Identity-based encryption

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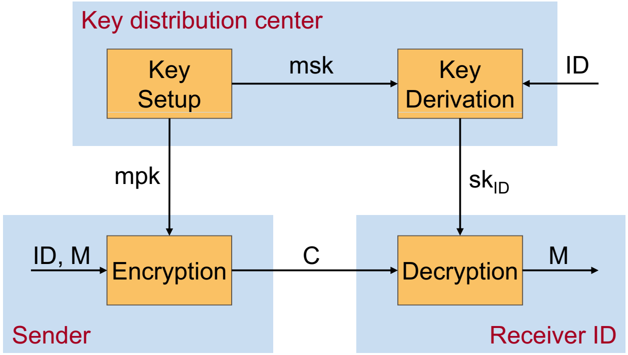
Identity-based encryption (IBE)

Goal: Allow senders to encrypt messages based on the receiver's identity.



IBE properties

- Generalization of public-key encryption
  - User public key can be an arbitrary string (e.g., email address)
  - One system-wide public key for all users
- Encryption can be performed using system-wide public key and users identity
- Users need to contact a key generation center to obtain their secret keys

Outline

Identity-based encryption (IBE)

An IBE scheme is defined by four algorithms:

- Setup(1^k):
  Outputs a master public key mpk and a master secret key msk.
- KeyDer(msk, id):
  Uses the master secret key msk to compute a secret key sk_{id} for the user with identity id.
- Enc(mpk, id, m):
  Generates a ciphertext C for identity id and message m using master public key mpk.
- Dec(C, sk_{id}):
  Allows the user in possession of sk_{id} to decrypt the ciphertext C to get back a message m.

IBE security notions

In the following slides, we consider two different types of attacks (*adaptive-identity* vs. *selective-identity*) and two security goals (*indistinguishability* and *anonymity*) notions of security for IBE schemes.

- **Indistinguishability**
  The adversary's goal is to distinguish $\mathsf{Enc}(mpk, id, m_0)$ from $\mathsf{Enc}(mpk, id, m_1)$ for values id_1, m_0,m_1 of its choice.
- **Anonymity**
  The adversary's goal is to distinguish $\mathsf{Enc}(mpk, id_0, m)$ from $\mathsf{Enc}(mpk, id_1, m)$ for values id_0, id_1, m of its choice.
- **Adaptive-identity chosen-plaintext attacks**
  In this model, the adversary is allowed to choose the challenge identity values at the time that it asks the challenge query.
- **Selective-identity chosen-plaintext attacks**
  In this model, the adversary has to choose the challenge identity values before seeing the public key.

IND-ID-CPA: Indistinguishability under chosen-plaintext attacks

- Let IBE = (Setup, KeyDer, Enc, Dec) be an identity-based encryption scheme.
- Let $\mathcal{A}$ be an adversary against the IND-ID-CPA security of IBE.

Game $\mathbf{Exp}^{\text{ind-cpa-}\beta}_{\mathcal{A},\mathsf{IBE}}(k)$

proc Initialize(k)
$(mpk, msk) \stackrel{R}{\leftarrow} \mathsf{Setup}(1^k)$
Return mpk

proc KeyDer(id)
$sk_{id} \stackrel{R}{\leftarrow} \mathsf{KeyDer}(msk, id)$
Return sk_{id}

proc LR(id^*, m_0^*, m_1^*)
$C^* \stackrel{R}{\leftarrow} \mathsf{Enc}(mpk, id^*, m_\beta^*)$
Return C^*

proc Finalize(β')
Return β'

The advantage of $\mathcal{A}$ against the IND-ID-CPA security of IBE is defined as

$$\mathbf{Adv}^{\text{ind-cpa}}_{\mathcal{A},\mathsf{IBE}}(k) = \Pr\left[\mathbf{Exp}^{\text{ind-cpa-1}}_{\mathcal{A},\mathsf{IBE}}(k) = 1\right] - \Pr\left[\mathbf{Exp}^{\text{ind-cpa-0}}_{\mathcal{A},\mathsf{IBE}}(k)) = 1\right]$$

IND-ID-CPA: An alternative definition

- Let IBE $= (\mathsf{Setup}, \mathsf{KeyDer}, \mathsf{Enc}, \mathsf{Dec})$ be an identity-based encryption scheme.
- Let $\mathcal{A}$ be an adversary against the IND-ID-CPA security of IBE.

Game $\mathbf{Exp}^{\mathrm{ind\text{-}cpa}}_{\mathcal{A},\mathsf{IBE}}(k)$

<u>**proc Initialize**(k)</u>

$\beta \stackrel{R}{\leftarrow} \{0,1\}$
$(mpk, msk) \stackrel{R}{\leftarrow} \mathsf{Setup}(1^k)$
Return mpk

<u>**proc KeyDer**(id)</u>

$sk_{id} \stackrel{R}{\leftarrow} \mathsf{KeyDer}(msk, id)$
Return sk_{id}

<u>**proc LR**(id^*, m_0^*, m_1^*)</u>

$C^* \stackrel{R}{\leftarrow} \mathsf{Enc}(mpk, id^*, m_\beta^*)$
Return C^*

<u>**proc Finalize**(β')</u>

Return $(\beta' = \beta)$

The advantage of $\mathcal{A}$ against the IND-ID-CPA security of IBE is defined as

$$\mathbf{Adv}^{\mathrm{ind\text{-}cpa}}_{\mathcal{A},\mathsf{IBE}}(k) = 2 \cdot \Pr\left[\mathbf{Exp}^{\mathrm{ind\text{-}cpa}}_{\mathcal{A},\mathsf{IBE}}(k) = \mathsf{true}\right] - 1$$

IND-sID-CPA: Indistinguishability under *selective-identity* chosen-plaintext attacks

- Let IBE $= (\mathsf{Setup}, \mathsf{KeyDer}, \mathsf{Enc}, \mathsf{Dec})$ be an identity-based encryption scheme.
- Let $\mathcal{A}$ be an adversary against the IND-sID-CPA security of IBE.

