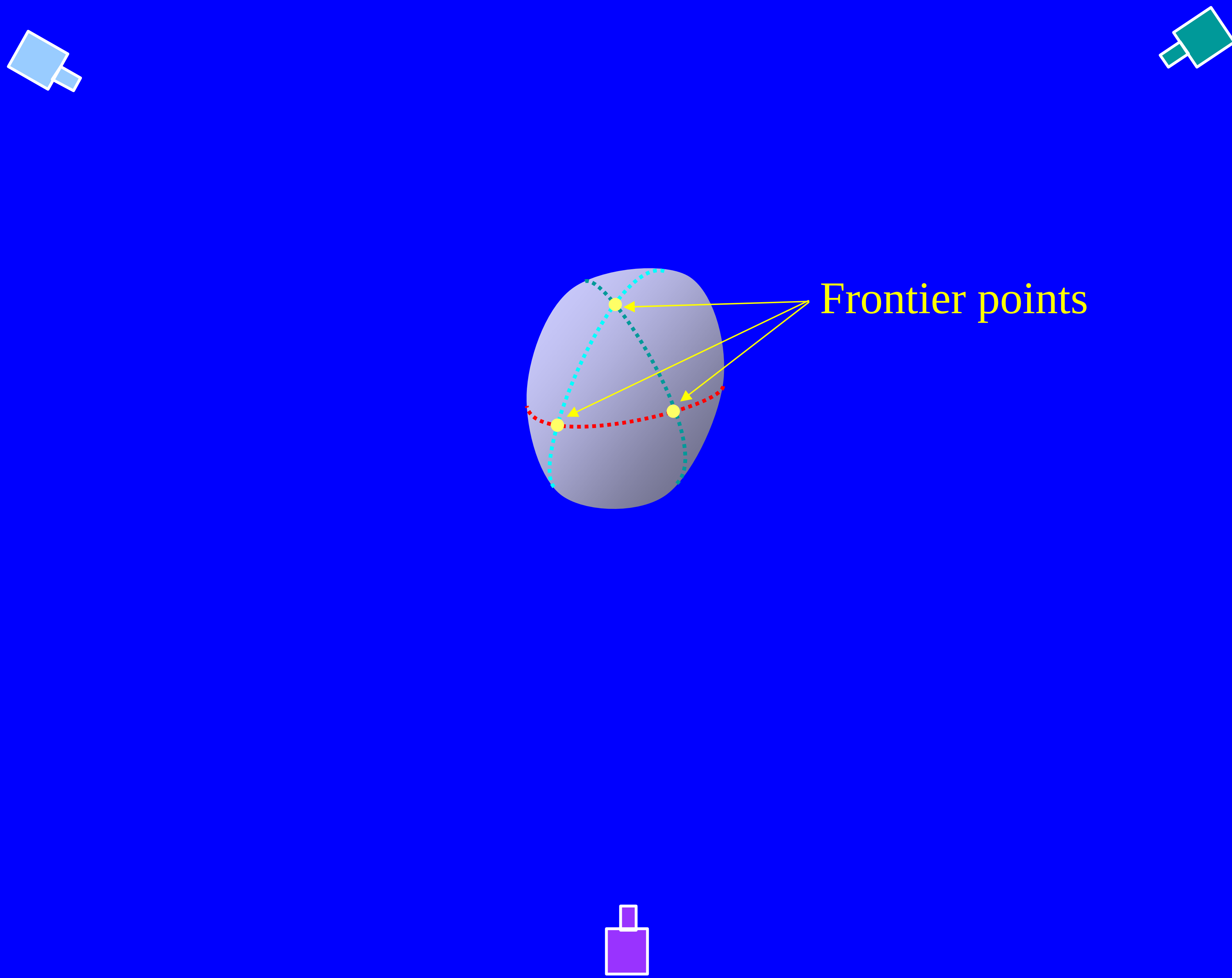
Lazebnik & Ponce (IJCV'06)

