Diffusion and Cascading Behavior in Random Networks

Marc Lelarge (INRIA-ENS)

Columbia University Joint CS/EE Networking Seminar

June 2, 2011.

(1) Diffusion Model

inspired from game theory

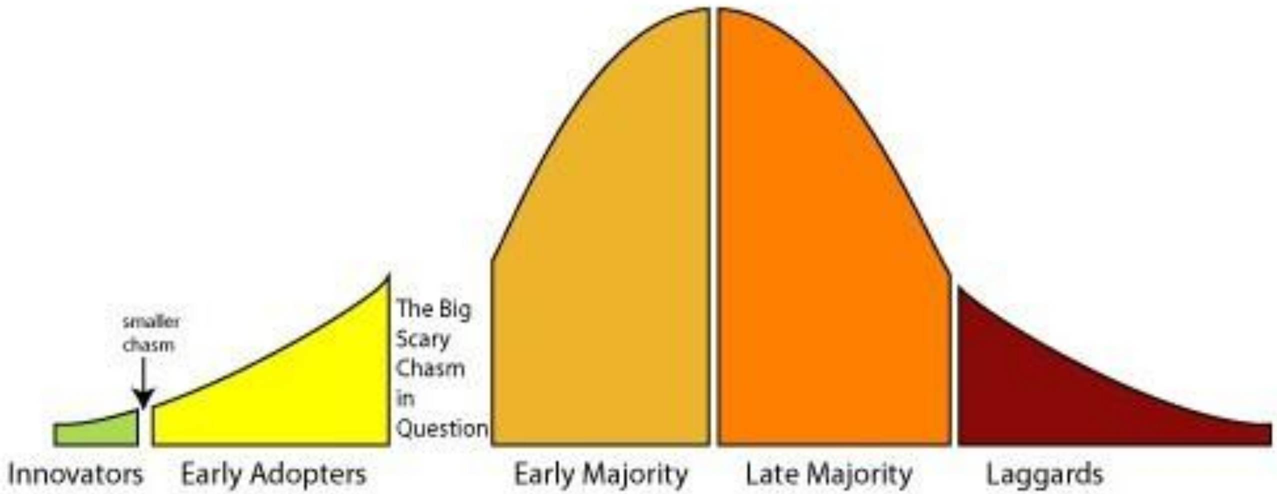
and statistical physics.

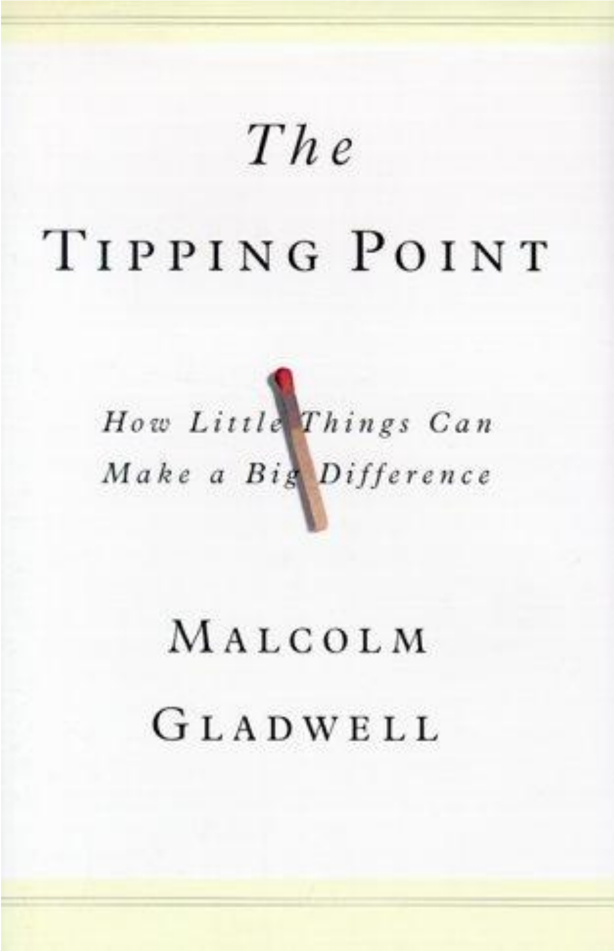
(2) Results

from a mathematical analysis.

(3) Heuristic

(0) Context





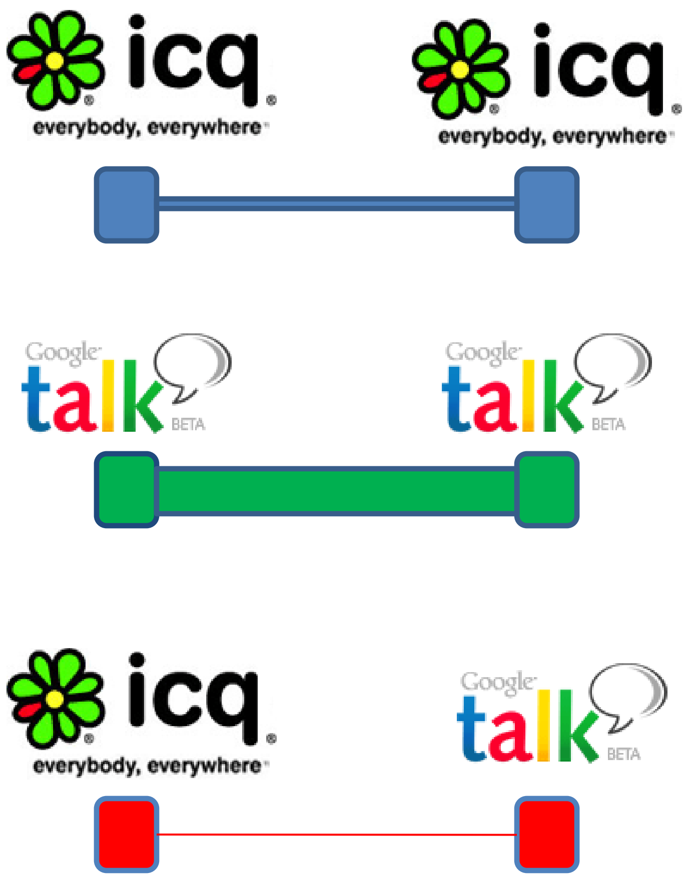
Crossing the Chasm
(Moore 1991)

