Efficient Control of Epidemics over Random Networks

Marc Lelarge (INRIA-ENS)

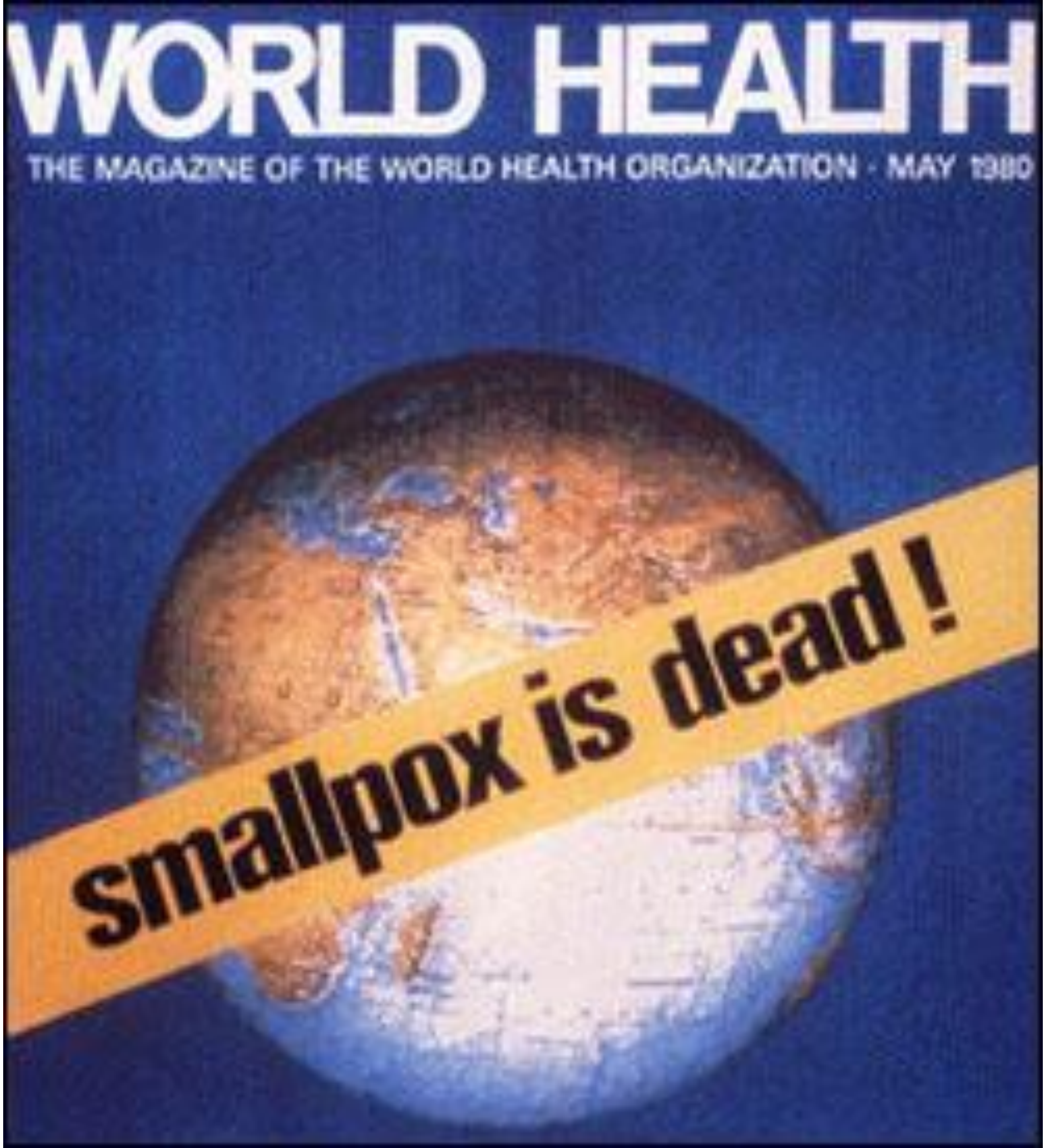
SIGMETRICS 09.

