Algorithms for Networked Information
TD2

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Exercise 1 Game Theory

1. In Indiana Jones and the Last Crusade, the hero's father was heavily injured by the nazis. The only way to save him is to bring him the Holy Grail. But Indy doesn't know which of the many chalices it is. The real Grail grants eternal life, but all the others will kill the person who drinks from it. In the film, Indy chooses one chalice and drinks to test whether he has the right one.
   Show that giving the chalice directly to his father is a dominant strategy.
2. Find all dominating strategies and Nash equilibria in the following game:

	L	M	H
t	3 0	2 6	1 1
m	3 2	1 0	7 0
b	3 5	2 4	1 3

Exercise 2 Expected Revenue

Suppose that there are two bidders in a Vickrey auction. Every bidder's estimation of the object is independently either 1 or 3 with probability 1/2.

1. What's the expected revenue for the seller?
2. How does the revenue change if we increase the number of bidders?

Exercise 3 Reserve Price

We now consider the case that the seller has its own evaluation $u \geq 0$ of the object. In this case, the seller announces a reserve price r before bidding starts. The object is sold only if there is a bid higher than r. In a second-price auction, the highest bidder pays the maximum of r and the second highest bid.

1. Show that being truthful remains a dominant strategy.
2. Let $u = 0$ with a single bidder having an evaluation uniformly distributed in $[0,1]$. What's the optimal reserve price? Generalize to the case $u \geq 0$.

Exercise 4 Collusion in a Vickrey Auction

Suppose that two bidders choose to collude and want to maximize the sum of their payoffs.

1. What are their optimal bids if there are no other bidders?
2. Do they change if there's a third bidder?

Exercise 5 Equilibria in First-Price Auctions

We now analyze a first-price auction in which the bidders have partial information on the evaluations of their competitors. Each bidder's evaluation is independently and uniformly drawn from $[0,1]$. Everyone knows their own evaluation and knows that the others' evaluations are drawn independently and uniformly from $[0,1]$.

A bidder's strategy is just a function $s(v) = b$ that takes as argument its private evaluation v and returns a bid $b \geq 0$. We make the following assumption:

(i) $s(\cdot)$ is strictly increasing and differntiable.

(ii) $s(v) \leq v$.

Because all bidders are identical, we make the symmetry assumption that everyone adopts the same strategy $s(\cdot)$.
We start with two bidders.

1. Under our assumptions, can the two bids be equal?
2. What's the expected payoff for each of the bidders as a function of their private evaluation v?

We now want to define the notion of equilibrium for our game. Because we consider only the symmetric case, a bidder can only lie about its private evaluation.

3. State the equilibrium condition.
4. Show that $s(v) = v/2$ is an equilibrium. Characterize all equilibria.
5. What's the seller's revenue? Would a second-price auction be more profitable?

We now consider the case of $n \geq 2$ bidders. The number n of players is public information to everybody.

6. Find all equilibria.
7. What's the seller's revenue? Would a second-price auction be more profitable?

We now turn to a form of auction where everybody pays their offer, independently of whether they actually get the object.

8. Find the expected payoff of a bidder. Find all equilibria.
9. Find the seller's revenue.