Game $\mathbf{Exp}^{\text{s-ind-cpa-}\beta}_{\mathcal{A},\mathsf{IBE}}(k)$

proc Initialize(k, id^*)	**proc LR**(m_0^*, m_1^*)
$(mpk, msk) \xleftarrow{R} \mathsf{Setup}(1^k)$	$C^* \xleftarrow{R} \mathsf{Enc}(mpk, id^*, m_\beta^*)$
Return mpk	Return C^*
proc KeyDer(id)	**proc Finalize**(β')
$sk_{id} \xleftarrow{R} \mathsf{KeyDer}(msk, id)$	Return β'
Return sk_{id}	

The advantage of $\mathcal{A}$ against the IND-sID-CPA security of IBE is defined as

$$\mathbf{Adv}^{\text{s-ind-cpa}}_{\mathcal{A},\mathsf{IBE}}(k) = \Pr\left[\mathbf{Exp}^{\text{s-ind-cpa-1}}_{\mathcal{A},\mathsf{IBE}}(k) = 1\right] - \Pr\left[\mathbf{Exp}^{\text{s-ind-cpa-0}}_{\mathcal{A},\mathsf{IBE}}(k)) = 1\right]$$

- Let IBE = (Setup, KeyDer, Enc, Dec) be an identity-based encryption scheme.
- Let $\mathcal{A}$ be an adversary against the IND-sID-CPA security of IBE.

Game $\mathbf{Exp}^{\text{s-ind-cpa}}_{\mathcal{A},\text{IBE}}(k)$

proc Initialize(k, id^*)

$\beta \stackrel{R}{\leftarrow} \{0,1\}$

$(mpk, msk) \stackrel{R}{\leftarrow} \text{Setup}(1^k)$

Return mpk

proc KeyDer(id)

$sk_{id} \stackrel{R}{\leftarrow} \text{KeyDer}(msk, id)$

Return sk_{id}

proc LR(m_0^*, m_1^*)

$C^* \stackrel{R}{\leftarrow} \text{Enc}(mpk, id^*, m_\beta^*)$

Return C^*

proc Finalize(β')

Return $(\beta' = \beta)$

The advantage of $\mathcal{A}$ against the IND-sID-CPA security of IBE is defined as

$$\mathbf{Adv}^{\text{s-ind-cpa}}_{\mathcal{A},\text{IBE}}(k) = 2 \cdot \Pr\left[\mathbf{Exp}^{\text{s-ind-cpa}}_{\mathcal{A},\text{IBE}}(k) = \text{true}\right] - 1$$

ANO-ID-CPA: Anonymity under chosen-plaintext attacks

- Let IBE $=$ (Setup, KeyDer, Enc, Dec) be an identity-based encryption scheme.
- Let $\mathcal{A}$ be an adversary against the ANO-ID-CPA security of IBE.

Game $\mathbf{Exp}^{\text{ano-cpa-}\beta}_{\mathcal{A},\mathsf{IBE}}(k)$

proc Initialize(k)

$(mpk, msk) \stackrel{R}{\leftarrow} \mathsf{Setup}(1^k)$
Return mpk

proc KeyDer(id)

$sk_{id} \stackrel{R}{\leftarrow} \mathsf{KeyDer}(msk, id)$
Return sk_{id}

proc LR(id_0^*, id_1^*, m^*)

$C^* \stackrel{R}{\leftarrow} \mathsf{Enc}(mpk, id_\beta^*, m^*)$
Return C^*

proc Finalize(β')

Return β'

The advantage of $\mathcal{A}$ against the ANO-ID-CPA security of IBE is defined as

$$\mathbf{Adv}^{\text{ano-cpa}}_{\mathcal{A},\mathsf{IBE}}(k) = \Pr\left[\mathbf{Exp}^{\text{ano-cpa-1}}_{\mathcal{A},\mathsf{IBE}}(k) = 1\right] - \Pr\left[\mathbf{Exp}^{\text{ano-cpa-0}}_{\mathcal{A},\mathsf{IBE}}(k)) = 1\right]$$

ANO-sID-CPA: Anonymity under *selective-identity* chosen-plaintext attacks

- Let IBE $= (\mathsf{Setup}, \mathsf{KeyDer}, \mathsf{Enc}, \mathsf{Dec})$ be an identity-based encryption scheme.
- Let $\mathcal{A}$ be an adversary against the ANO-sID-CPA security of IBE.

Game $\mathbf{Exp}^{\text{s-ano-cpa-}\beta}_{\mathcal{A},\mathsf{IBE}}(k)$

proc Initialize(k)

$(mpk, msk) \xleftarrow{R} \mathsf{Setup}(1^k, id_0^*, id_1^*)$
Return mpk

proc KeyDer(id)

$sk_{id} \xleftarrow{R} \mathsf{KeyDer}(msk, id)$
Return sk_{id}

proc LR(m^*)

$C^* \xleftarrow{R} \mathsf{Enc}(mpk, id_\beta^*, m^*)$
Return C^*

proc Finalize(β')

Return β'

The advantage of $\mathcal{A}$ against the ANO-sID-CPA security of IBE is defined as

$$\mathbf{Adv}^{\text{s-ano-cpa}}_{\mathcal{A},\mathsf{IBE}}(k) = \Pr\left[\mathbf{Exp}^{\text{s-ano-cpa-1}}_{\mathcal{A},\mathsf{IBE}}(k) = 1\right] - \Pr\left[\mathbf{Exp}^{\text{s-ano-cpa-0}}_{\mathcal{A},\mathsf{IBE}}(k)) = 1\right]$$

Outline

Computational Diffie-Hellman (CDH)

- Let $\mathbb{G}$ be a finite cyclic group of prime order p.
- Let $\mathcal{A}$ be an adversary against the CDH problem in a group $\mathbb{G}$.

Game $\mathbf{Exp}_{\mathbb{G}}^{\mathrm{cdh}}(\mathcal{A})$

proc Initialize($\mathbb{G}$)	**proc Finalize(Z)**
$g \stackrel{R}{\leftarrow} \mathbb{G}^*$ $x \stackrel{R}{\leftarrow} \mathbb{Z}_p^*$; $X \leftarrow g^x$ $y \stackrel{R}{\leftarrow} \mathbb{Z}_p^*$; $Y \leftarrow g^y$ Return $(\mathbb{G}, g, X, Y)$	Return $(Z = g^{xy})$

The advantage of $\mathcal{A}$ against the CDH problem is defined as

$$\mathbf{Adv}_{\mathbb{G}}^{\mathrm{cdh}}(\mathcal{A}) = \Pr\left[\mathbf{Exp}_{\mathbb{G}}^{\mathrm{cdh}}(\mathcal{A}) = \mathrm{true}\right]$$

Decisional Diffie-Hellman (DDH)

- Let $\mathbb{G}$ be a finite cyclic group of prime order p.
- Let $\mathcal{A}$ be an adversary against the CDH problem in a group $\mathbb{G}$.

