Network Economics: two examples

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(A) Diffusion in Social Networks

(B) Economics of Information Security

(1) Diffusion Model

inspired from game theory
and statistical physics.

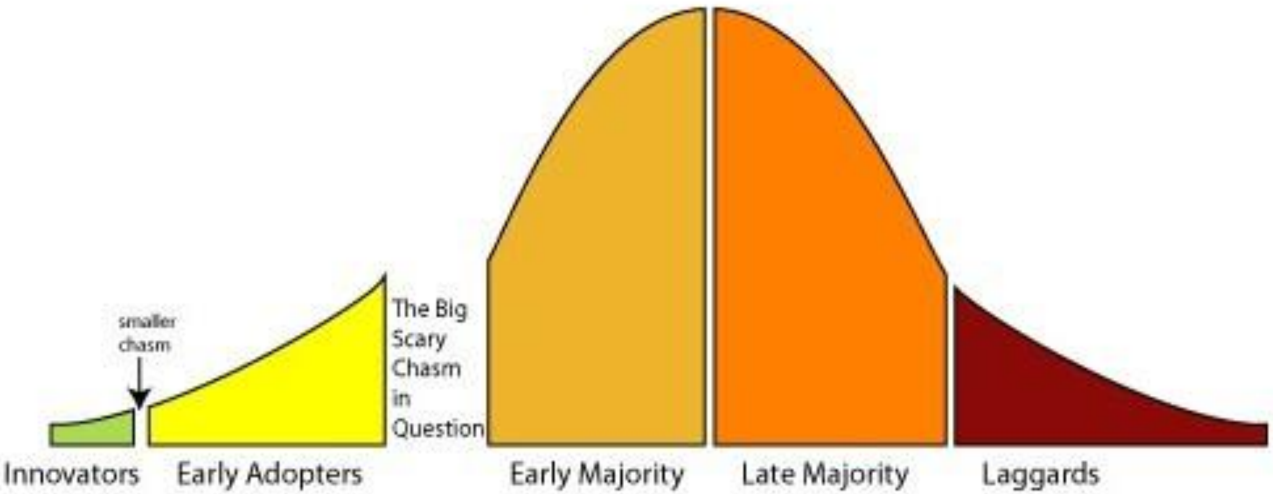
(2) Results

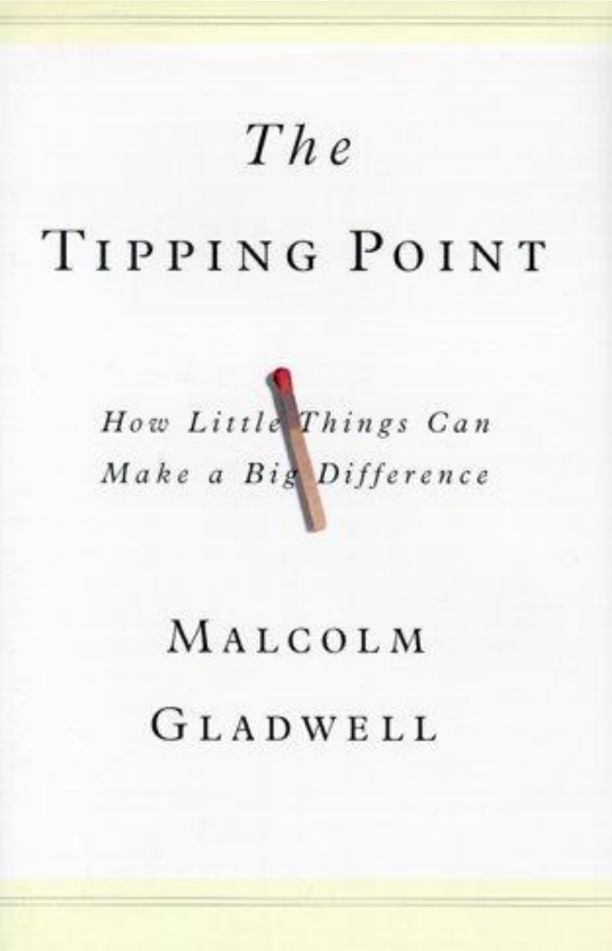
from a mathematical analysis.

(3) Adding Clustering

joint work with Emilie Coupechoux

(0) Context





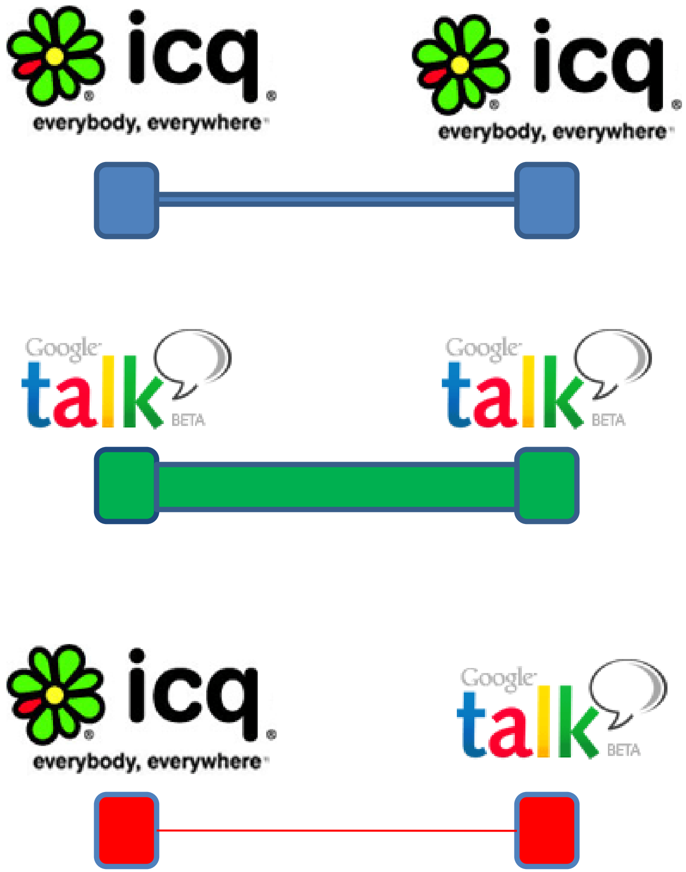
Crossing the Chasm
(Moore 1991)

(1) Diffusion Model

(2) Results

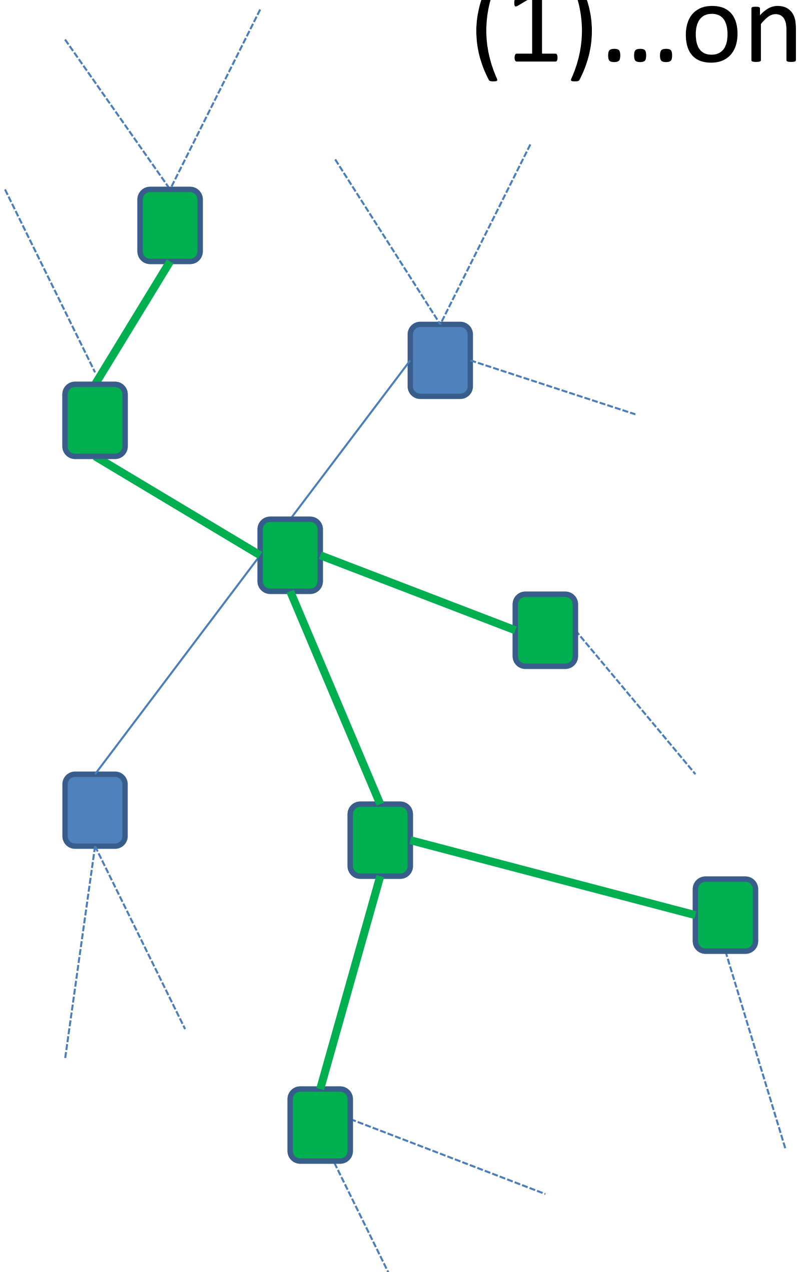
(3) Adding Clustering

(1) Coordination game…



- Both receive payoff q.
- Both receive payoff $1-q>q$.
- Both receive nothing.

(1)...on a network.



