Diffusion and Cascading Behavior in Random Networks

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(1) Diffusion Model

inspired from game theory

and statistical physics.

(2) Results

from a mathematical analysis.
(0) Context

Crossing the Chasm
  (Moore 1991)
(1) Diffusion Model

(2) Results
• Both receive payoff \( q \).

• Both receive payoff \( 1-q>q \).

• Both receive nothing.
(1)...on a network.

- Everybody start with icq.
- Total payoff = sum of the payoffs with each neighbor.
- A seed of nodes switches to talk.

(Morris 2000)
(1) Threshold Model

- State of agent $i$ is represented by
  \[ X_i = \begin{cases} 
  0 & \text{if } \text{icq} \\
  1 & \text{if } \text{talk} 
  \end{cases} \]

- Switch from $\text{icq}$ to $\text{talk}$ if:
  \[ \sum_{j \sim i} X_j \geq q d_i \]
(1) Model for the network?

Statistical physics: bootstrap percolation.
(1) Model for the network?
(1) Random Graphs

- Random graphs with given degree sequence introduced by Molloy and Reed (1995).
- Examples:
  - Erdös-Rényi graphs, $G(n, \lambda/n)$.
  - Graphs with power law degree distribution.
- We are interested in large population asymptotics.
- Average degree is $\lambda$. 
(1) Diffusion Model
\[ q = \text{relative threshold} \]
\[ \lambda = \text{average degree} \]

(2) Results
(1) Diffusion Model

$q = \text{relative threshold}$

$\lambda = \text{average degree}$

(2) Results
(2) Some experiments

Seed = one node, $\lambda=3$ and $q=0.24$
(source: the Technoverse blog)
(2) Some experiments

Seed = one node, $\lambda=3$ and $1/q>4$
(source: the Technooverse blog)
(2) Some experiments

Seed = one node, \( \lambda=3 \) and \( q=0.24 \) (or \( 1/q>4 \))
(source: the Technoverse blog)
(2) Contagion threshold

In accordance with (Watts 2002)
(2) A new Phase Transition
(2) Pivotal players

- Giant component of players requiring only one neighbor to switch.

Tipping point: Diffusion like standard epidemic

- Chasm: Pivotal players = Early adopters
(2) $q$ above contagion threshold

- New parameter: size of the seed as a fraction of the total population $0 < \alpha < 1$.
- Monotone dynamic $\rightarrow$ only one final state.
(2) Minimal size of the seed, $q > 1/4$

Tipping point: Connectivity helps

Chasm: Connectivity hurts
Conclusion

• Simple tractable model:
  – Threshold rule introduces local dependencies
  – Random network: heterogeneity of population

• 2 regimes:
  – Low connectivity: tipping point
  – High connectivity: chasm

• More results in the paper:
  – Heterogeneity of thresholds, active/inactive links, equilibria of the game and coexistence.
Thanks!

- Diffusion and Cascading Behavior in Random Networks. Available at http://www.di.ens.fr/~lelarge