Game $\mathbf{Exp}_{\mathbb{G}}^{\mathrm{ddh}\text{-}0}(\mathcal{A})$	**Game $\mathbf{Exp}_{\mathbb{G}}^{\mathrm{ddh}\text{-}1}(\mathcal{A})$**
proc Initialize$(\mathbb{G})$	**proc Initialize**$(\mathbb{G})$
$g \xleftarrow{R} \mathbb{G}^*$	$g \xleftarrow{R} \mathbb{G}^*$
$x \xleftarrow{R} \mathbb{Z}_p^*$; $X \leftarrow g^x$	$x \xleftarrow{R} \mathbb{Z}_p^*$; $X \leftarrow g^x$
$y \xleftarrow{R} \mathbb{Z}_p^*$; $Y \leftarrow g^y$	$y \xleftarrow{R} \mathbb{Z}_p^*$; $Y \leftarrow g^y$
$z \leftarrow ab \mod p$; $Z \leftarrow g^z$	$z \xleftarrow{R} \mathbb{Z}_p^*$; $Z \leftarrow g^z$
Return $(\mathbb{G}, g, X, Y, Z)$	Return $(\mathbb{G}, g, X, Y, Z)$
proc Finalize(β')	**proc Finalize**(β')
Return $(\beta' = 1)$	Return $(\beta' = 1)$

The advantage of $\mathcal{A}$ in solving the DDH problem is defined as

$$\mathbf{Adv}_{\mathbb{G}}^{\mathrm{ddh}}(\mathcal{A}) = \Pr\left[\mathbf{Exp}_{\mathbb{G}}^{\mathrm{ddh}\text{-}0}(\mathcal{A}) = \mathsf{true}\right] - \Pr\left[\mathbf{Exp}_{\mathbb{G}}^{\mathrm{ddh}\text{-}1}(\mathcal{A}) = \mathsf{true}\right]$$

Bilinear Diffie-Hellman (BDH)

- Let $\mathcal{G}$ be a *pairing parameter generator*.
- Let $\mathcal{A}$ be an adversary against the BDH problem relative to $\mathcal{G}$.

Game $\mathbf{Exp}^{\mathrm{bdh}}_{\mathcal{G},k}(\mathcal{A})$

proc Initialize(1^k)	**proc Finalize**(Z)
$(\mathbb{G}, \mathbb{G}_T, p, \hat{e}) \stackrel{R}{\leftarrow} \mathcal{G}(1^k)$	Return $(Z = \hat{e}(g,g)^{abc})$
$g \stackrel{R}{\leftarrow} \mathbb{G}^*$	
$a \stackrel{R}{\leftarrow} \mathbb{Z}_p^*$; $A \leftarrow g^a$	
$b \stackrel{R}{\leftarrow} \mathbb{Z}_p^*$; $B \leftarrow g^b$	
$c \stackrel{R}{\leftarrow} \mathbb{Z}_p^*$; $C \leftarrow g^b$	
Return $(\mathbb{G}, g, A, B, C)$	

The advantage of $\mathcal{A}$ against the BDH problem relative to $\mathcal{G}$ is defined as

$$\mathbf{Adv}^{\mathrm{bdh}}_{\mathcal{G},k}(\mathcal{A}) = \Pr\left[\mathbf{Exp}^{\mathrm{bdh}}_{\mathcal{G},k}(\mathcal{A}) = \mathrm{true}\right]$$

Bilinear Decisional Diffie-Hellman (BDDH)

- Let $\mathcal{G}$ be a *pairing parameter generator*.
- Let $\mathcal{A}$ be an adversary against the BDDH problem relative to $\mathcal{G}$.

Game $\mathbf{Exp}^{\mathrm{bddh\text{-}0}}_{\mathcal{G},k}(\mathcal{A})$	Game $\mathbf{Exp}^{\mathrm{bddh\text{-}1}}_{\mathcal{G},k}(\mathcal{A})$
proc Initialize(1^k)	**proc Initialize**(1^k)
$(\mathbb{G}, \mathbb{G}_T, p, \hat{e}) \xleftarrow{R} \mathcal{G}(1^k)$	$(\mathbb{G}, \mathbb{G}_T, p, \hat{e}) \xleftarrow{R} \mathcal{G}(1^k)$
$g \xleftarrow{R} \mathbb{G}^*$	$g \xleftarrow{R} \mathbb{G}^*$
$a \xleftarrow{R} \mathbb{Z}_p^*$; $A \leftarrow g^a$	$a \xleftarrow{R} \mathbb{Z}_p^*$; $A \leftarrow g^a$
$b \xleftarrow{R} \mathbb{Z}_p^*$; $B \leftarrow g^b$	$b \xleftarrow{R} \mathbb{Z}_p^*$; $B \leftarrow g^b$
$c \xleftarrow{R} \mathbb{Z}_p^*$; $C \leftarrow g^b$	$c \xleftarrow{R} \mathbb{Z}_p^*$; $C \leftarrow g^b$
$z \leftarrow abc \mod p$; $Z \leftarrow \hat{e}(g,g)^z$	$z \xleftarrow{R} \mathbb{Z}_p^*$; $Z \leftarrow \hat{e}(g,g)^z$
Return $(\mathbb{G}, g, A, B, C, Z)$	Return $(\mathbb{G}, g, A, B, C, Z)$
proc Finalize(β')	**proc Finalize**(β')
Return $(\beta' = 1)$	Return $(\beta' = 1)$

The advantage of $\mathcal{A}$ in solving the BDDH problem is defined as

$$\mathbf{Adv}^{\mathrm{bddh}}_{\mathcal{G},k}(\mathcal{A}) = \Pr\left[\mathbf{Exp}^{\mathrm{bddh\text{-}0}}_{\mathcal{G},k}(\mathcal{A}) = \mathsf{true}\right] - \Pr\left[\mathbf{Exp}^{\mathrm{bddh\text{-}1}}_{\mathcal{G},k}(\mathcal{A}) = \mathsf{true}\right]$$

