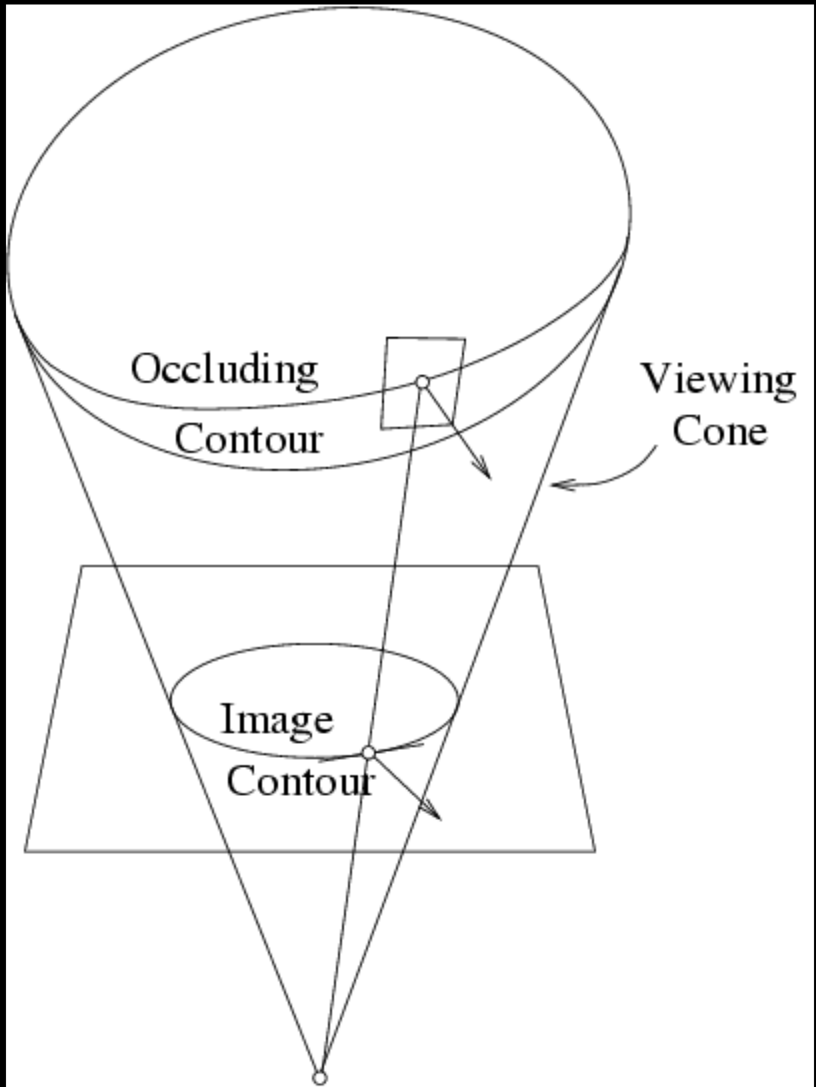
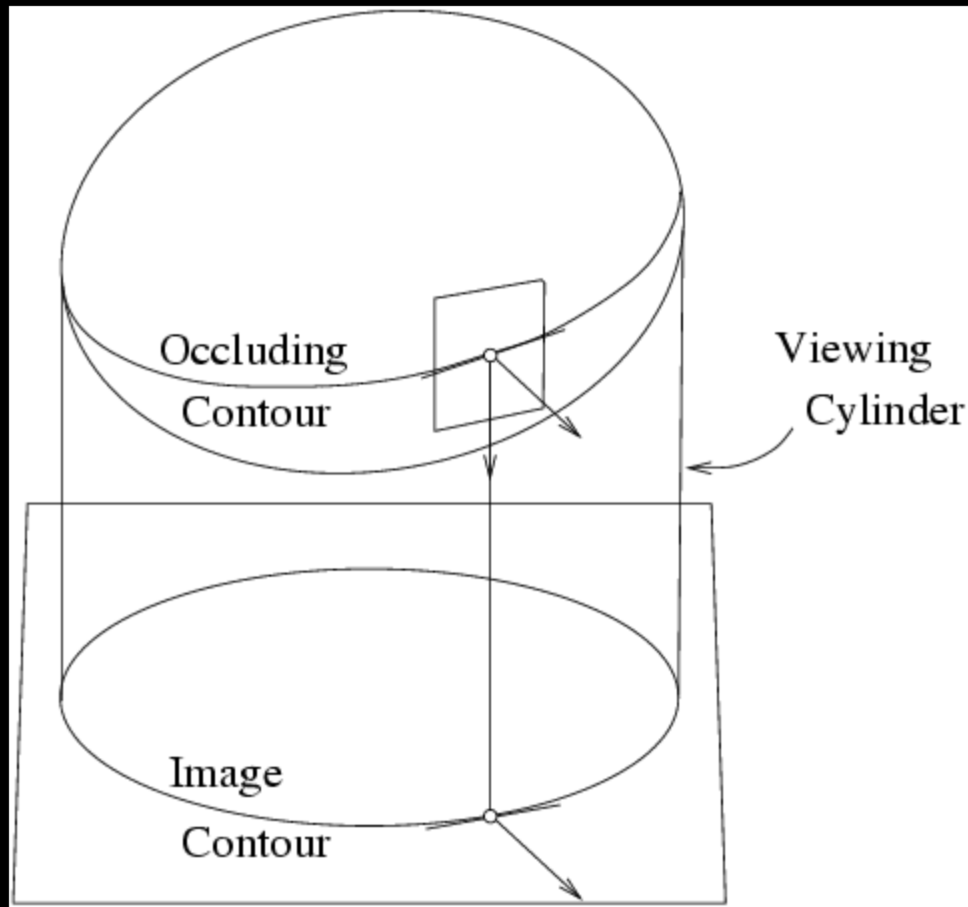
SMOOTH SURFACES AND THEIR OUTLINES

- Elements of Differential Geometry
- The second fundamental form
- Koenderink's Theorem
- Aspect graphs
- More differential geometry
- A catalogue of visual events
- Computing the aspect graph

- http://www.di.ens.fr/~ponce/geomvis/lect6.pptx
- http://www.di.ens.fr/~ponce/geomvis/lect6.pdf

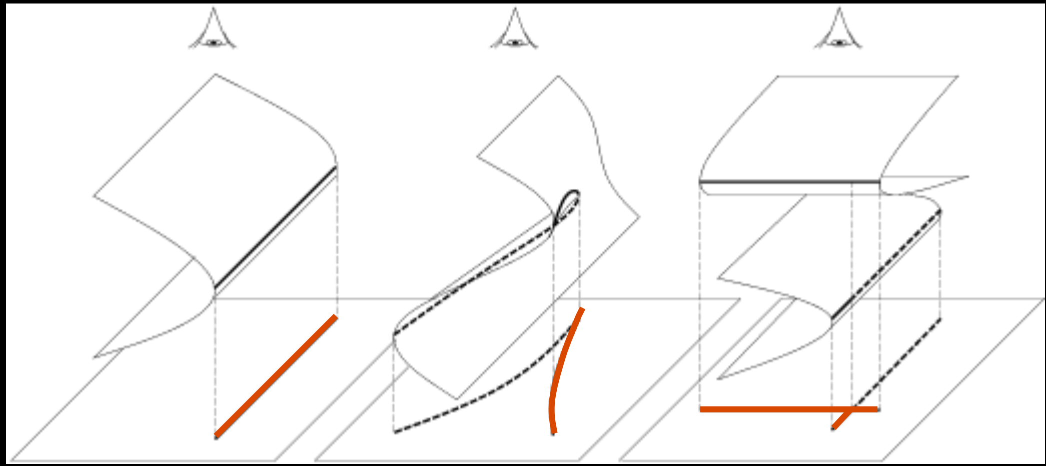
Smooth Shapes and their Outlines





Can we say anything about a 3D shape from the shape of its contour?

What are the contour stable features??



folds cusps T-junctions

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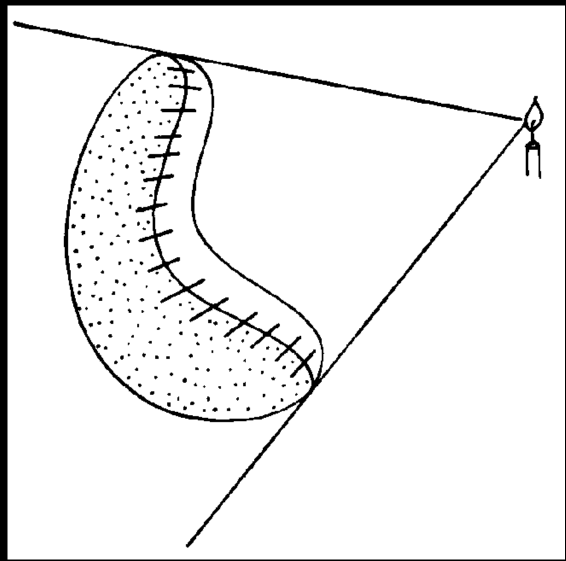
Shadows are like silhouettes..



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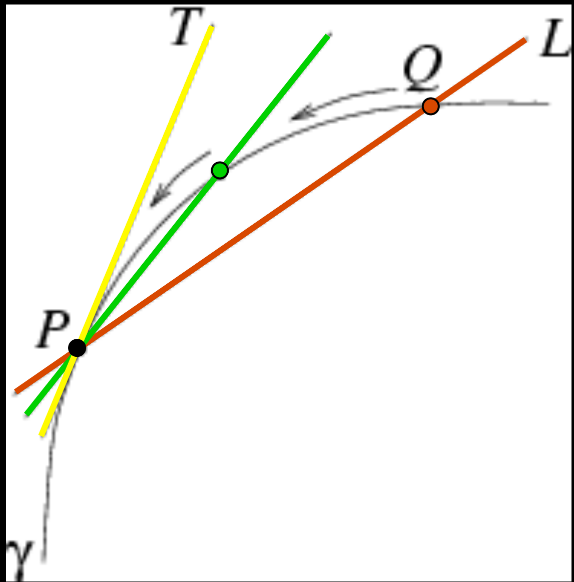
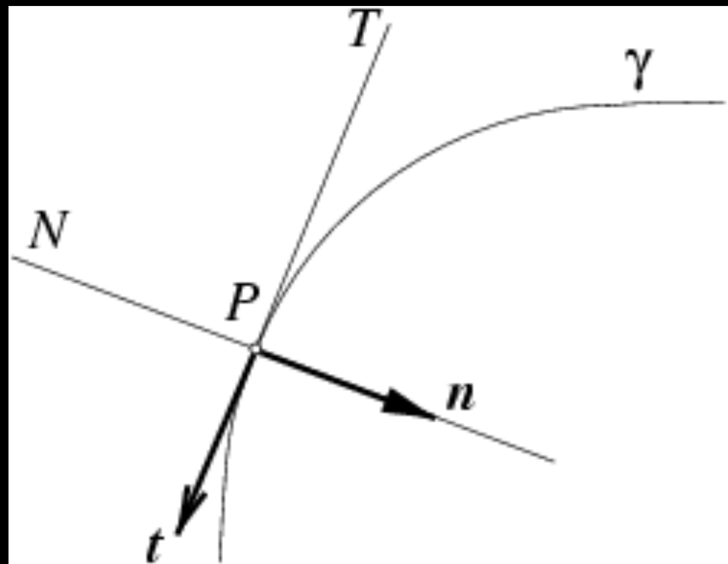
Differential geometry: geometry in the small

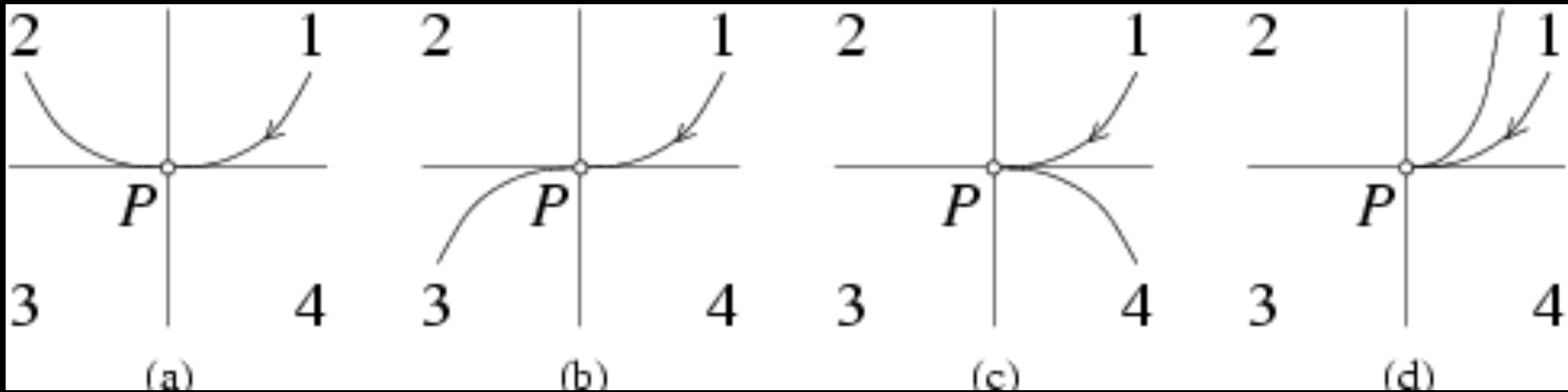


A tangent is the limit of a sequence of secants.

The normal to a curve is perpendicular to the tangent line.



What can happen to a curve in the vicinity of a point?



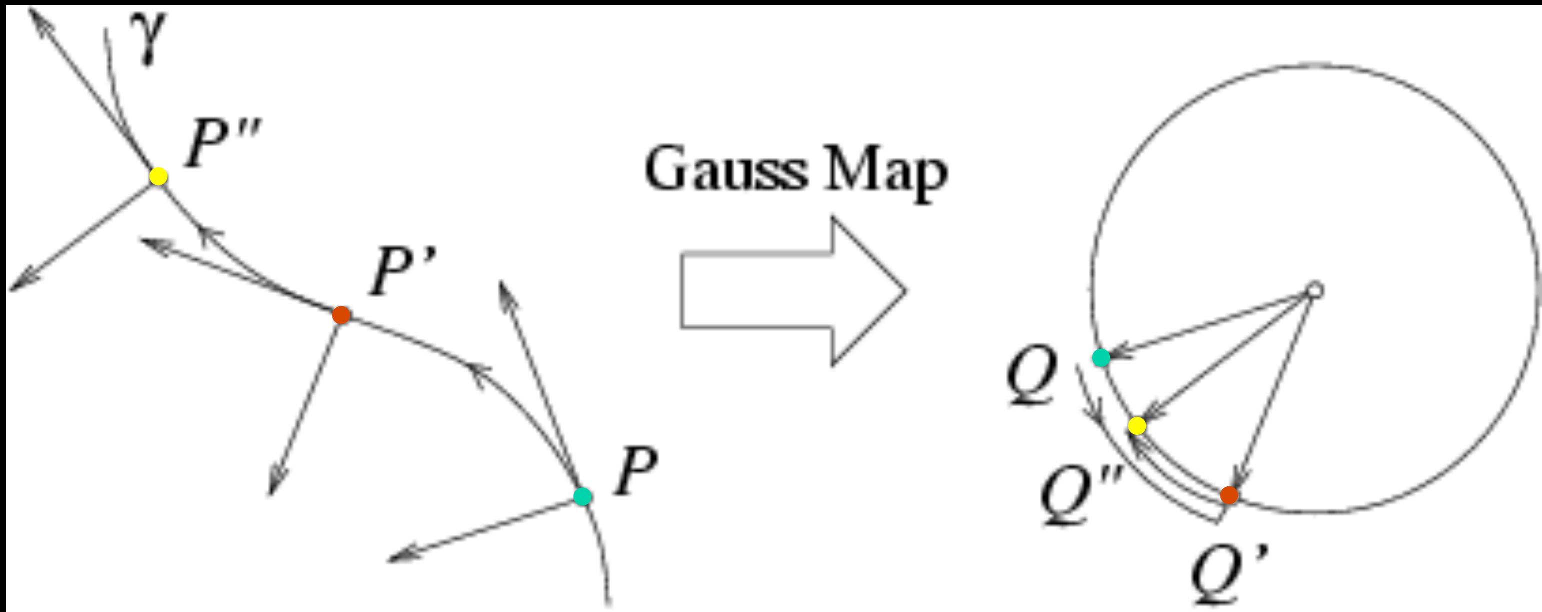
(a) Regular point;

(b) inflection;

(c) cusp of the first kind;

(d) cusp of the second kind.

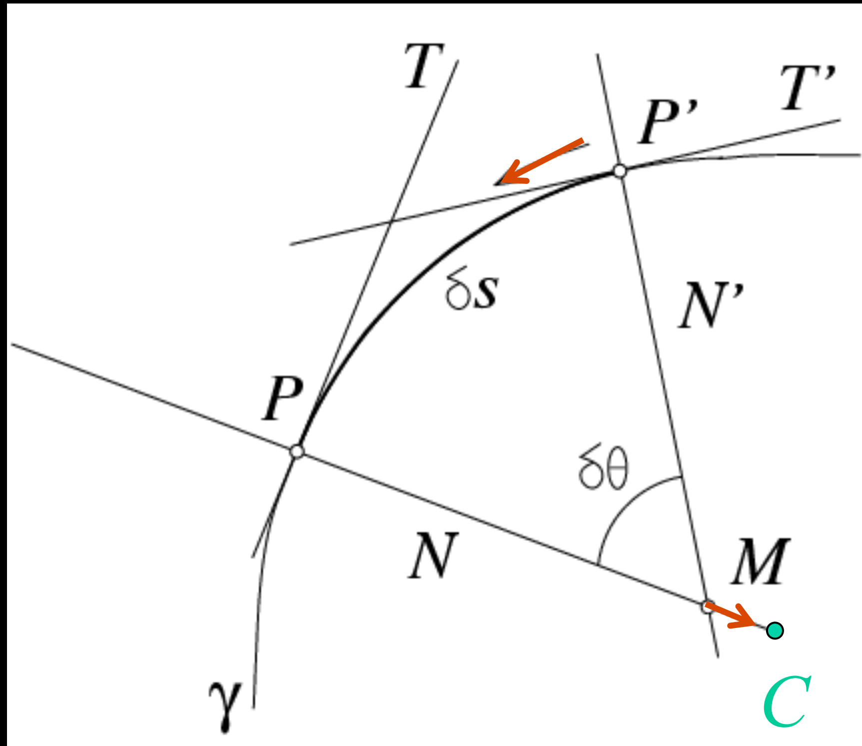
The Gauss Map



- It maps points on a curve onto points on the unit circle.

