Network Economics: two examples

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(A) Diffusion in Social Networks

(B) Economics of Information Security
(1) Diffusion Model
inspired from game theory
and statistical physics.

(2) Results
from a mathematical analysis.

(3) Adding Clustering
joint work with Emilie Coupechoux
(0) Context

Crossing the Chasm
(Moore 1991)
(1) Diffusion Model

(2) Results

(3) Adding Clustering
(1) Coordination game...

- Both receive payoff $q$.  
- Both receive payoff $1-q > q$.  
- Both receive nothing.
(1)...on a network.

- Everybody start with ICQ.
- Total payoff = sum of the payoffs with each neighbor.
- A seed of nodes switches to \textit{Blume 95, Morris 00}.
(1) Threshold Model

- State of agent $i$ is represented by

$$X_i = \begin{cases} 
0 & \text{if } \text{icq} \\
1 & \text{if } \text{talk}
\end{cases}$$

- Switch from \text{icq} to \text{talk} if:

$$\sum_{j \sim i} X_j \geq qd_i$$
(1) Model for the network?

- $p = 0.04$
- $p = 0.05$
- $p = 0.08$

Statistical physics: bootstrap percolation.
(1) Model for the network?
(1) Random Graphs

- Random graphs with given degree sequence introduced by (Molloy and Reed, 95).
- Examples:
  - Erdös-Rényi graphs, $G(n, \lambda/n)$.
  - Graphs with power law degree distribution.
- We are interested in large population asymptotics.
  - Average degree is $\lambda$.
  - No clustering: $C=0$. 
(1) Diffusion Model
q = relative threshold
\( \lambda \) = average degree

(2) Results

(3) Adding Clustering
(1) Diffusion Model
\[ q = \text{relative threshold} \]
\[ \lambda = \text{average degree} \]

(2) Results

(3) Adding Clustering
(2) Contagion (Morris 00)

• Does there exist a finite groupe of players such that their action under best response dynamics spreads contagiously everywhere?

• Contagion threshold: $q_c = \text{largest } q \text{ for which contagious dynamics are possible.}$

• Example: interaction on the line

\[ q_c = \frac{1}{2} \]
(2) Another example: d-regular trees

\[ q_c = \frac{1}{d} \]
(2) Some experiments

Seed = one node, $\lambda=3$ and $q=0.24$

(source: the Technoaverse blog)
(2) Some experiments

Seed = one node, $\lambda = 3$ and $1/q > 4$

(source: the Technoverse blog)
(2) Some experiments

Seed = one node, $\lambda=3$ and $q=0.24$ (or $1/q>4$)
(source: the Technoverse blog)
(2) Contagion threshold

In accordance with (Watts 02)

Contagion threshold

No cascade

Global cascades

Mean degree
(2) A new Phase Transition
(2) Pivotal players

- Giant component of players requiring only one neighbor to switch: deg < 1/q.

Tipping point:
Diffusion like standard epidemic

Chasm:
Pivotal players = Early adopters
(2) q above contagion threshold

• New parameter: size of the seed as a fraction of the total population $0 < \alpha < 1$.

• Monotone dynamic $\rightarrow$ only one final state.
(2) Minimal size of the seed, $q > 1/4$

Tipping point: Connectivity helps

Chasm: Connectivity hurts

Mean degree
(2) $q > 1/4$, low connectivity

Connectivity helps the diffusion.
(2) $q > 1/4$, high connectivity

Connectivity inhibits the global cascade, but once it occurs, it facilitates its diffusion.
(2) Equilibria for $q < q_c$

- Trivial equilibria: all 0 / all 1
- Initial seed applies best-response, hence can switches back. If the dynamic converges, it is an equilibrium.
- Robustness of all 0 equilibrium?
- Initial seed = 2 pivotal neighbors
  -> pivotal equilibrium
(2) Strength of Equilibria for $q < q_c$

Mean number of trials to switch from all 0 to pivotal equilibrium

In Contrast with (Montanari, Saberi 10)
Their results for $q \approx 1/2$
(2) Coexistence for $q < q_c$

Size giant component

Connected Players 0

Players 1

Coexistence
(1) Diffusion Model

(2) Results

(3) Adding Clustering
joint work with Emilie Coupechoux
(3) Simple model with tunable clustering

- Clustering coefficient:

\[ C = \frac{3 \text{ number of triangles}}{\text{number of connected triples}} \]

- Adding cliques:
(3) Contagion threshold with clustering

Clustering inhibits contagion

Clustering helps contagion

No cascade

Global cascades

Positive clustering

No clustering
(3) Side effect of clustering!