Outline

Boneh-Boyen IBE scheme (BB1)

Setup(1^k):
$(\mathbb{G}, \mathbb{G}_{\mathrm{T}}, p, \hat{e}) \overset{R}{\leftarrow} \mathcal{G}(1^k)$
$g \overset{R}{\leftarrow} \mathbb{G}$
$a \overset{R}{\leftarrow} \mathbb{Z}_p \ ; \ A \leftarrow g^a$
$b \overset{R}{\leftarrow} \mathbb{Z}_p \ ; \ B \leftarrow g^b$
$h \overset{R}{\leftarrow} \mathbb{Z}_p \ ; \ H \leftarrow g^h$
$mpk \leftarrow (g, A, B, H, \mathbb{G}, \mathbb{G}_{\mathrm{T}}, p, \hat{e})$
$msk \leftarrow g^{ab}$
return (mpk, msk)

Enc(mpk, id, m):
$t \overset{R}{\leftarrow} \mathbb{Z}_p \ ; \ C_1 \leftarrow g^t$
$C_2 \leftarrow \left(B^{id} H\right)^t$
$K \leftarrow \hat{e}(A, B)^t$
$C_3 \leftarrow m \cdot K$
return (C_1, C_2, C_3)

KeyDer(msk, id):
$r \overset{R}{\leftarrow} \mathbb{Z}_p$
$usk_1 \leftarrow g^r$
$usk_2 \leftarrow msk \cdot \left(B^{id} H\right)^r$
return (usk_1, usk_2)

Dec(usk, C):
parse usk as (usk_1, usk_2)
parse C as (C_1, C_2, C_3)
$K' \leftarrow \hat{e}(usk_2, C_1)/\hat{e}(usk_1, C_2)$
$m' \leftarrow C_3/K'$
return m'

Correctness of BB1 IBE scheme

For a valid ciphertext, we have:

$$
\begin{aligned}
K' &= \hat{e}(usk_2, C_1)/\hat{e}(usk_1, C_2) \\
&= \hat{e}(msk \cdot (B^{id}H)^r, g^t)/\hat{e}(g^r, (B^{id}H)^t) \\
&= \hat{e}(g^{ab} \cdot (B^{id}H)^r, g^t)/\hat{e}(g^r, (B^{id}H)^t) \\
&= \hat{e}(g^{ab}, g^t) \cdot \hat{e}((B^{id}H)^r, g^t)/\hat{e}(g^r, (B^{id}H)^t) \\
&= \hat{e}(g^a, g^b)^t \cdot \hat{e}((B^{id}H), g)^{rt}/\hat{e}(g, (B^{id}H))^{rt} \\
&= \hat{e}(A, B)^t \\
&= K
\end{aligned}
$$

BDDH security of BB1 IBE scheme

Theorem

Let

- BB1 *refer to the Boneh-Boyen IBE scheme described above,*
- $\mathcal{G}$ *be a pairing parameter generator, and*
- $\mathcal{A}$ *be an adversary against* $\mathrm{IND\text{-}sID\text{-}CPA}$ *security of* BB1, *making at most a single query to the* **LR** *procedure.*

Then, there exists an adversary $\mathcal{B}$ *against the BDDH problem relative to* $\mathcal{G}$, *whose running time is that of* $\mathcal{A}$ *and such that*

$$\mathbf{Adv}^{\mathrm{s\text{-}ind\text{-}cpa}}_{\mathcal{A},\mathsf{BB1}}(k) \leq 2 \cdot \mathbf{Adv}^{\mathrm{bddh}}_{\mathcal{G},k}(\mathcal{B}).$$

Security proof of BB1 scheme

Proof will define a sequence of five games $(\mathsf{G}_0, \ldots, \mathsf{G}_4)$.
For simplicity, we assume that $mpk = (g, A, B, H)$ and omit the other values. We also omit the pairing parameter generation in procedure **Initialize**.

- G_0 This game is the real attack game against BB1.
- G_1 We change the computation of H so that $B^{id^*} H = g^{\alpha}$ for a random α.
- G_2 We change the simulation of the key derivation procedure **KeyDer** so that the game answers these queries without the knowledge of the master secret key.
- G_3 We change the simulation of the **LR** procedure so that $C_2^* = C_1^{*\alpha}$. That is, we don't need to know t to compute it.
- G_4 We change the simulation of the **LR** procedure so that K is chosen uniformly at random.

Game G_0

Game $G_0^{\mathcal{A}}$

proc Initialize(k, id^*)

$\beta \xleftarrow{R} \{0,1\}$
$g \xleftarrow{R} \mathbb{G}$
$a \xleftarrow{R} \mathbb{Z}_p$; $A \leftarrow g^a$
$b \xleftarrow{R} \mathbb{Z}_p$; $B \leftarrow g^b$
$h \xleftarrow{R} \mathbb{Z}_p$; $H \leftarrow g^h$
$mpk \leftarrow (g, A, B, H)$
$msk \leftarrow g^{ab}$
Return mpk

proc Finalize(β')

Return $(\beta' = \beta)$

proc LR(m_0^*, m_1^*)

$t \xleftarrow{R} \mathbb{Z}_p$; $C_1^* \leftarrow g^t$
$C_2^* \leftarrow (B^{id^*} H)^t$
$K \leftarrow \hat{e}(A, B)^t$
$C_3^* \leftarrow m_\beta^* \cdot K$
Return (C_1^*, C_2^*, C_3^*)

proc KeyDer(id)

$r \xleftarrow{R} \mathbb{Z}_p$; $usk_1 \leftarrow g^r$
$usk_2 \leftarrow g^{ab} \cdot (B^{id} H)^r$
Return (usk_1, usk_2)

Game $G_1^{\mathcal{A}}$

proc Initialize(k, id^*)

$\beta \stackrel{R}{\leftarrow} \{0,1\}$

$g \stackrel{R}{\leftarrow} \mathbb{G}$

$a \stackrel{R}{\leftarrow} \mathbb{Z}_p \; ; \; A \leftarrow g^a$

$b \stackrel{R}{\leftarrow} \mathbb{Z}_p \; ; \; B \leftarrow g^b$

$\boxed{\alpha \stackrel{R}{\leftarrow} \mathbb{Z}_p \; ; \; H \leftarrow B^{-id^*} g^{\alpha}}$