- Everybody start with icq everybody, everywhere
- Total payoff = sum of the payoffs with each neighbor.
- A seed of nodes switches to Google talk BETA

(Blume 95,
Morris 00)

(1) Threshold Model

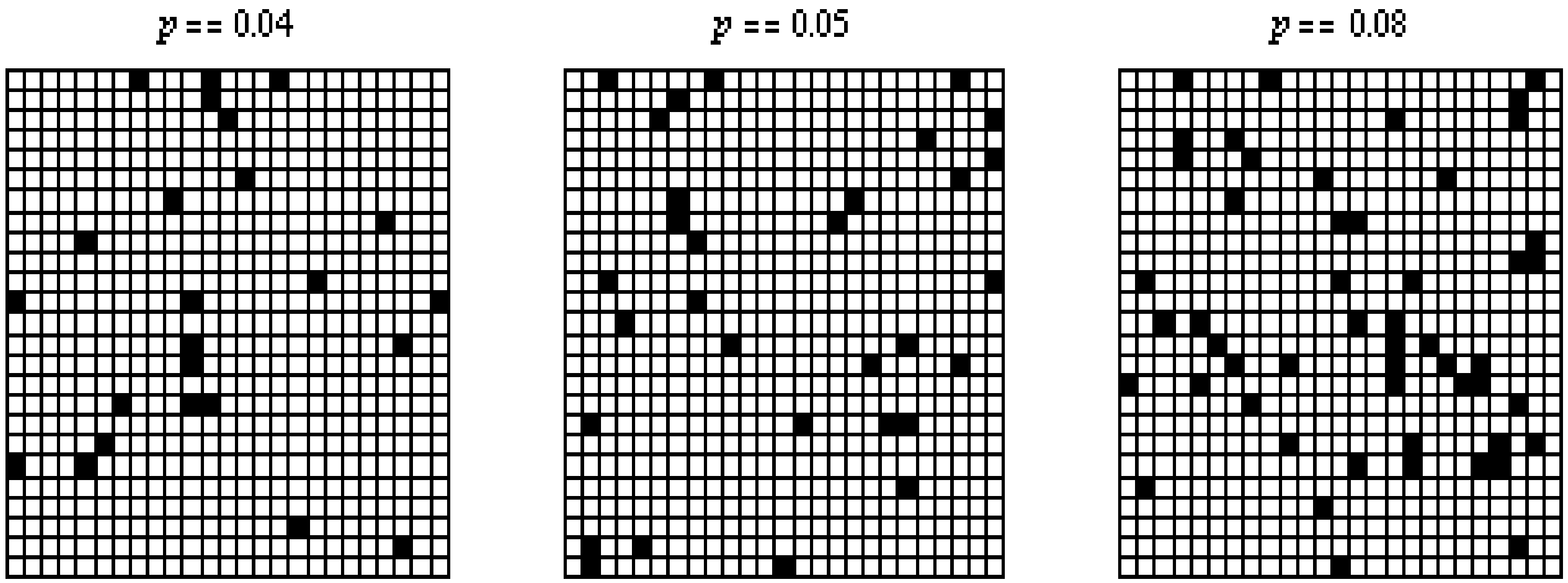
- State of agent i is represented by

$$X_i = \begin{cases} 0 & \text{if icq} \\ 1 & \text{if Google talk} \end{cases}$$

- Switch from icq to Google talk if:

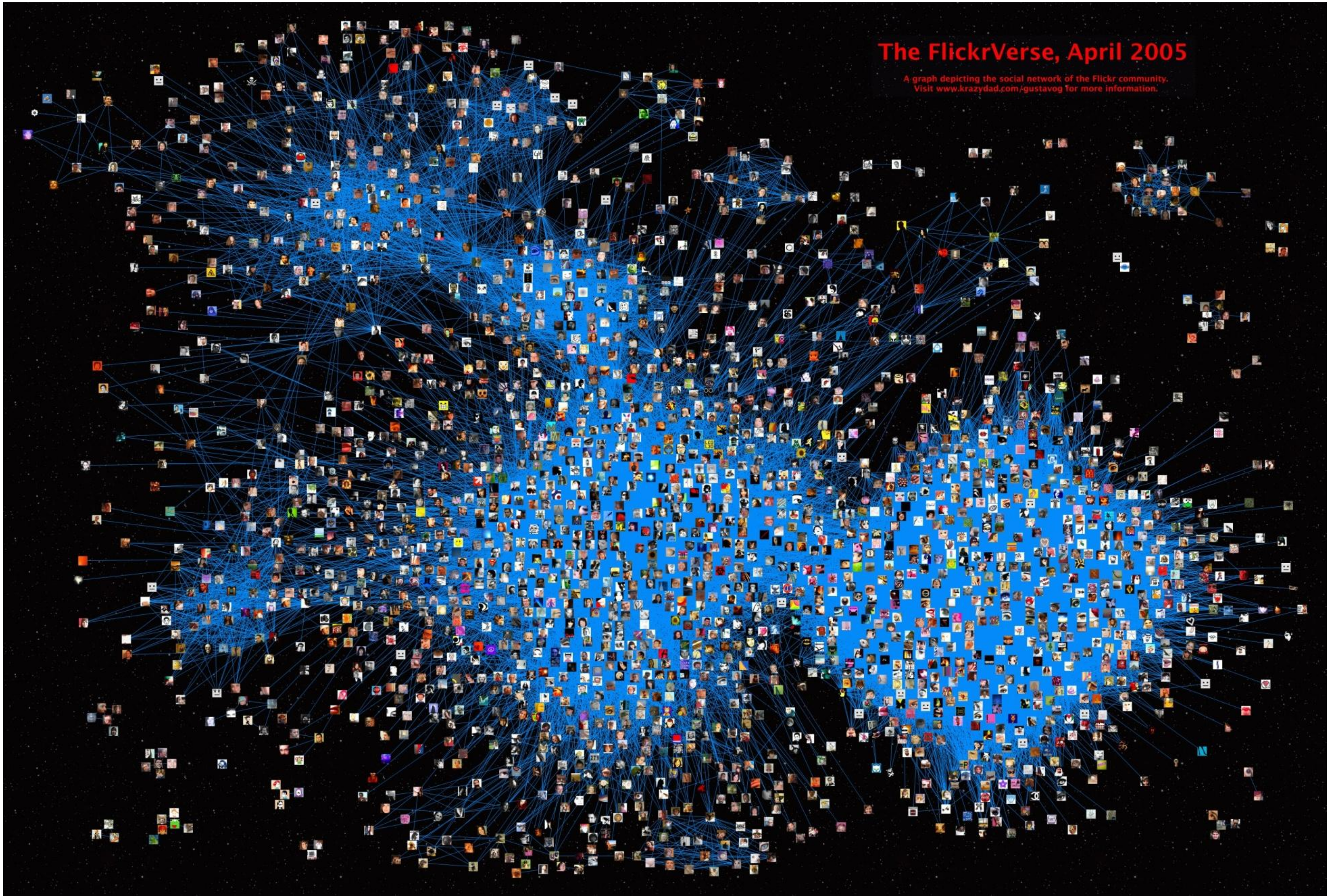
$$\sum_{j \sim i} X_j \geq q d_i$$

(1) Model for the network?



Statistical physics: bootstrap percolation.

(1) Model for the network?



(1) Random Graphs

- Random graphs with given degree sequence introduced by (Molloy and Reed, 95).
- Examples:
  - Erdös-Réyni graphs, $G(n,\lambda/n)$.
  - Graphs with power law degree distribution.
- We are interested in large population asymptotics.
- Average degree is λ.
- No clustering: $C=0$.

(1) Diffusion Model

q = relative threshold

λ = average degree

(2) Results

(3) Adding Clustering

(1) Diffusion Model

q = relative threshold

λ = average degree

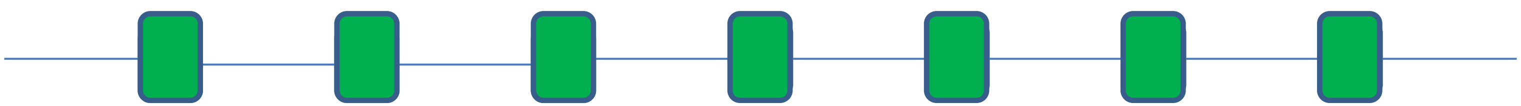
(2) Results

(3) Adding Clustering

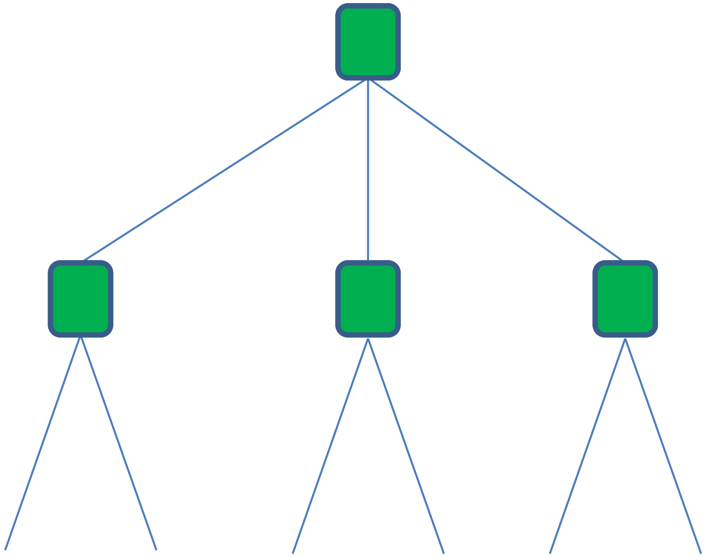
(2) Contagion (Morris 00)