(1) Diffusion Model

(2) Results

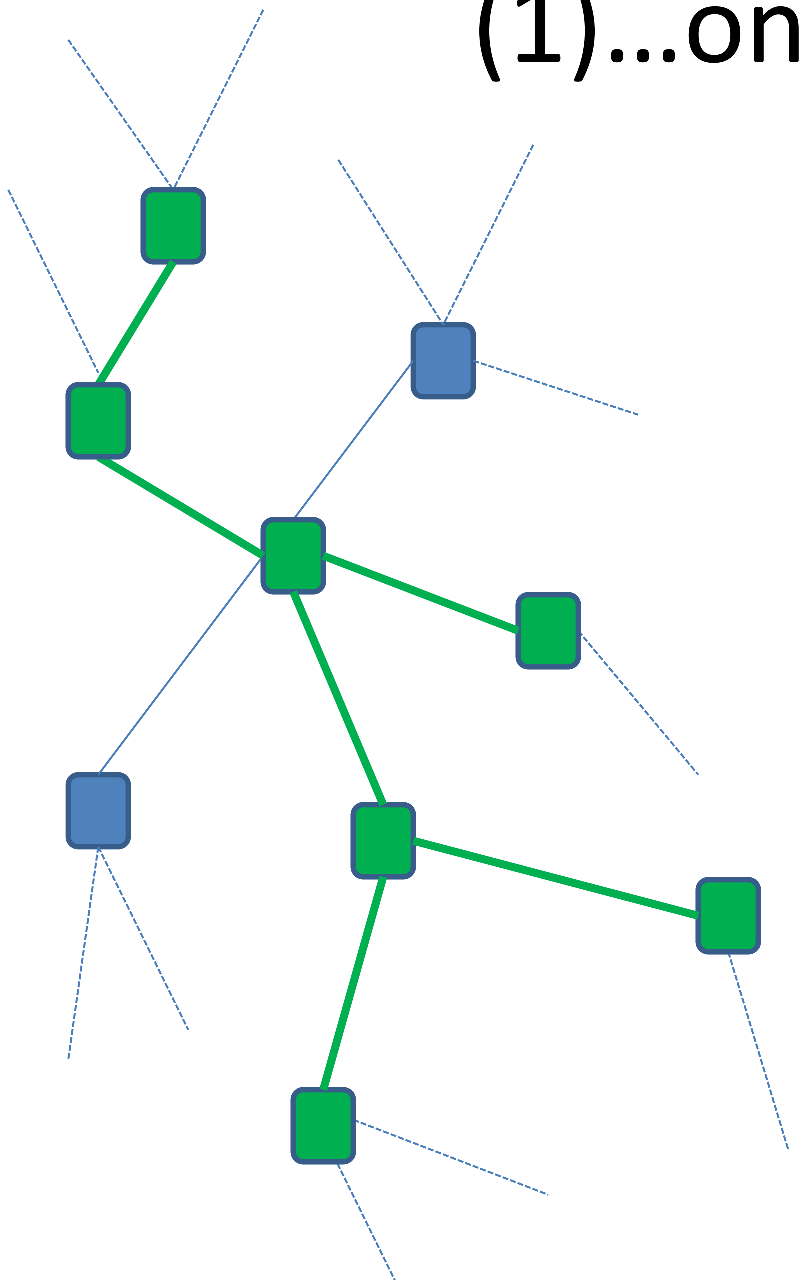
(3) Heuristic

(1) Coordination game…



- Both receive payoff q.
- Both receive payoff 1-q>q.
- Both receive nothing.

(1)…on a network.



- Everybody start with icq everybody, everywhere
- Total payoff = sum of the payoffs with each neighbor.
- A seed of nodes switches to Google talk BETA

(Morris 2000)

(1) Threshold Model

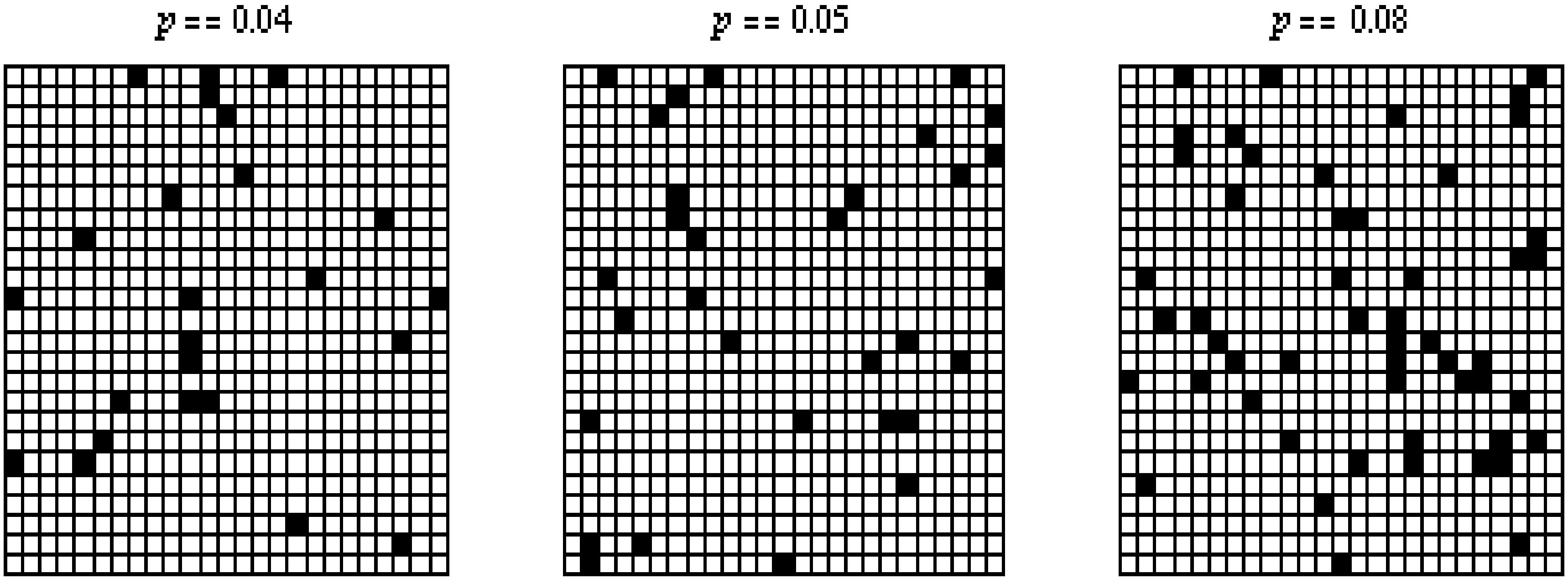
- State of agent i is represented by

$$X_i = \begin{cases} 0 & \text{if icq} \\ 1 & \text{if Google talk} \end{cases}$$

- Switch from icq to Google talk if:

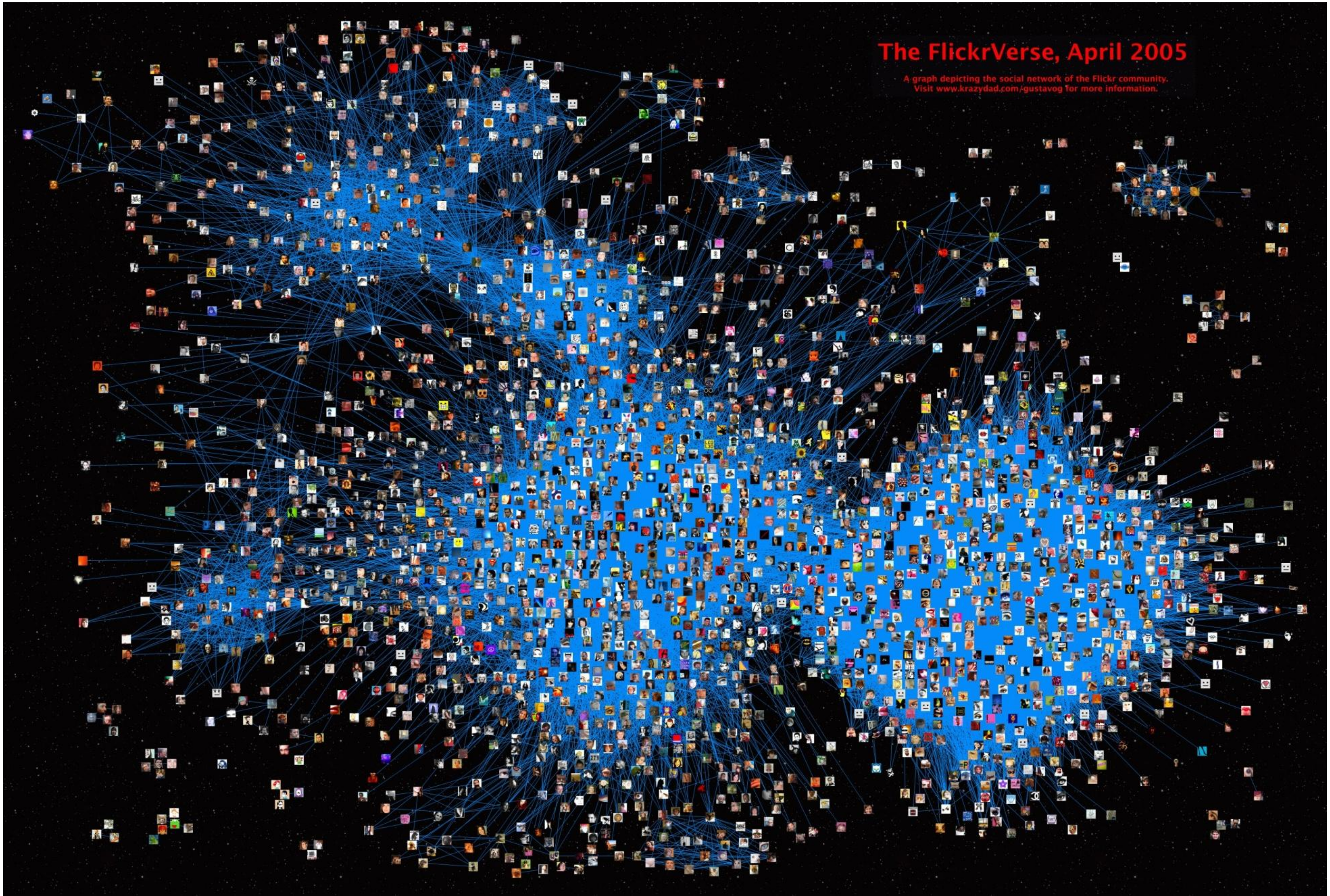
$$\sum_{j \sim i} X_j \geq q d_i$$

(1) Model for the network?



Statistical physics: bootstrap percolation.

(1) Model for the network?



(1) Random Graphs

- Random graphs with given degree sequence introduced by Molloy and Reed (1995).
- Examples:
  - Erdös-Réyni graphs, $G(n,\lambda/n)$.
  - Graphs with power law degree distribution.
- We are interested in large population asymptotics.
- Average degree is λ.

(1) Diffusion Model

q = relative threshold

λ = average degree

(2) Results

(3) Heuristic

(1) Diffusion Model

q = relative threshold

λ = average degree

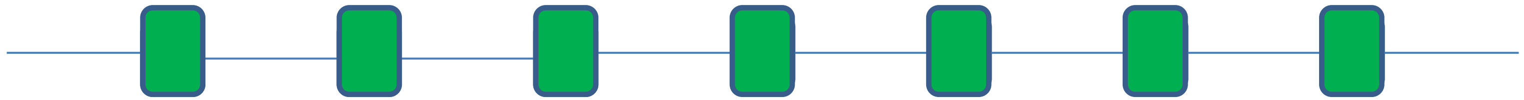
(2) Results

(3) Heuristic

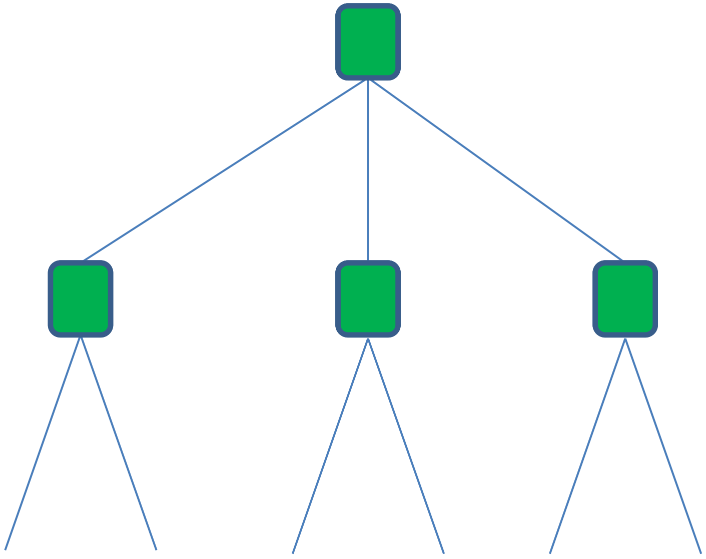
(2) Contagion (Morris 2000)

- Does there exist a finite groupe of players such that their action under best response dynamics spreads contagiously everywhere?
- Contagion threshold: q_c = largest q for which contagious dynamics are possible.
- Example: interaction on the line