$mpk \leftarrow (g, A, B, H)$

$msk \leftarrow g^{ab}$

Return mpk

proc Finalize(β')

Return $(\beta' = \beta)$

proc LR(m_0^*, m_1^*)

$t \stackrel{R}{\leftarrow} \mathbb{Z}_p \; ; \; C_1^* \leftarrow g^t$

$C_2^* \leftarrow (B^{id^*} H)^t$

$K \leftarrow \hat{e}(A, B)^t$

$C_3^* \leftarrow m_\beta^* \cdot K$

Return (C_1^*, C_2^*, C_3^*)

proc KeyDer(id)

$r \stackrel{R}{\leftarrow} \mathbb{Z}_p \; ; \; usk_1 \leftarrow g^r$

$usk_2 \leftarrow g^{ab} \cdot (B^{id} H)^r$

Return (usk_1, usk_2)

Game G_2

Game $G_2^{\mathcal{A}}$

proc Initialize(k, id^*)

$\beta \stackrel{R}{\leftarrow} \{0,1\}$
$g \stackrel{R}{\leftarrow} \mathbb{G}$
$a \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $A \leftarrow g^a$
$b \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $B \leftarrow g^b$
$\alpha \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $H \leftarrow B^{-id^*} g^{\alpha}$
$mpk \leftarrow (g, A, B, H)$
$msk \leftarrow g^{ab}$
Return mpk

proc Finalize(β')

Return $(\beta' = \beta)$

proc LR(m_0^*, m_1^*)

$t \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $C_1^* \leftarrow g^t$
$C_2^* \leftarrow (B^{id^*} H)^t$
$K \leftarrow \hat{e}(A, B)^t$
$C_3^* \leftarrow m_\beta^* \cdot K$
Return (C_1^*, C_2^*, C_3^*)

proc KeyDer(id)

$\tilde{r} \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $usk_1 \leftarrow g^{\tilde{r}} A^{-1/(id-id^*)}$
$usk_2 \leftarrow A^{-\alpha/(id-id^*)} \cdot (B^{id} H)^{\tilde{r}}$
Return (usk_1, usk_2)

Game G_3

Game $G_3^{\mathcal{A}}$

proc Initialize(k, id^*)

$\beta \stackrel{R}{\leftarrow} \{0,1\}$
$g \stackrel{R}{\leftarrow} \mathbb{G}$
$a \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $A \leftarrow g^a$
$b \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $B \leftarrow g^b$
$\alpha \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $H \leftarrow B^{-id^*} g^{\alpha}$
$mpk \leftarrow (g, A, B, H)$
$msk \leftarrow g^{ab}$
Return mpk

proc Finalize(β')

Return $(\beta' = \beta)$

proc LR(m_0^*, m_1^*)

$t \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $C_1^* \leftarrow g^t$
$\boxed{C_2^* \leftarrow {C_1^*}^{\alpha}}$
$K \leftarrow \hat{e}(A, B)^t$
$C_3^* \leftarrow m_\beta^* \cdot K$
Return (C_1^*, C_2^*, C_3^*)

proc KeyDer(id)

$\tilde{r} \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $usk_1 \leftarrow g^{\tilde{r}} A^{-1/(id-id^*)}$
$usk_2 \leftarrow A^{-\alpha/(id-id^*)} \cdot (B^{id} H)^{\tilde{r}}$
Return (usk_1, usk_2)

Game G_4

Game $G_4^{\mathcal{A}}$

proc Initialize(k, id^*)

$\beta \xleftarrow{R} \{0,1\}$
$g \xleftarrow{R} \mathbb{G}$
$a \xleftarrow{R} \mathbb{Z}_p$; $A \leftarrow g^a$
$b \xleftarrow{R} \mathbb{Z}_p$; $B \leftarrow g^b$
$\alpha \xleftarrow{R} \mathbb{Z}_p$; $H \leftarrow B^{-id^*} g^{\alpha}$
$mpk \leftarrow (g, A, B, H)$
$msk \leftarrow g^{ab}$
Return mpk

proc Finalize(β')

Return $(\beta' = \beta)$

proc LR(m_0^*, m_1^*)

$t \xleftarrow{R} \mathbb{Z}_p$; $C_1^* \leftarrow g^t$
$C_2^* \leftarrow {C_1^*}^{\alpha}$
$\boxed{K \xleftarrow{R} \mathbb{G}}$
$C_3^* \leftarrow m_\beta^* \cdot K$
Return (C_1^*, C_2^*, C_3^*)

proc KeyDer(id)

$\tilde{r} \xleftarrow{R} \mathbb{Z}_p$; $usk_1 \leftarrow g^{\tilde{r}} A^{-1/(id-id^*)}$
$usk_2 \leftarrow A^{-\alpha/(id-id^*)} \cdot (B^{id} H)^{\tilde{r}}$
Return (usk_1, usk_2)

Probability analysis

Claim 1 $\mathbf{Adv}^{\text{s-ind-cpa}}_{\mathcal{A},\mathsf{BB1}}(k) = 2 \cdot \Pr\left[\, \mathsf{G}_0^{\mathcal{A}} = \mathsf{true} \,\right] - 1$

Claim 2 $\Pr\left[\, \mathsf{G}_1^{\mathcal{A}} = \mathsf{true} \,\right] = \Pr\left[\, \mathsf{G}_0^{\mathcal{A}} = \mathsf{true} \,\right]$

Claim 3 $\Pr\left[\, \mathsf{G}_2^{\mathcal{A}} = \mathsf{true} \,\right] = \Pr\left[\, \mathsf{G}_1^{\mathcal{A}} = \mathsf{true} \,\right]$

Claim 4 $\Pr\left[\, \mathsf{G}_3^{\mathcal{A}} = \mathsf{true} \,\right] = \Pr\left[\, \mathsf{G}_2^{\mathcal{A}} = \mathsf{true} \,\right]$

Claim 5 $\left|\Pr\left[\, \mathsf{G}_4^{\mathcal{A}} = \mathsf{true} \,\right] - \Pr\left[\, \mathsf{G}_3^{\mathcal{A}} = \mathsf{true} \,\right]\right| \leq \mathbf{Adv}^{\text{bddh}}_{\mathcal{G},k}(\mathcal{B})$

Claim 6 $\Pr\left[\, \mathsf{G}_4^{\mathcal{A}} = \mathsf{true} \,\right] = 1/2$

It's straightforward to verify that the security theorem follows from the claims above.