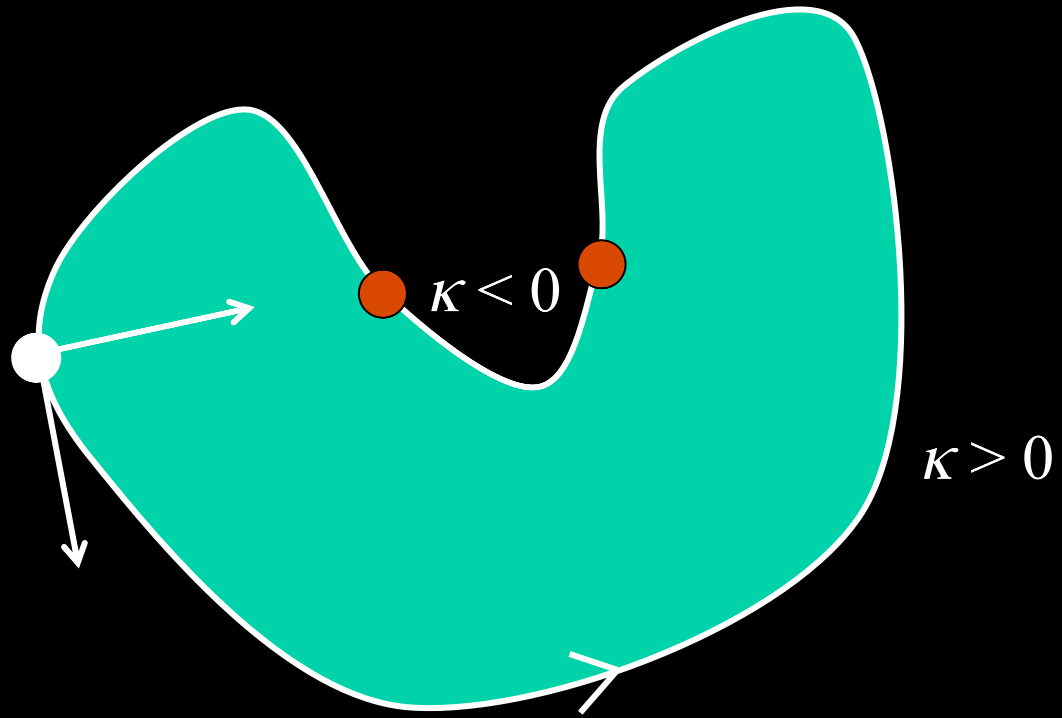
- The direction of traversal of the Gaussian image reverts at inflections: it folds there.

The curvature



- C is the center of curvature;
- $R = CP$ is the radius of curvature;
- $\kappa = \lim \delta\theta/\delta s = 1/R$ is the curvature.

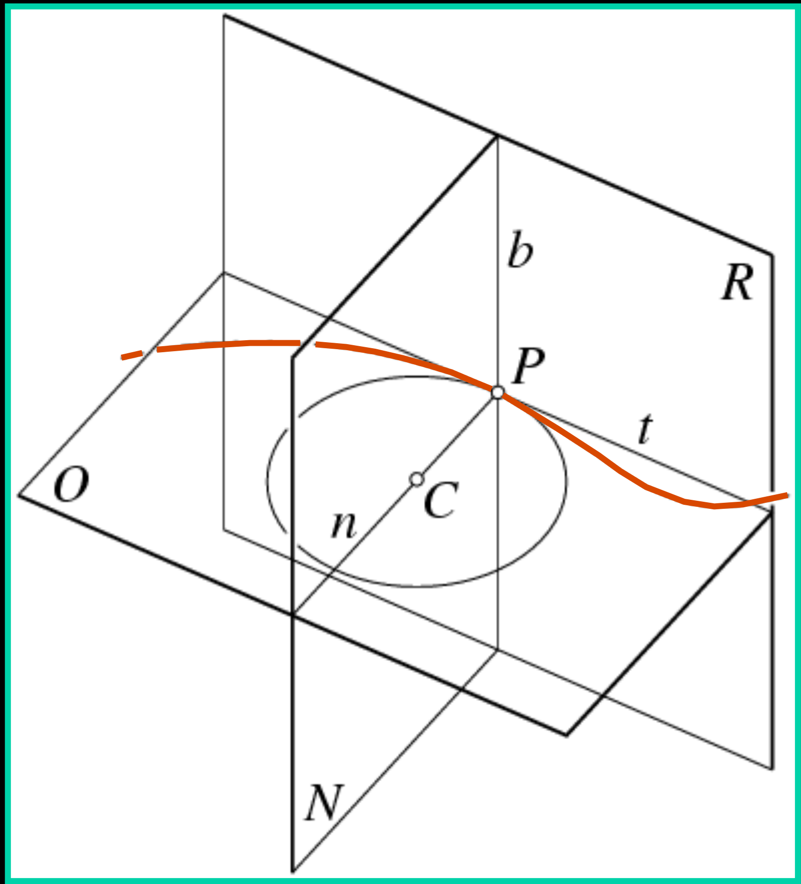
Closed curves admit a canonical orientation..



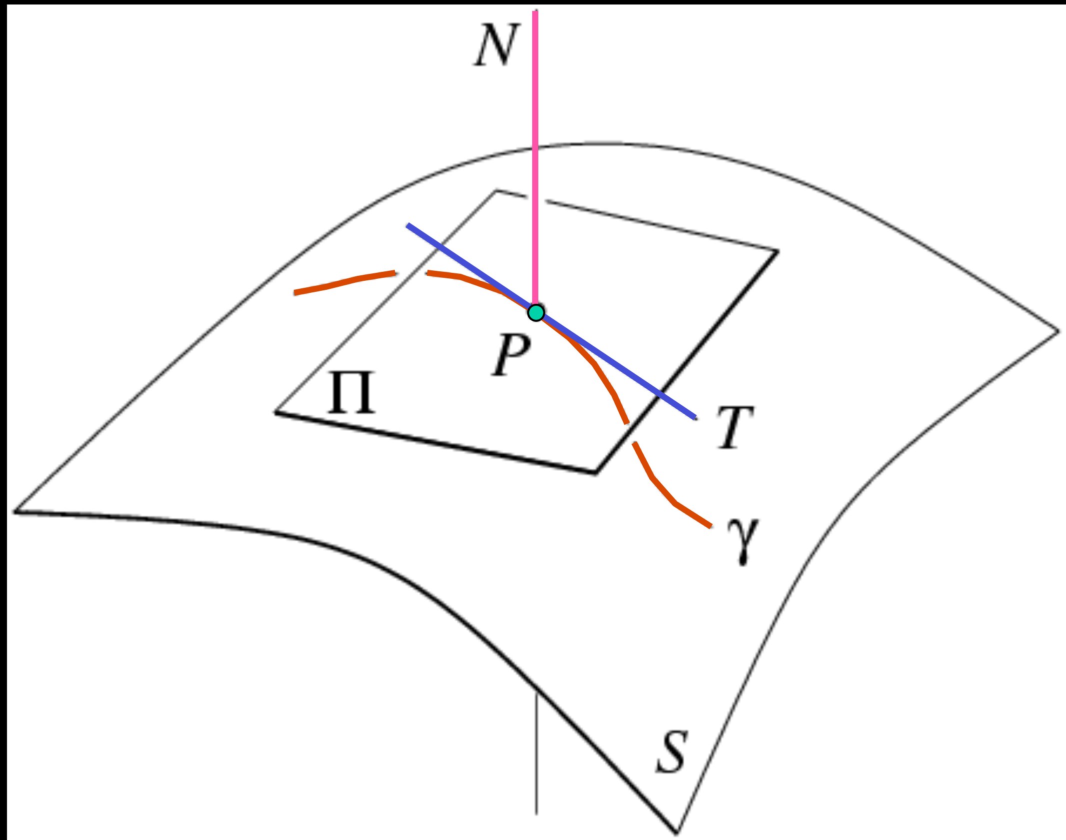
$\kappa = d\theta / ds$ ← derivative of the Gauss map

$d\mathbf{t}/ds = \kappa\, \mathbf{n}$

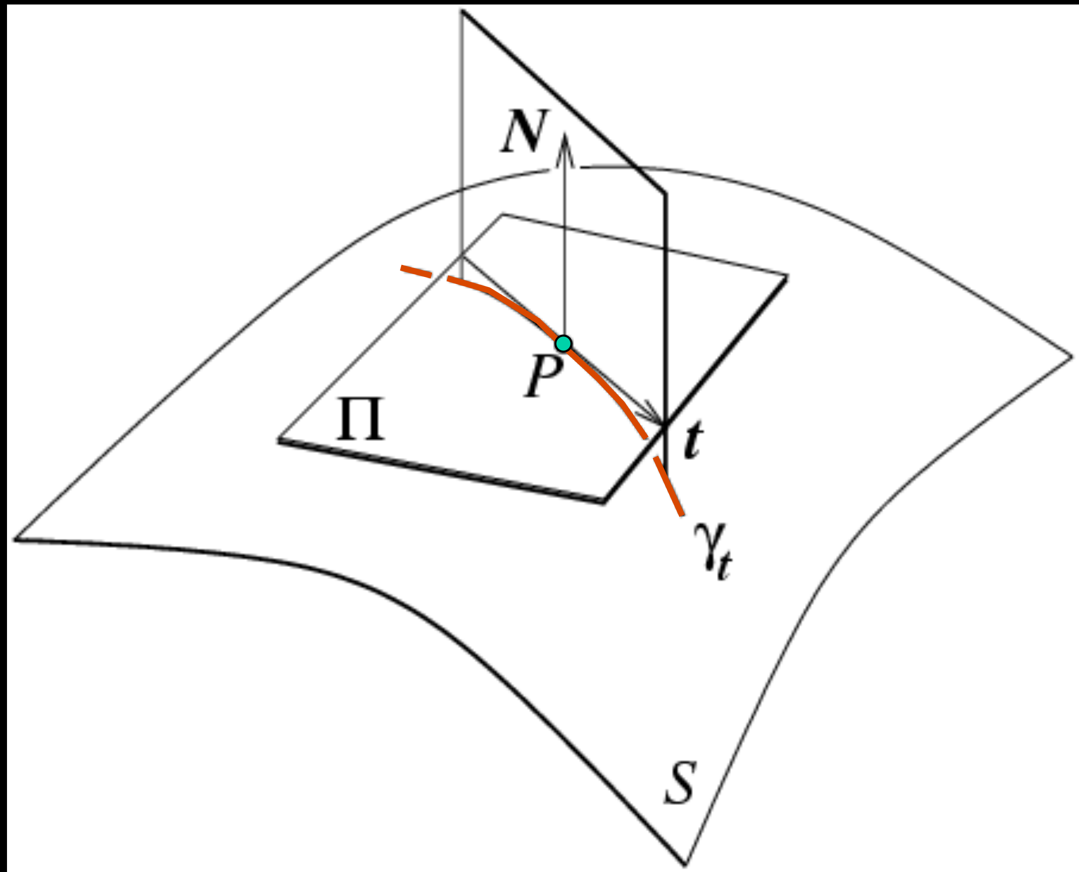
Twisted curves are more complicated animals..



A smooth surface, its tangent plane and its normal.



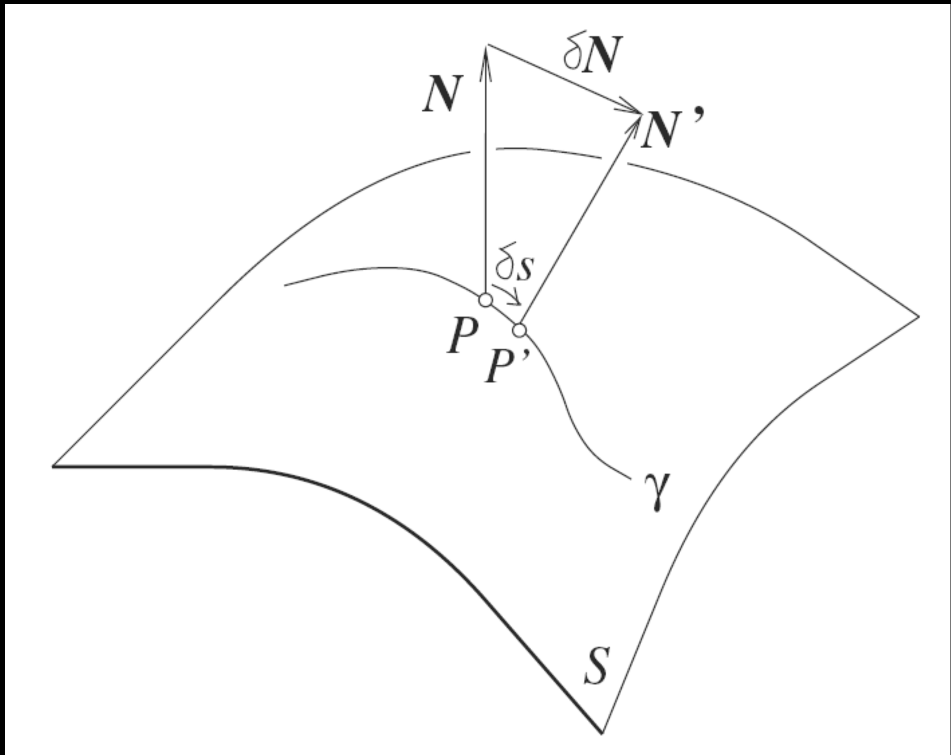
Normal sections and normal curvatures



Principal curvatures:
minimum value κ_1
maximum value κ_2

Gaussian curvature:
$K = \kappa_1 \kappa_2$

The differential of the Gauss map



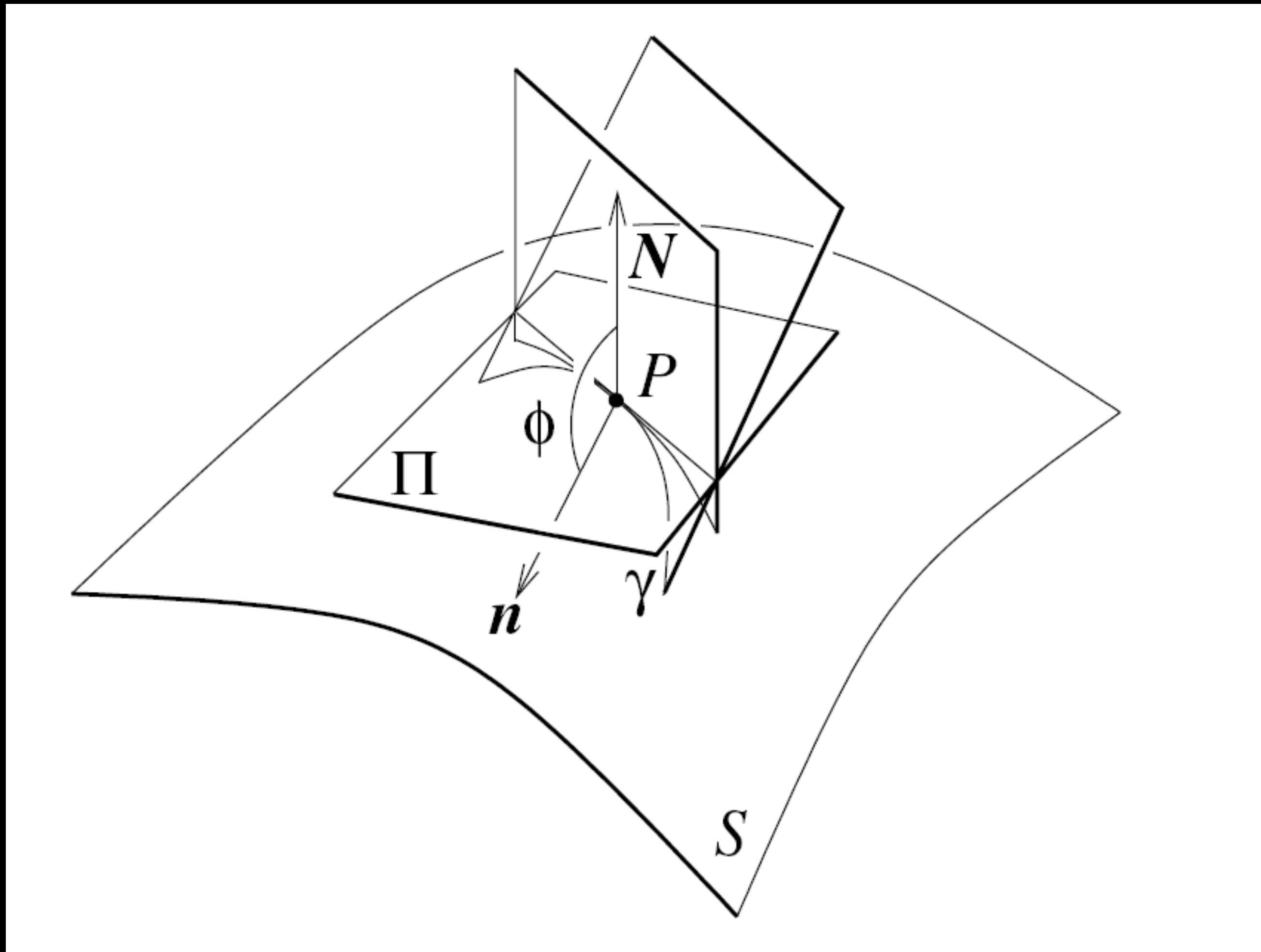
$$dN(t) = \lim_{\delta s \to 0} \frac{1}{\delta s} \delta N$$

Second fundamental form:

$II(u, v) = u^T dN(v)$

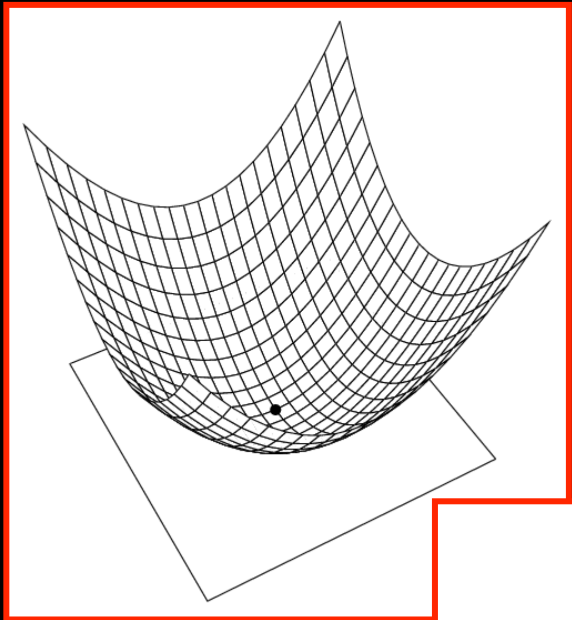
(II is symmetric.)

- The normal curvature is $\kappa_t = II(t, t)$.
- Two directions are said to be conjugated when $II(u, v) = 0$.



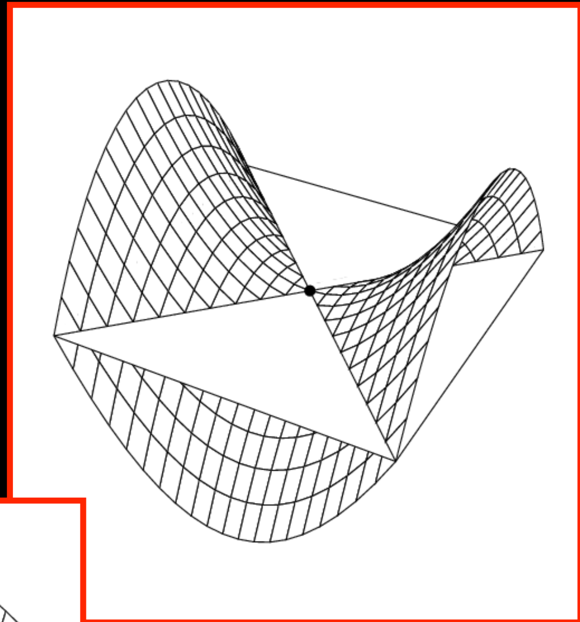
Meusnier's theorem: $\kappa_t = - \kappa \cos\varphi$.