$$q_c = \frac{1}{2}$$

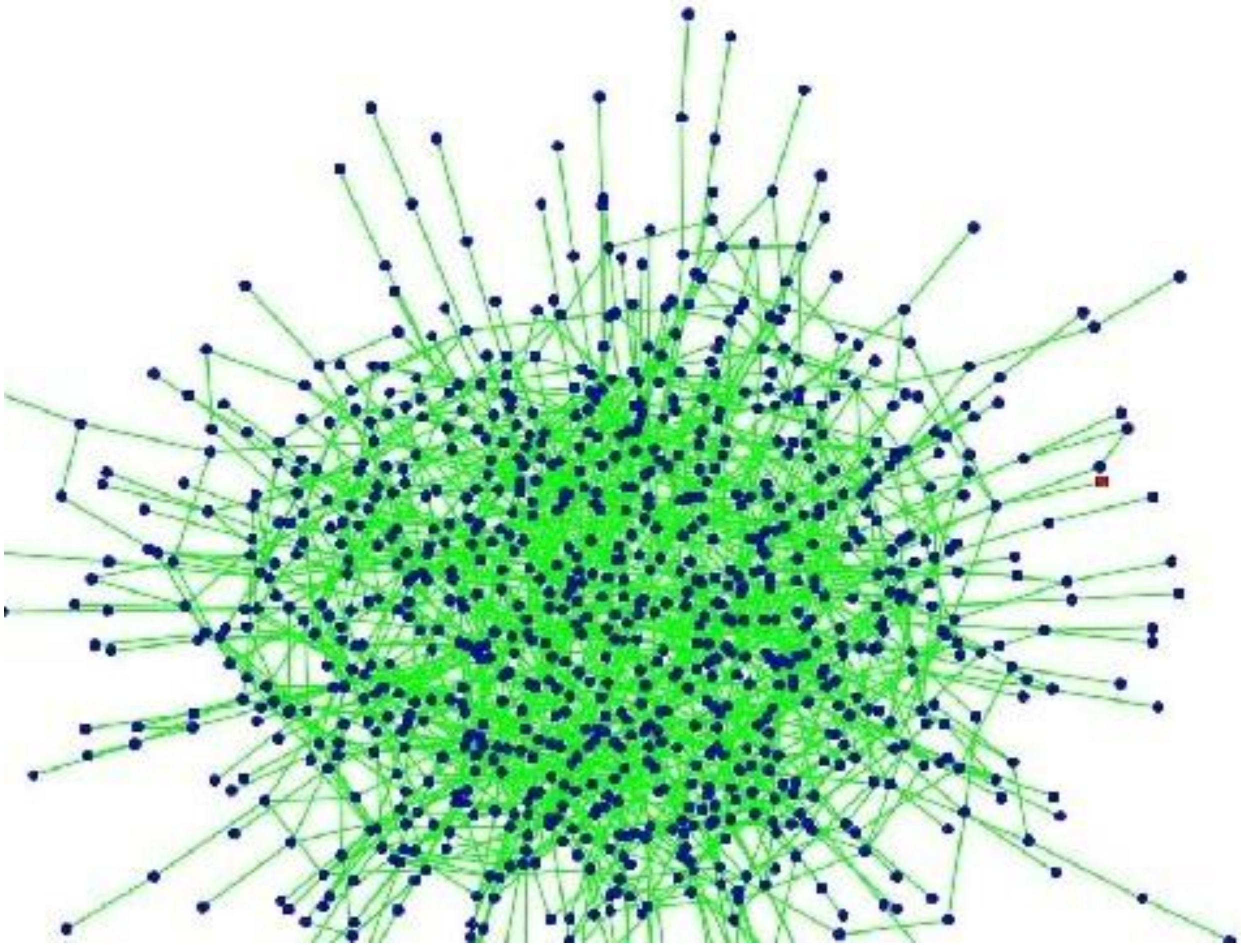


(2)Another example: d-regular trees



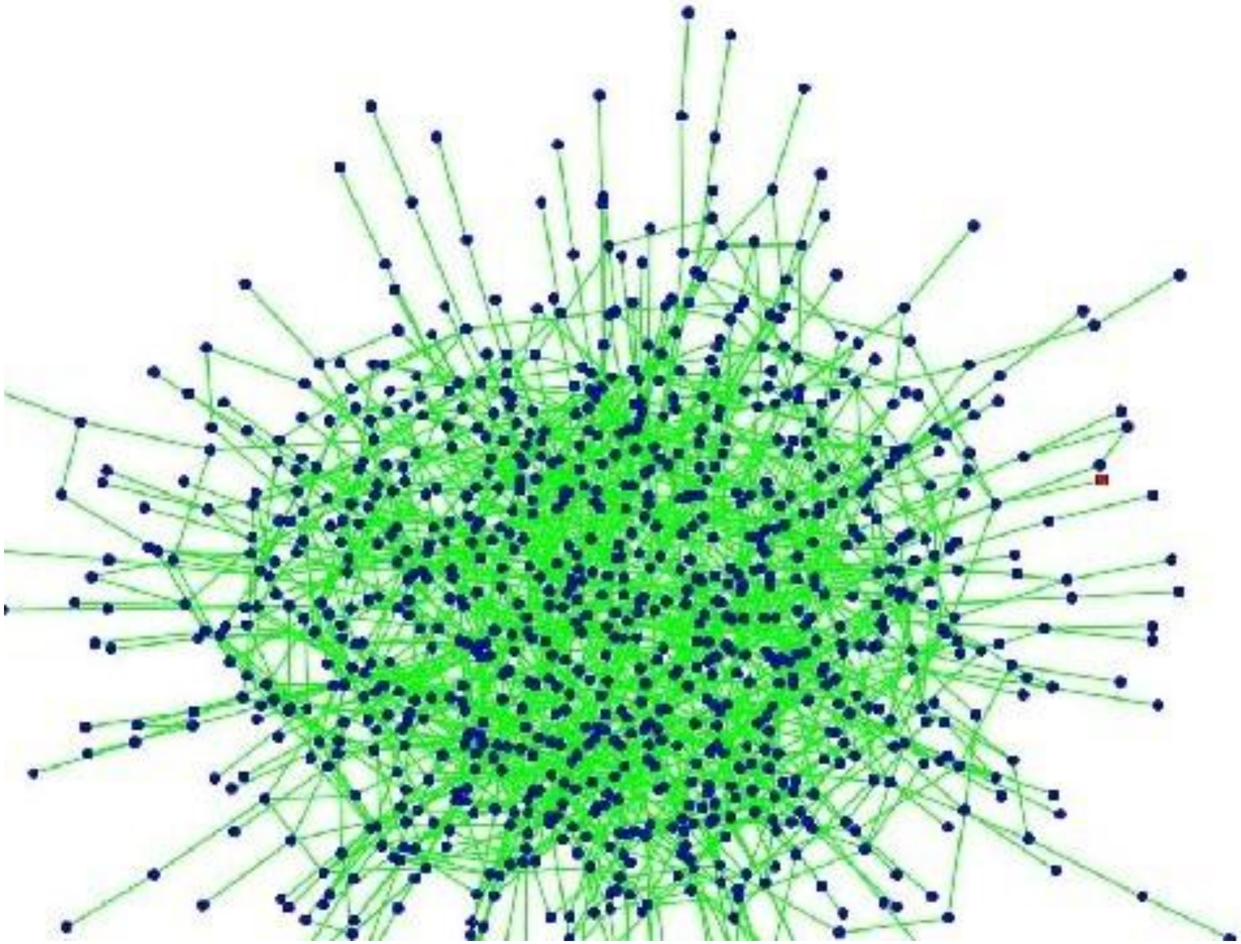
$$q_c = \frac{1}{d}$$

(2) Some experiments



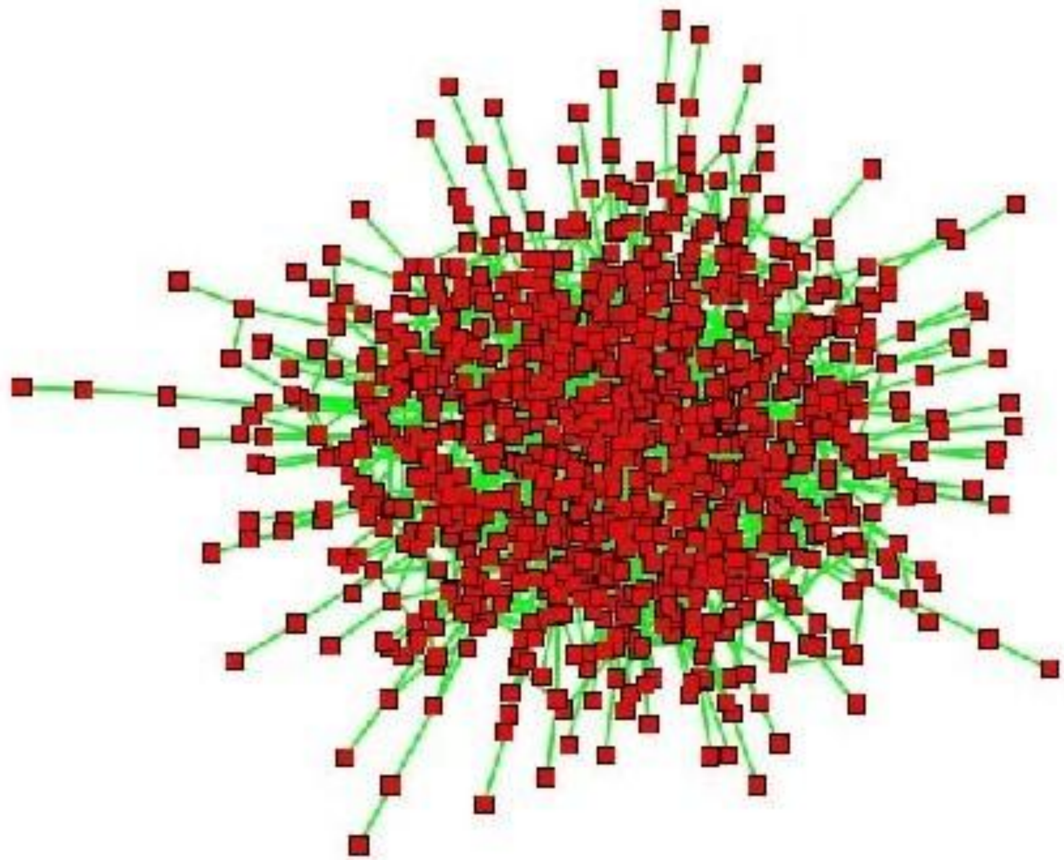
Seed = one node, λ=3 and q=0.24
(source: the Technoverse blog)