Proof of Claims 1, 2, 4, and 6

- Claim 1 follows the security definition.
- Claim 2 follows from the fact that H is still uniformly distributed in $\mathbb{G}$.
- Claim 4 follows from the fact that C_2^* is still being correctly computed.

$$\begin{aligned} C_2^* &= (B^{id^*} H)^t \\ &= (B^{id^*} B^{-id^*} g^{\alpha})^t \\ &= g^{\alpha t} \\ &= C_1^{*\alpha} \end{aligned}$$

- Claim 6 follows from the fact that $\mathcal{A}$ has no information about β in G_4.

Proof of Claim 3

Claim 3 follows from the fact that (usk_1, usk_2) is still a valid random secret key for user id, where r is being implicitly set to $\tilde{r} - a/(id - id^*)$.

$$
\begin{aligned}
usk_1 &= g^r = g^{\tilde{r}-a/(id-id^*)} \\
&= g^{\tilde{r}} g^{-a/(id-id^*)} \\
&= g^{\tilde{r}} A^{-1/(id-id^*)} \\
usk_2 &= g^{ab}(B^{id}H)^r \\
&= g^{ab}(B^{id}H)^{-a/(id-id^*)}(B^{id}H)^{\tilde{r}} \\
&= g^{ab}(B^{id}B^{-id^*}g^{\alpha})^{-a/(id-id^*)}(B^{id}H)^{\tilde{r}} \\
&= g^{ab}(g^{b(id-id^*)}g^{\alpha})^{-a/(id-id^*)}(B^{id}H)^{\tilde{r}} \\
&= g^{ab}g^{-ab}g^{-a\alpha/(id-id^*)}(B^{id}H)^{\tilde{r}} \\
&= A^{-\alpha/(id-id^*)}(B^{id}H)^{\tilde{r}}
\end{aligned}
$$

Proof of Claim 5

In order to prove Claim 5, we need to build an adversary $\mathcal{B}$ against the BDDH problem.

- Let $(\mathbb{G}, g, A, B, C, Z)$ be the input of $\mathcal{B}$.
- To simulate procedure **Initialize**, $\mathcal{B}$ chooses α at random, sets $H = B^{-id^*} g^\alpha$ and returns $mpk = (g, A, B, H)$ as the public key.
- When simulating procedure **LR**, $\mathcal{B}$ sets $C_1^* = C$, $C_2^* = C_1^{*\alpha}$, and $K = Z$.
- $\mathcal{B}$ simulates procedures **KeyDer** and **Finalize** exactly as in G_3.
- When $\mathcal{B}$ is being executed in Game $\mathbf{Exp}^{\mathrm{bddh\text{-}0}}_{\mathcal{G},k}(\mathcal{B})$, $\mathcal{B}$ simulates G_3 to $\mathcal{A}$. That is, $\Pr\left[\, \mathsf{G}_3^{\mathcal{A}} = \mathsf{true} \,\right] = \Pr\left[\, \mathbf{Exp}^{\mathrm{bddh\text{-}0}}_{\mathcal{G},k}(\mathcal{B}) = \mathsf{true} \,\right]$.
- When $\mathcal{B}$ is being executed in Game $\mathbf{Exp}^{\mathrm{bddh\text{-}1}}_{\mathcal{G},k}(\mathcal{B})$, $\mathcal{B}$ simulates G_4 to $\mathcal{A}$. That is, $\Pr\left[\, \mathsf{G}_4^{\mathcal{A}} = \mathsf{true} \,\right] = \Pr\left[\, \mathbf{Exp}^{\mathrm{bddh\text{-}1}}_{\mathcal{G},k}(\mathcal{B}) = \mathsf{true} \,\right]$.
- The claim follows.

Boneh-Franklin BasicIdent IBE scheme

- Let $\mathcal{G}$ be a *pairing parameter generator*.
- Let $H : \{0,1\}^* \to \mathbb{G}^*$ be a random oracle.

Setup(1^k):
$(\mathbb{G}, \mathbb{G}_T, p, \hat{e}) \stackrel{R}{\leftarrow} \mathcal{G}(1^k)$
$g \stackrel{R}{\leftarrow} \mathbb{G}$; $s \stackrel{R}{\leftarrow} \mathbb{Z}_p^*$; $S \leftarrow g^s$
$msk \leftarrow s$
$mpk \leftarrow ((\mathbb{G}, \mathbb{G}_T, p, \hat{e}), S, H)$
return (mpk, msk)

KeyDer(msk, id):
$Q_{id} \leftarrow H(id)$
$usk \leftarrow Q_{id}^s$
return (usk)

Enc(mpk, id, m):
$r \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $C_1 \leftarrow g^r$
$Q_{id} \leftarrow H(id)$; $K \leftarrow (\hat{e}(S, Q_{id}))^r$
$C_2 \leftarrow m \cdot K$
return (C_1, C_2)

Dec(usk, C):
parse C as (C_1, C_2)
$K \leftarrow \hat{e}(C_1, usk)$
$m' \leftarrow C_2 / K$
return m'

BDDH security of Boneh-Franklin BasicIdent IBE scheme

Theorem

Let

- BF *refer to the Boneh-Franklin* BasicIdent *IBE scheme in the previous slide,*
- $\mathcal{G}$ *be a pairing parameter generator, and*
- $\mathcal{A}$ *be an adversary against* IND-ID-CPA *security of* BF*, making at most* q_H *queries to the random oracle* H *and at most a single query to the* **LR** *procedure.*

Then, there exists an adversary $\mathcal{B}$ *against the BDDH problem relative to* $\mathcal{G}$*, whose running time is that of* $\mathcal{A}$ *and such that*

$$\mathbf{Adv}^{\text{ind-cpa}}_{\mathcal{A},\mathsf{BF}}(k) \leq 2 \cdot q_H \cdot \mathbf{Adv}^{\text{bddh}}_{\mathcal{G},k}(\mathcal{B}).$$

Security proof of BF scheme

Proof will define a sequence of five games ($G_0, \ldots, G_4$).
For simplicity, we assume that $mpk = S$ and omit the other values. We also omit the pairing parameter generation in procedure **Initialize**.