The local shape of a smooth surface



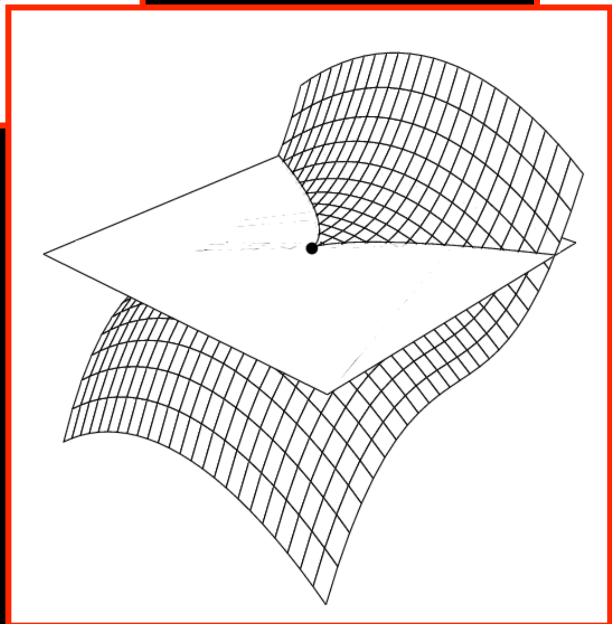
Elliptic point

$K > 0$



Hyperbolic point

$K < 0$



Parabolic point $K = 0$

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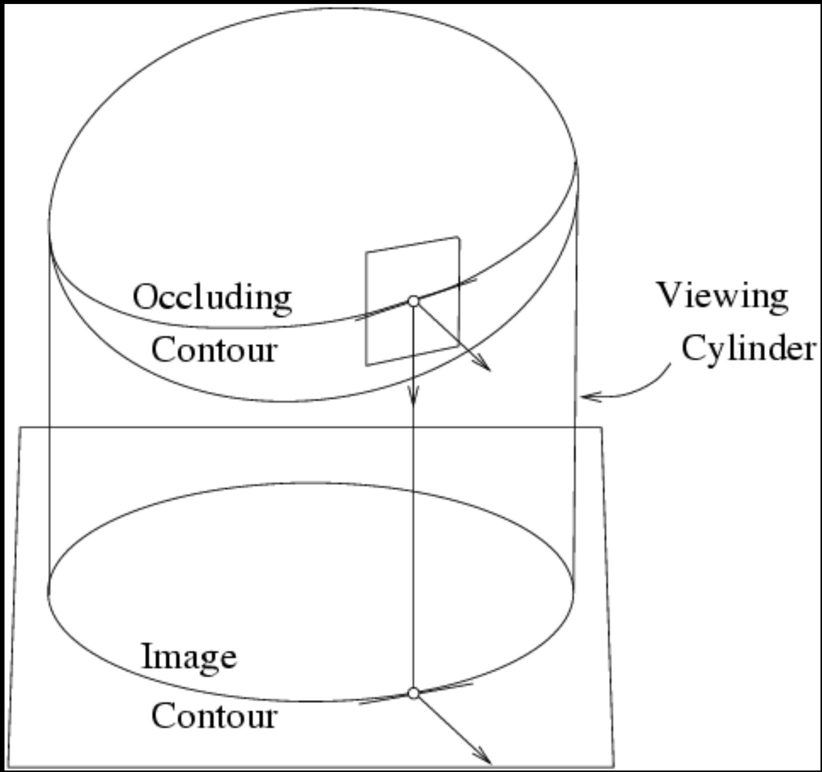
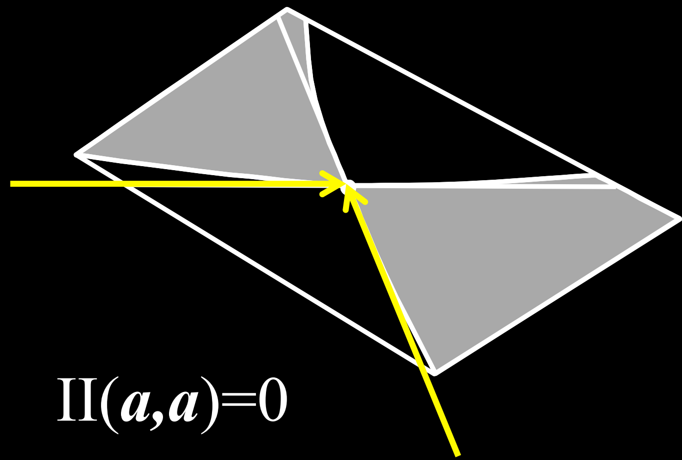
Parabolic lines marked on the Apollo Belvedere by Felix Klein



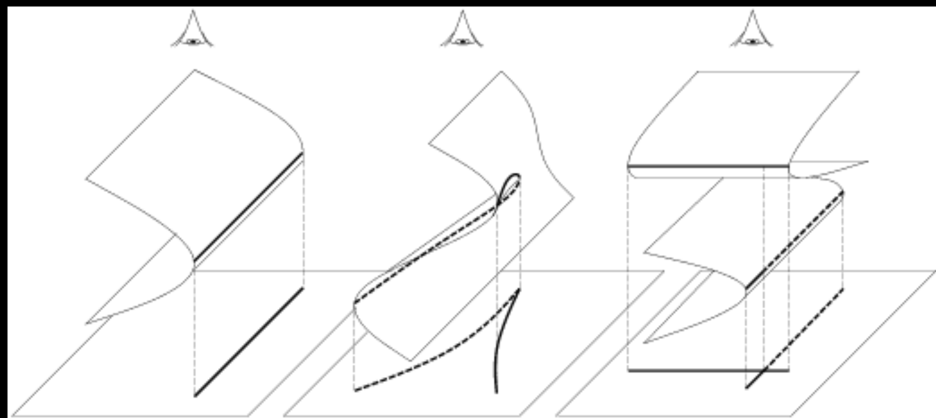
$$N \,.\, v = 0 \Rightarrow \mathrm{II}(\, t \,,\, v\,)=0$$

Asymptotic directions:

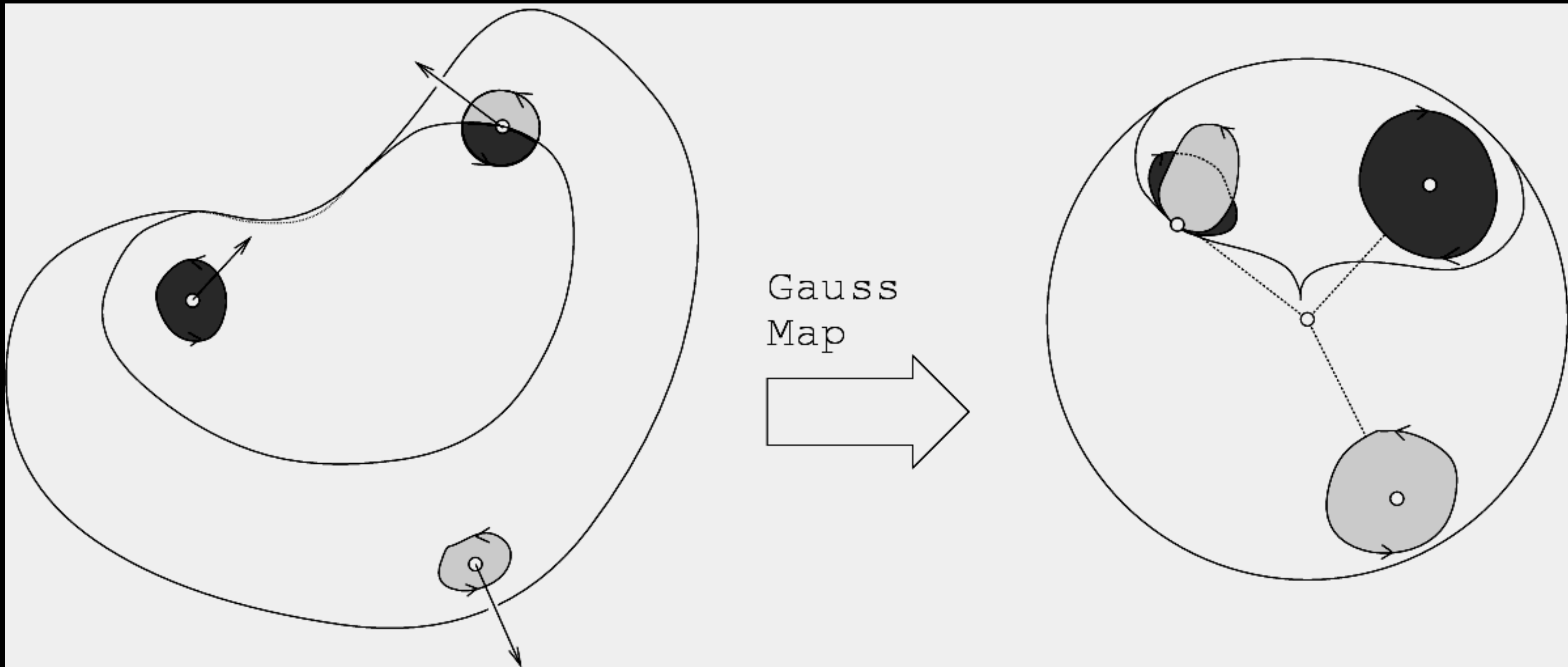


$\mathrm{II}(a,a)=0$

The contour cusps when
when a viewing ray grazes
the surface along an
asymptotic direction $v=a$.



The Gauss map



The Gauss map folds at parabolic points.

$K = dA'/dA$

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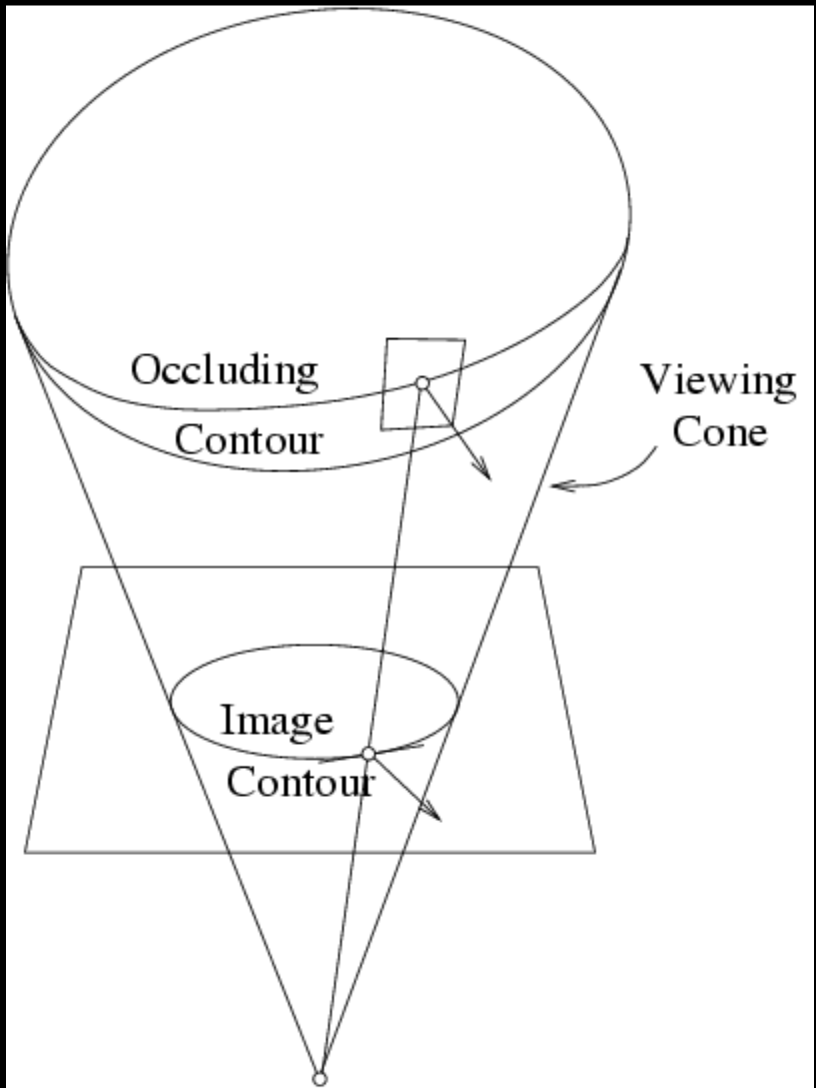
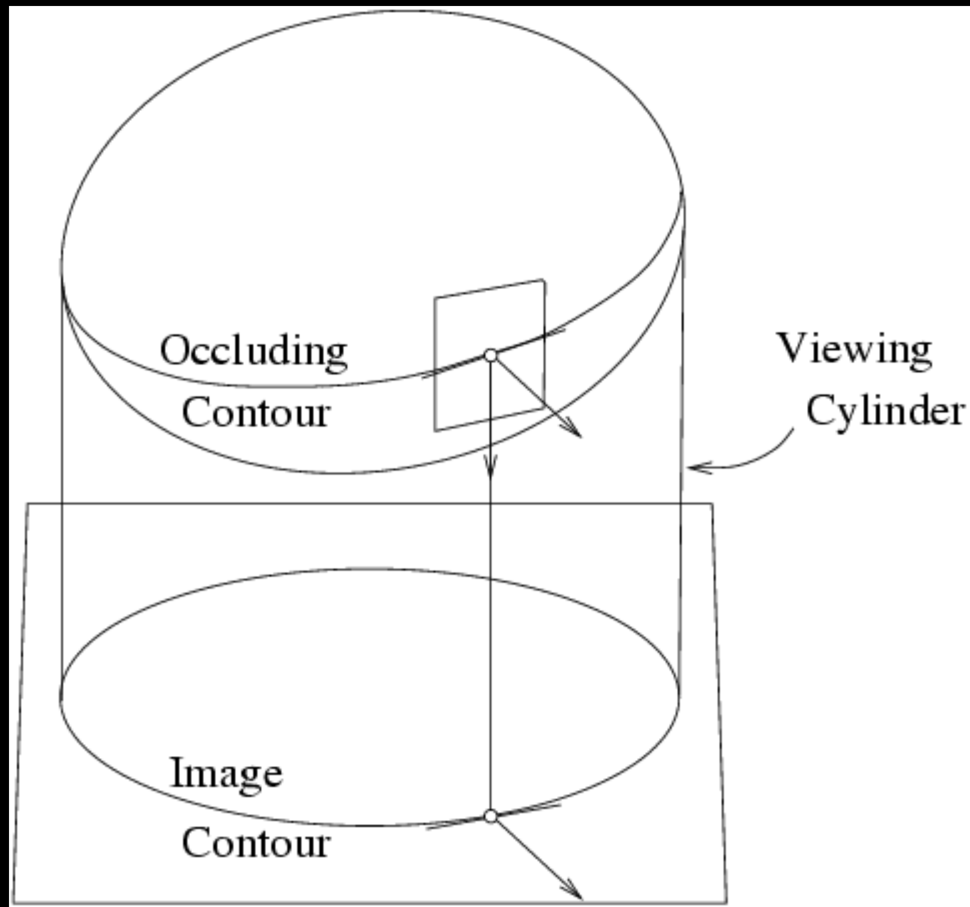
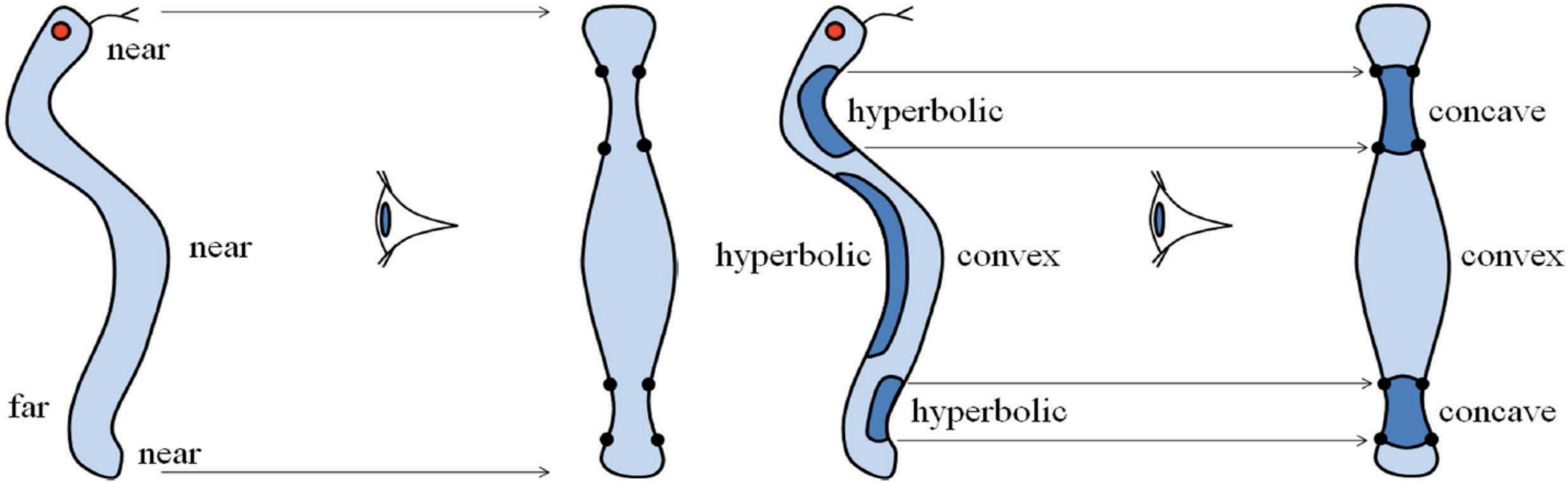
Smooth Shapes and their Outlines