(2) Some experiments



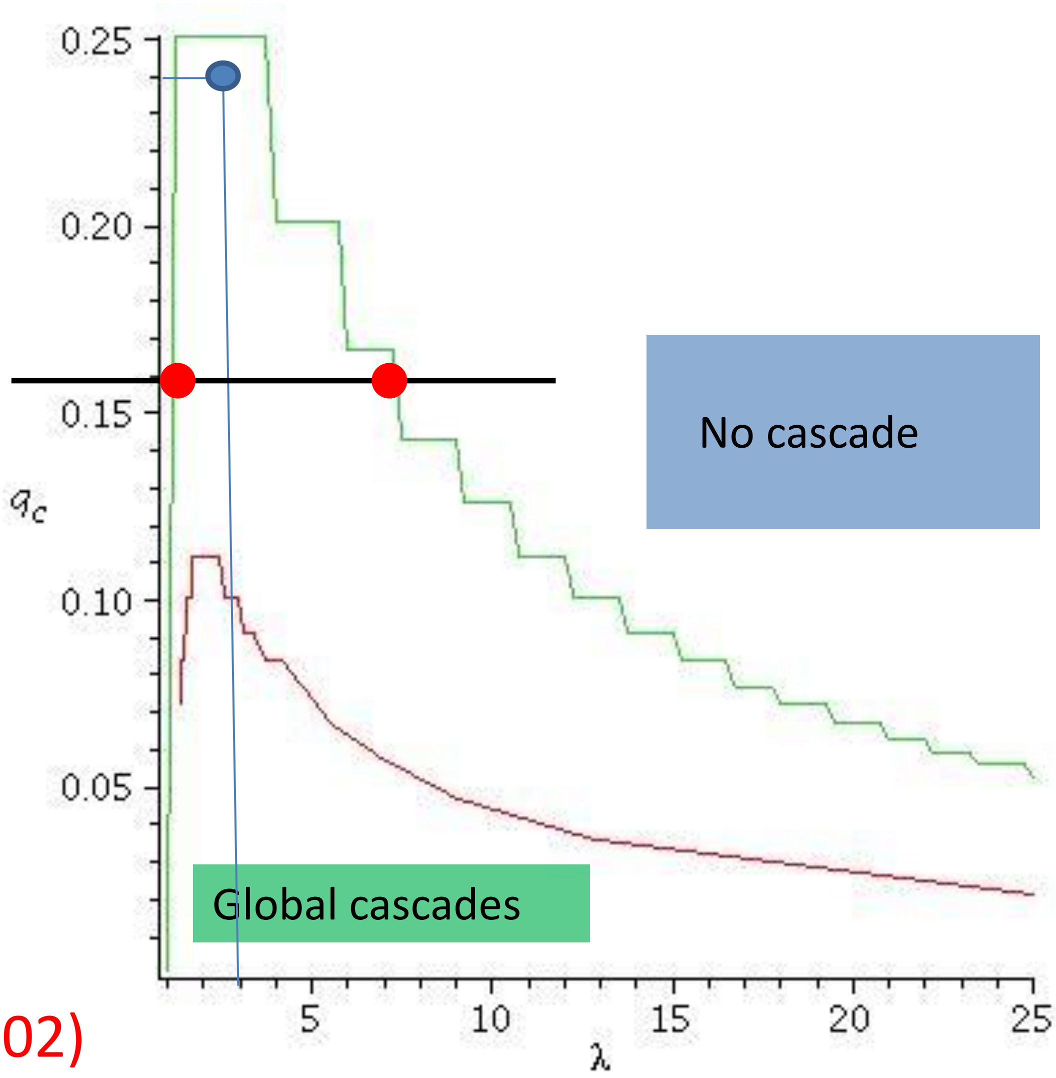
Seed = one node, λ=3 and 1/q>4
(source: the Technoverse blog)

(2) Some experiments



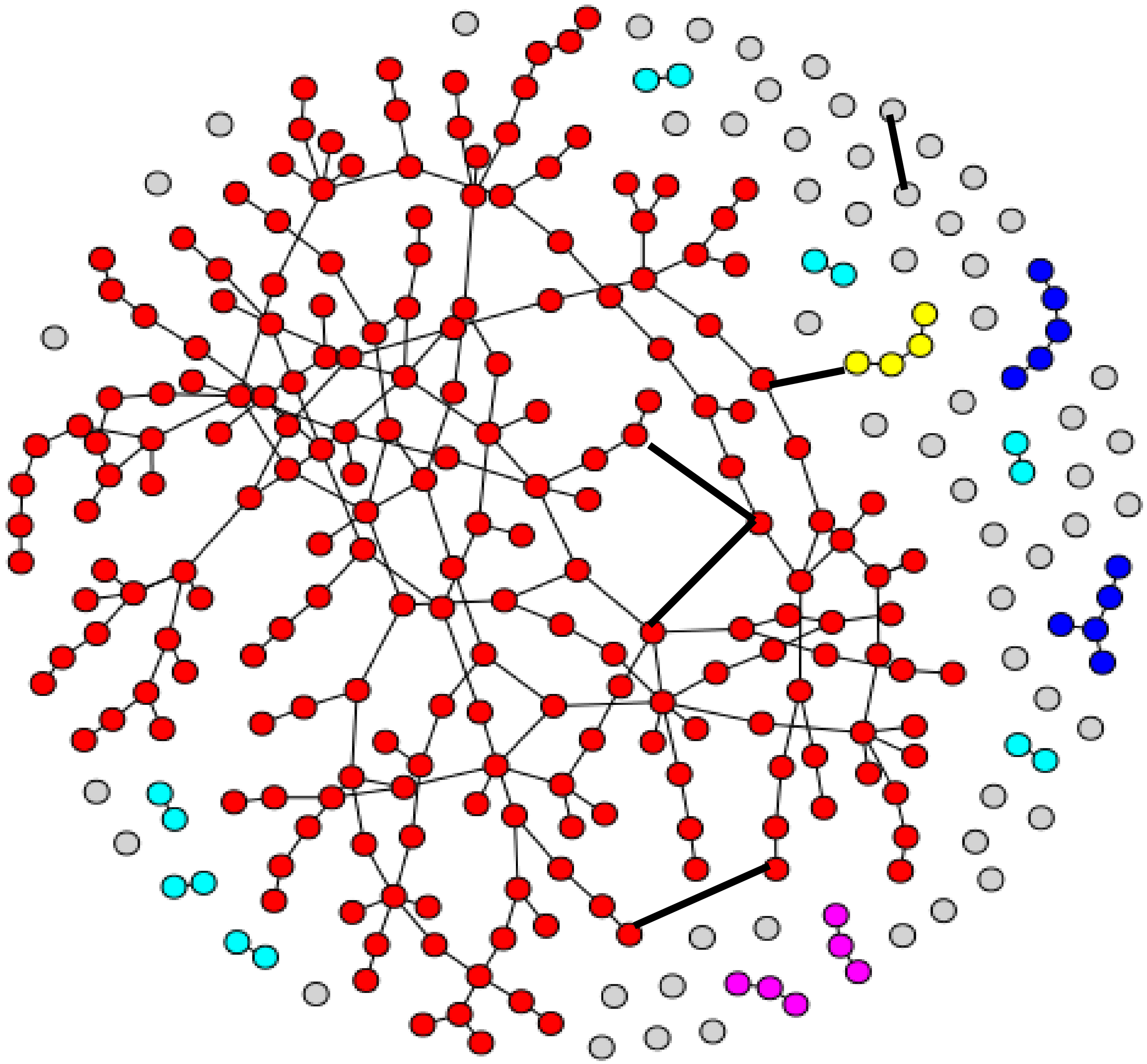
Seed = one node, λ=3 and q=0.24 (or 1/q>4)
(source: the Technoverse blog)