- Does there exist a finite groupe of players such that their action under best response dynamics spreads contagiously everywhere?
- Contagion threshold: q_c = largest q for which contagious dynamics are possible.
- Example: interaction on the line

$$q_c = \frac{1}{2}$$

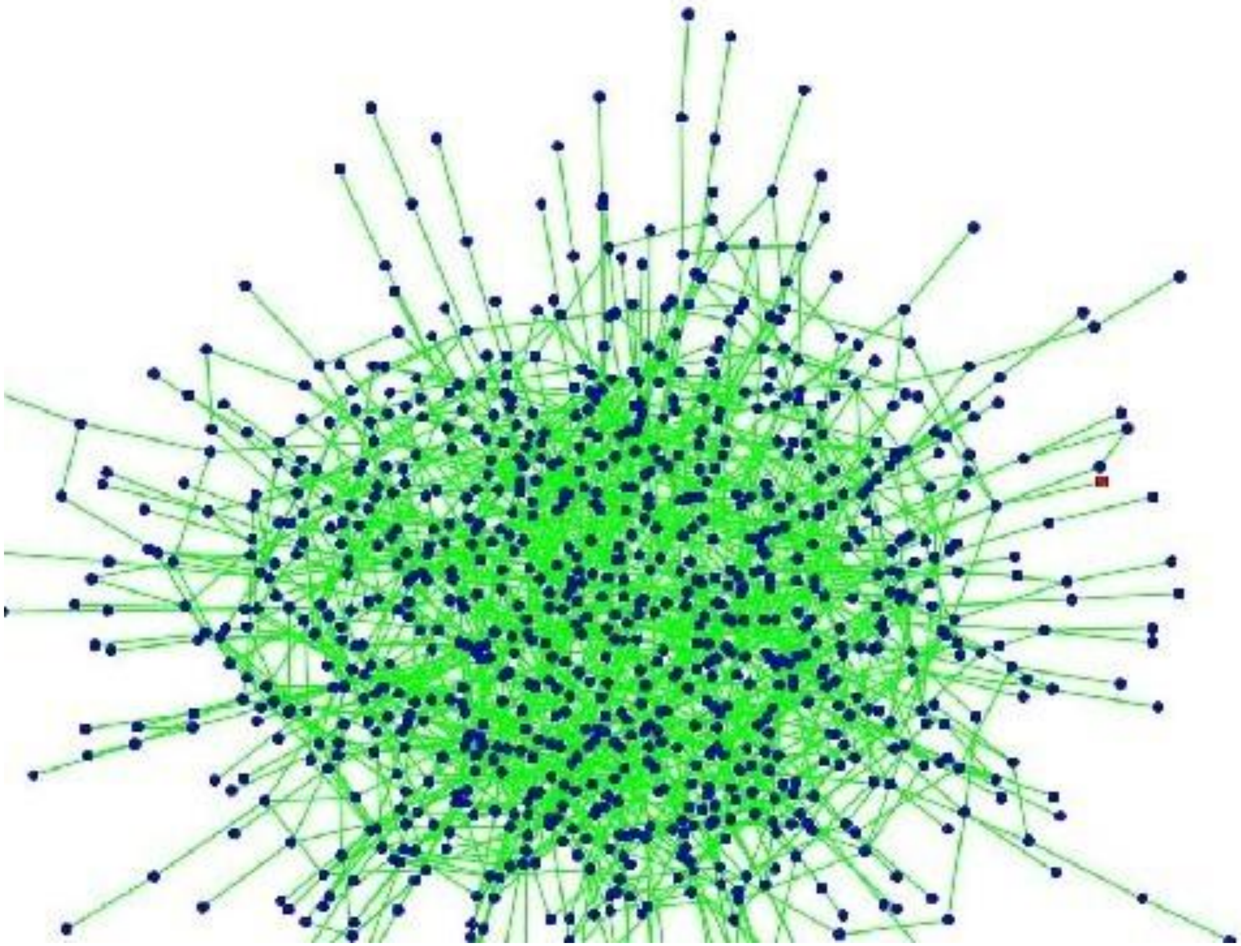


(2)Another example: d-regular trees



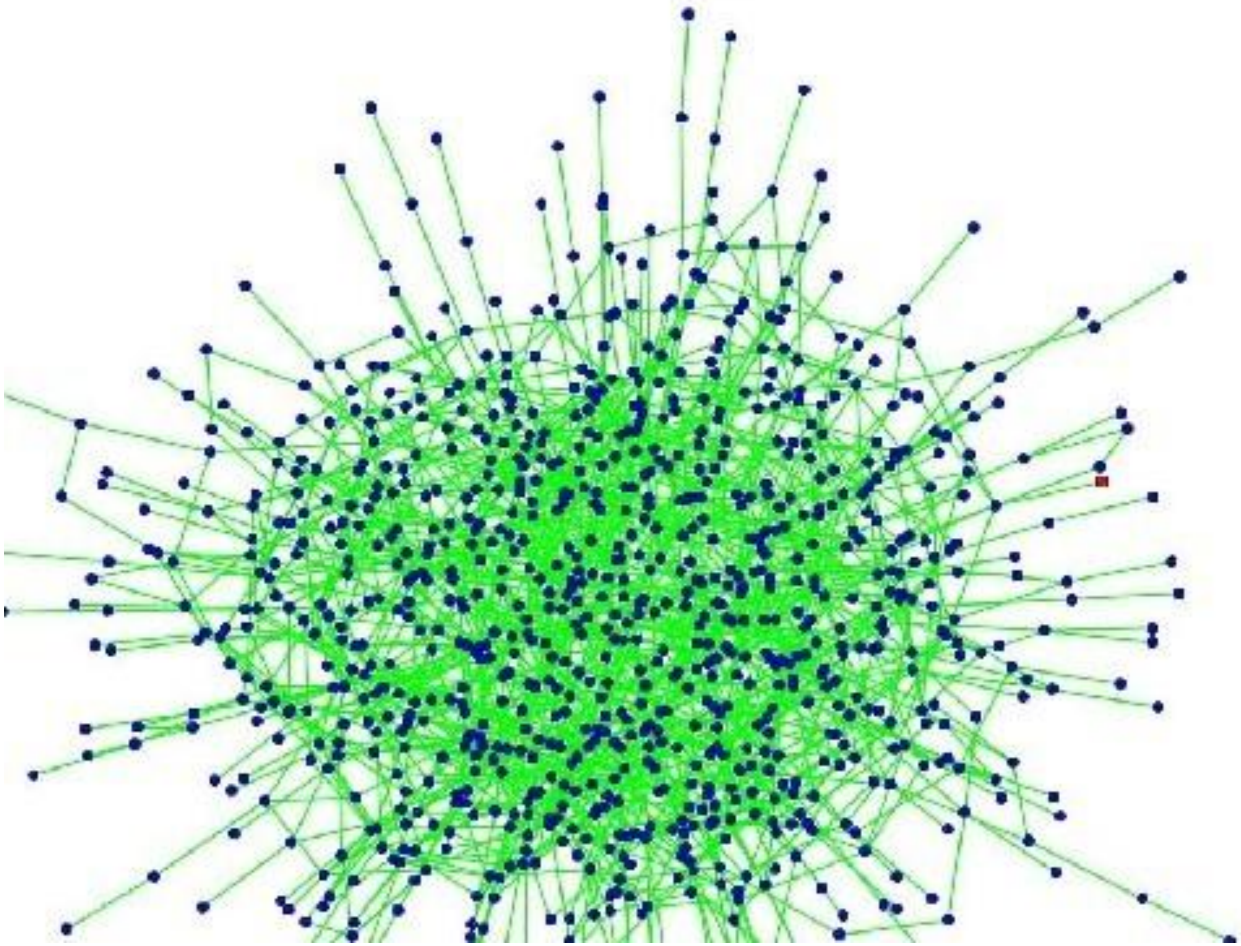
$$q_c = \frac{1}{d}$$

(2) Some experiments



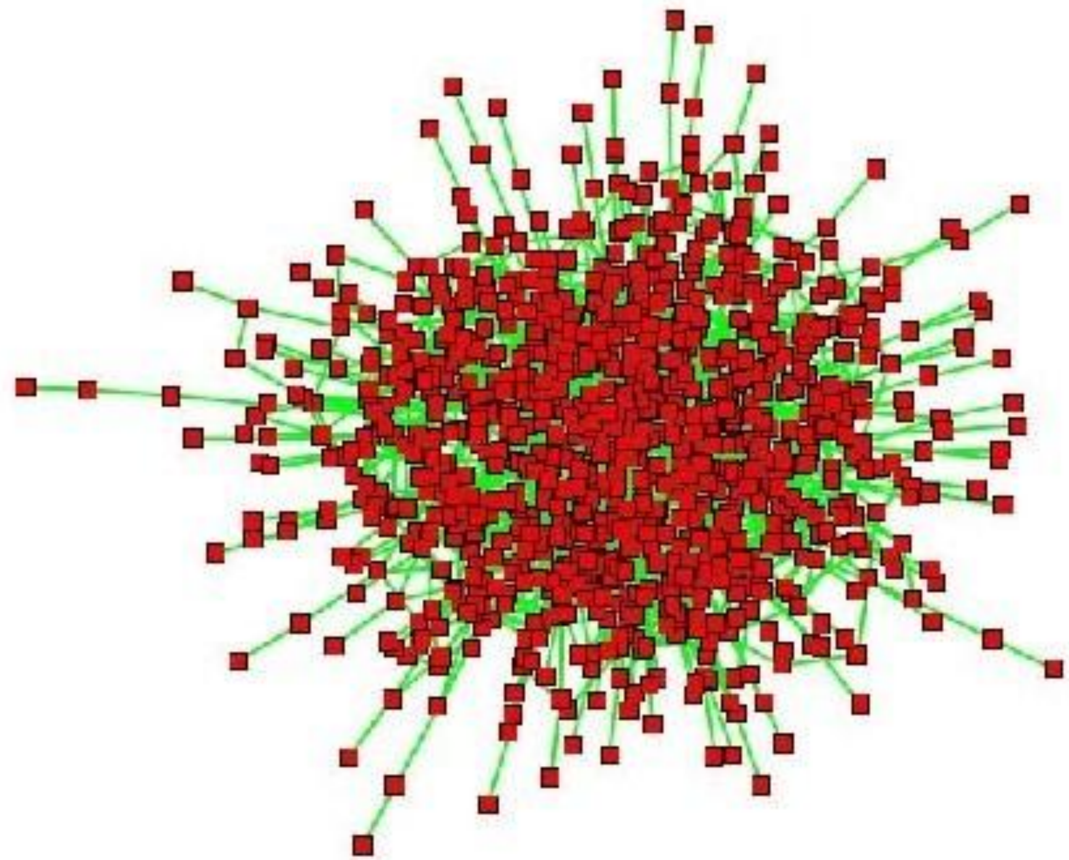
Seed = one node, λ=3 and q=0.24
(source: the Technoverse blog)