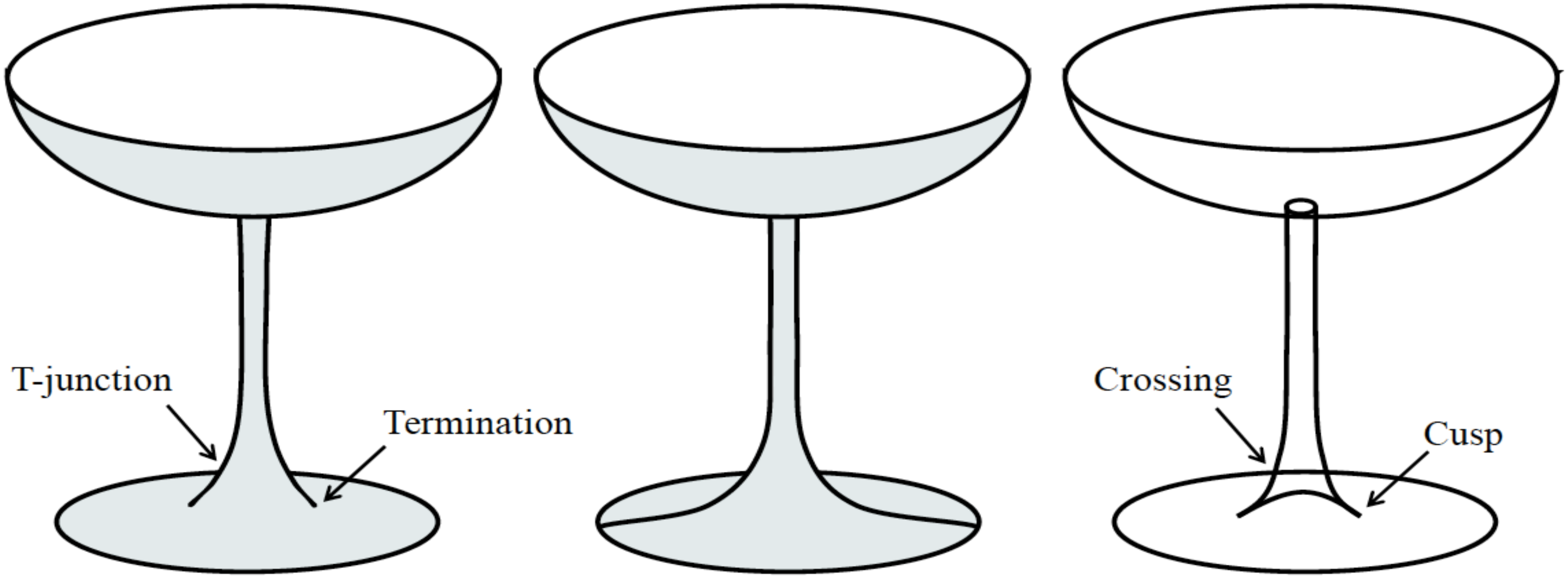


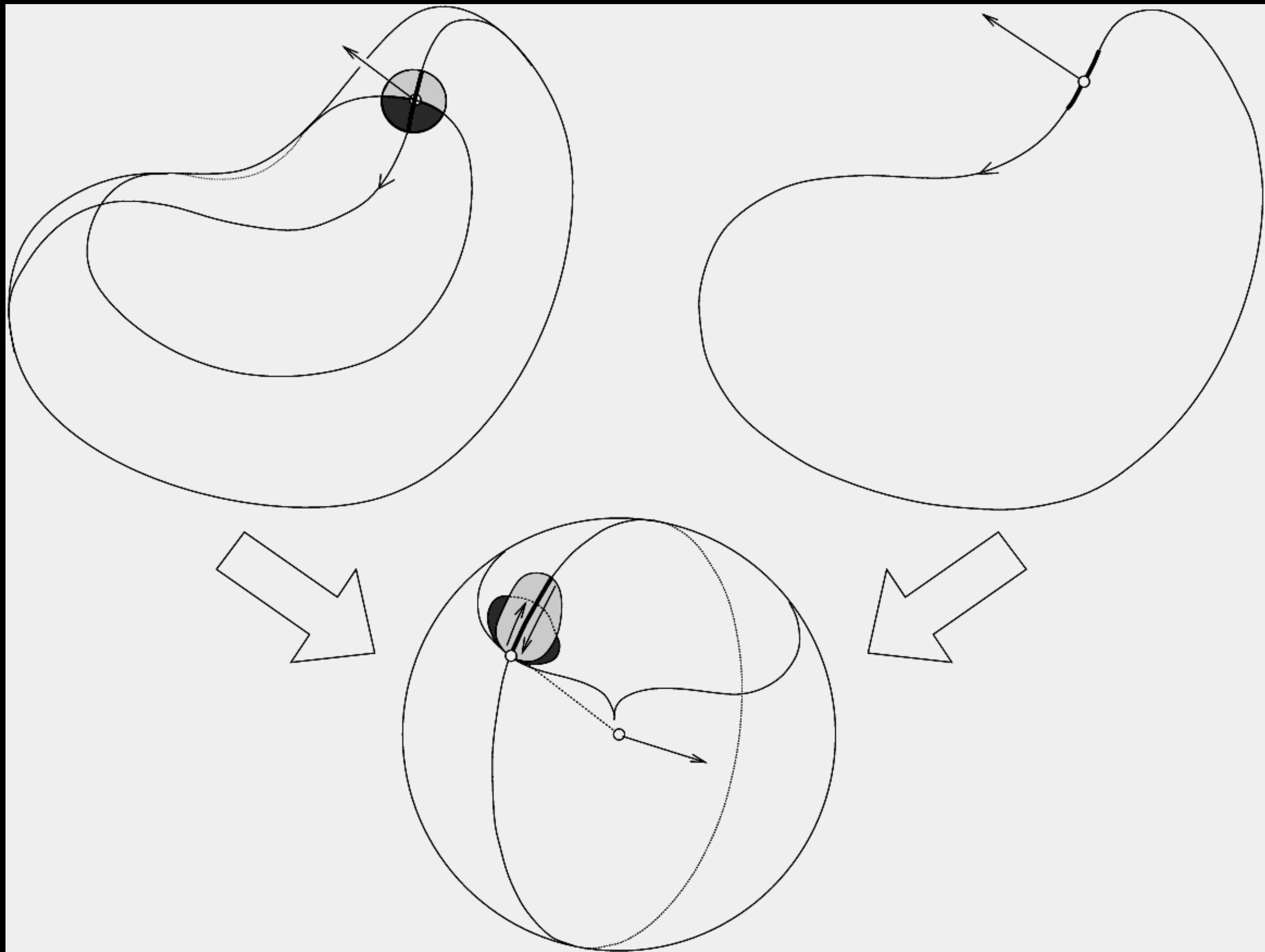


Can we say anything about a 3D shape from the shape of its contour?



After Marr (1977) and Koenderink (1984).





Theorem [Koenderink, 1984]: the inflections of the silhouette are the projections of parabolic points.

Koenderink's Theorem (1984)

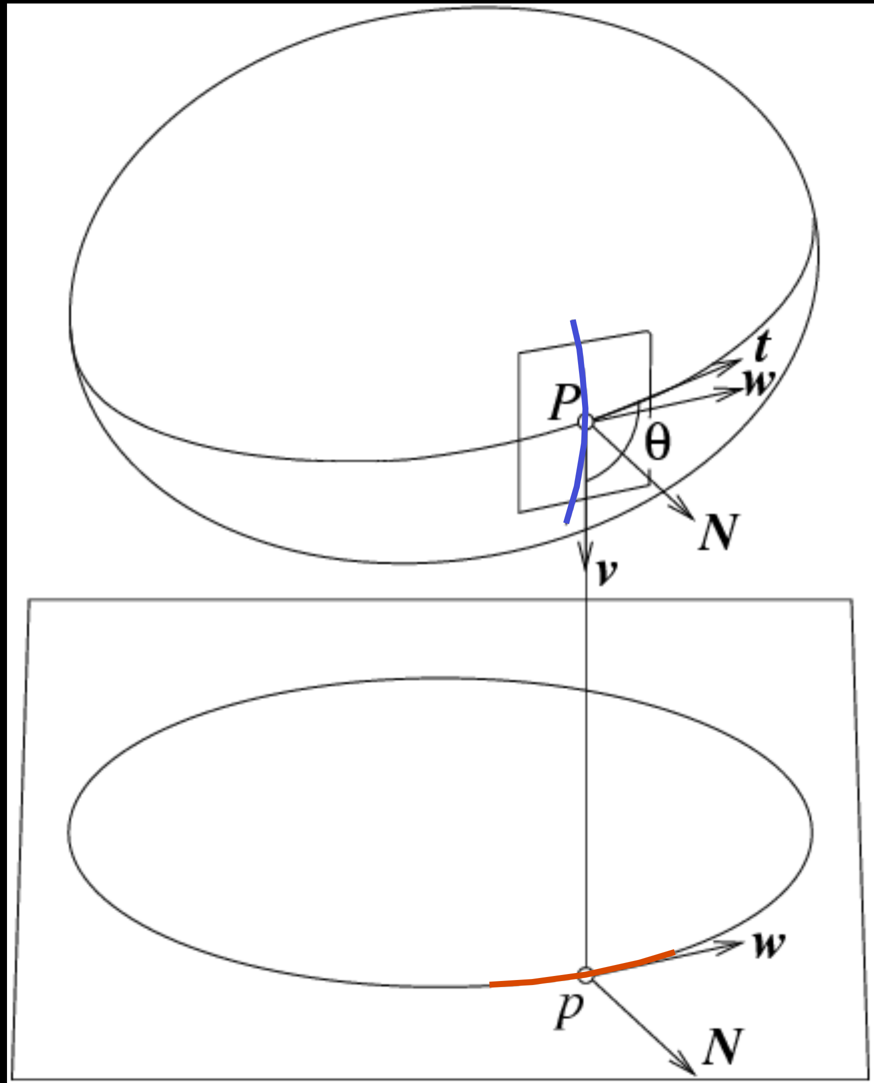
$$K = \kappa_r \kappa_c$$

Note: $\kappa_r > 0.$

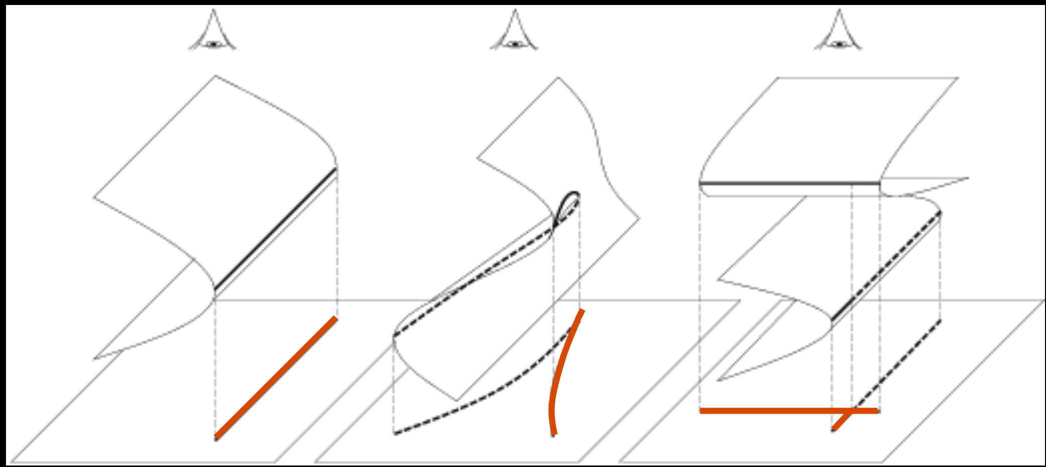
Corollary: K and κ_c have the same sign!

Proof: Based on the idea that, given two conjugated directions,

$$K \sin^2\theta = \kappa_u \kappa_v$$



What are the contour stable features??



folds cusps T-junctions

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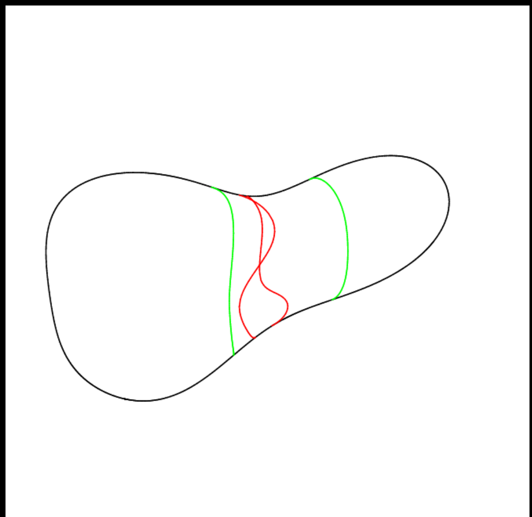
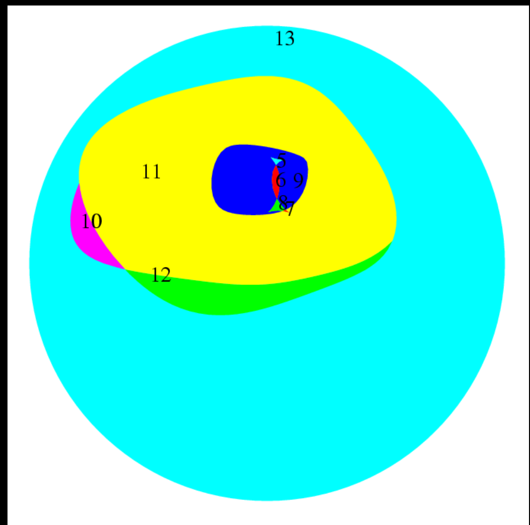
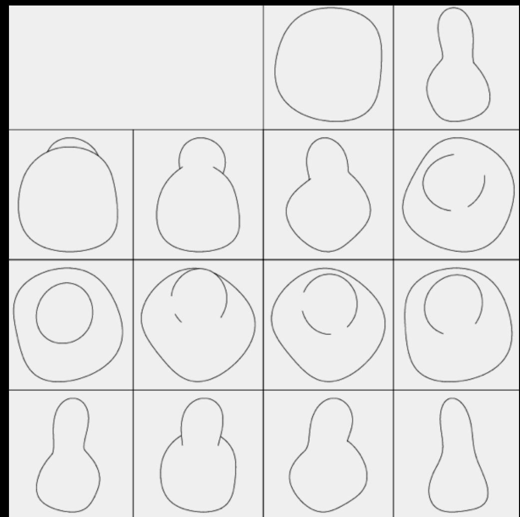
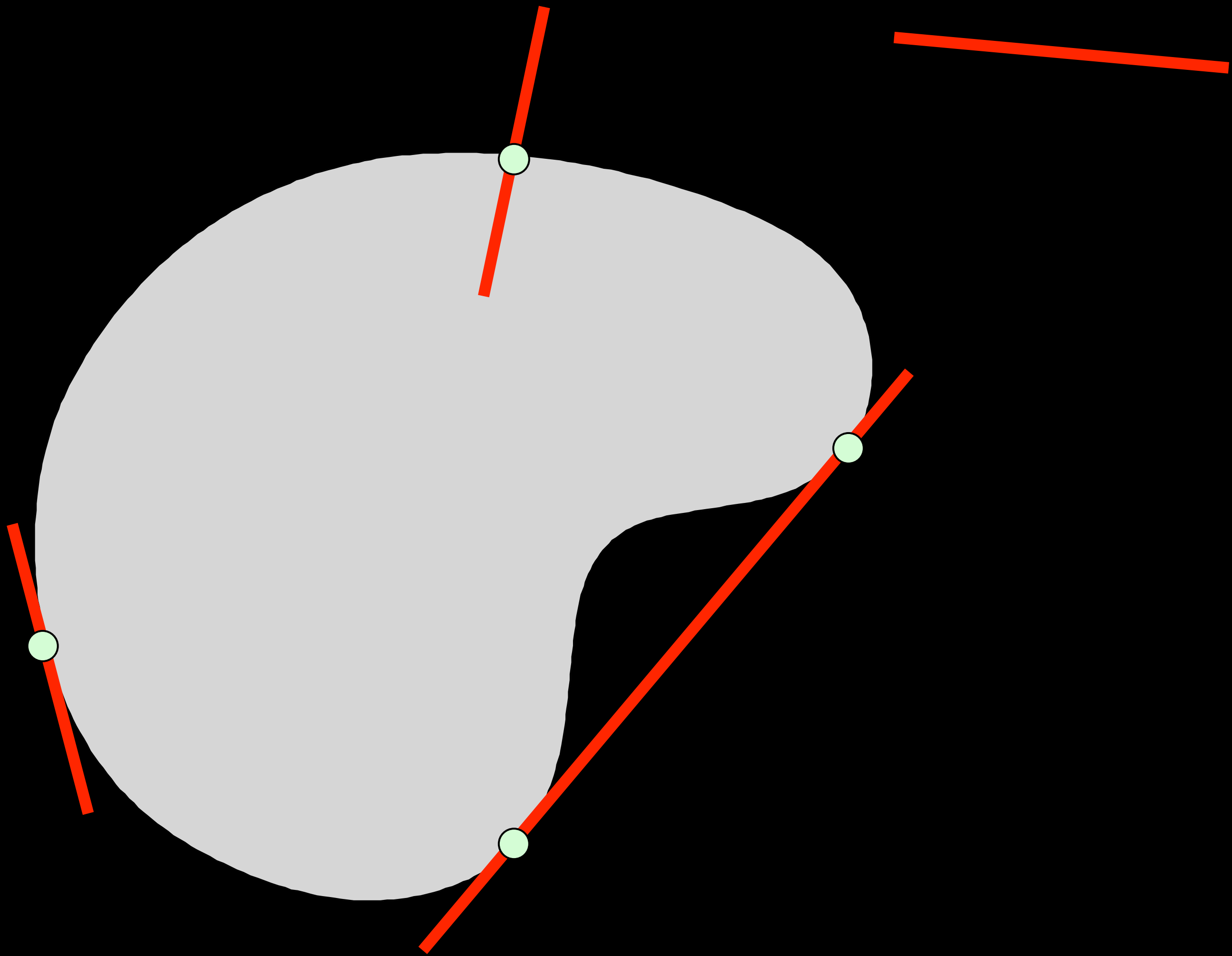
How does the appearance of an object change with viewpoint?