(2) Contagion threshold



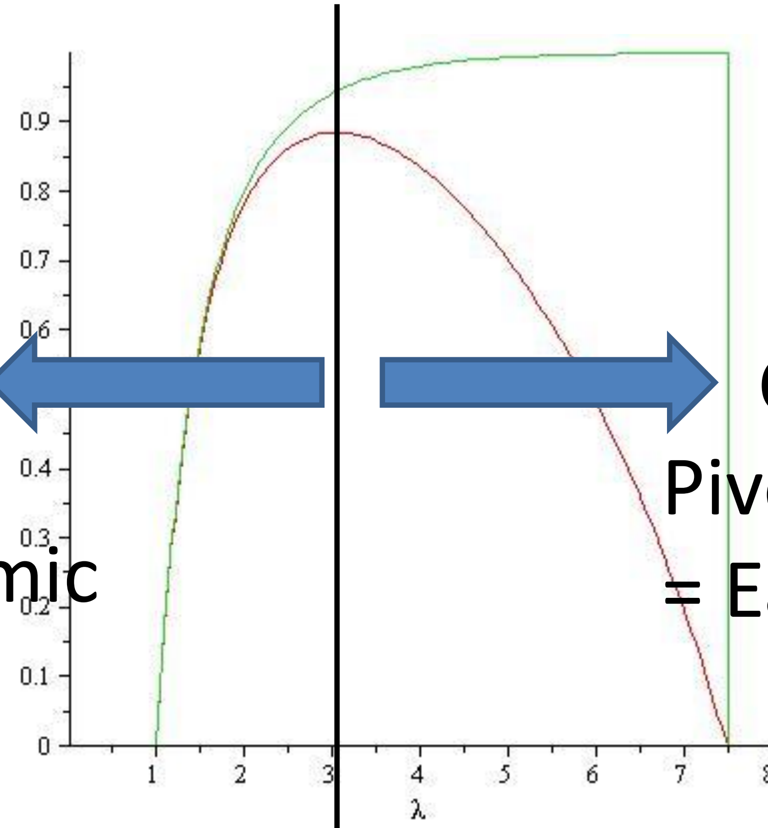
In accordance with (Watts 2002)

(2) A new Phase Transition



(2) Pivotal players

- Giant component of players requiring only one neighbor to switch.

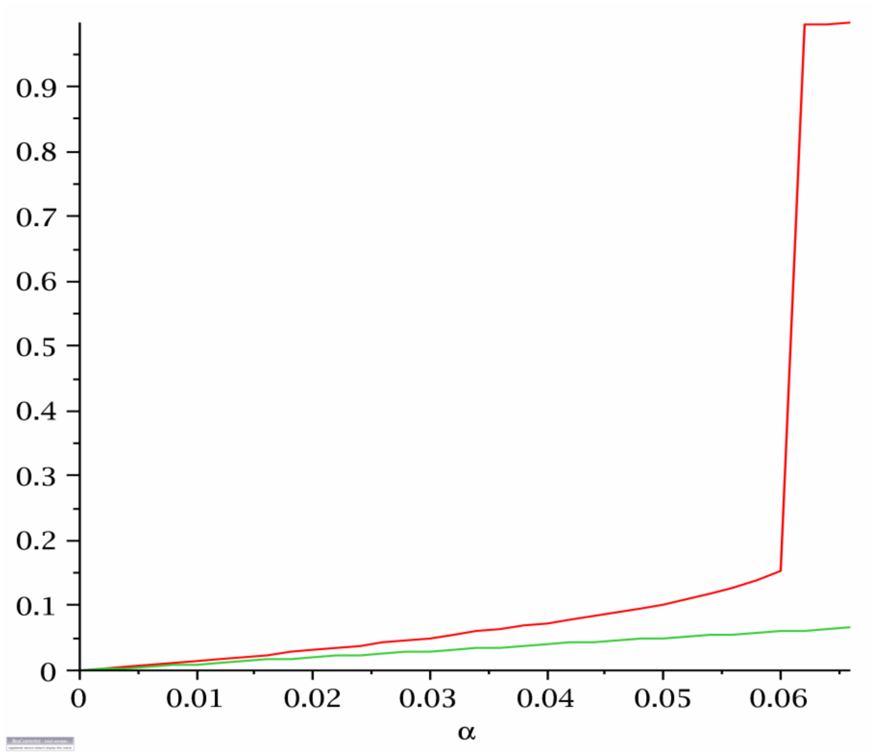


Tipping point:
Diffusion like
standard epidemic

Chasm :
Pivotal players
= Early adopters

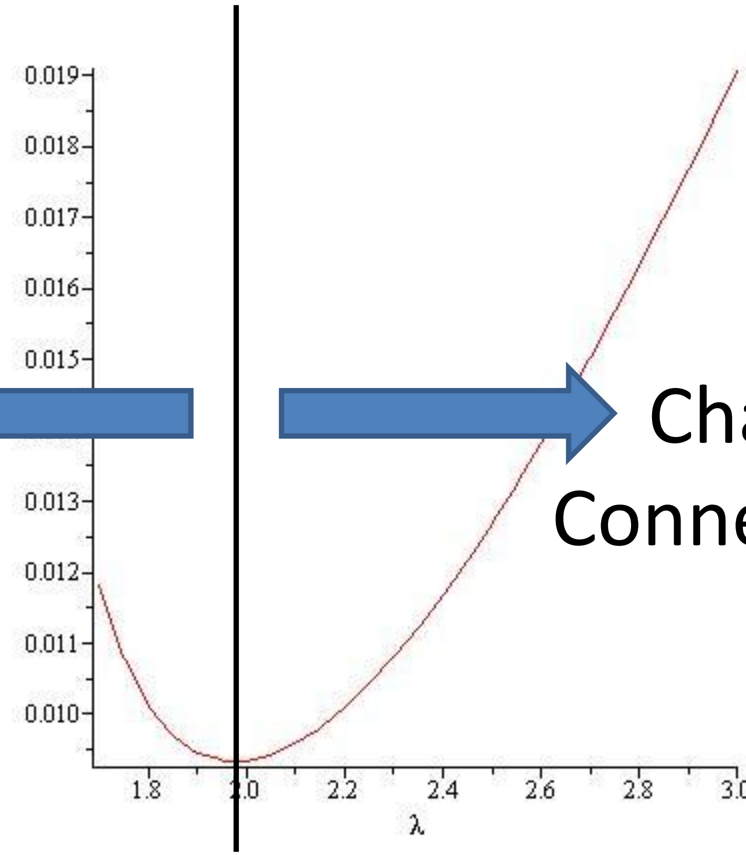
(2) q above contagion threshold

- New parameter: size of the seed as a fraction of the total population $0 < \alpha < 1$.
- Monotone dynamic → only one final state.