Fraction of pivotal players and size of the cascade
Conclusion for (A)

• Simple tractable model:
  – Threshold rule
  – Random network: heterogeneity of population
  – Tunable degree/clustering

• 1 notion: **Pivotal Players** and 2 regimes:
  – Low connectivity: tipping point / clustering hurts
  – High connectivity: chasm / clustering helps activation

• **Open problems:**
  – Size of optimal seed? Dynamics of the diffusion? More than 2 states?
(A) Diffusion in Social Networks

(B) Economics of Information Security
(1) Network Security Games

(2) Complete Information

(3) Incomplete Information
(1) Network Security as a Public Good

- System security often depends on the effort of many individuals, making security a public good. Hirshleifer (83), Varian (02).

- Weakest link: security depends on the minimum effort.

- Best shot: security depends on the maximum effort.
(1) Local Best Response

\[ X_i = \begin{cases} 
1 & \text{if Secure} \\
0 & \text{if Not protected} 
\end{cases} \]

- Weakest link:
  \[ X_i = \min_{j \sim i} X_j \]

- Best shot:
  \[ X_i = \min_{j \sim i} (1 - X_j) \]
(1) Network Security Games

(2) Complete Information

(3) Incomplete Information
(2) Complete Information

- Weakest link
  
  \[ X_i = \min_{j \sim i} X_j \]

  Only trivial equilibria: all 0/ all 1.

- Best shot
  
  \[ X_i = \min_{j \sim i} (1 - X_j) \]
(2) Slight extensions WL

- Weakest link:
  \[ X_i = \min_{j \sim i} X_j = 1(\sum_{j \sim i} X_j \geq d_i) \]

- Weakest link with parameter K:
  \[ X_i = 1(\sum_{j \sim i} X_j \geq d_i - K) \]

- Change of variables:
  \[ Y_i = 1 - X_i = 1(\sum_{j \sim i} Y_j \geq K + 1) \]

  Bootstrap percolation!

- Richer structure of equilibria.
(2) Slight extensions BS

• Best shot:
\[ X_i = \min_{j \sim i} (1 - X_j) = 1(\sum_{j \sim i} X_j \leq 0) \]

• Best shot with parameter K:
\[ X_i = 1(\sum_{j \sim i} X_j \leq K) \]

• Equilibria: Maximal Independent Set of order K+1. Bramoullé Kranton (07)
(2) Best Shot with parameter $K=1$
(1) Network Security Games

(2) Complete Information

(3) Incomplete Information
(3) Incomplete Information

• Bayesian game. Galeotti et al. (10)
• Type = degree $d$
• Neighbors’ actions are i.i.d. $\text{Bern}(\gamma)$
• Network externality function:

$$h(\gamma, d) = P(\text{loss} | X_i = 0) - P(\text{loss} | X_i = 1)$$

corresponds to the price, an agent with degree $d$ is ready to invest in security.
(3) Weakest link

- Network externality function:
  \[ h(\gamma, d) = E\left[ \min_{i=1}^{d} \text{Bern}_i(\gamma) \right] - 0 \]
  \[ = \gamma^d \]

- This function being increasing, incentives are aligned in the population.

- This gives a Coordination problem for the game. Lelarge (12)
(3) Best shot

- Network externality function:

\[
h(\gamma, d) = 1 - E\left[ \max_{i=1}^{d} \text{Bern}_i(\gamma) \right] \\
= (1 - \gamma)^d
\]

- This function being decreasing, incentives are not aligned in the population.

- This gives a Free rider problem for the game. Lelarge (12)
Conclusion (B)

• **Information structure** of the game is crucial: network externality function when incomplete information.

• Technology is not enough! There is a need to design *economic incentives* to ensure the deployment of security technologies.

• **Open problems:**
  – Dynamics? mean field games? more quantitative results? Local and global interactions?
Thank you!