A simple problem

- Model for an epidemic on a graph:
  - Spread of worms, email viruses…
  - Diffusion of information, gossip…
- Strategic players: Attacker / Defender
  - Defender plays first by vaccinating nodes.
  - Viral marketing.
- Information available: none for the Defender!
- Defense has to be decentralized.

A hopeless goal?

Some hope…


WORLD HEALTH
THE MAGAZINE OF THE WORLD HEALTH ORGANIZATION · MAY 1980
smallpox is dead !

Acquaintance vaccination

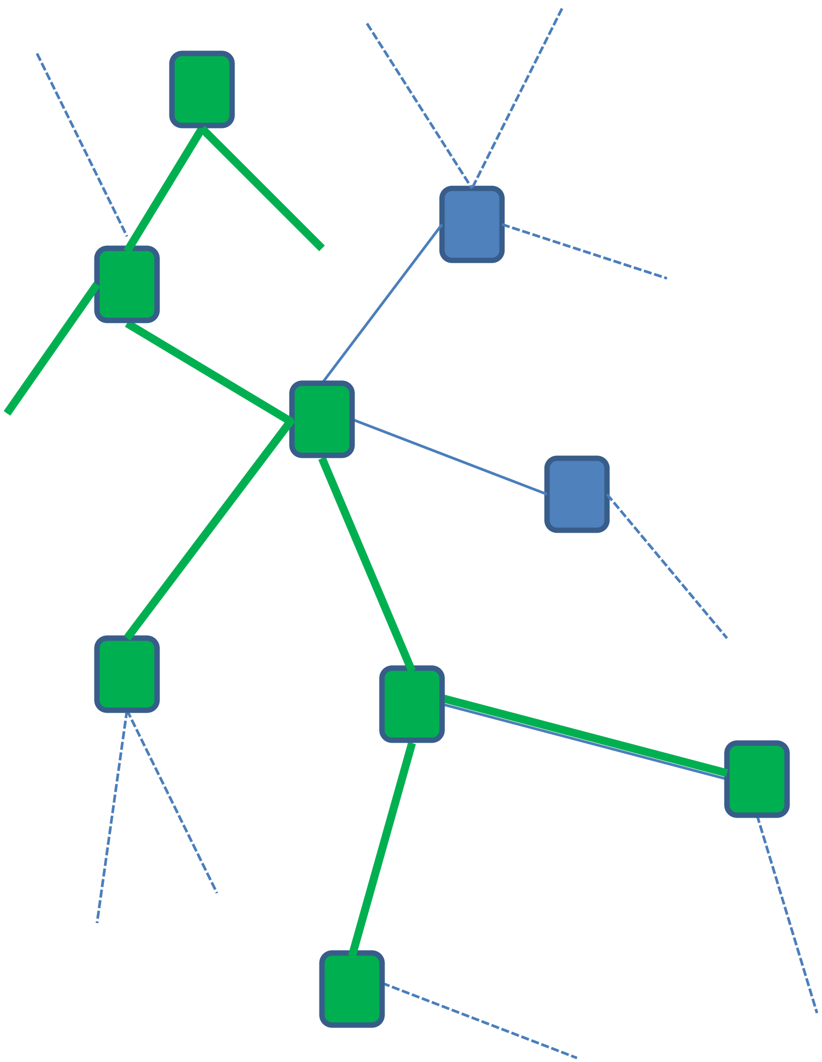
- Proposed by Cohen, Havlin and ben-Avraham in Phys.Rev.Let. 2003.
- Sample each node uniformly and inoculate a neighbor of this node taken at random.
- Why does it work?
  - Sampling with a bias toward high-degree nodes.
- Rigorous analysis and analytic formulas for various epidemics.

(1) Percolated Threshold Model

(2) Analytic Results

(3) Toolbox

(1) Percolated Threshold Model



- Bond percolation with proba. π
- Symmetric threshold epidemic:

$$\sum_{j \sim i} X_j \geq K_i(d_i)$$

- Seed of active nodes

(1) Versatile model for epidemics

- Null threshold = contact process
- No bond percolation = bootstrap percolation
- Some easy general results:
  - Monotonicity: only transition passive to active.
  - In a finite graph, there is only one possible final state for the epidemic.
- I will concentrate on properties of the final state, for large random graphs.