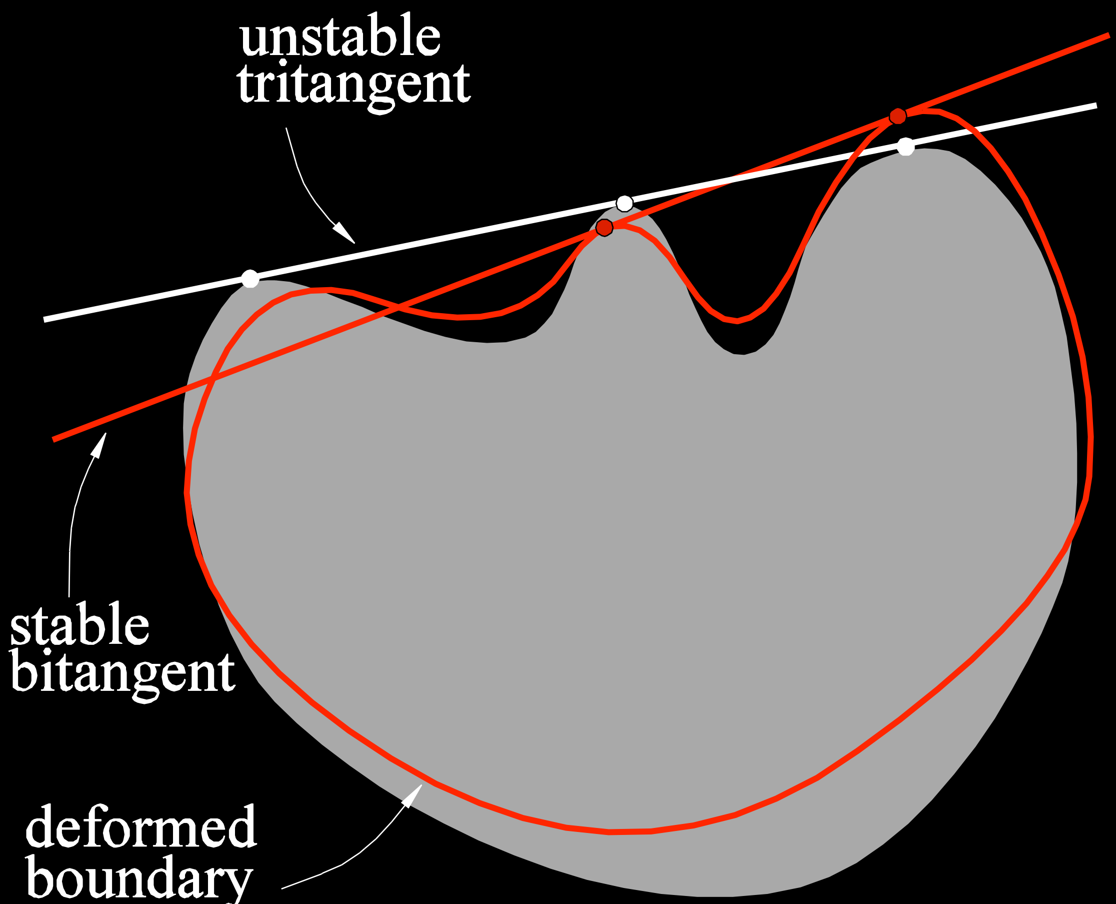




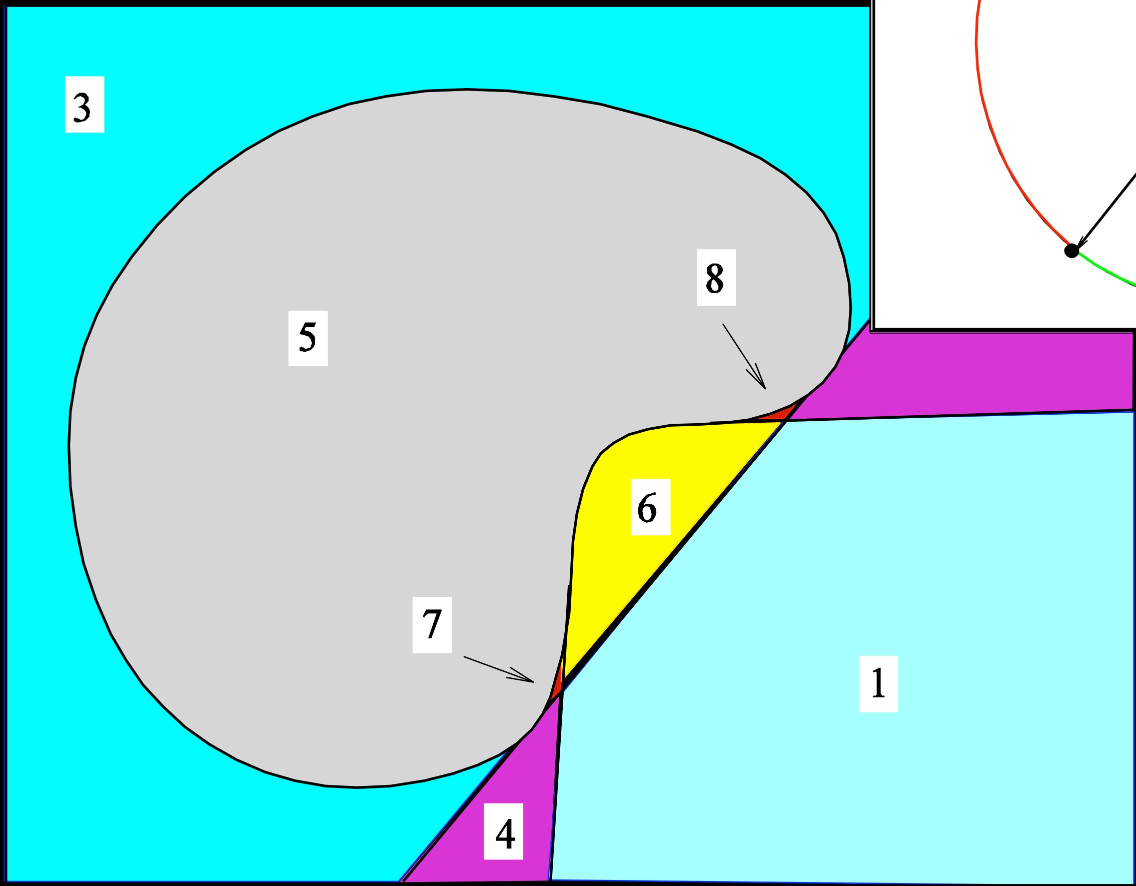
Contacts between lines and smooth curves

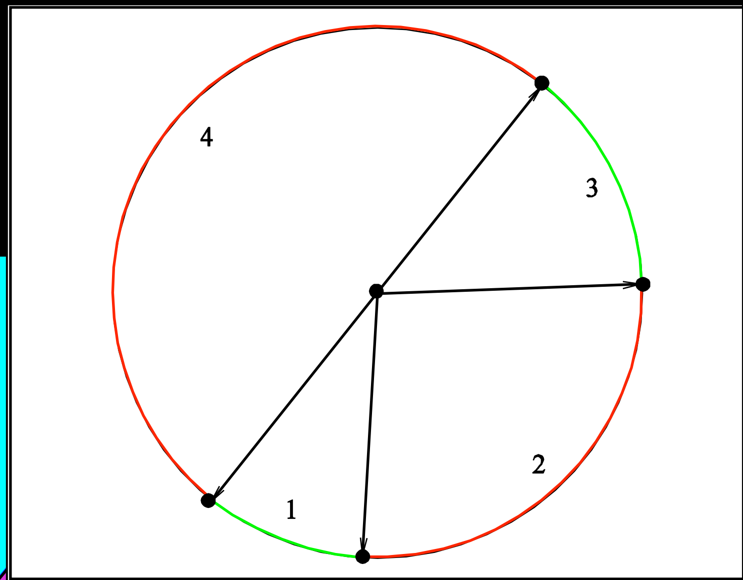


Exceptional and Generic Curves



The Aspect Graph
In Flatland





The Geometry of the Gauss Map

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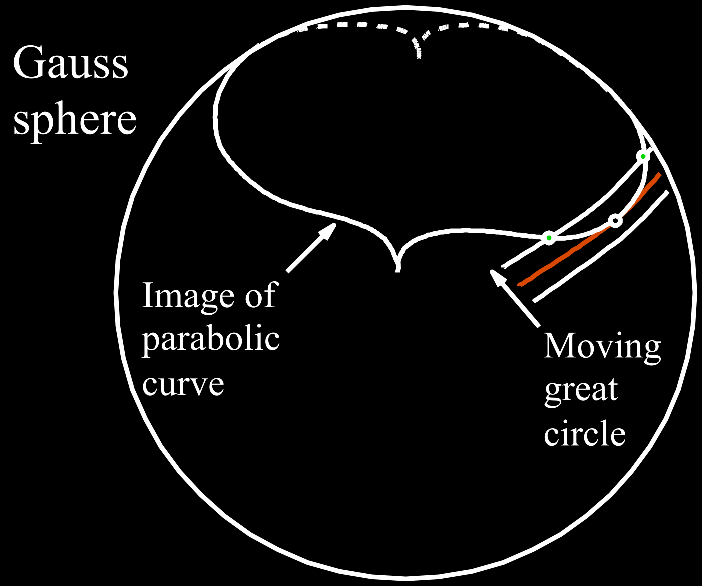
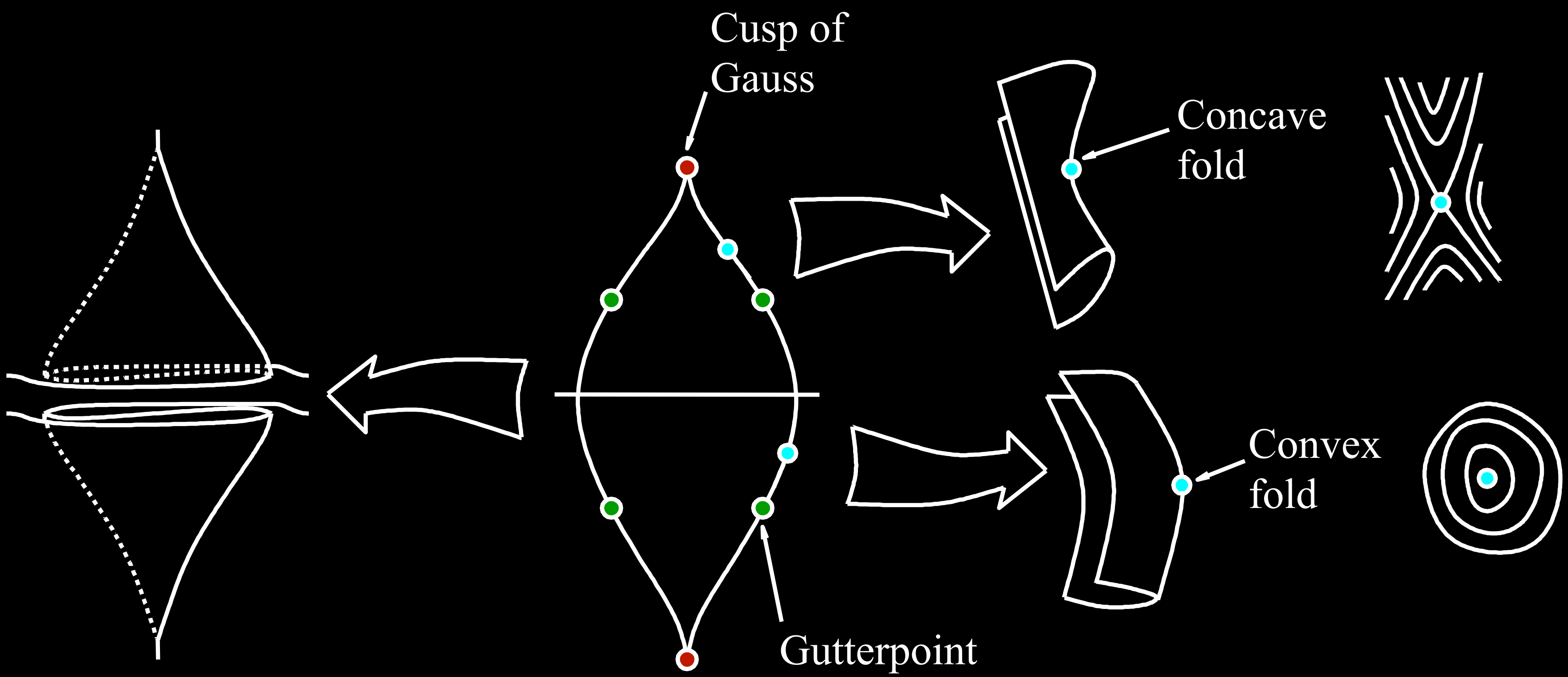
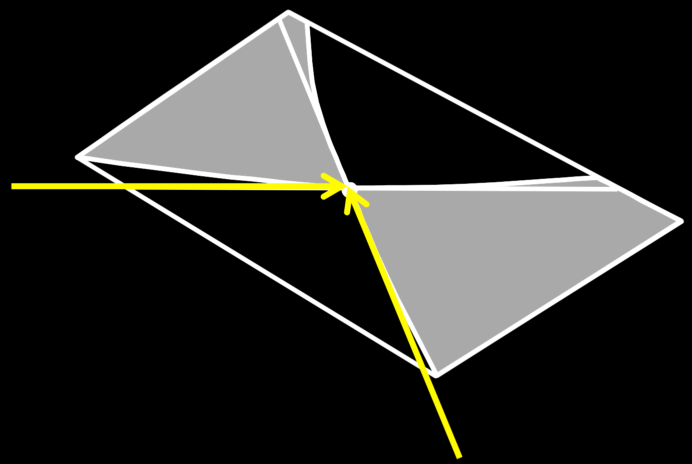
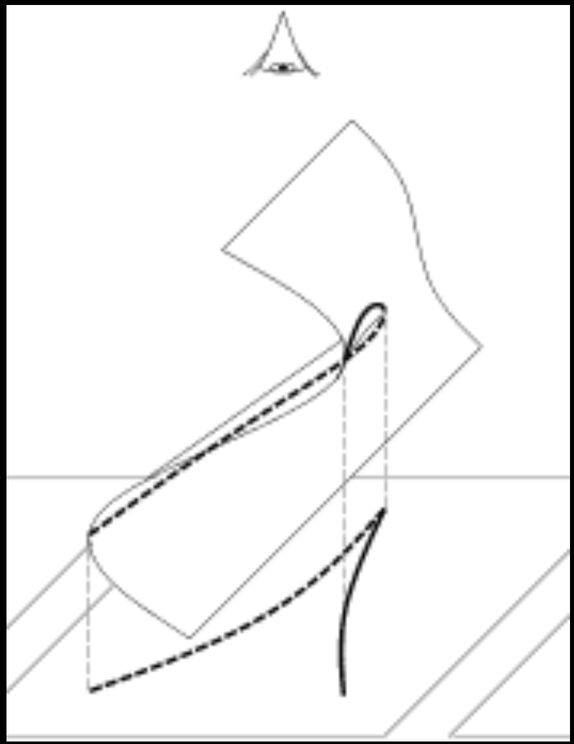
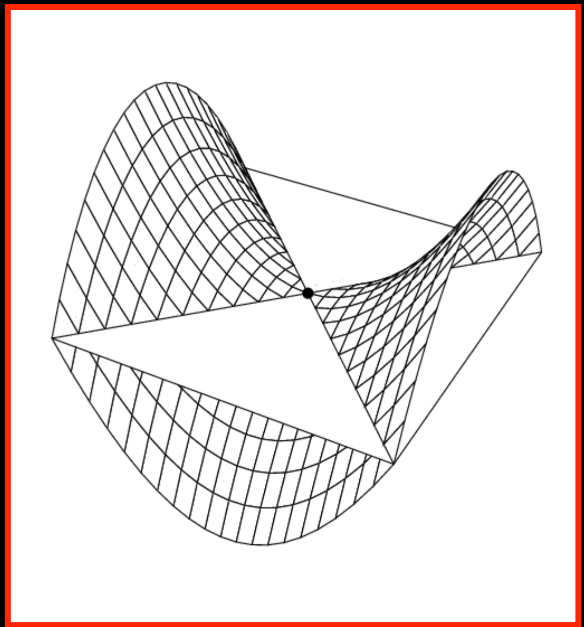




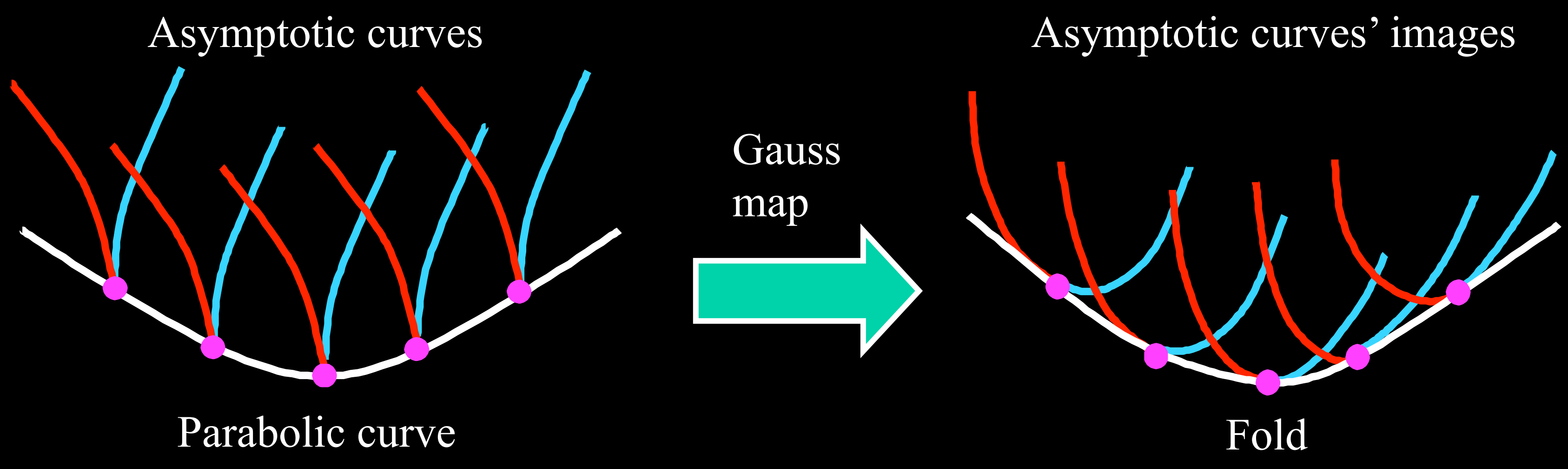
Asymptotic directions at ordinary hyperbolic points







The integral curves of the asymptotic directions form two families of asymptotic curves (red and blue)



- Asymptotic directions are self conjugate: $\boldsymbol{a} \,.\, dN(\boldsymbol{a}) = 0$

- At a parabolic point $dN(\boldsymbol{a}) = 0$, so for any curve

  $\boldsymbol{t} \,.\, \mathrm{d}N(\boldsymbol{a}) = \boldsymbol{a} \,.\, dN(\boldsymbol{t}) = 0$

- In particular, the Gaussian images of the asymptotic and parabolic curves are both orthogonal to $\boldsymbol{a}$.