(2) Some experiments



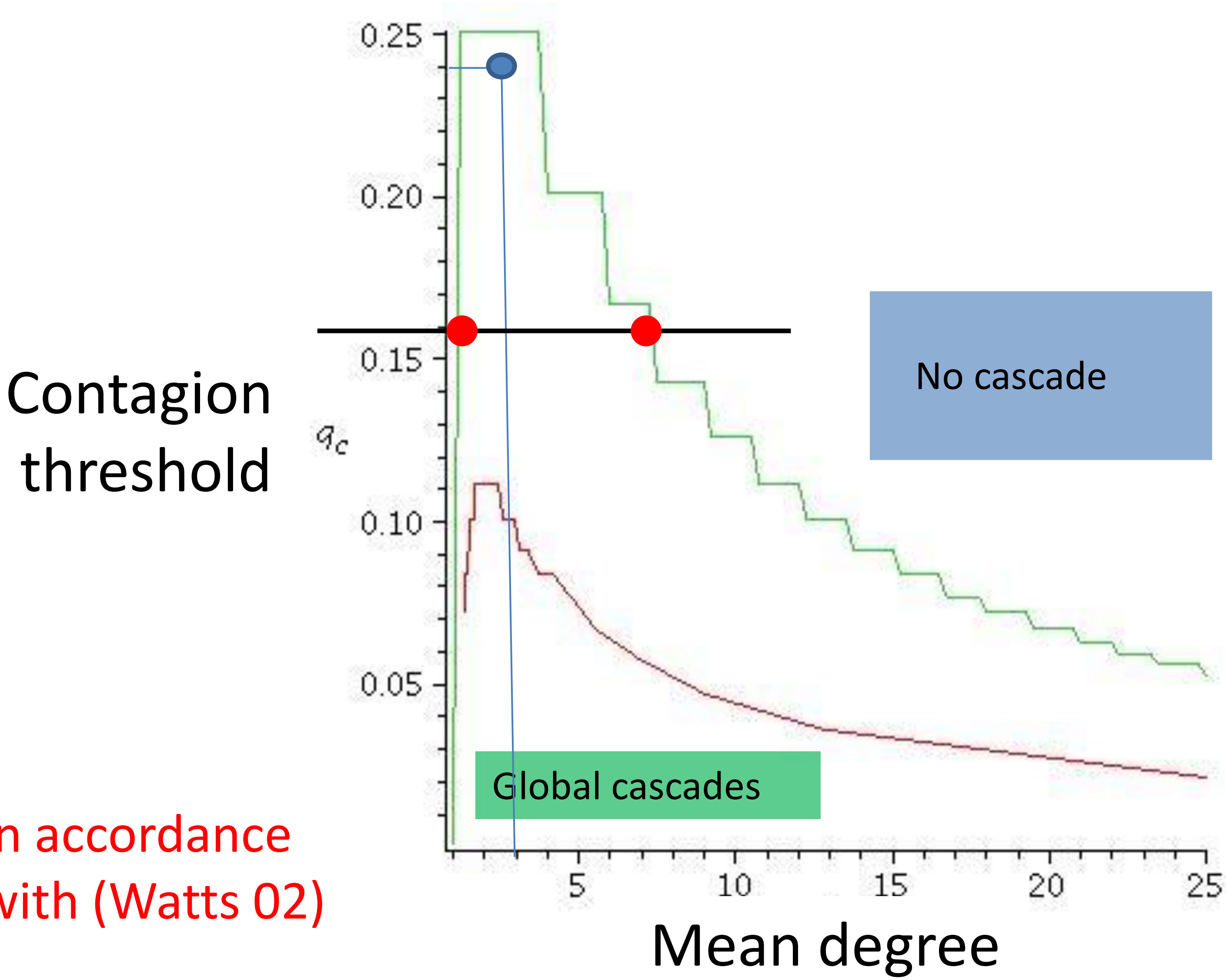
Seed = one node, λ=3 and 1/q>4
(source: the Technoverse blog)

(2) Some experiments



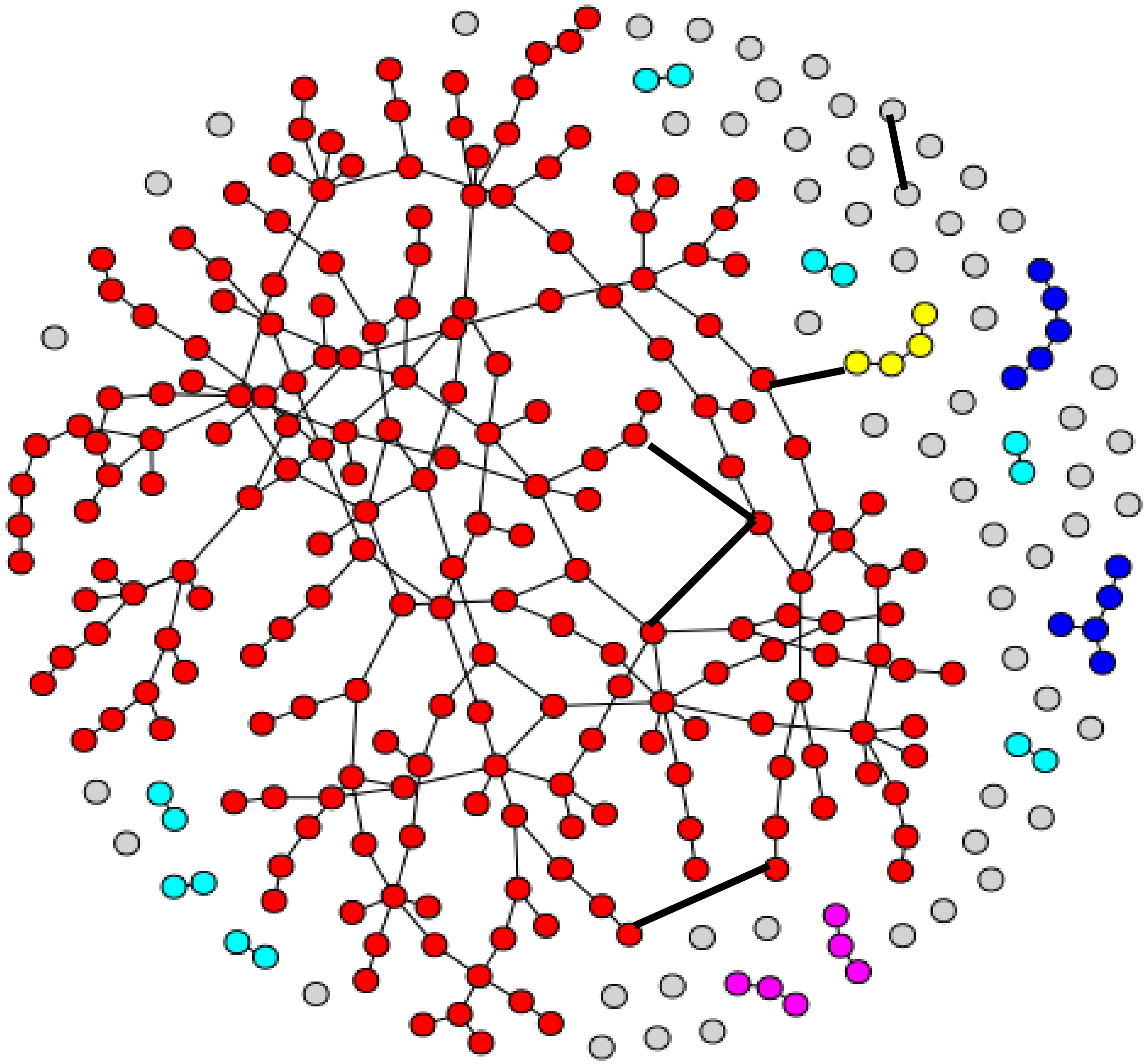
Seed = one node, λ=3 and q=0.24 (or 1/q>4)
(source: the Technoverse blog)

(2) Contagion threshold



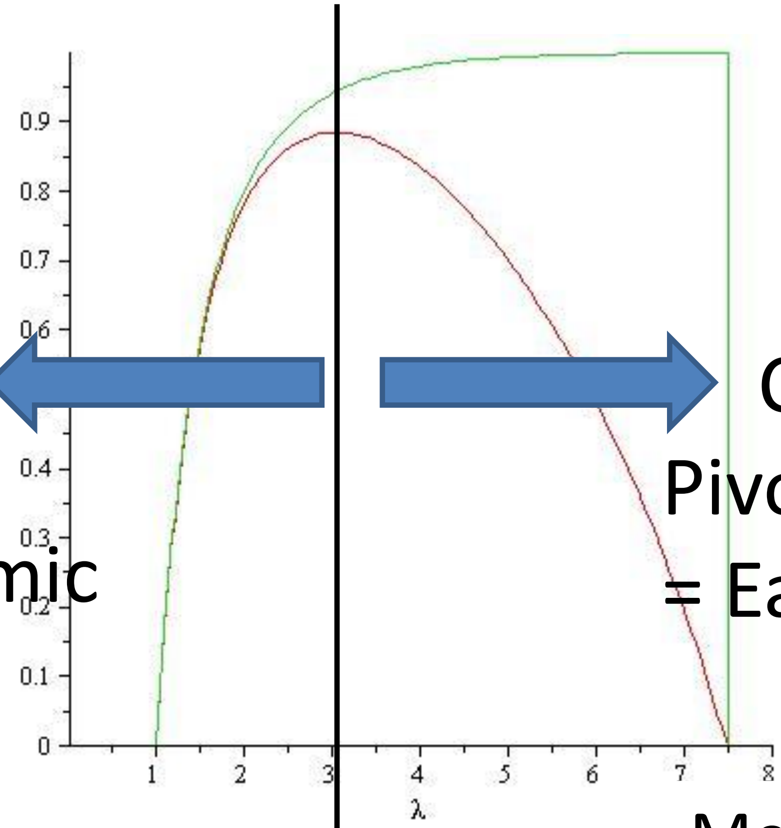
In accordance with (Watts 02)

(2) A new Phase Transition



(2) Pivotal players

- Giant component of players requiring only one neighbor to switch: deg <1/q.