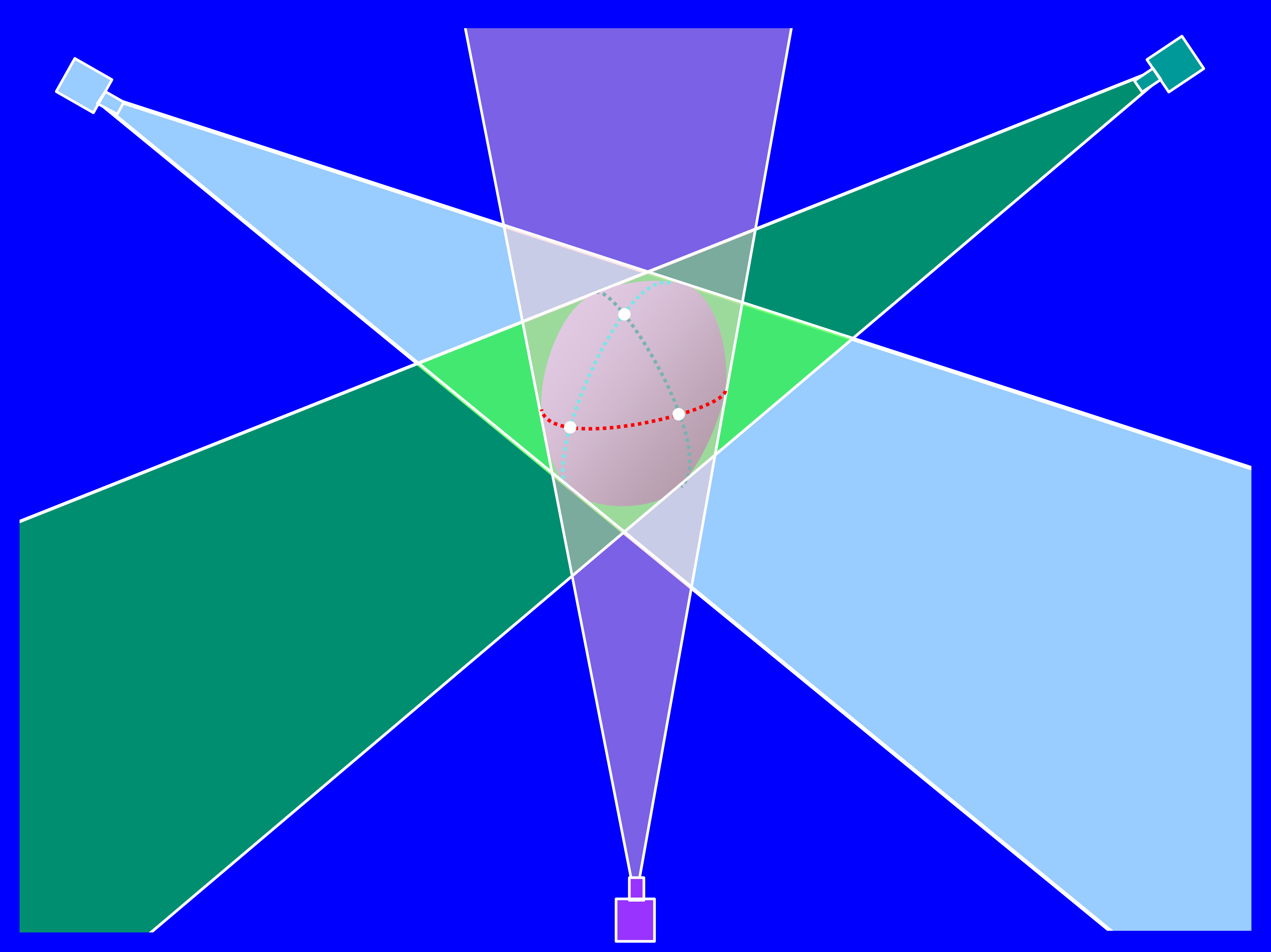


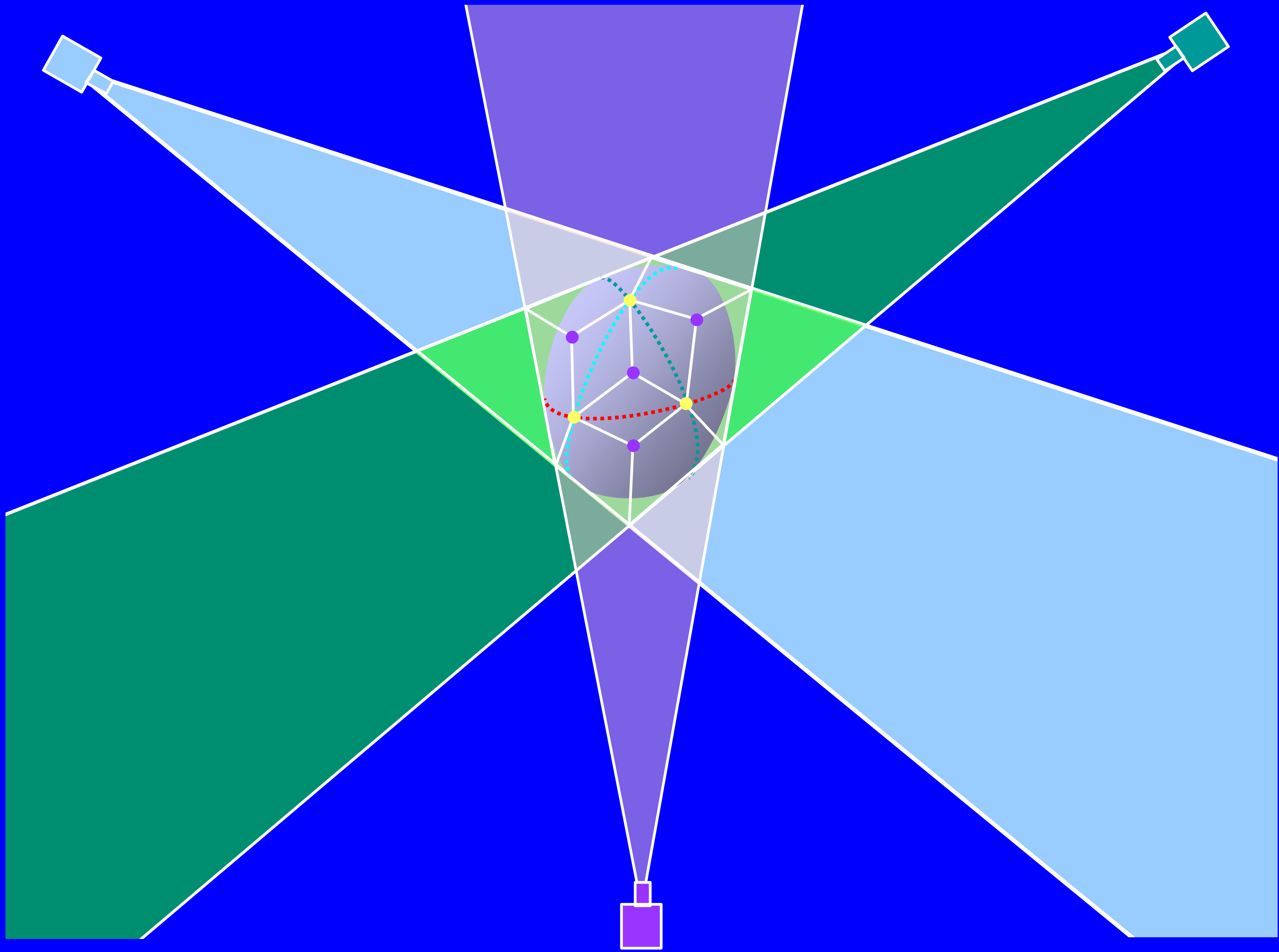
Projective visual hulls

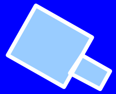
Lazebnik, Furukawa & Ponce (2005)

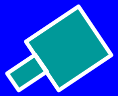