The Geometry of the Gauss Map

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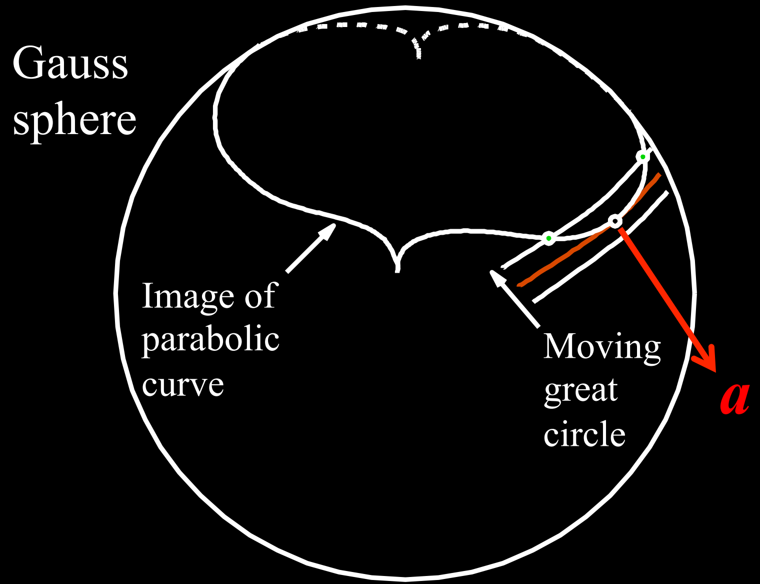
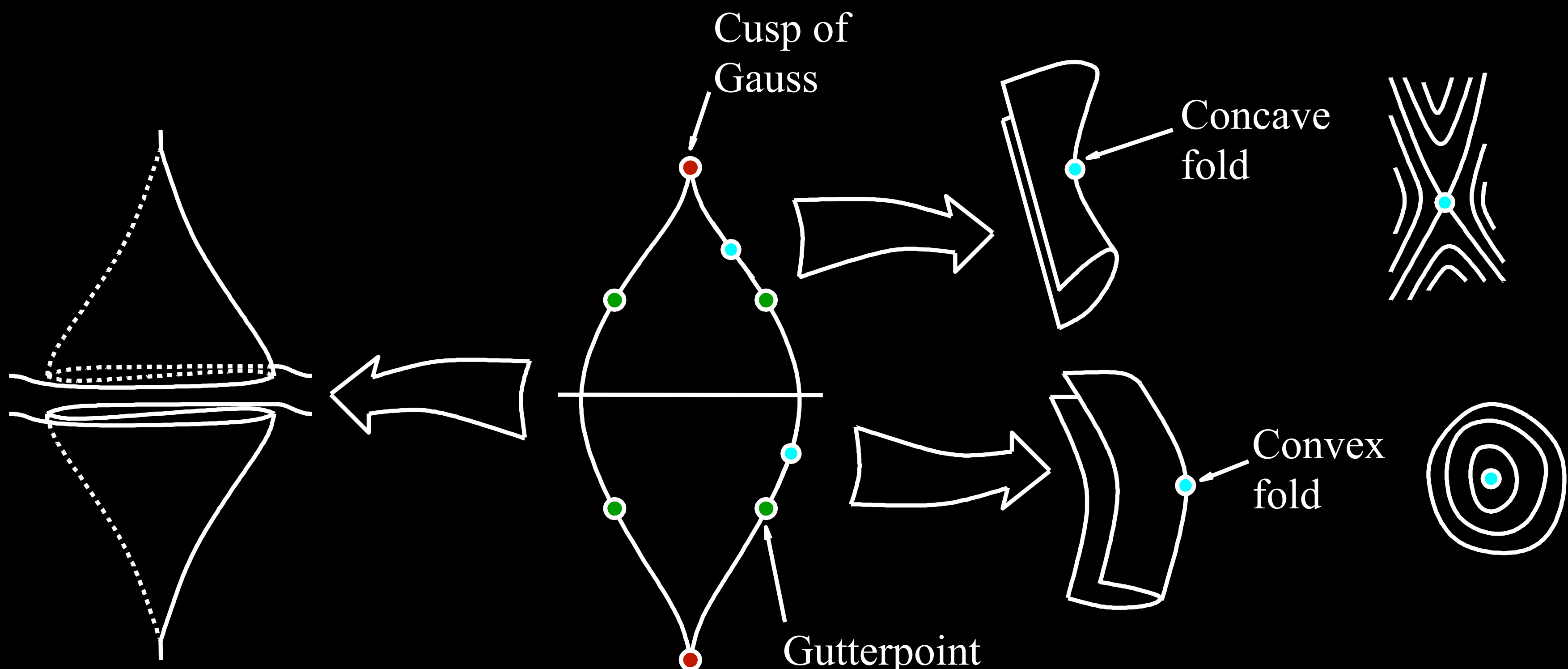
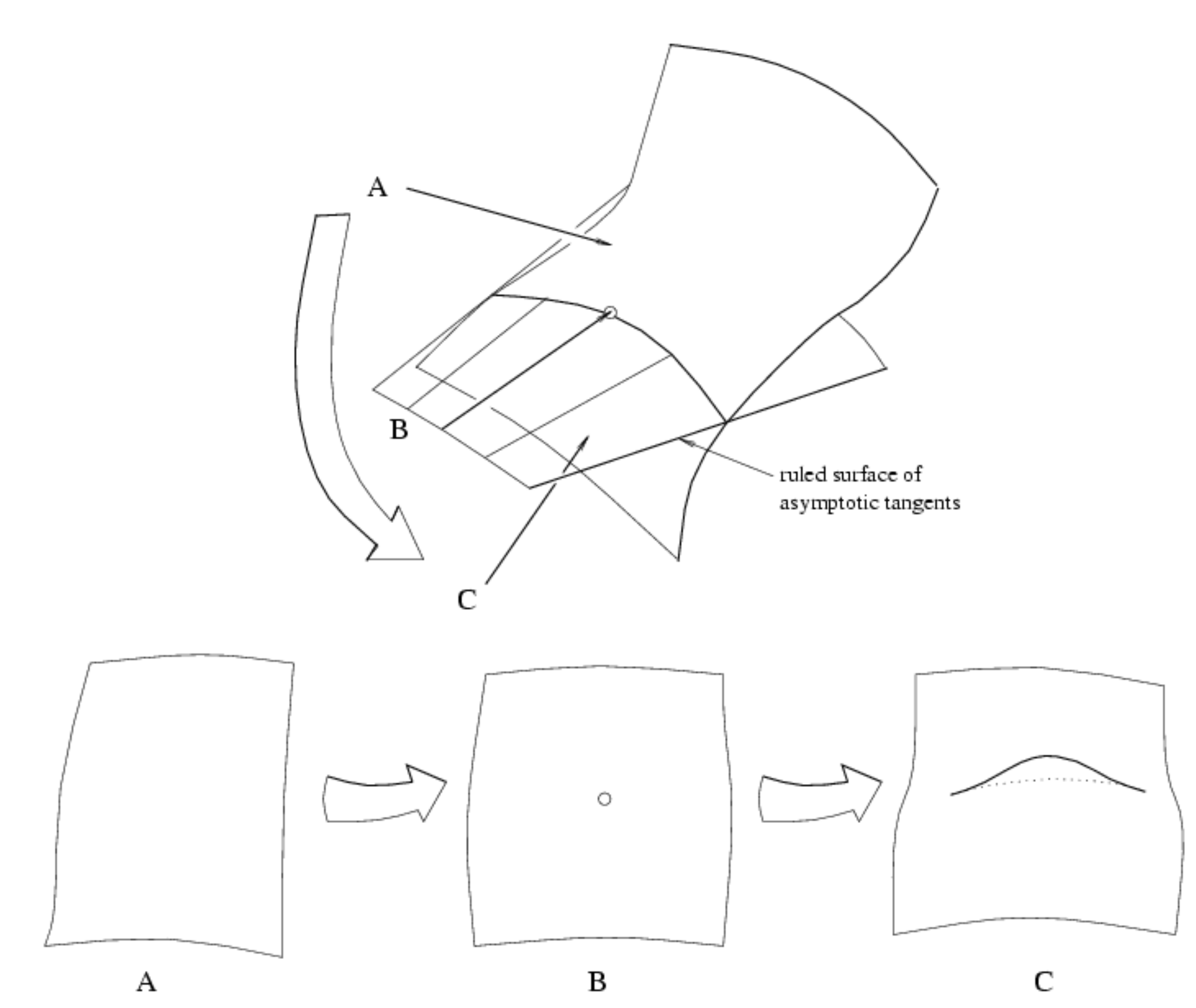




The Lip Event

$$\boldsymbol{v} \,.\, d\boldsymbol{N}\,(\boldsymbol{a}) = 0 \Rightarrow \boldsymbol{v} \approx \boldsymbol{a}$$



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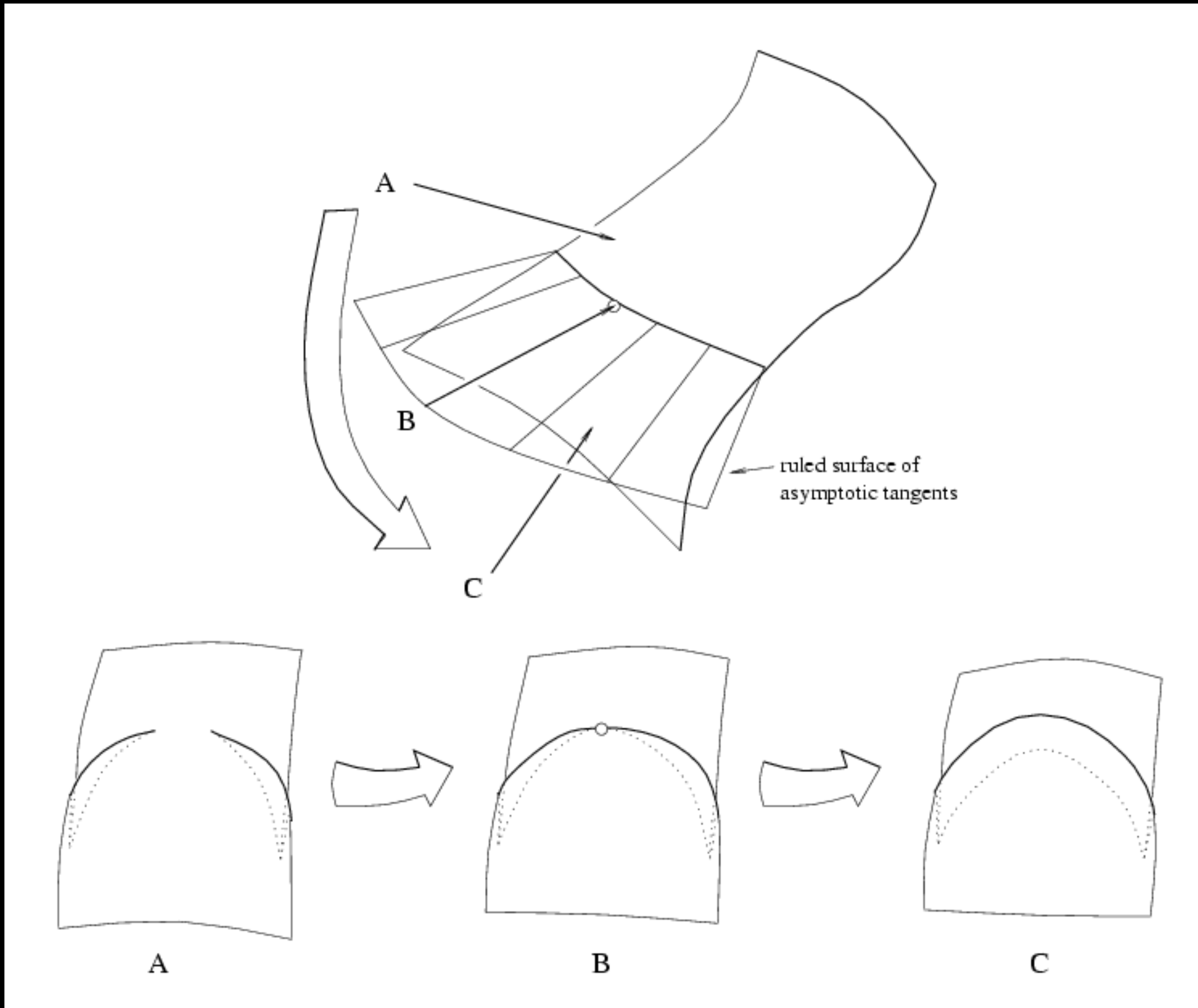
The Beak-to-Beak Event

$$\boldsymbol{v} \cdot dN(\boldsymbol{a}) = 0 \Rightarrow \boldsymbol{v} \approx \boldsymbol{a}$$



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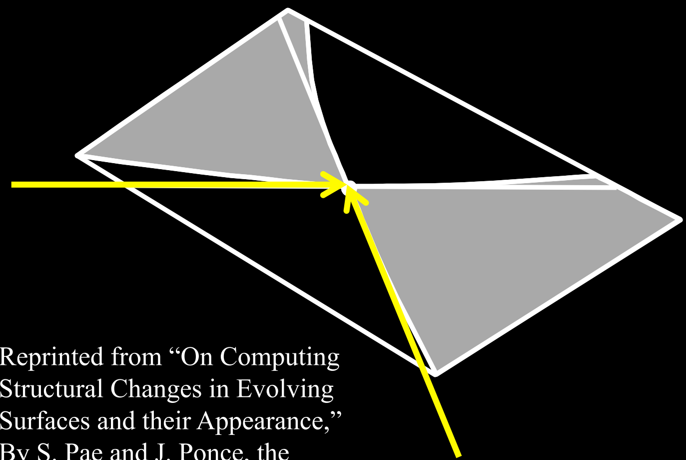
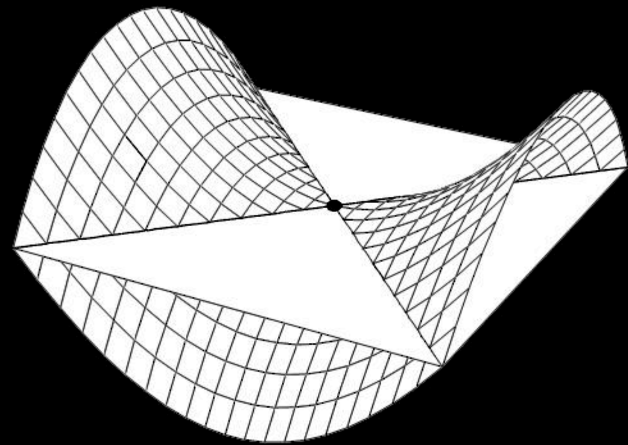
Ordinary Hyperbolic Point





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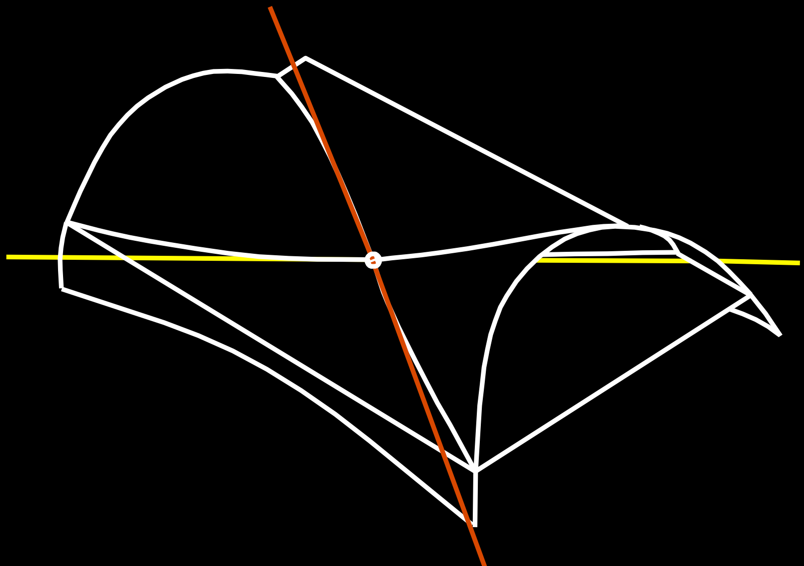
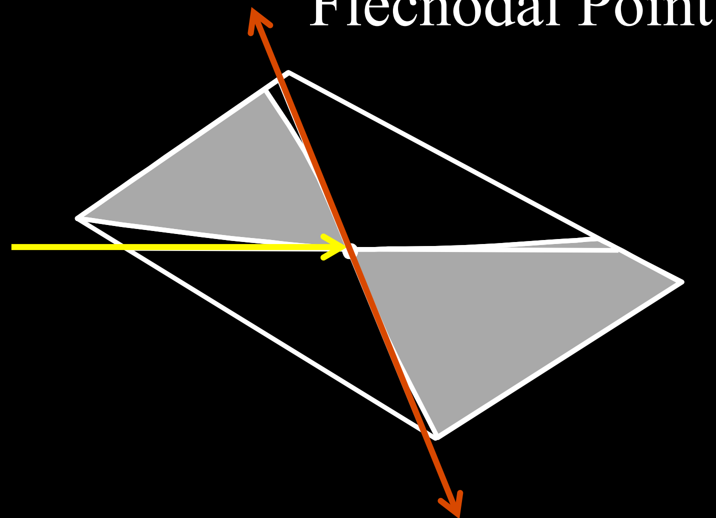
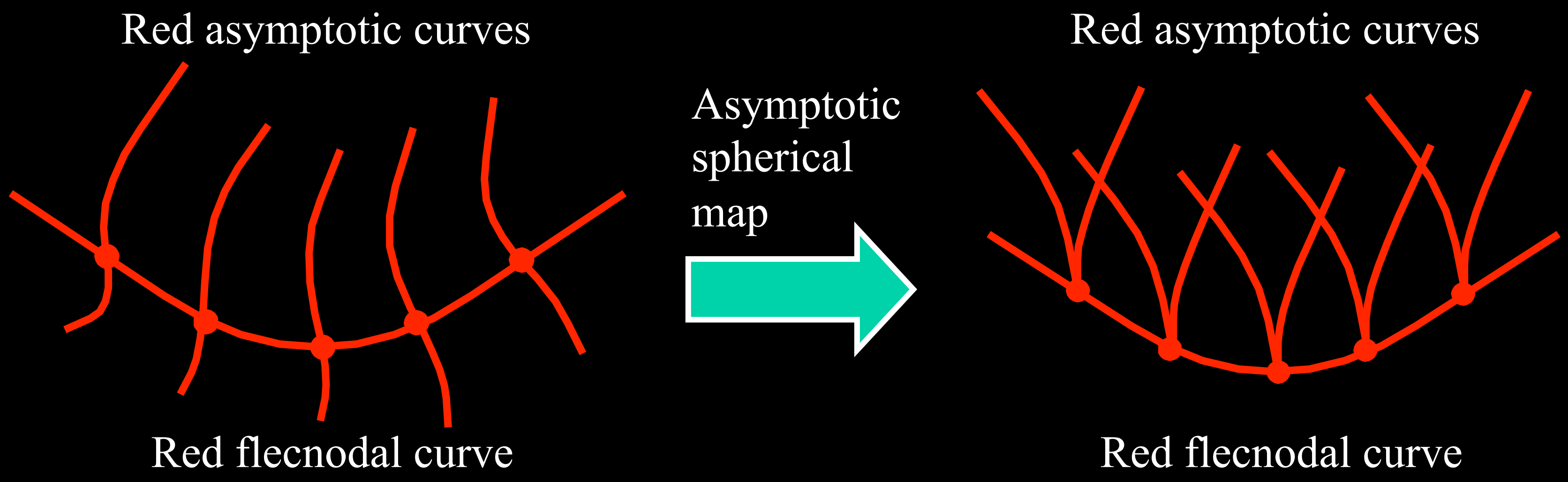