(1) Vaccination and Attack

- Perfect vaccine: remove vaccinated population from the graph (site percolation).
- Acquaintance vaccination: Sample each node uniformly and inoculate a neighbor of this node taken at random.
- Degree based attack: randomly attack a node with a probability depending on its degree.

(1) Percolated Threshold Model

(2) Analytic Results

- Explicit formulas for the epidemic spread with vaccination and attack given in the paper!

(3) Toolbox

(2) Cascade Condition

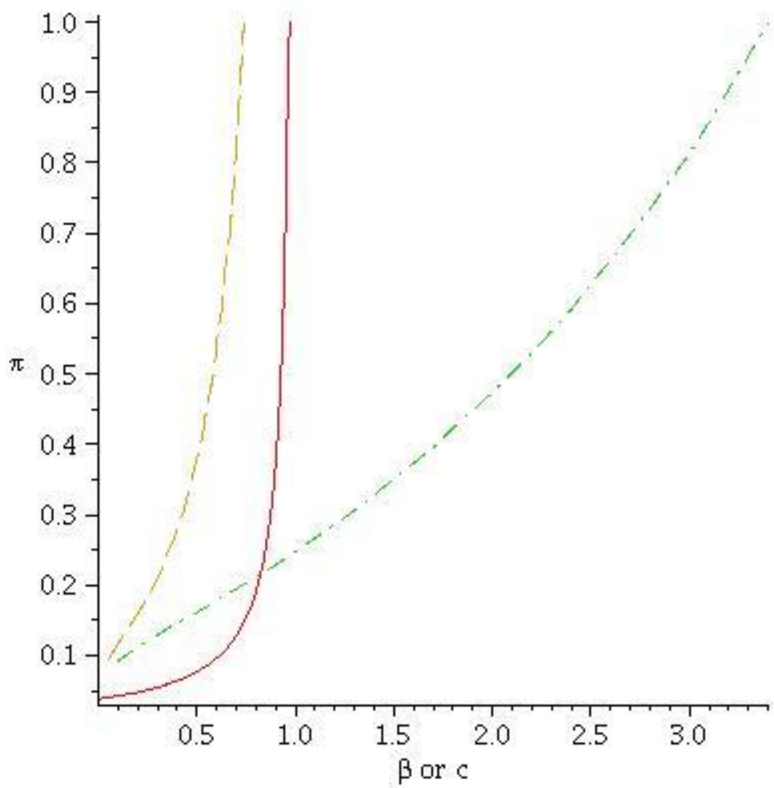
- Random graph with degree distribution: D

  (configuration model: Molloy-Reed 95)
- Bond percolation: π and threshold: $K(d)$.
- When can a single active node have a global impact?