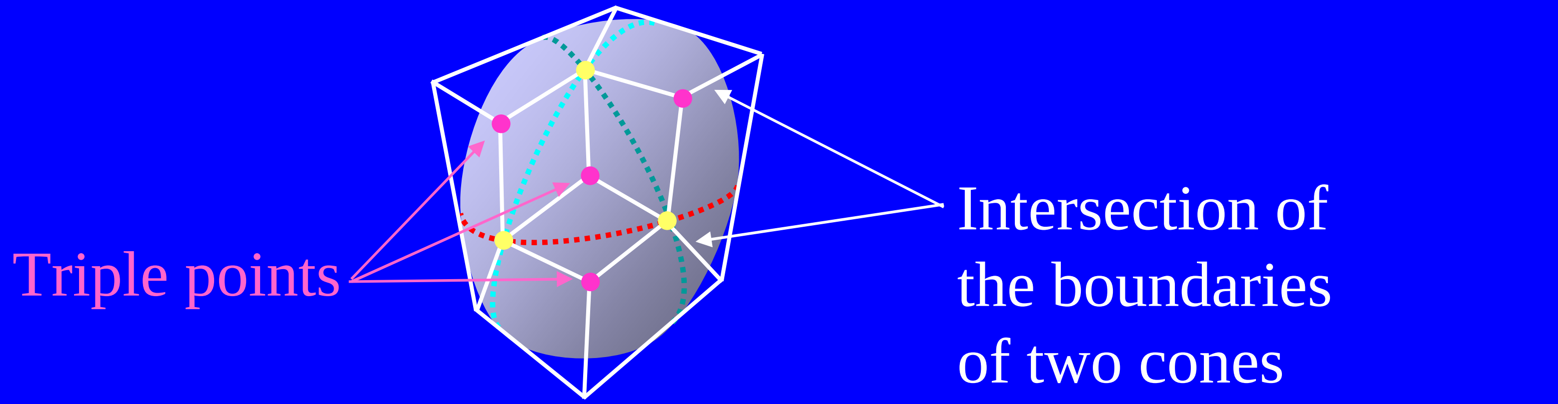

Frontier points











The visual hull

Baumgart (1974); Laurentini (1995); Petitjean (1998); Matusik et al. (2001); Lazebnik, Boyer & Ponce (2001); Franco & Boyer (2005).