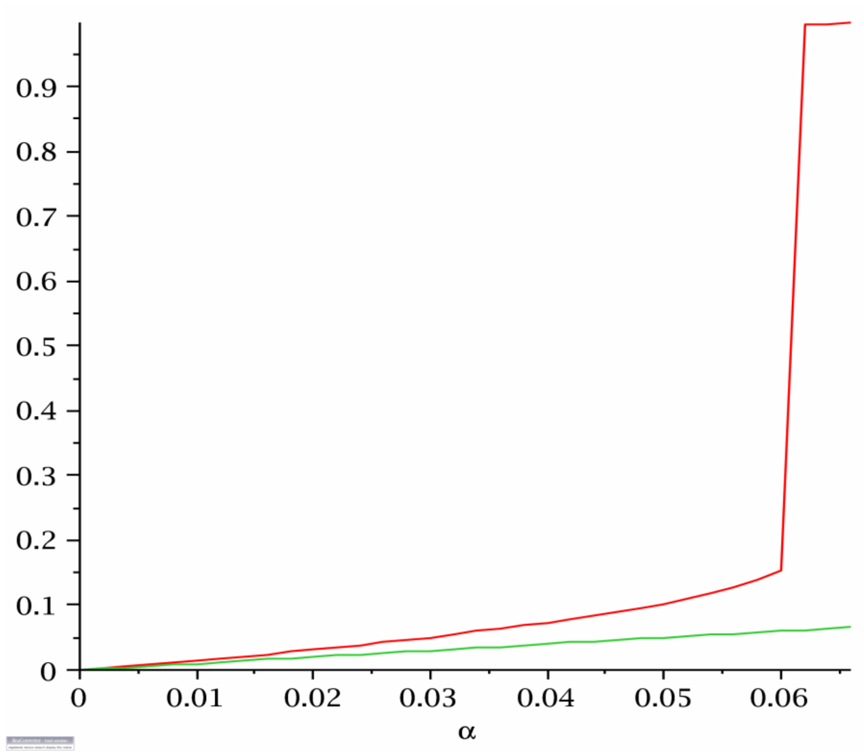
Tipping point: Diffusion like standard epidemic

Chasm : Pivotal players = Early adopters

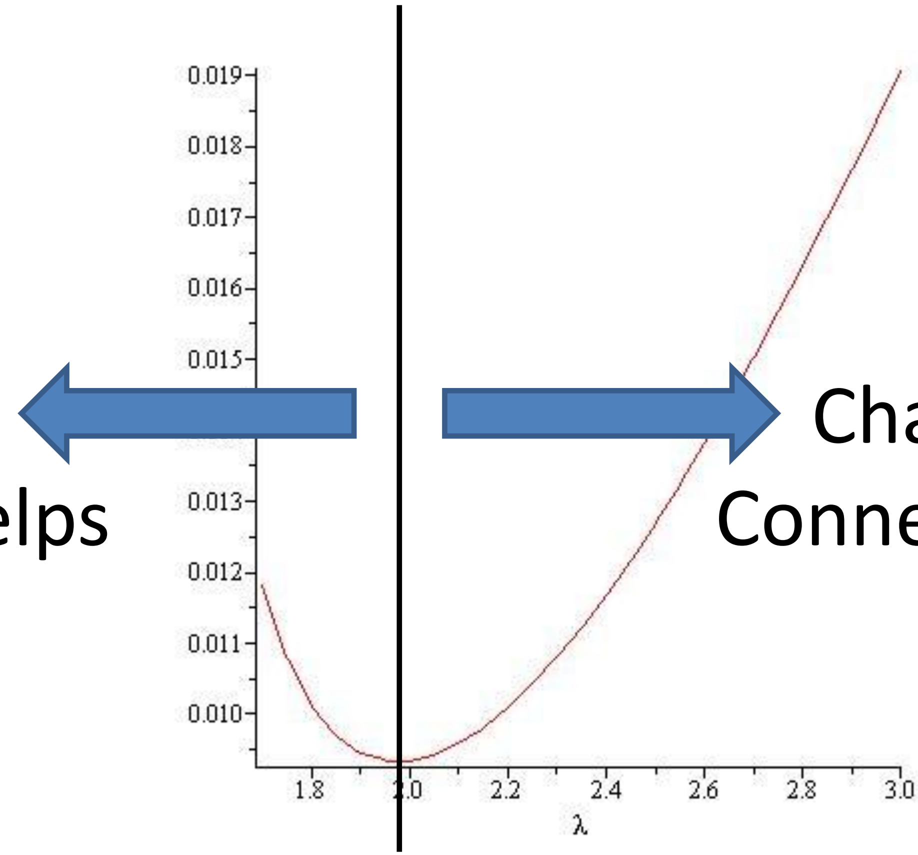
Mean degree

(2) q above contagion threshold

- New parameter: size of the seed as a fraction of the total population $0 < \alpha < 1$.
- Monotone dynamic → only one final state.



(2)Minimal size of the seed, q>1/4



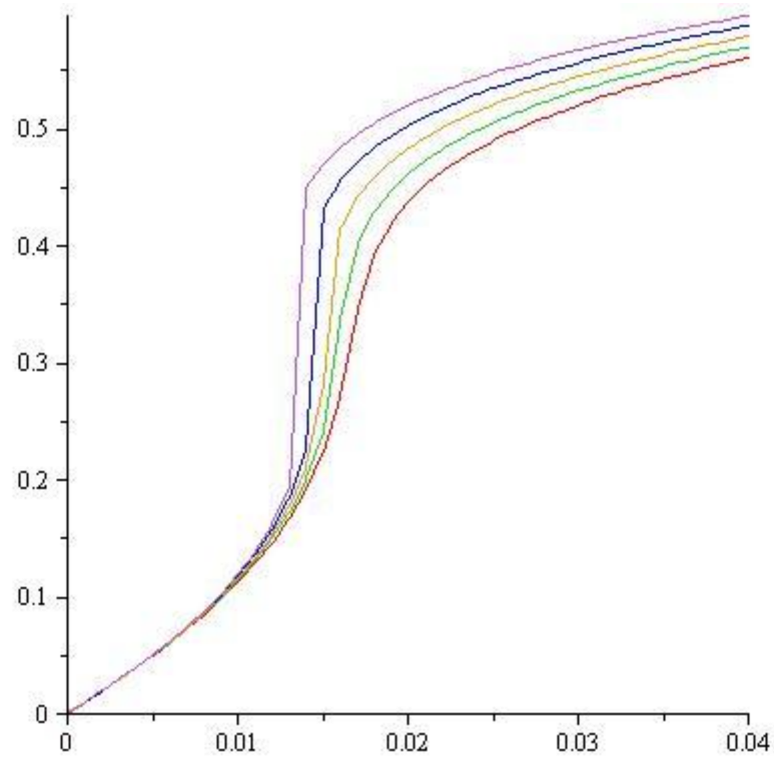
Tipping point:
Connectivity helps

Chasm :
Connectivity hurts

Mean degree

(2) q>1/4, low connectivity

Size of the contagion

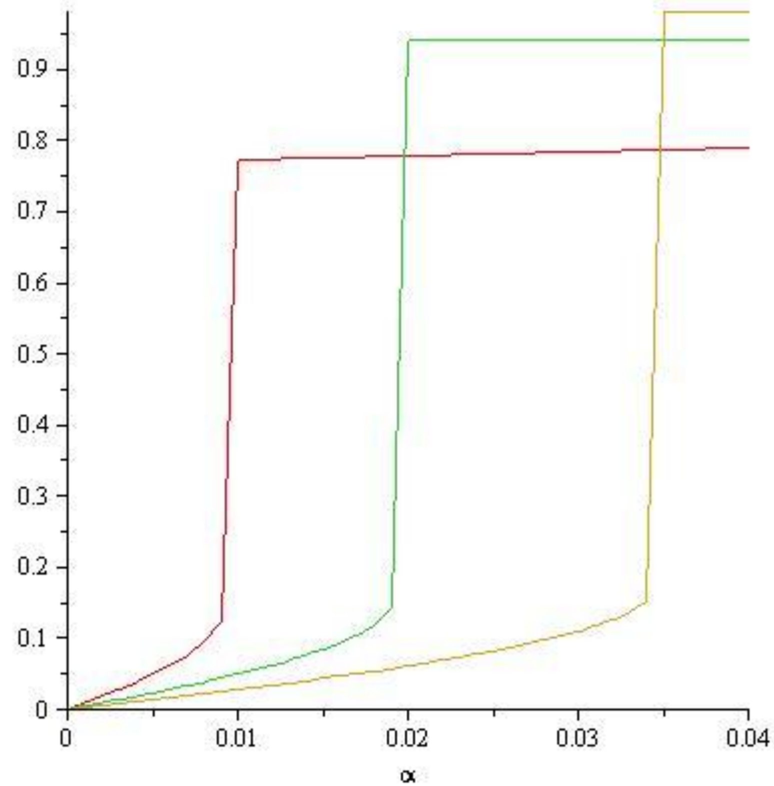


Size of the seed

Connectivity helps the diffusion.