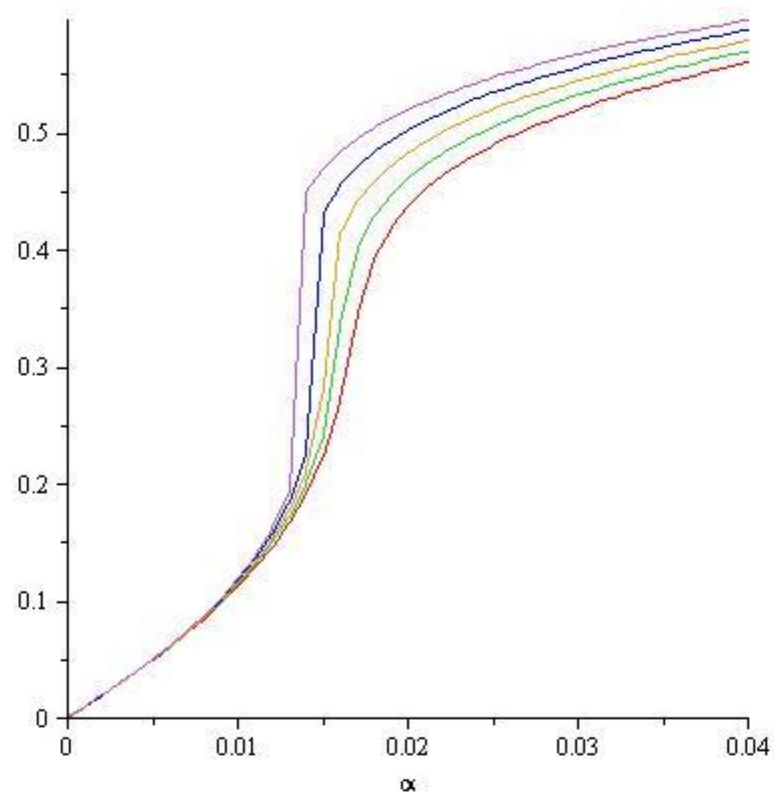
(2)Minimal size of the seed, q>1/4

Tipping point:
Connectivity helps



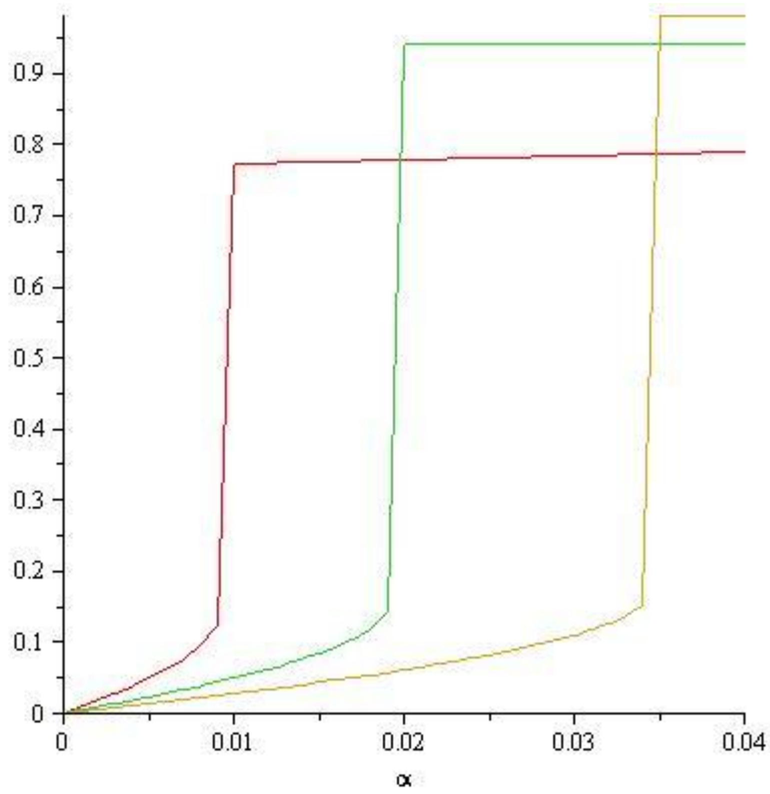
Chasm :
Connectivity hurts

(2) q>1/4, low connectivity



Connectivity helps the diffusion.

(2) q>1/4, high connectivity



Connectivity inhibits the global cascade, but once it occurs, it facilitates its diffusion.

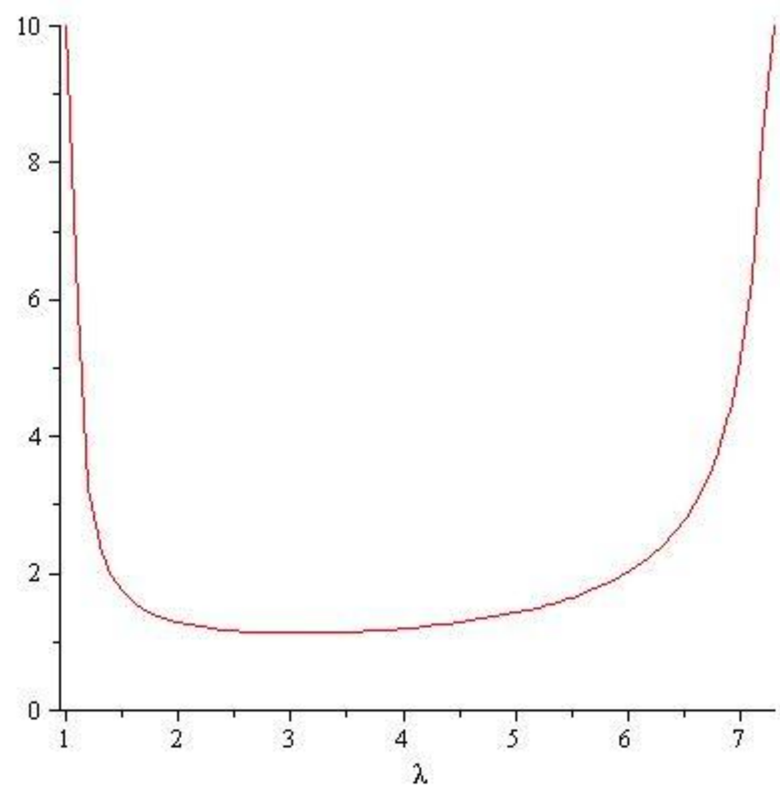
(2) Equilibria for $q<q_c$

- Trivial equilibria: all A / all B
- Initial seed applies best-response, hence can switches back. If the dynamic converges, it is an equilibrium.
- Robustness of all A equilibrium?
- Initial seed = 2 pivotal neighbors