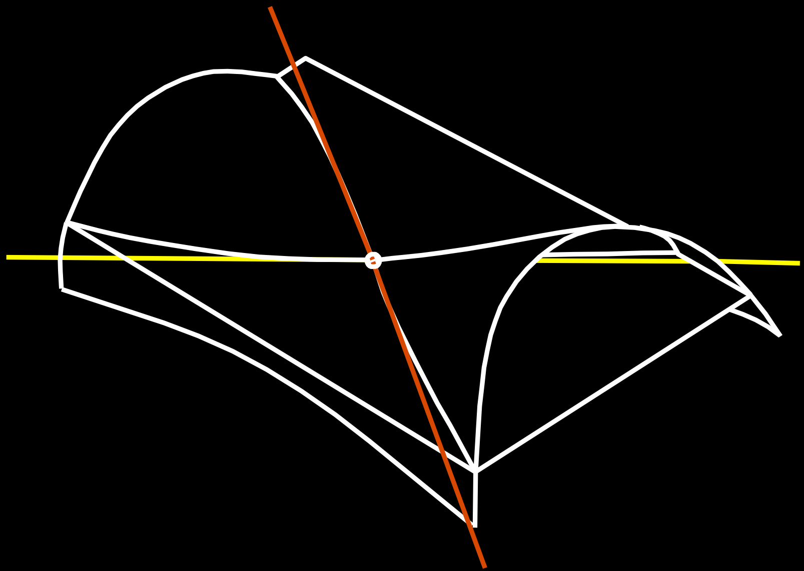
Flecnodal Point

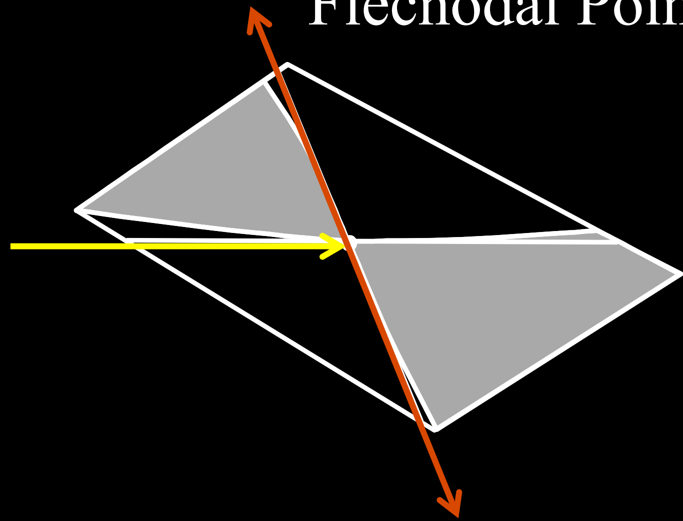


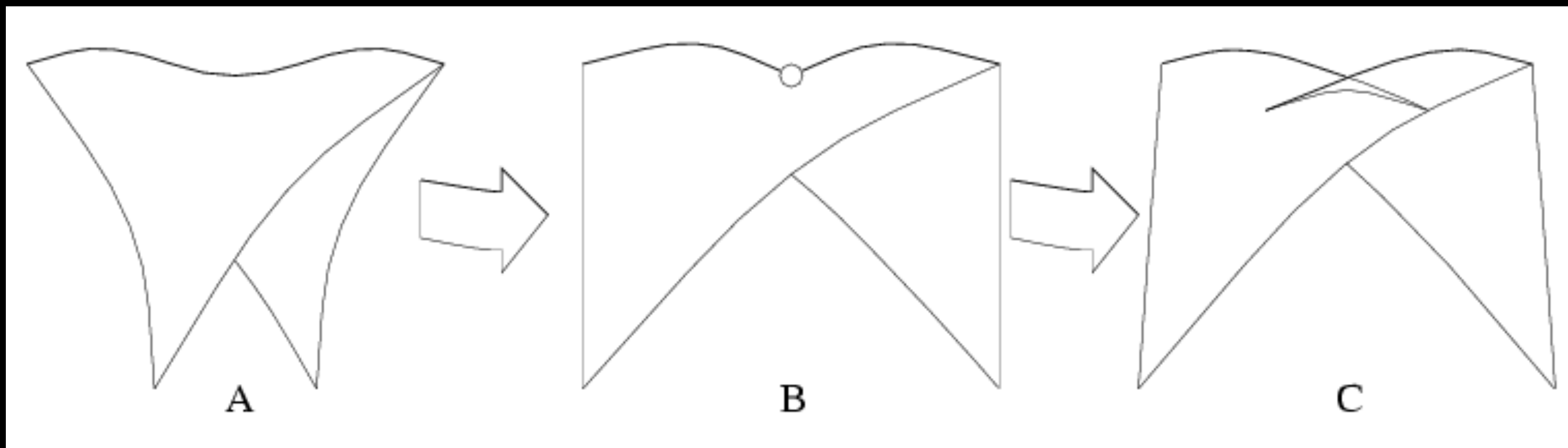


Cusp pairs appear or disappear as one crosses the fold of the asymptotic spherical map.
This happens at asymptotic directions along parabolic curves, and asymptotic directions along flecnodal curves.

The Swallowtail Event







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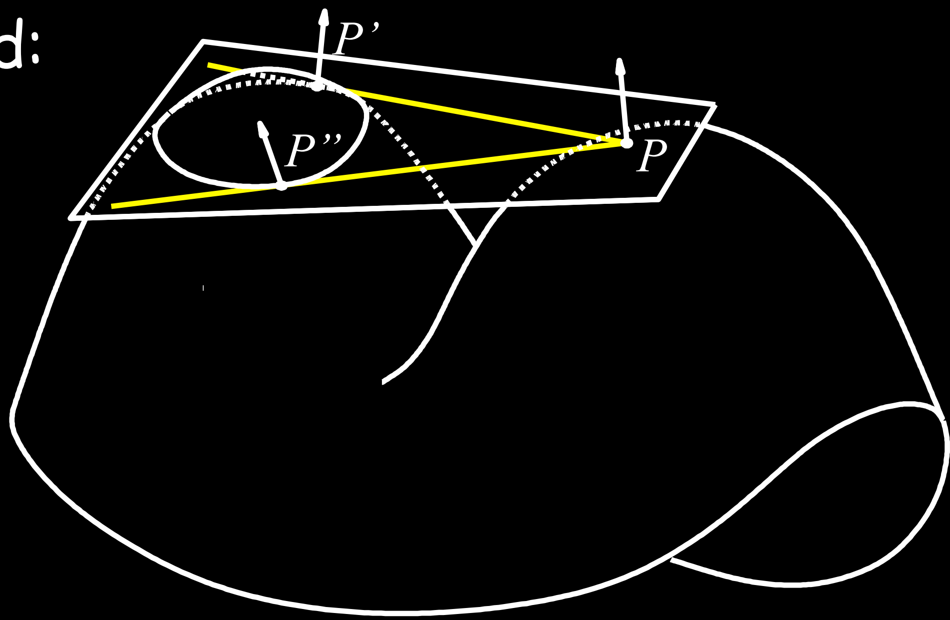
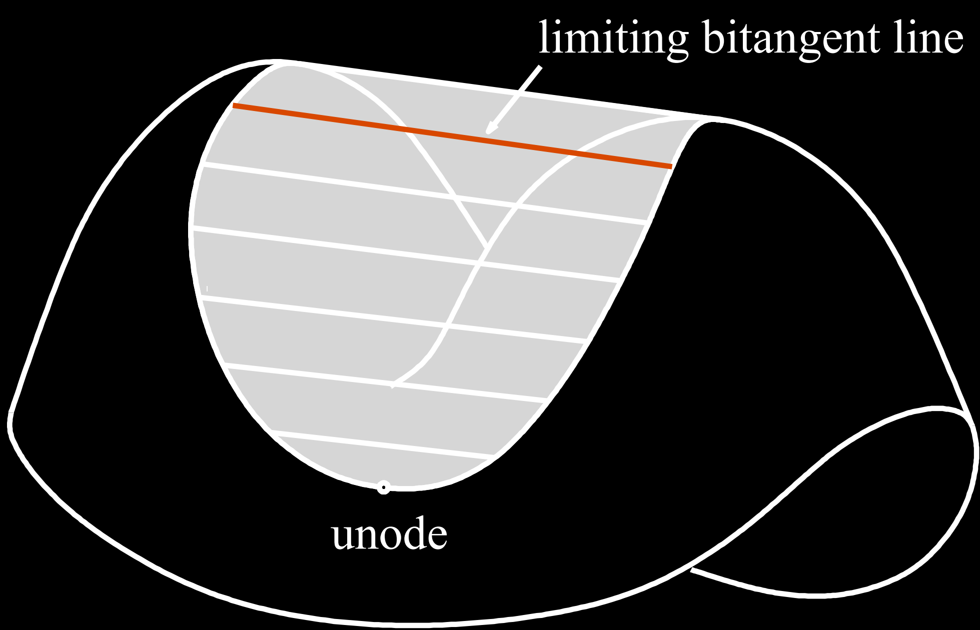
The Bitangent Ray Manifold:

Ordinary bitangents..





..and exceptional (limiting) ones.

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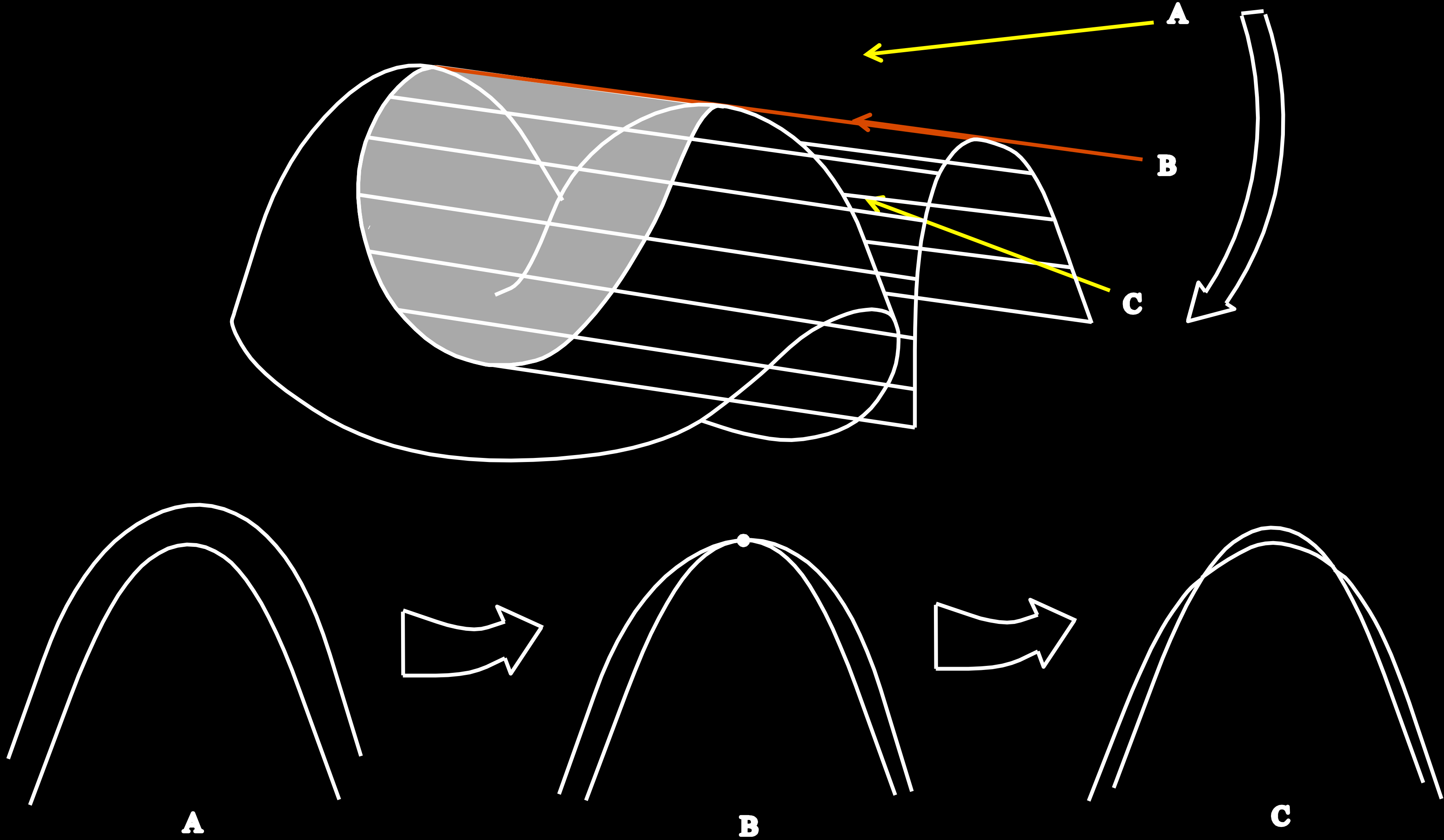
The Tangent Crossing Event



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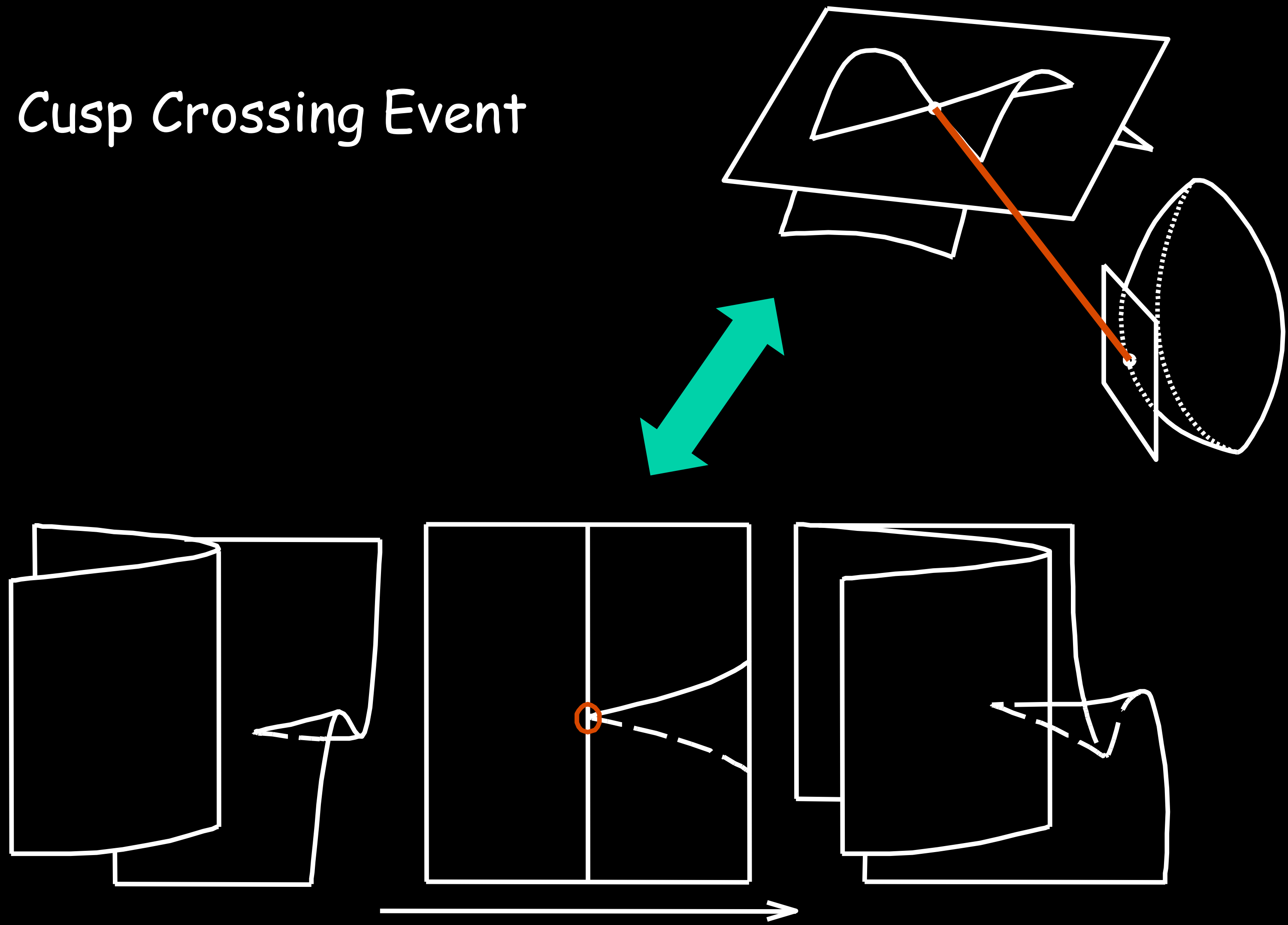
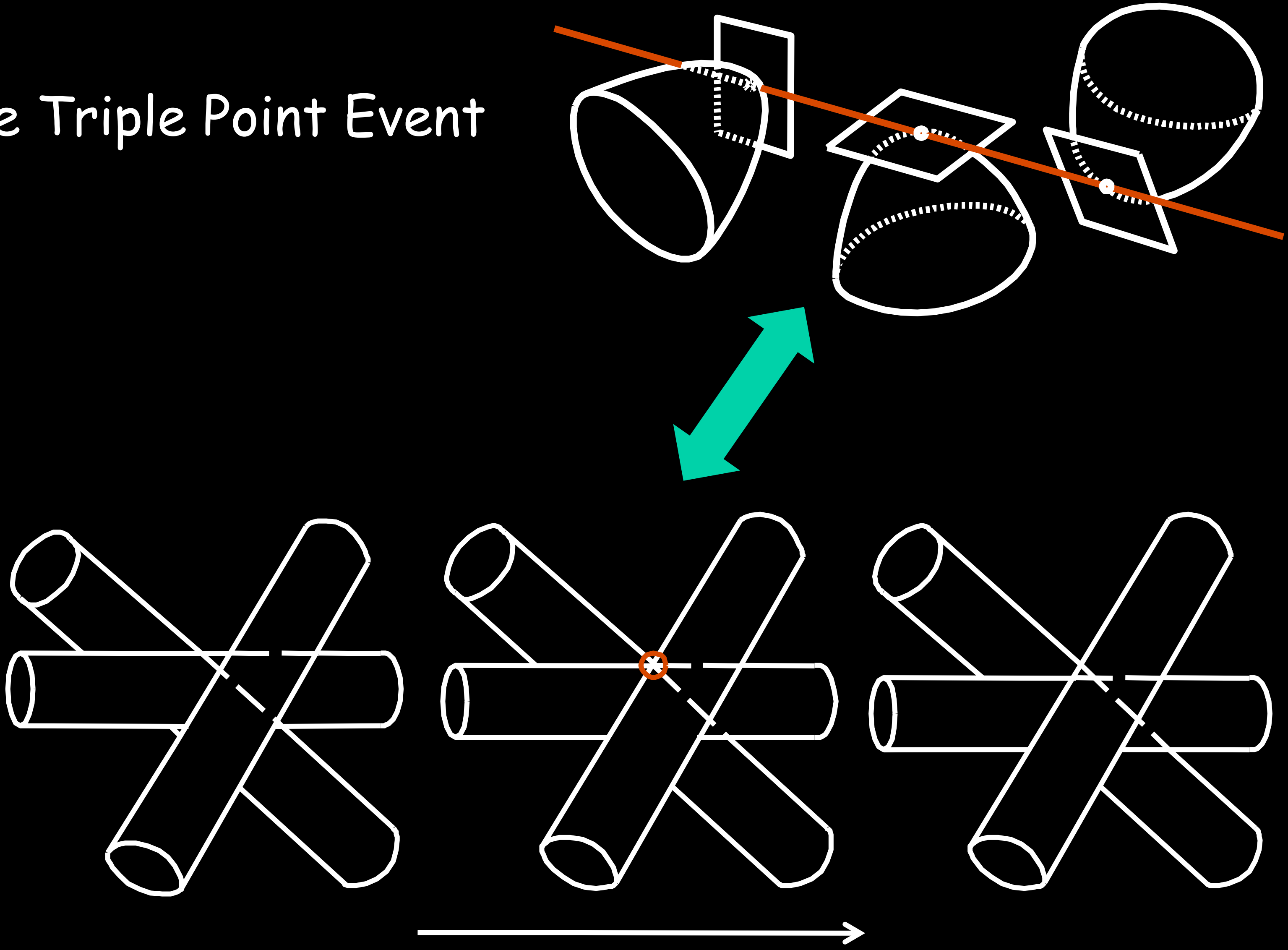
The Cusp Crossing Event



After "Computing Exact Aspect Graphs of Curved Objects: Algebraic Surfaces," by S. Petitjean, J. Ponce, and D.J. Kriegman, the International Journal of Computer Vision, 9(3):231-255 (1992). © 1992 Kluwer Academic Publishers.