(2) q>1/4, high connectivity

Size of the contagion



Size of the seed

Connectivity inhibits the global cascade,
but once it occurs, it facilitates its diffusion.

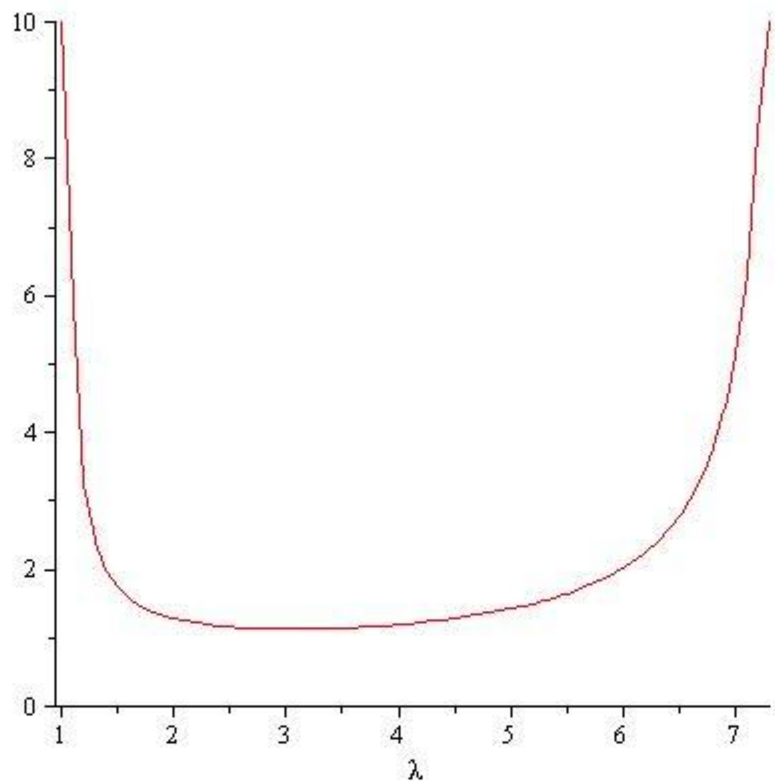
(2) Equilibria for $q<q_c$

- Trivial equilibria: all 0 / all 1
- Initial seed applies best-response, hence can switches back. If the dynamic converges, it is an equilibrium.
- Robustness of all 0 equilibrium?
- Initial seed = 2 pivotal neighbors

  –> pivotal equilibrium

(2) Strength of Equilibria for $q<q_c$

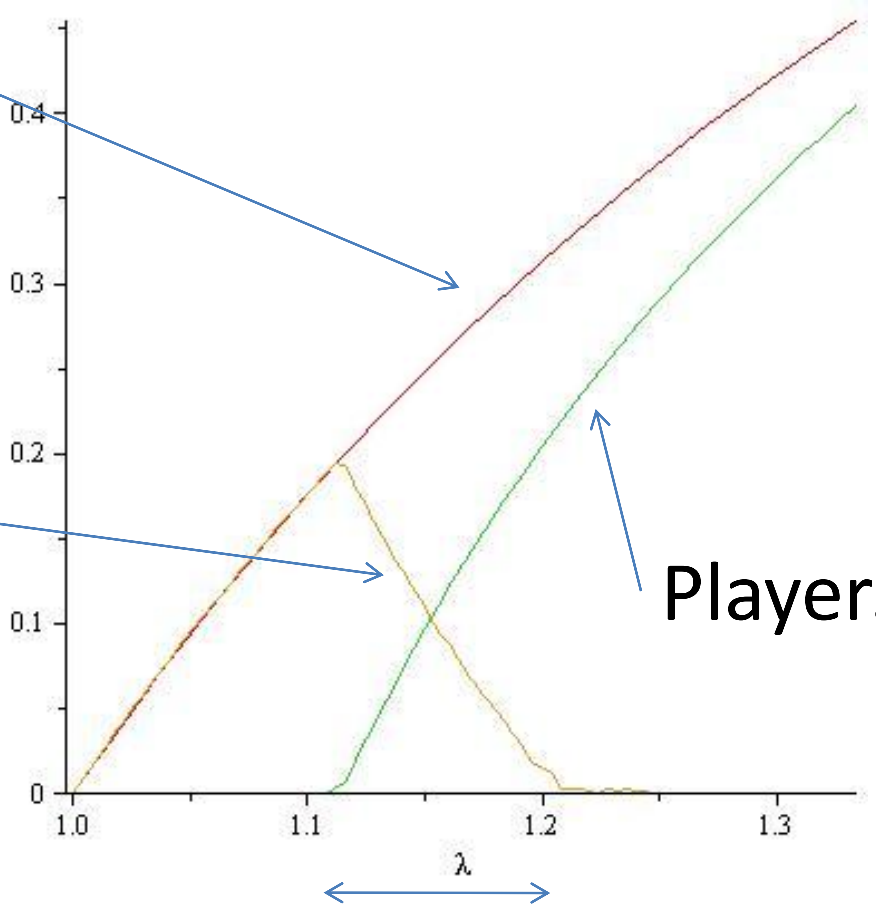
Mean number of trials to switch from all 0 to pivotal equilibrium



Mean degree

In Contrast with (Montanari , Saberi 10) Their results for q≈1/2

(2) Coexistence for $q<q_c$



Size giant component

Connected Players 0

Players 1

Coexistence

(1) Diffusion Model

(2) Results

(3) Adding Clustering

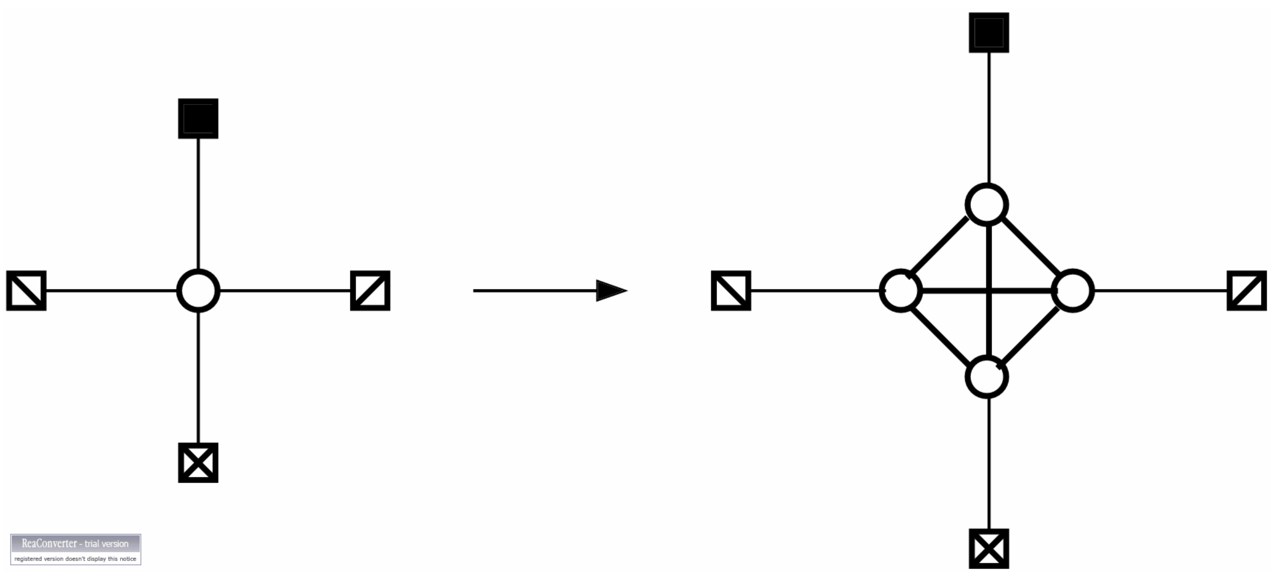
joint work with Emilie Coupechoux

(3) Simple model with tunable clustering

- Clustering coefficient:

$$C = \frac{3 \text{ number of triangles}}{\text{number of connected triples}}$$

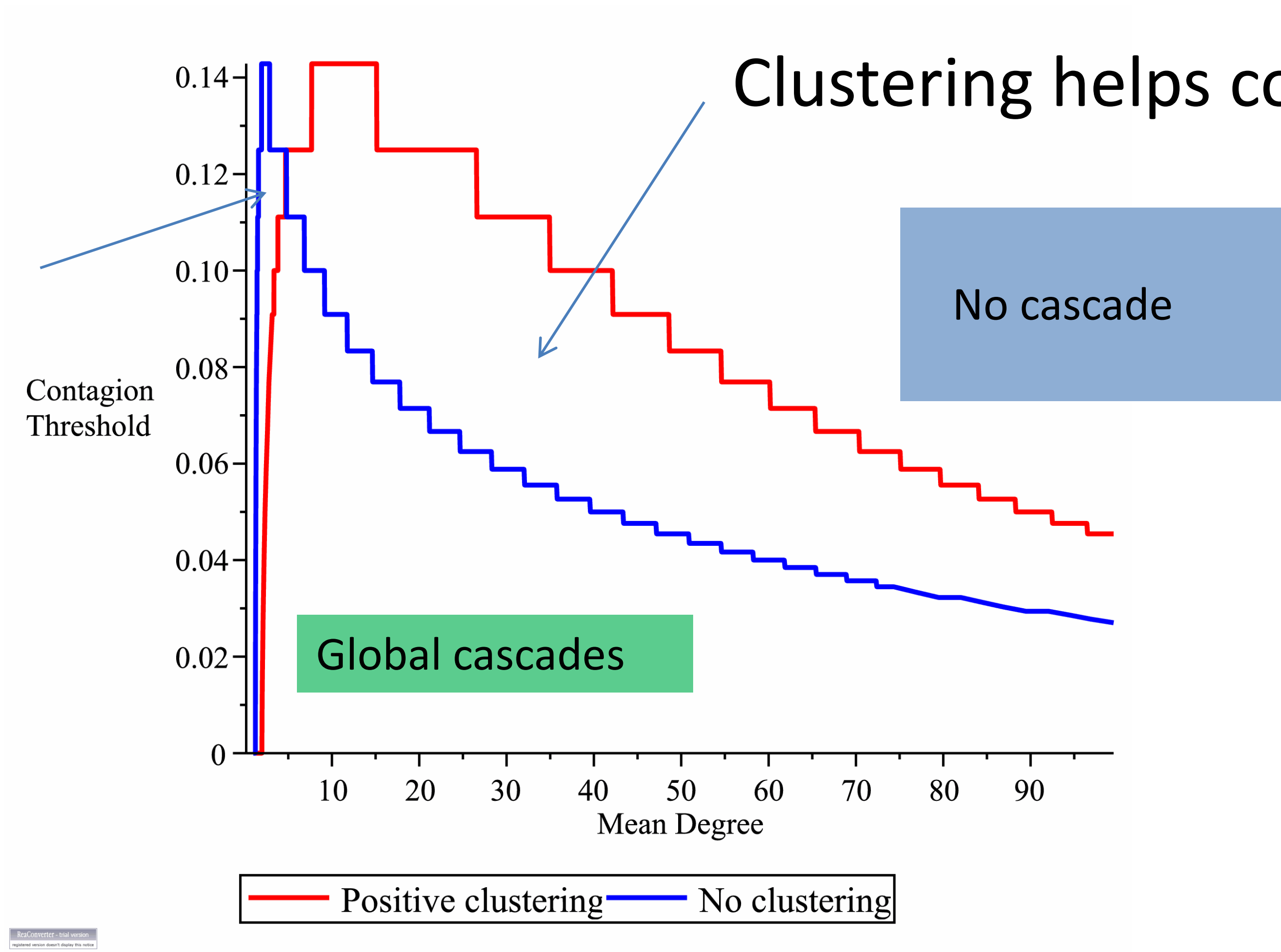
- Adding cliques:



(3) Contagion threshold with clustering

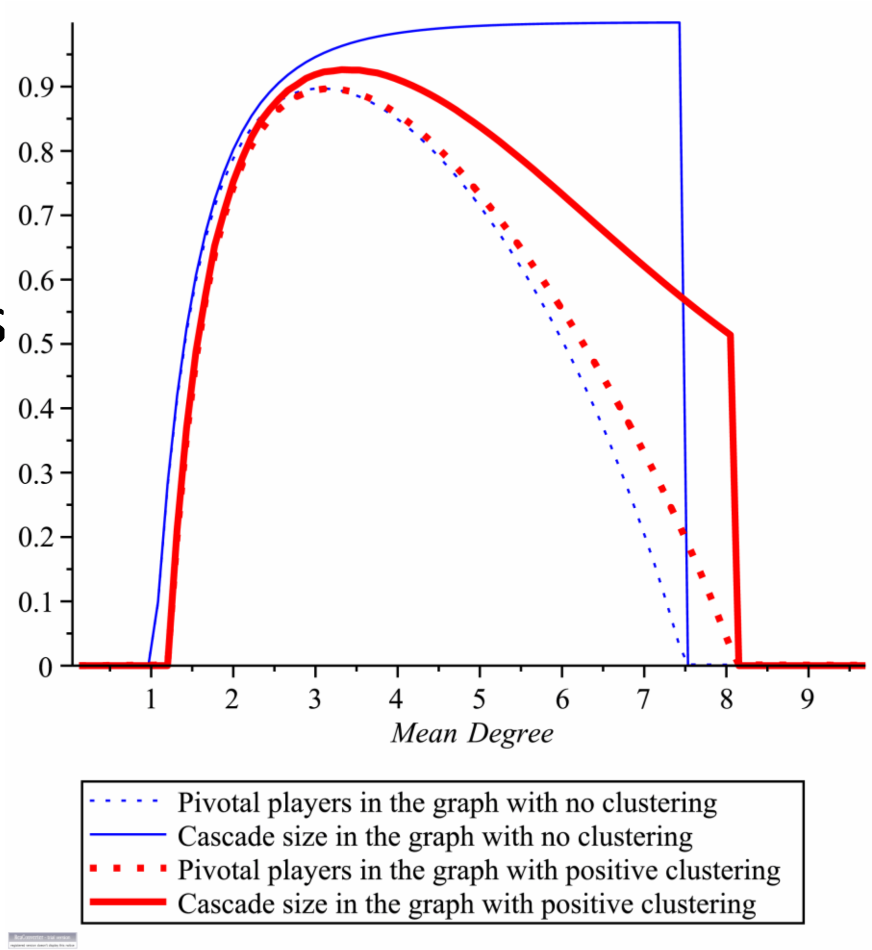
Clustering inhibits contagion

Clustering helps contagion



(3) Side effect of clustering!

Fraction of pivotal players and size of the cascade



Conclusion for (A)

- Simple tractable model:
  - Threshold rule
  - Random network : heterogeneity of population
  - Tunable degree/clustering
- 1 notion: Pivotal Players and 2 regimes:
  - Low connectivity: tipping point / clustering hurts
  - High connectivity: chasm / clustering helps activation
- Open problems:
  - Size of optimal seed? Dynamics of the diffusion? More than 2 states?

(A) Diffusion in Social Networks

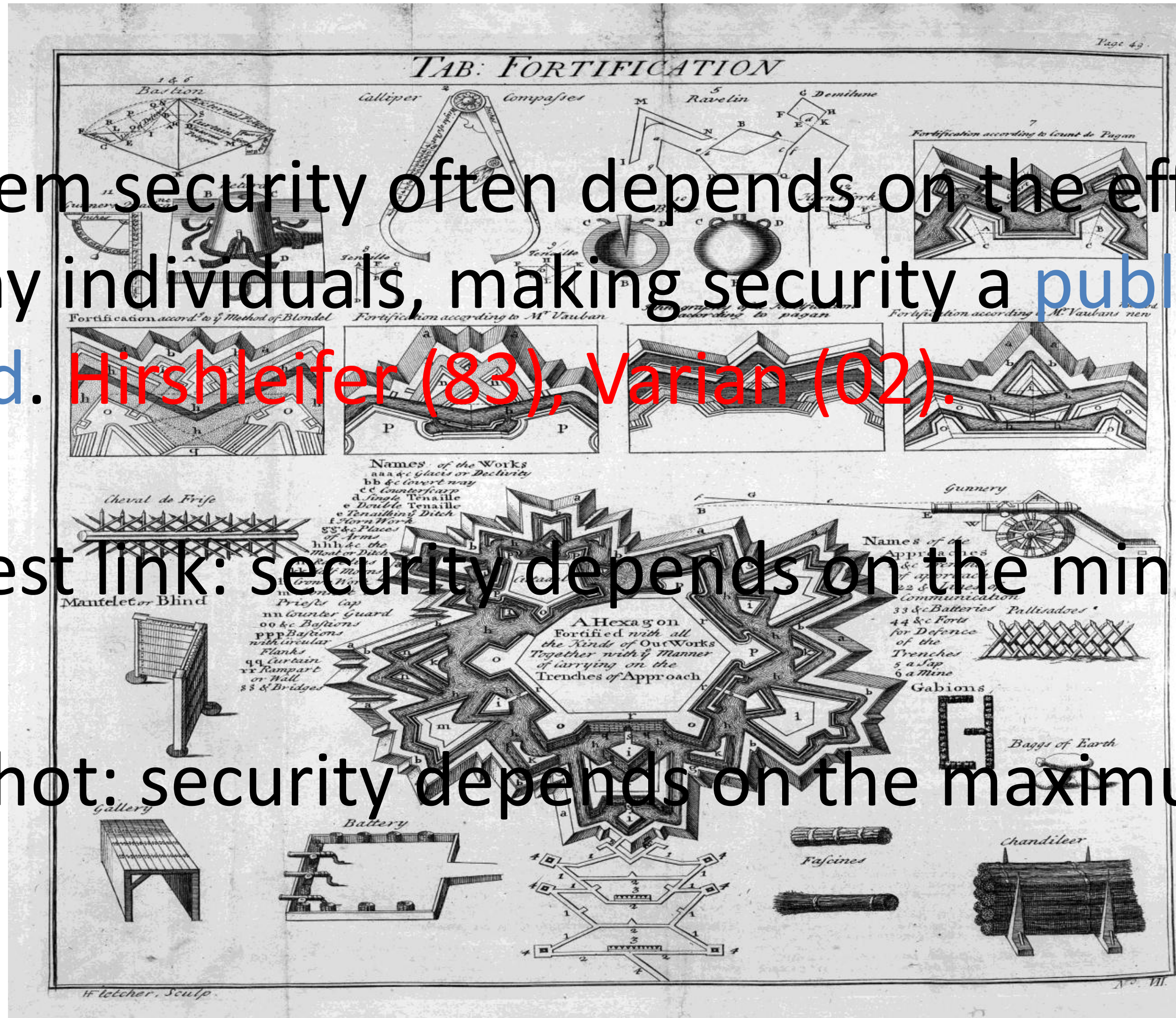
(B) Economics of Information Security

(1) Network Security Games

(2) Complete Information

(3) Incomplete Information

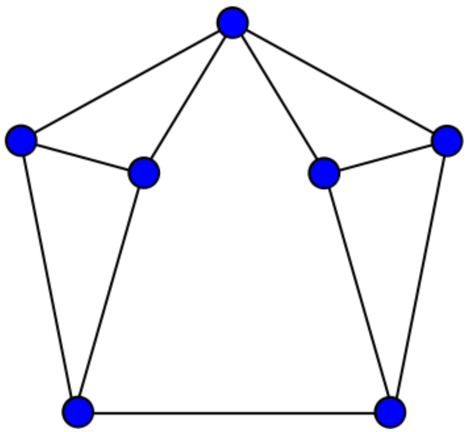
(1) Network Security as a Public Good



- System security often depends on the effort of many individuals, making security a public good. Hirshleifer (83), Varian (02).