Oriented projective structure

Lazebnik & Ponce (2003)

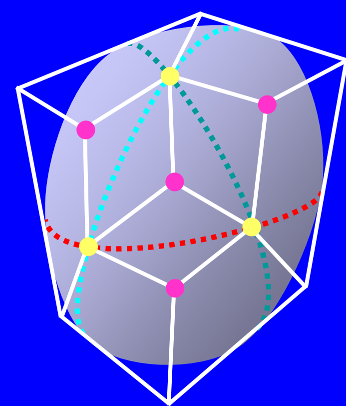
Aspect graphs

Koenderink & Van Doorn (1976)

Visibility complexes

Pocchiola & Vegter (1993); Durand et al. (1997)

Stolfi (1991); Laveau & Faugeras (1994)



Aspect graphs

Koenderink & Van Doorn (1976)

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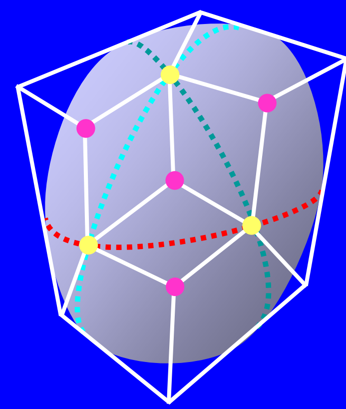