The Triple Point Event



After "Computing Exact Aspect Graphs of Curved Objects: Algebraic Surfaces," by S. Petitjean, J. Ponce, and D.J. Kriegman, the International Journal of Computer Vision, 9(3):231-255 (1992). © 1992 Kluwer Academic Publishers.

Tracing Visual Events

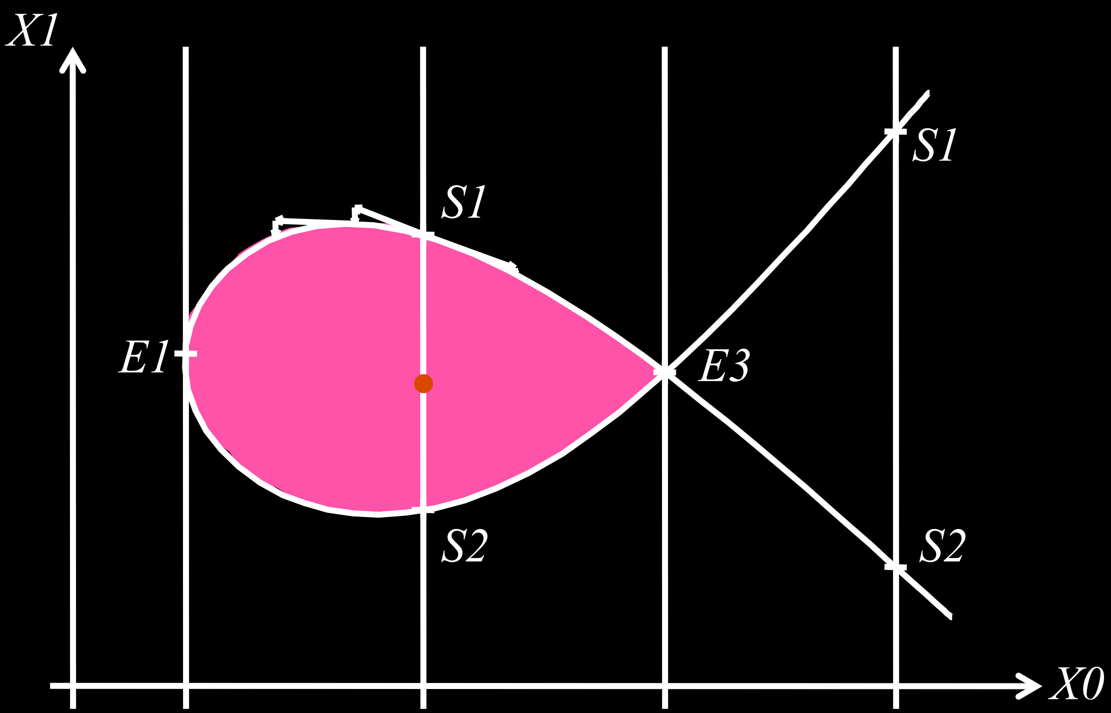
Computing the Aspect Graph

$$F(x,y,z)=0$$

$$P_1(x_1,\ldots,x_n)=0$$

...

$$P_n(x_1,\ldots,x_n)=0$$



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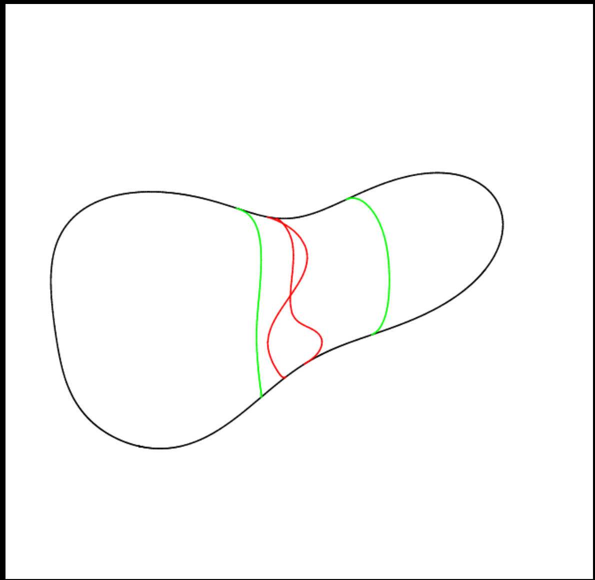
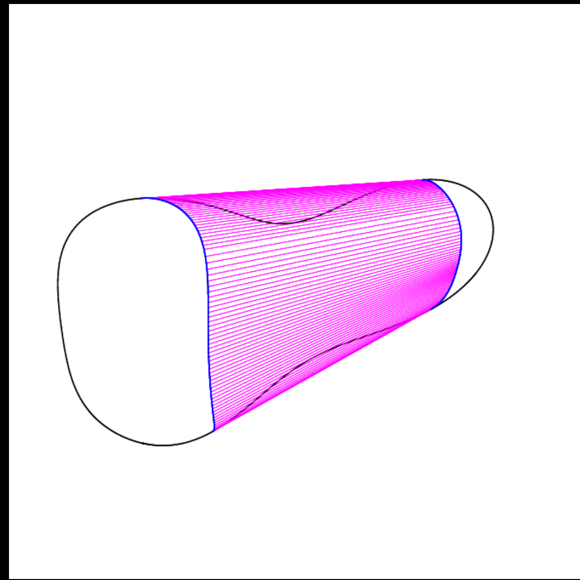
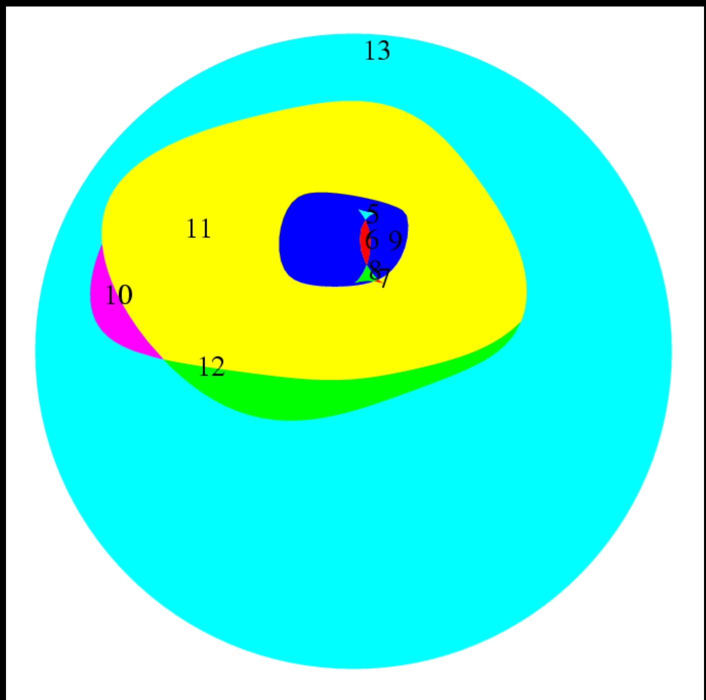
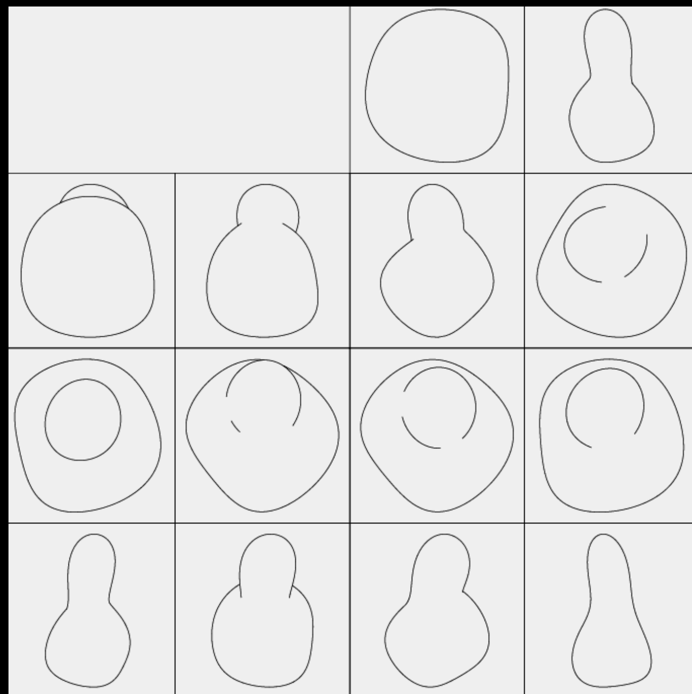
- Curve Tracing
- Cell Decomposition

An Example

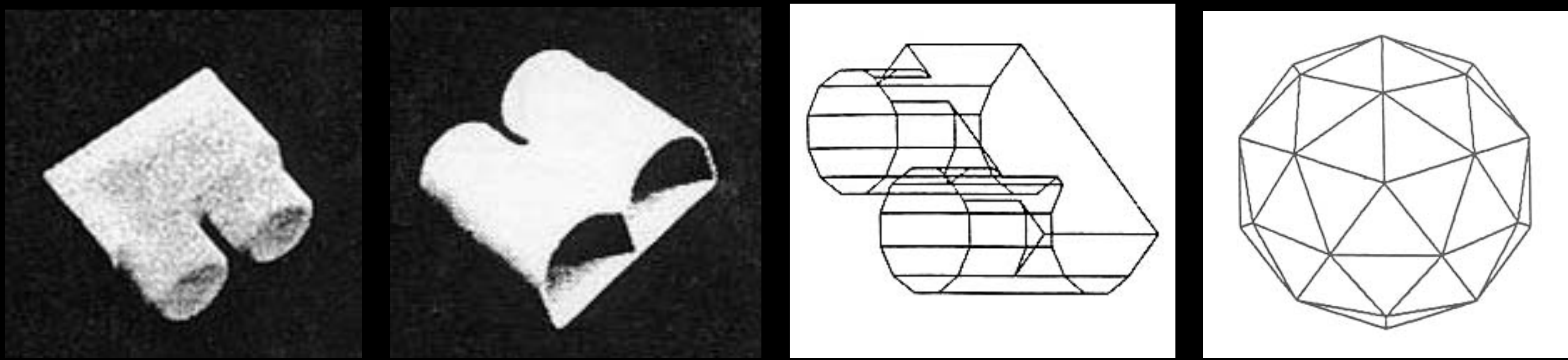


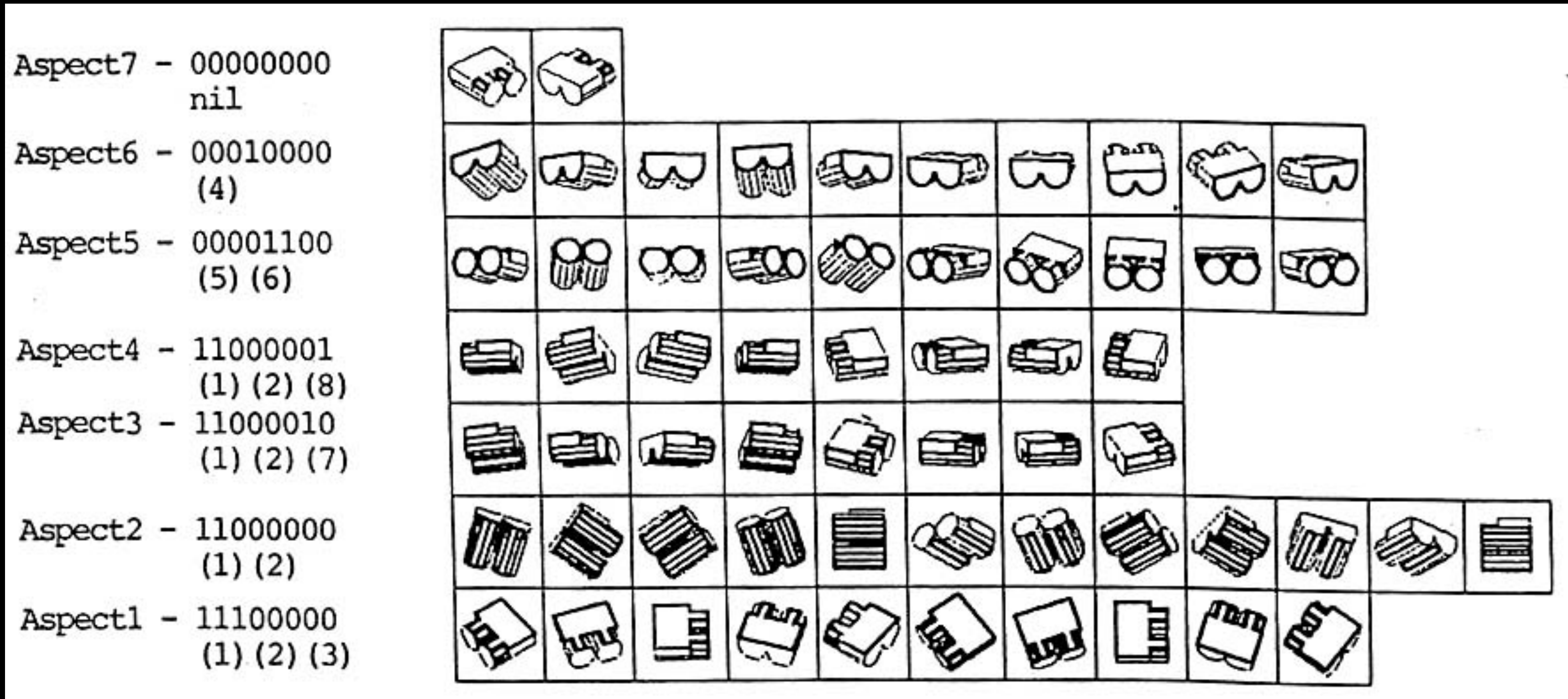






Approximate Aspect Graphs (Ikeuchi & Kanade, 1987)





Reprinted from "Automatic Generation of Object Recognition Programs," by K. Ikeuchi and T. Kanade, Proc. of the IEEE, 76(8):1016-1035 (1988).
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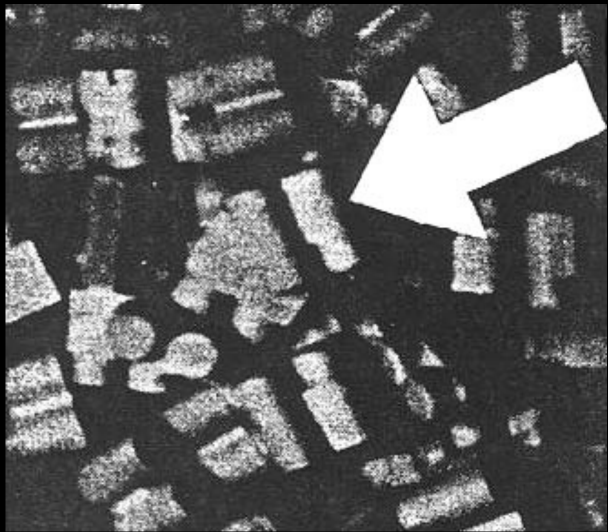
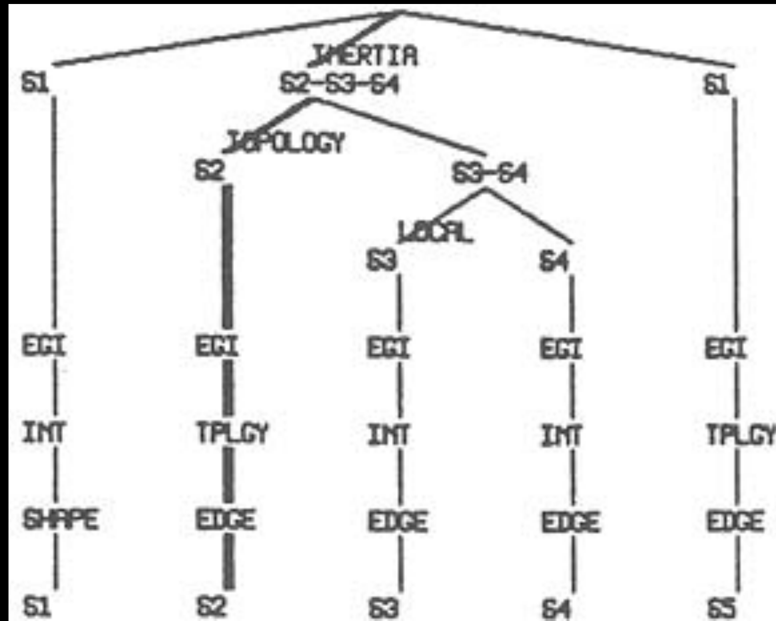
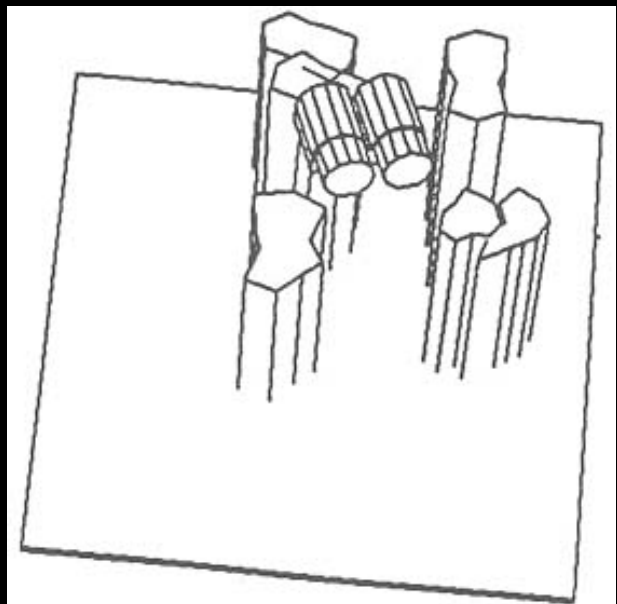
Approximate Aspect Graphs II: Object Localization (Ikeuchi & Kanade, 1987)







Reprinted from "Precompiling a Geometrical Model into an Interpretation Tree for Object Recognition in Bin-Picking Tasks," by K. Ikeuchi, Proc. DARPA Image Understanding Workshop, 1987.