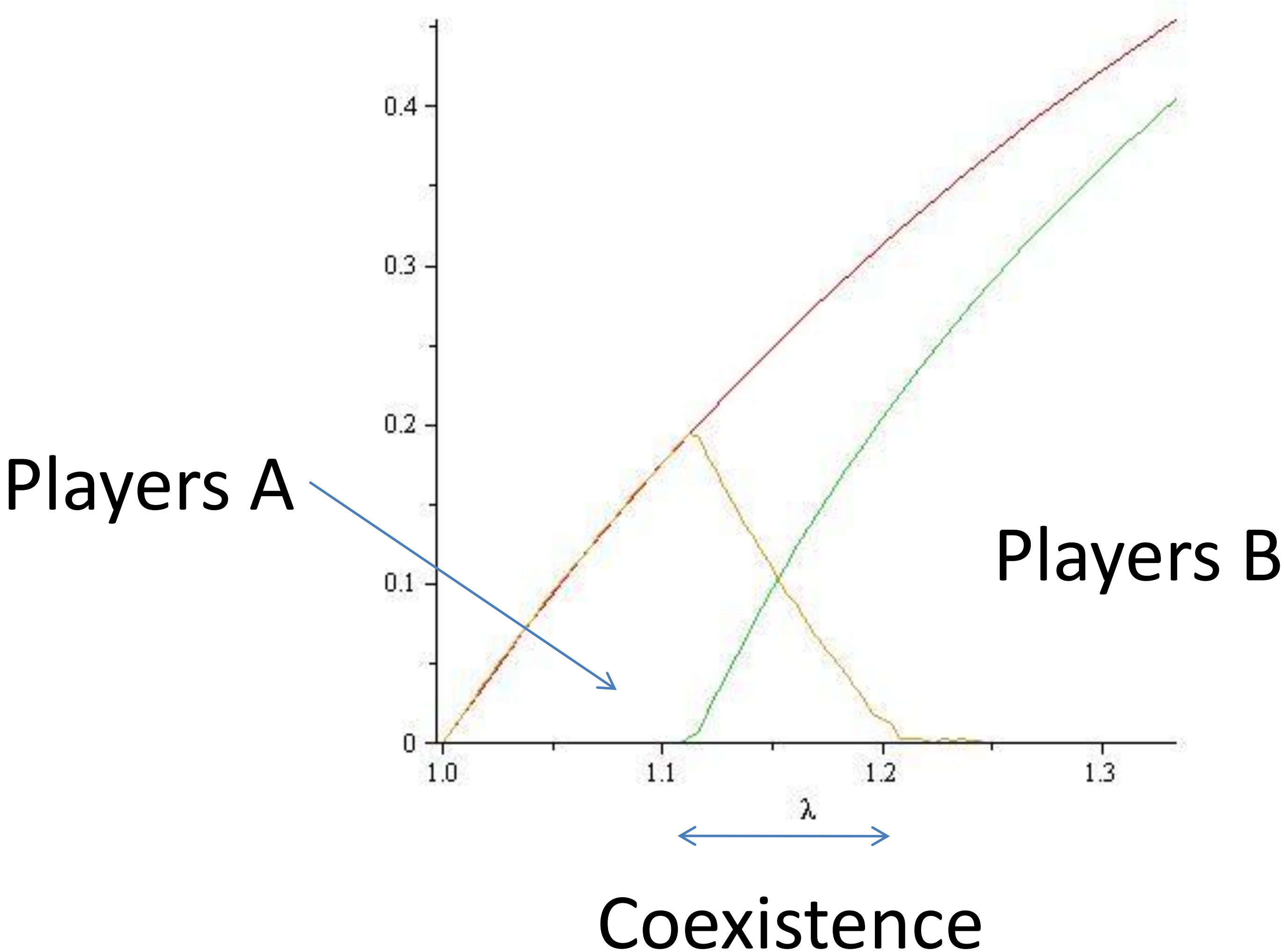
  –> pivotal equilibrium

(2) Strength of Equilibria for $q<q_c$

Mean number of trials to switch from all A to pivotal equilibrium



(2) Coexistence for $q<q_c$

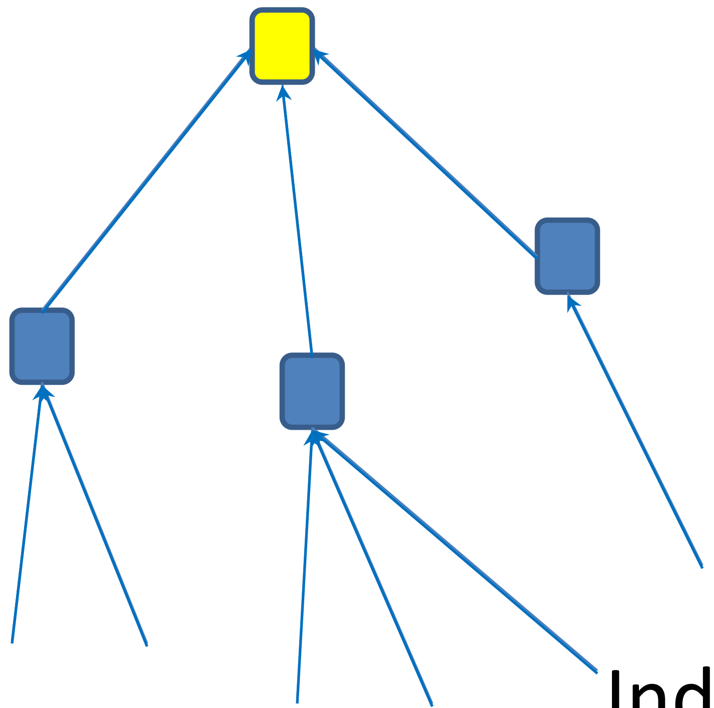


(1) Diffusion Model

(2) Results

(3) Heuristic

(3) Locally tree-like



Independent computations on trees

(3) Branching Process Approximation

- Local structure of G = random tree
- Recursive Distributional Equation (RDE) or:

$$Y_i = \begin{cases} 1 & \text{if infected from 'below'} \\ 0 & \text{otherwise.} \end{cases}$$

$$Y_i = 1 - (1 - \sigma_i) \mathbb{1}\left(\sum_{\ell \to i} Y_\ell \le q d_i \right)$$

(3) Solving the RDE

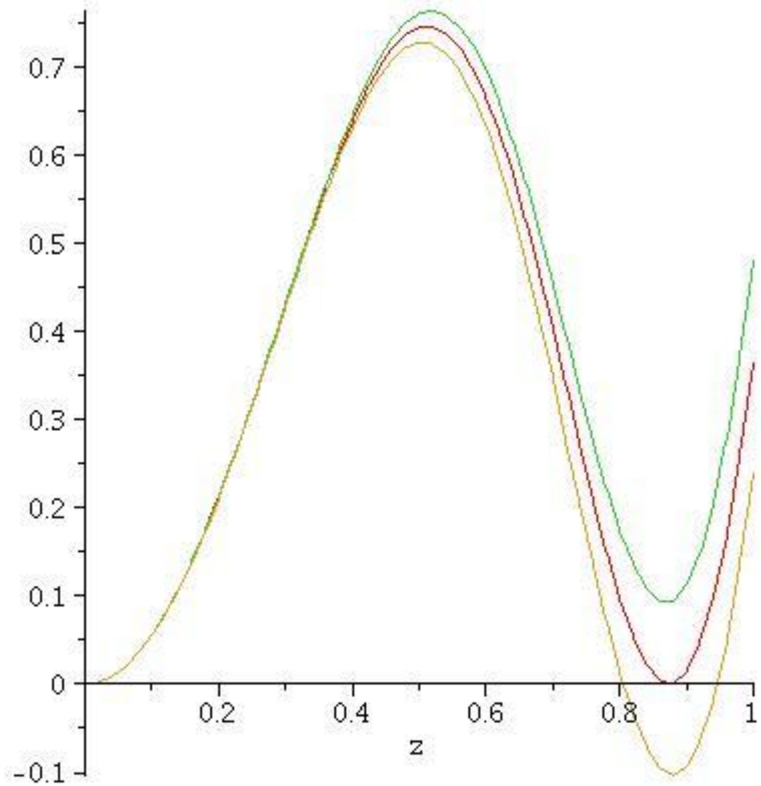
$$Y \stackrel{d}{=} 1 - (1-\sigma)\mathbb{1}\left(\sum_{\ell=1}^{\hat{D}-1} Y_\ell \leq q\hat{D}\right)$$

$$z = \mathbb{P}(Y = 0)$$

$$\lambda z^2 = (1-\alpha)h(z)$$

$$h(z) = \sum_{s,r \geq s - \lfloor qs \rfloor} r p_s \binom{s}{r} z^r (1-z)^{s-r}$$

(3) Phase transition in one picture



$$z^* = \max\{z \in [0,1],\ \lambda z^2 - (1-\alpha)h(z) = 0\}$$

Conclusion

- Simple tractable model:
  - Threshold rule introduces local dependencies
  - Random network : heterogeneity of population
- 2 regimes:
  - Low connectivity: tipping point
  - High connectivity: chasm
- More results in the paper:
  - heterogeneity of thresholds, active/inactive links, rigorous proof.

Thank you!

- Diffusion and Cascading Behavior in Random Networks. Available at http://www.di.ens.fr/~lelarge