Aspect graphs

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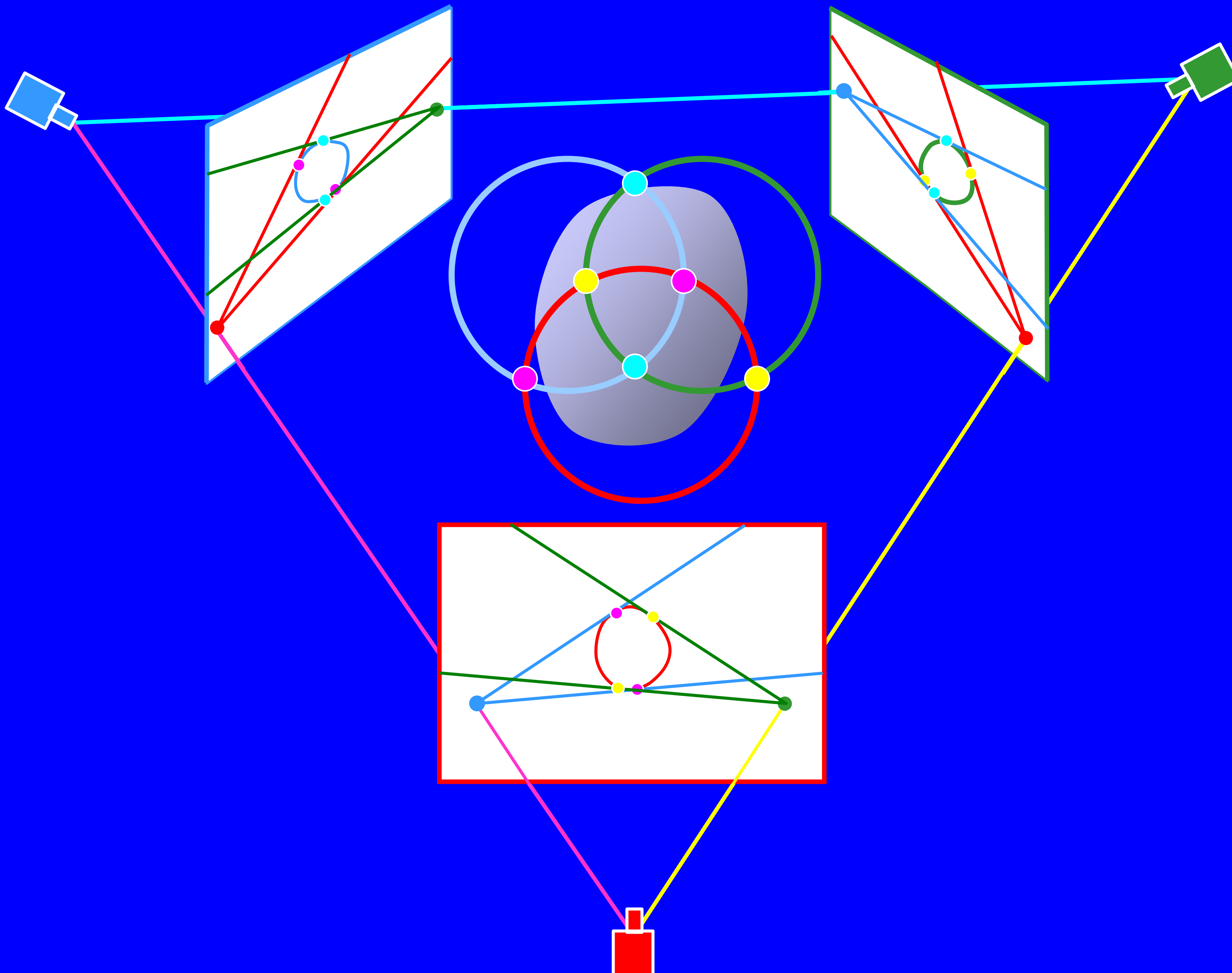
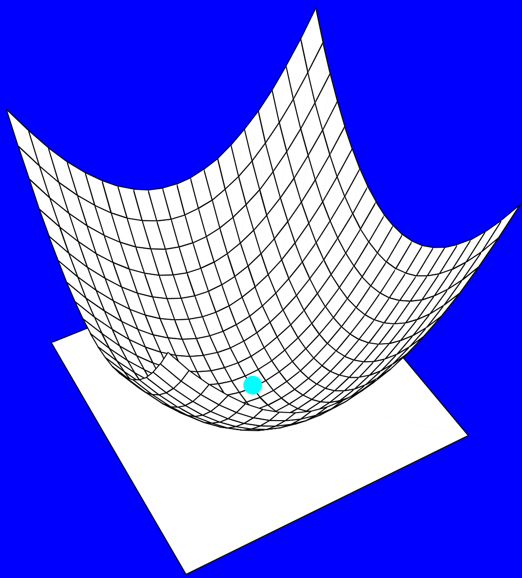
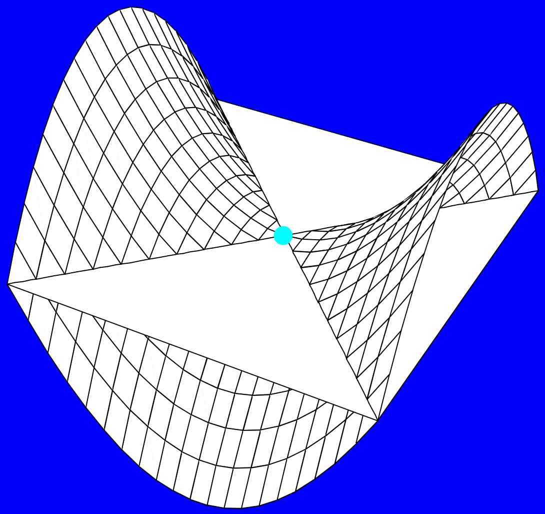
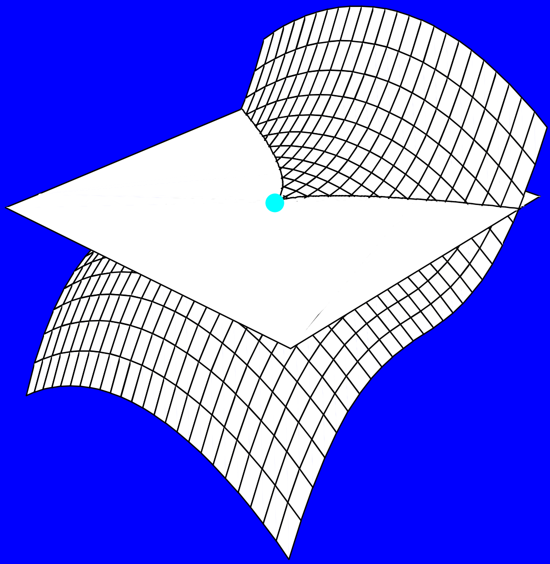