- Weakest link: security depends on the minimum effort.
- Best shot: security depends on the maximum effort.

(1) Local Best Response



$$X_i = \begin{cases} 1 & \text{if Secure} \\ 0 & \text{if Not protected} \end{cases}$$

- Weakest link:

$$X_i = \min_{j \sim i} X_j$$

- Best shot:

$$X_i = \min_{j \sim i}(1 - X_j)$$

(1) Network Security Games

(2) Complete Information

(3) Incomplete Information

(2) Complete Information

- Weakest link

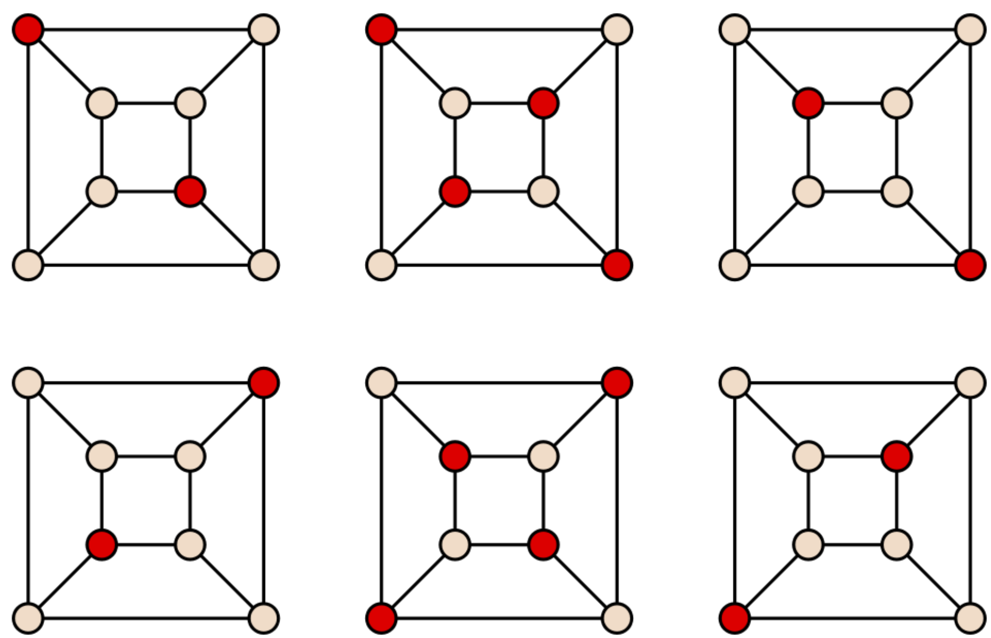
$$X_i = \min_{j \sim i} X_j$$

Only trivial equilibria:

all 0/ all 1.

- Best shot

$$X_i = \min_{j \sim i}(1 - X_j)$$



(2) Slight extensions WL

- Weakest link:

$$X_i = \min_{j\sim i} X_j = \mathbf{1}(\sum_{j\sim i} X_j \geq d_i)$$

- Weakest link with parameter K:

$$X_i = \mathbf{1}(\sum_{j\sim i} X_j \geq d_i - K)$$

- Change of variables:

$$Y_i = 1 - X_i = \mathbf{1}(\sum_{j\sim i} Y_j \geq K + 1)$$

Bootstrap percolation!

- Richer structure of equilibria.

(2) Slight extensions BS

- Best shot:

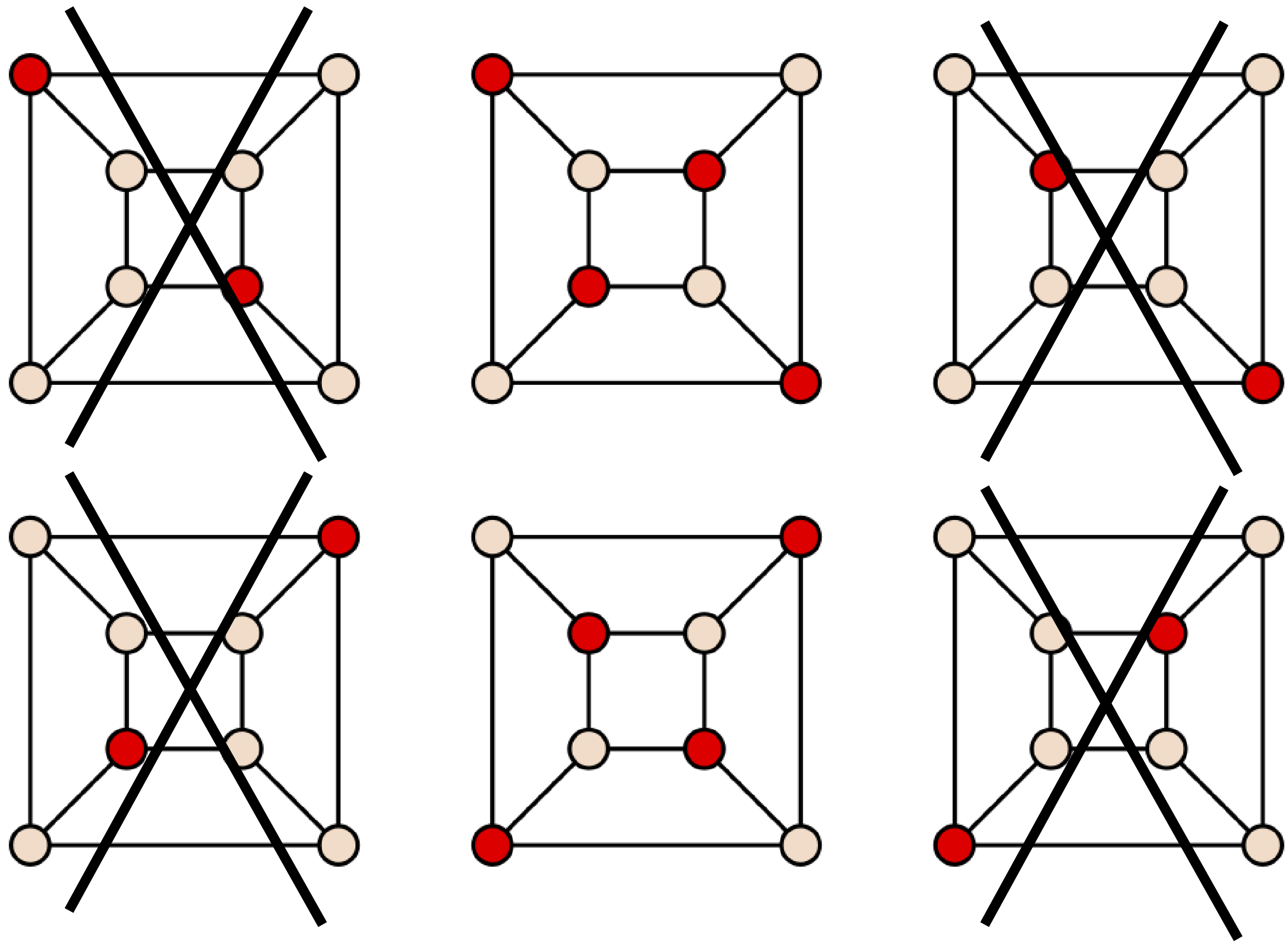
$$X_i = \mathsf{min}_{j \sim i}(1 - X_j) = \mathbf{1}(\sum_{j \sim i} X_j \leq 0)$$

- Best shot with parameter K:

$$X_i = \mathbf{1}(\sum_{j \sim i} X_j \leq K)$$

- Equilibria: Maximal Independent Set of order K+1. Bramoullé Kranton (07)

(2) Best Shot with parameter K=1



(1) Network Security Games

(2) Complete Information

(3) Incomplete Information

(3) Incomplete Information

- Bayesian game. Galeotti et al. (10)
- Type = degree d
- Neighbors' actions are i.i.d. Bern(γ)
- Network externality function:

$$h(\gamma, d) = P(\text{loss}|X_i = 0) - P(\text{loss}|X_i = 1)$$

corresponds to the price, an agent with degree d is ready to invest in security.

(3) Weakest link

- Network externality function:

$$h(\gamma, d) = E[\min_{i=1}^{d} \text{Bern}_i(\gamma)] - 0$$
$$= \gamma^d$$

- This function being increasing, incentives are aligned in the population.
- This gives a Coordination problem for the game. Lelarge (12)

(3) Best shot

- Network externality function:

$$\begin{aligned} h(\gamma, d) &= 1 - E[\max_{i=1}^{d} \mathrm{Bern}_i(\gamma)] \\ &= (1-\gamma)^d \end{aligned}$$

- This function being decreasing, incentives are not aligned in the population.
- This gives a Free rider problem for the game. Lelarge (12)

Conclusion (B)

- Information structure of the game is crucial: network externality function when incomplete information.
- Technology is not enough! There is a need to design economic incentives to ensure the deployment of security technologies.
- Open problems:
  - Dynamics? mean field games? more quantitative results? Local and global interactions?

Thank you!