$$\pi\mathbb{E}[D(D-1)\mathbb{1}(K(D)=0)] > \mathbb{E}[D]$$

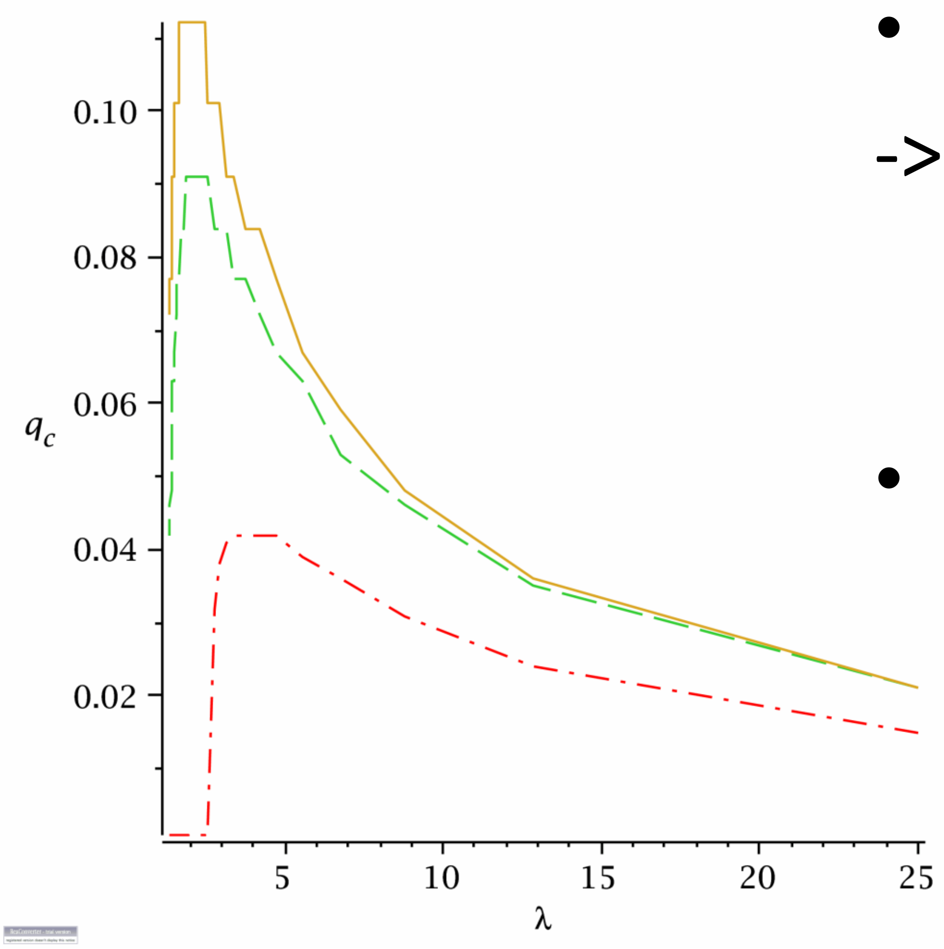
- $K \equiv 0$ Epidemic contagion threshold.

(2) Vaccination for the contact process



- Epidemic threshold as a function of vaccinated population.
- If $\mathbb{E}[D^2] = \infty$, uniform vaccination is useless. Acquaintance vaccination can stop epidemic!

(2) Vaccination for threshold model



- Threshold $K(d) = qd$

-> become active when fraction of active neighbors $\geq q$

- Contagion threshold as a function of mean degree.