Elliptical



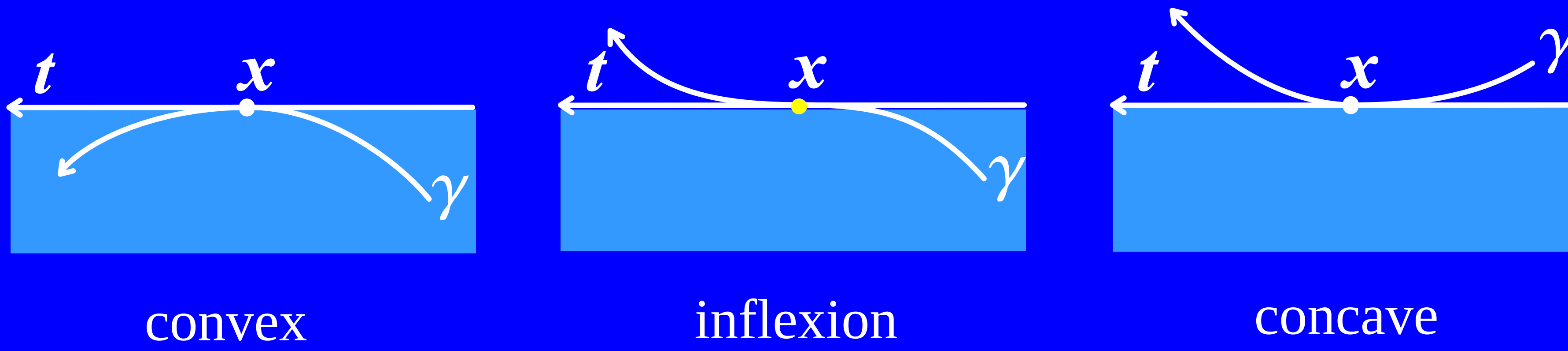
Hyperbolic



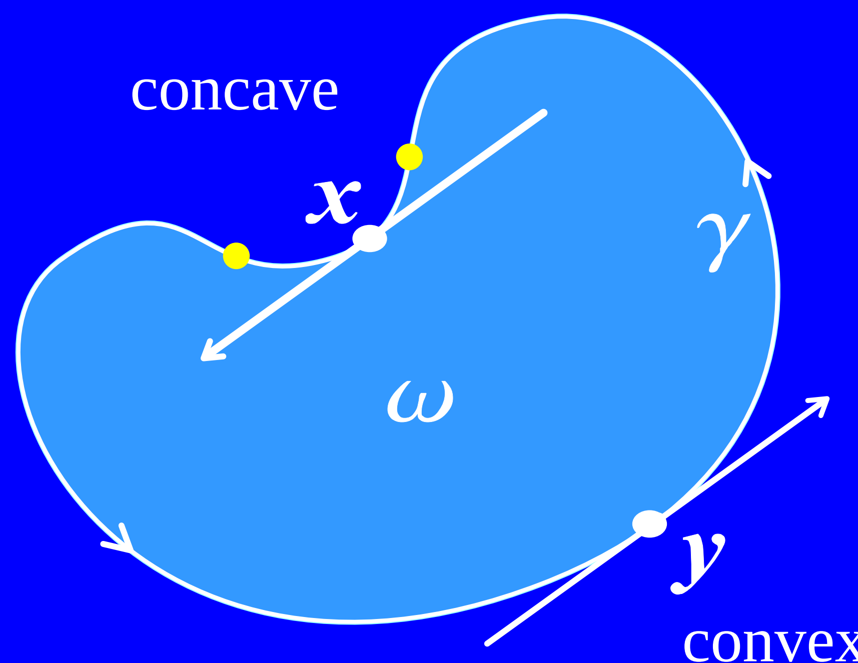
Parabolic

$$K = ln - m^2$$

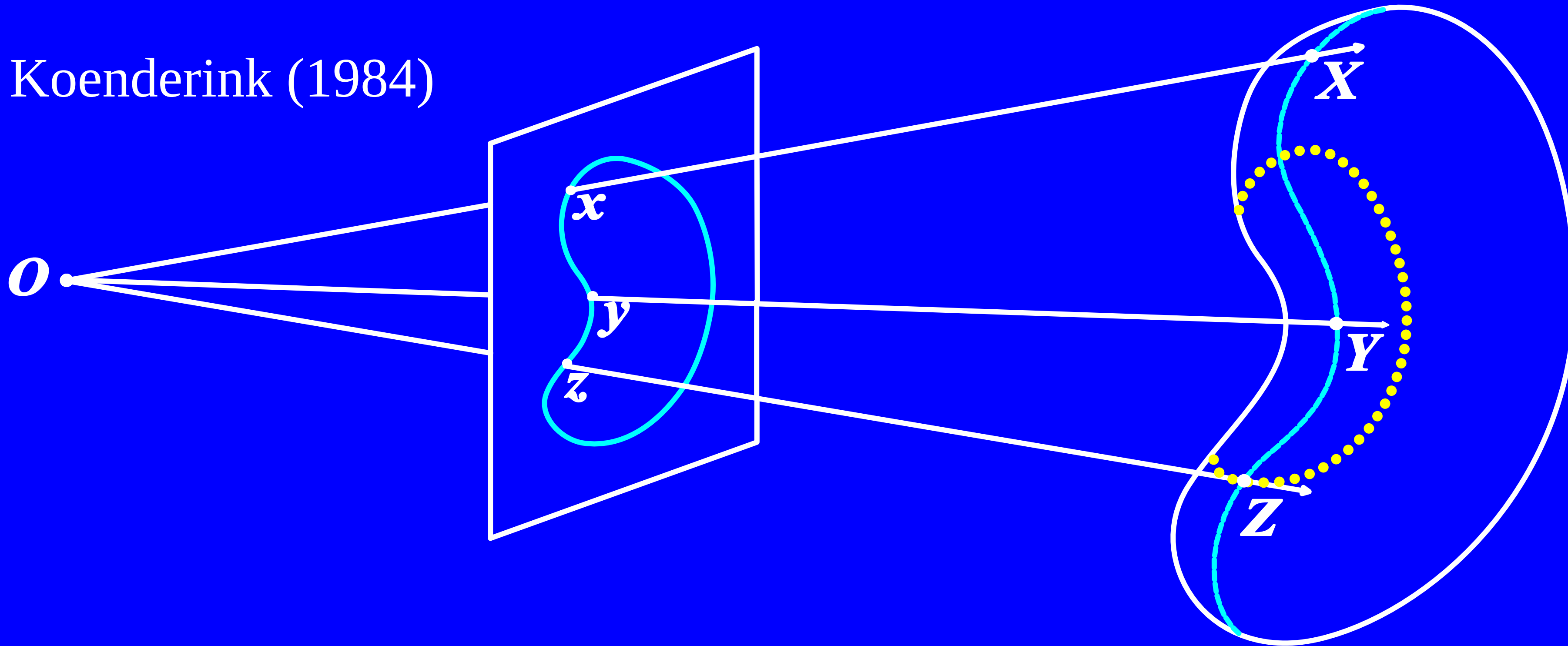
$$\begin{cases} l = |\mathbf{X}, \mathbf{X}_u, \mathbf{X}_v, \mathbf{X}_{uu}| \\ m = |\mathbf{X}, \mathbf{X}_u, \mathbf{X}_v, \mathbf{X}_{uv}| \\ n = |\mathbf{X}, \mathbf{X}_u, \mathbf{X}_v, \mathbf{X}_{vv}| \end{cases}$$