- G_0 This game is the real attack game against BF.
- G_1 We guess the hash query involved in the challenge query and abort if the guess is incorrect, returning a random output for the game.
- G_2 We change the simulation of the random oracle procedure **H** so that the game knows the discrete log of $H(id)$ for any identity other than the challenge.
- G_3 We change the simulation of the key derivation procedure **KeyDer** so that the game answers these queries without the knowledge of the master secret key.
- G_4 In this game, we change the simulation of the **LR** procedure so that K is chosen uniformly at random.

Game G_0

Game $G_0^{\mathcal{A}}$

proc Initialize(k)

$\beta \stackrel{R}{\leftarrow} \{0,1\}$

$\Lambda_H \leftarrow \varepsilon$; $ctr \leftarrow 0$

$s \stackrel{R}{\leftarrow} \mathbb{Z}_p^*$; $S \leftarrow g^s$

$msk \leftarrow s$

$mpk \leftarrow S$

Return mpk

proc KeyDer(id)

if $(ctr, id, Y, y) \notin \Lambda_H$, $\mathbf{H}(id)$

$usk \leftarrow H(id)^s$

Return usk

proc H(id)

if $(ctr, id, Y, y) \in \Lambda_H$, return Y

$ctr \leftarrow ctr + 1$; $Y \stackrel{R}{\leftarrow} \mathbb{G}$

$\Lambda_H \leftarrow \Lambda_H \cup \{(ctr, id, Y, y)\}$

Return (C_1^*, C_2^*)

proc LR(id^*, m_0^*, m_1^*)

$r \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $C_1^* \leftarrow g^r$

$K \leftarrow \hat{e}(S, H(id^*))^r$

$C_2^* \leftarrow m_\beta^* \cdot K$

Return (C_1^*, C_2^*)

proc Finalize(β')

Return $(\beta' = \beta)$

Game $G_1^{\mathcal{A}}$

proc Initialize(k)

$\beta \stackrel{R}{\leftarrow} \{0,1\}$

$\boxed{i^* \stackrel{R}{\leftarrow} \{1,\ldots,q_H\}}$

$\Lambda_H \leftarrow \varepsilon$; $ctr \leftarrow 0$

$s \stackrel{R}{\leftarrow} \mathbb{Z}_p^*$; $S \leftarrow g^s$

$msk \leftarrow s$

$mpk \leftarrow S$

Return mpk

proc KeyDer(id)

if $(ctr, id, Y, y) \notin \Lambda_H$, $\mathbf{H}(id)$

$usk \leftarrow H(id)^s$

Return usk

proc H(id)

if $(ctr, id, Y, y) \in \Lambda_H$, return Y

$ctr \leftarrow ctr + 1$; $Y \stackrel{R}{\leftarrow} \mathbb{G}$

$\boxed{\text{if } i^* = ctr \text{ and } id \neq id^*, \text{ abort}}$

$\Lambda_H \leftarrow \Lambda_H \cup \{(ctr, id, Y, y)\}$

Return (C_1^*, C_2^*)

proc LR(id^*, m_0^*, m_1^*)

$r \stackrel{R}{\leftarrow} \mathbb{Z}_p$; $C_1^* \leftarrow g^r$

$K \leftarrow \hat{e}(S, H(id^*))^r$

$C_2^* \leftarrow m_\beta^* \cdot K$

Return (C_1^*, C_2^*)

Game $G_2^{\mathcal{A}}$

proc Initialize(k)

$\beta \xleftarrow{R} \{0,1\}$
$i^* \xleftarrow{R} \{1,\ldots,q_H\}$
$\Lambda_H \leftarrow \varepsilon$; $ctr \leftarrow 0$
$s \xleftarrow{R} \mathbb{Z}_p^*$; $S \leftarrow g^s$
$msk \leftarrow s$
$mpk \leftarrow S$
Return mpk

proc KeyDer(id)

if $(ctr, id, Y, y) \notin \Lambda_H$, $\mathbf{H}(id)$
$usk \leftarrow H(id)^s$
Return usk

proc H(id)

if $(ctr, id, Y, y) \in \Lambda_H$, return Y
$ctr \leftarrow ctr + 1$; $\boxed{y \xleftarrow{R} \mathbb{Z}_p \ ; \ Y \leftarrow g^y}$
if $i^* = ctr$ and $id \neq id^*$, abort
$\Lambda_H \leftarrow \Lambda_H \cup \{(ctr, id, Y, y)\}$
Return (C_1^*, C_2^*)

proc LR(id^*, m_0^*, m_1^*)

$r \xleftarrow{R} \mathbb{Z}_p$; $C_1^* \leftarrow g^r$
$K \leftarrow \hat{e}(S, H(id^*))^r$
$C_2^* \leftarrow m_\beta^* \cdot K$
Return (C_1^*, C_2^*)

Game G_3

Game $G_3^{\mathcal{A}}$

proc Initialize(k)

$\beta \xleftarrow{R} \{0,1\}$
$i^* \xleftarrow{R} \{1,\ldots,q_H\}$
$\Lambda_H \leftarrow \varepsilon\ ;\ ctr \leftarrow 0$
$s \xleftarrow{R} \mathbb{Z}_p^*\ ;\ S \leftarrow g^s$
$msk \leftarrow s$
$mpk \leftarrow S$
Return mpk

proc KeyDer(id)

if $(ctr, id, Y, y) \notin \Lambda_H$, $\mathbf{H}(id)$
read $(ctr, id, Y, y) \in \Lambda_H$
$usk \leftarrow S^y$
Return usk

proc H(id)

if $(ctr, id, Y, y) \in \Lambda_H$, return Y
$ctr \leftarrow ctr + 1\ ;\ y \xleftarrow{R} \mathbb{Z}_p\ ;\ Y \leftarrow g^y$
if $i^* = ctr$ and $id \neq id^*$, abort
$\Lambda_H \leftarrow \Lambda_H \cup \{(ctr, id, Y, y)\}$
Return (C_1^*, C_2^*)

proc LR(id^*, m_0^*, m_1^*)

$r \xleftarrow{R} \mathbb{Z}_p\ ;\ C_1^* \leftarrow g^r$
$K \leftarrow \hat{e}(S, H(id^*))^r$
$C_2^* \leftarrow m_\beta^* \cdot K$
Return (C_1^*, C_2^*)

Game G_4

Game $G_4^{\mathcal{A}}$

proc Initialize(k)