(1) Percolated Threshold Model

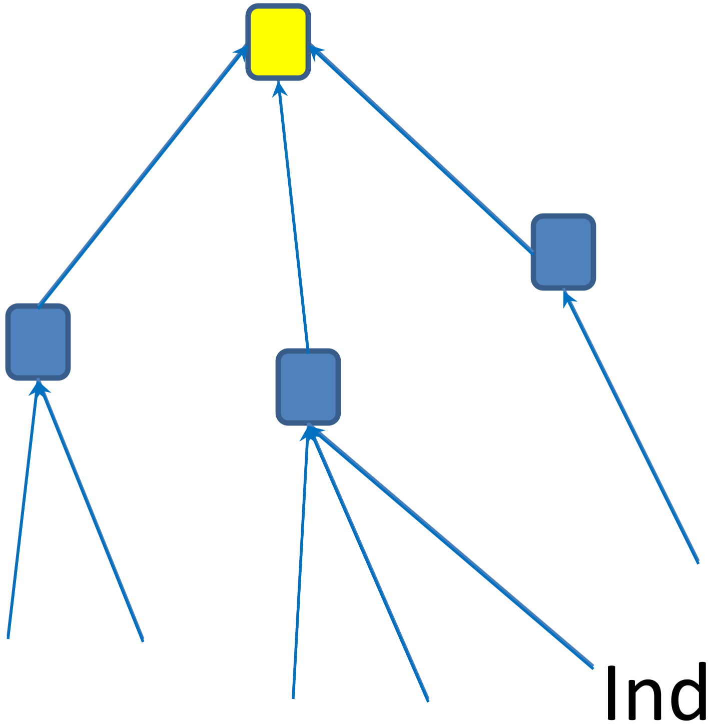
(2) Analytic Results

(3) Toolbox

(3) Good use

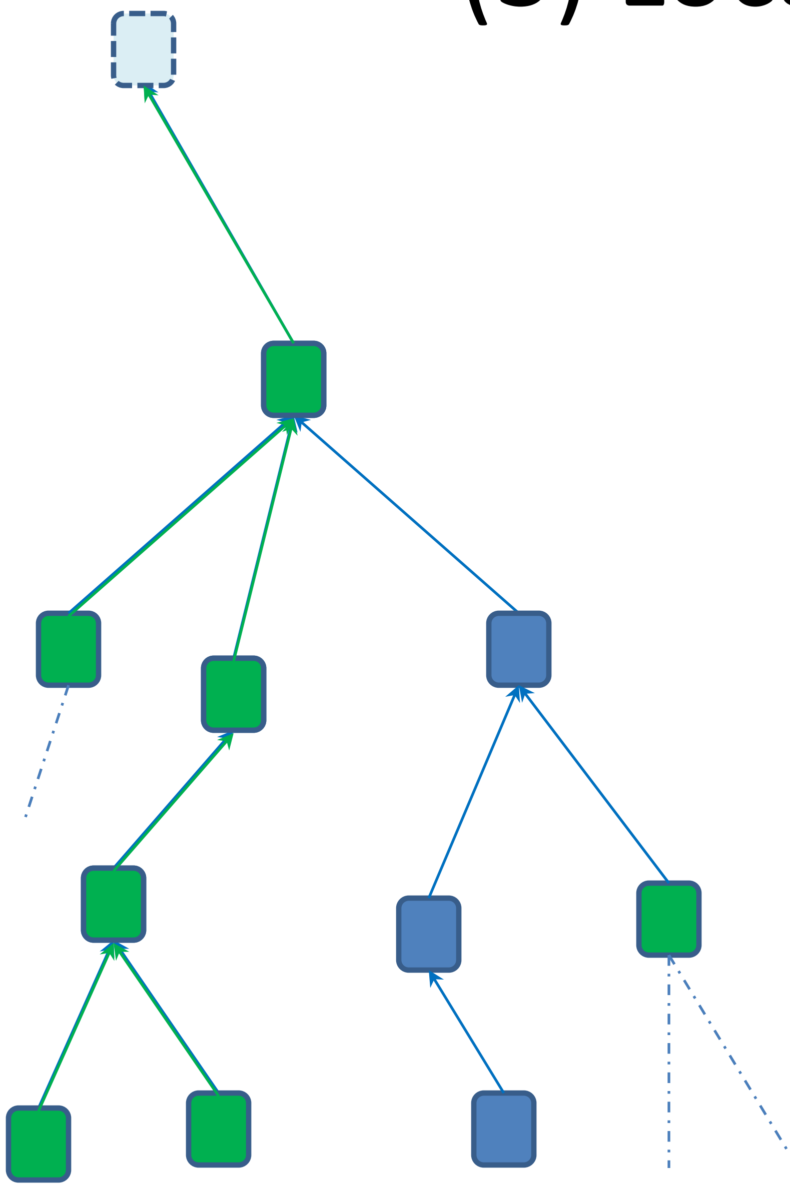
- (i) Theorem for the epidemic spread for a general graph with degree sequence D and degree based attack.
- (ii) Proposition: how vaccination modifies the distribution of D
- (iii) Show that we can apply the first Theorem on the graph obtained after vaccination.
- The steps (i) and (ii) are independent and can be used in different context…

(3) Locally tree-like



Independent computations on trees

(3) Locally tree-like



- Ignore the trees below any infected node.
- Propagate the epidemic bottom up.
- For the root, add a virtual healthy node above it.

To conclude

- Acquaintance vaccination is impressively effective!
- Economics of epidemics: incentives for vaccination.

  Economics of Malware: Epidemic Risks Model, Network Externalities and Incentives, M.L., WEIS09
- Applications to viral marketing, gossip:

  Duncan Watts \ Kempe, Kleinberg, Tardos

  Diffusion of innovations on random networks: Understanding the chasm, M.L., WINE 08.

  On Modeling Product Advertisement in Social Networks, Bridge Zhao, Yongkun Li, John C. S. Lui, Dah Ming Chiu (poster session)
- Technical details : www.di.ens.fr/~lelarge/