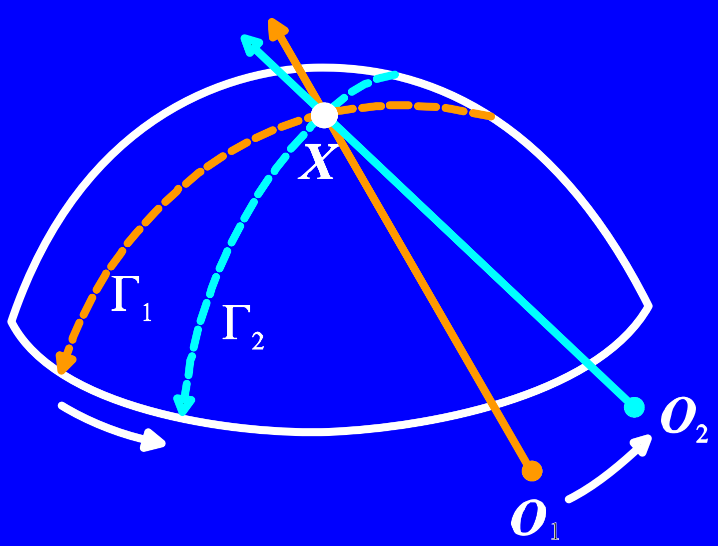


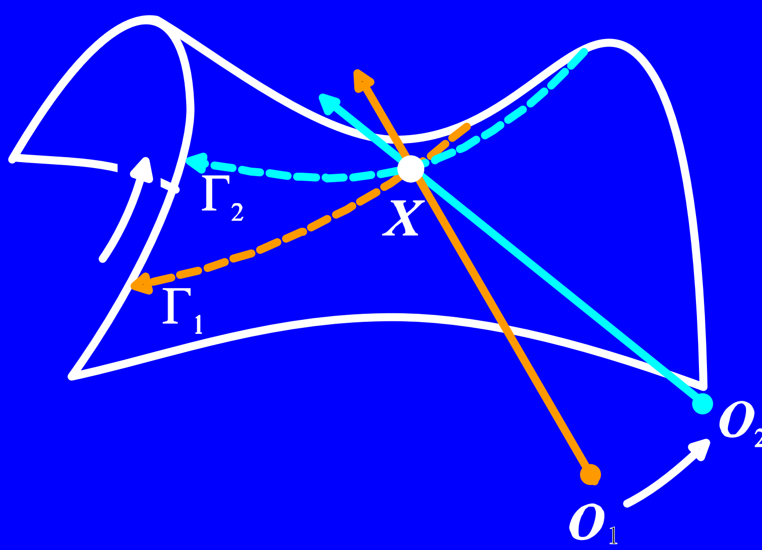
$$\kappa = \mid x, x', x'' \mid$$



Koenderink (1984)







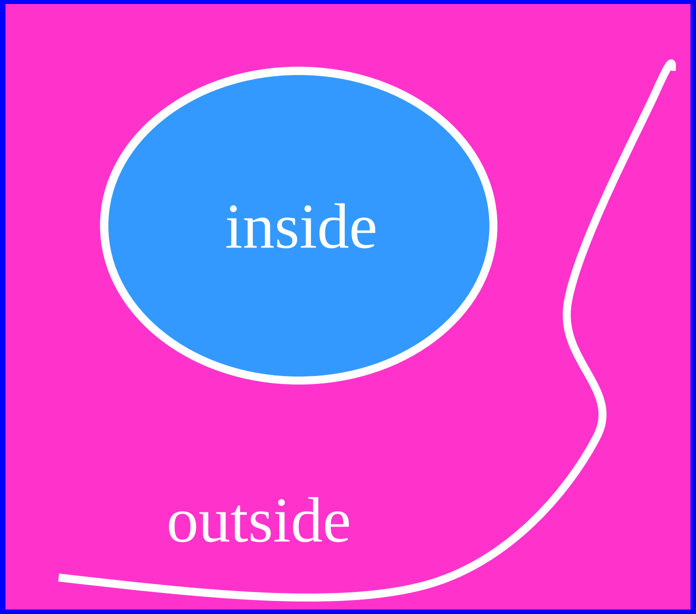
Projective visual hulls

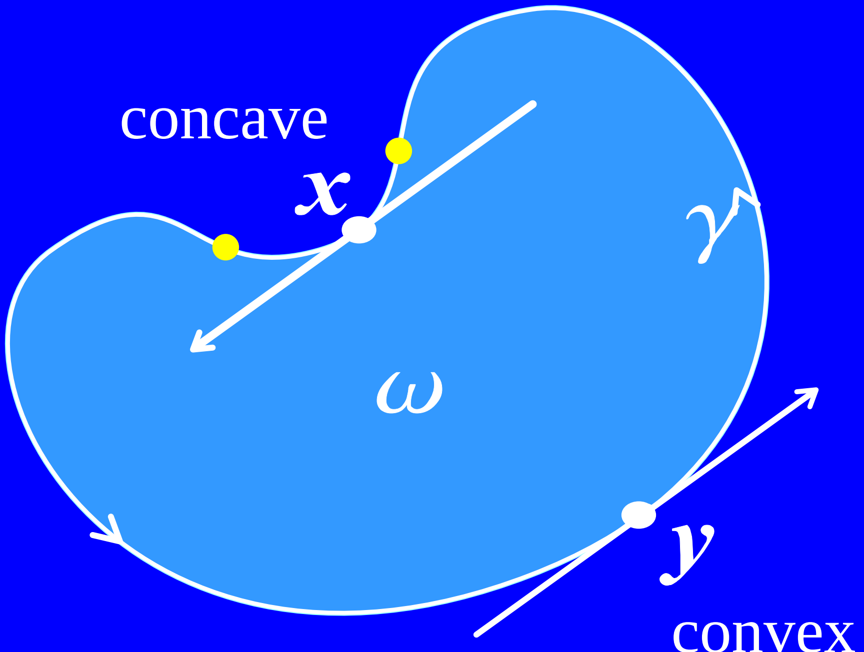
Lazebnik, Furukawa & Ponce (2004)

Affine structure and motion

Furukawa, Sethi, Kriegman & Ponce (2004)

What about plain projective geometry?





With X. Goaoc, S. Lazard,
S. Petitjean, M. Teillaud.

What about polyhedral approximations of smooth surfaces?

With X. Goaoc and S. Lazard.