$\beta \xleftarrow{R} \{0,1\}$
$i^* \xleftarrow{R} \{1,\ldots,q_H\}$
$\Lambda_H \leftarrow \varepsilon$; $ctr \leftarrow 0$
$s \xleftarrow{R} \mathbb{Z}_p^*$; $S \leftarrow g^s$
$msk \leftarrow s$
$mpk \leftarrow S$
Return mpk

proc KeyDer(id)

if $(ctr, id, Y, y) \notin \Lambda_H$, $\mathbf{H}(id)$
read $(ctr, id, Y, y) \in \Lambda_H$
$usk \leftarrow S^y$
Return usk

proc H(id)

if $(ctr, id, Y, y) \in \Lambda_H$, return Y
$ctr \leftarrow ctr + 1$; $y \xleftarrow{R} \mathbb{Z}_p$; $Y \leftarrow g^y$
if $i^* = ctr$ and $id \neq id^*$, abort
$\Lambda_H \leftarrow \Lambda_H \cup \{(ctr, id, Y, y)\}$
Return (C_1^*, C_2^*)

proc LR(id^*, m_0^*, m_1^*)

$r \xleftarrow{R} \mathbb{Z}_p$; $C_1^* \leftarrow g^r$
$\boxed{K \xleftarrow{R} \mathbb{G}_T}$
$C_2^* \leftarrow m_\beta^* \cdot K$
Return (C_1^*, C_2^*)

Probability analysis

Claim 1 $\mathbf{Adv}^{\text{ind-cpa}}_{\mathcal{A},\mathsf{BF}}(k) = 2 \cdot \Pr\left[\, \mathsf{G}_0^{\mathcal{A}} = \mathsf{true} \,\right] - 1$

Claim 2 $\Pr\left[\, \mathsf{G}_1^{\mathcal{A}} = \mathsf{true} \,\right] = (1 - 1/q_H) \cdot 1/2 + 1/q_H \cdot \Pr\left[\, \mathsf{G}_0^{\mathcal{A}} = \mathsf{true} \,\right]$

Claim 3 $\Pr\left[\, \mathsf{G}_2^{\mathcal{A}} = \mathsf{true} \,\right] = \Pr\left[\, \mathsf{G}_1^{\mathcal{A}} = \mathsf{true} \,\right]$

Claim 4 $\Pr\left[\, \mathsf{G}_3^{\mathcal{A}} = \mathsf{true} \,\right] = \Pr\left[\, \mathsf{G}_2^{\mathcal{A}} = \mathsf{true} \,\right]$

Claim 5 $\left|\Pr\left[\, \mathsf{G}_4^{\mathcal{A}} = \mathsf{true} \,\right] - \Pr\left[\, \mathsf{G}_3^{\mathcal{A}} = \mathsf{true} \,\right]\right| \leq \mathbf{Adv}^{\text{bddh}}_{\mathcal{G},k}(\mathcal{B})$

Claim 6 $\Pr\left[\, \mathsf{G}_4^{\mathcal{A}} = \mathsf{true} \,\right] = 1/2$

It's straightforward to verify that the security theorem follows from the claims above.

Proof of claims

- Claim 1 follows the security definition.
- Claims 3 and 4 are true because the changes made to the games do not affect their outcome.
- Claims 2 follows from the fact that the output of the game is chosen uniformly at random when aborting.

$$
\begin{aligned}
&\Pr\left[\,\mathsf{G}_1^{\mathcal{A}} = \text{true}\,\right] \\
&= \Pr\left[\,\mathsf{G}_1^{\mathcal{A}} = \text{true} \wedge \mathrm{abort}\,\right] + \Pr\left[\,\mathsf{G}_1^{\mathcal{A}} = \text{true} \wedge \overline{\mathrm{abort}}\,\right] \\
&= \Pr\left[\,\mathsf{G}_1^{\mathcal{A}} = \text{true} | \mathrm{abort}\,\right] \Pr\left[\,\mathrm{abort}\,\right] + \\
&\quad \Pr\left[\,\mathsf{G}_1^{\mathcal{A}} = \text{true} | \overline{\mathrm{abort}}\,\right] \Pr\left[\,\overline{\mathrm{abort}}\,\right] \\
&= 1/2 \cdot (1 - 1/q_H) + \Pr\left[\,\mathsf{G}_1^{\mathcal{A}} = \text{true} | \overline{\mathrm{abort}}\,\right] \cdot 1/q_H \\
&= 1/2 \cdot (1 - 1/q_H) + \Pr\left[\,\mathsf{G}_0^{\mathcal{A}} = \text{true}\,\right] \cdot 1/q_H
\end{aligned}
$$

Proof of claims (cont.)

- In order to prove Claim 5, we need to build an adversary $\mathcal{B}$ against the BDDH problem.
  - Let $(\mathbb{G}, g, A, B, C, Z)$ be the input of $\mathcal{B}$.
  - $\mathcal{B}$ sets $mpk = A$, $C_1^* = B$, $H(id^*) = C$, and $K = Z$. Everything else in the simulation is performed as in G_3.
  - When $\mathcal{B}$ is being executed in Game $\mathbf{Exp}^{\mathrm{bddh}\text{-}0}_{\mathcal{G},k}(\mathcal{B})$, $\mathcal{B}$ simulates G_3 to $\mathcal{A}$. That is, $\Pr\left[\mathsf{G}_3^{\mathcal{A}} = \mathsf{true}\right] = \Pr\left[\mathbf{Exp}^{\mathrm{bddh}\text{-}0}_{\mathcal{G},k}(\mathcal{B}) = \mathsf{true}\right]$.
  - When $\mathcal{B}$ is being executed in Game $\mathbf{Exp}^{\mathrm{bddh}\text{-}1}_{\mathcal{G},k}(\mathcal{B})$, $\mathcal{B}$ simulates G_4 to $\mathcal{A}$. That is, $\Pr\left[\mathsf{G}_4^{\mathcal{A}} = \mathsf{true}\right] = \Pr\left[\mathbf{Exp}^{\mathrm{bddh}\text{-}1}_{\mathcal{G},k}(\mathcal{B}) = \mathsf{true}\right]$.
  - The claim follows.
- Claim 6 follows from the fact that $\mathcal{A}$ has no information about β in G_4 and that the output of the game is chosen uniformly at random when aborting.

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