Diffusion and Cascading Behavior in Random Networks

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WIDS MIT, May 31, 2011.

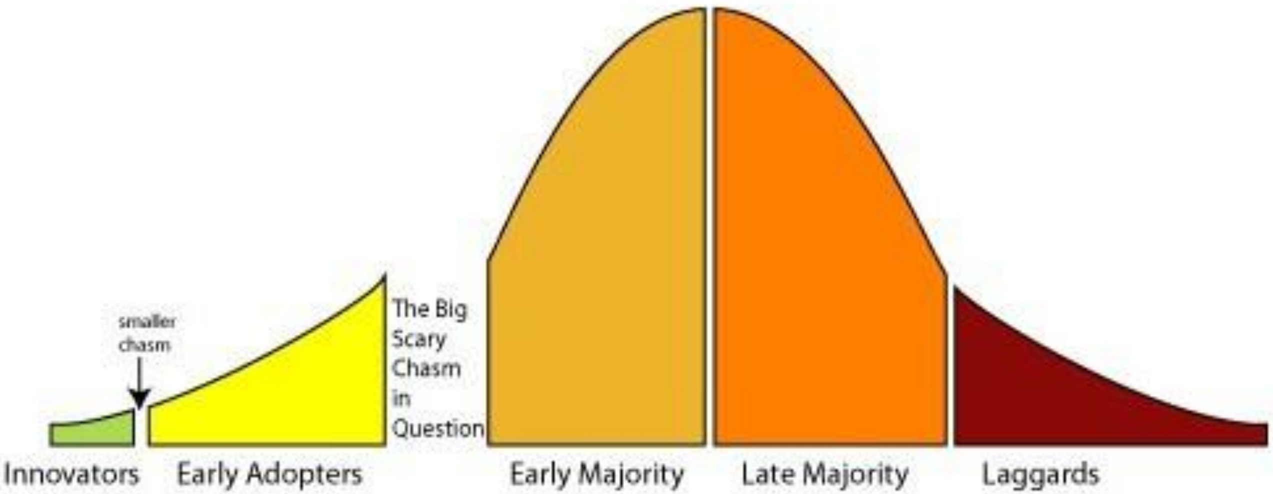
(1) Diffusion Model

inspired from game theory
and statistical physics.

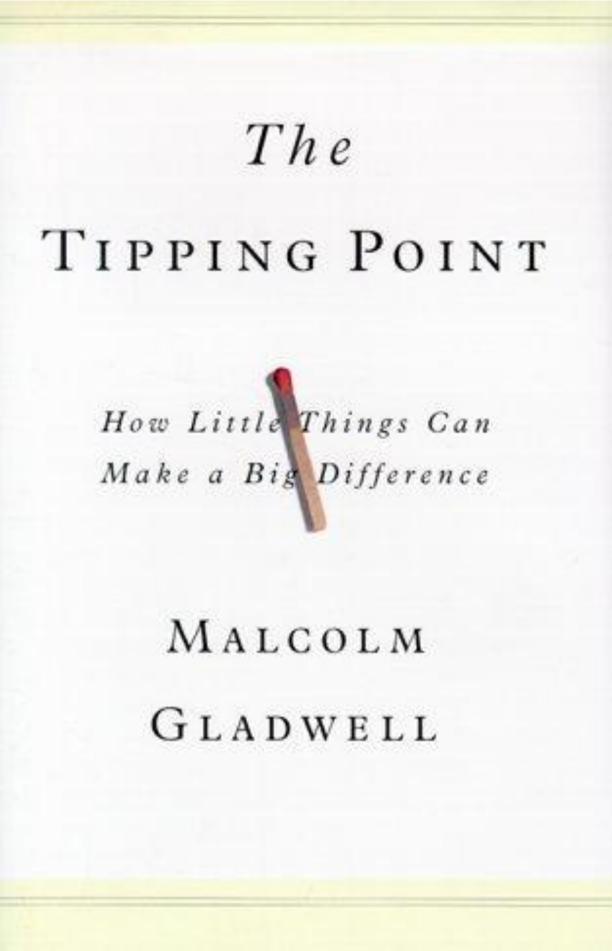
(2) Results

from a mathematical analysis.

(0) Context



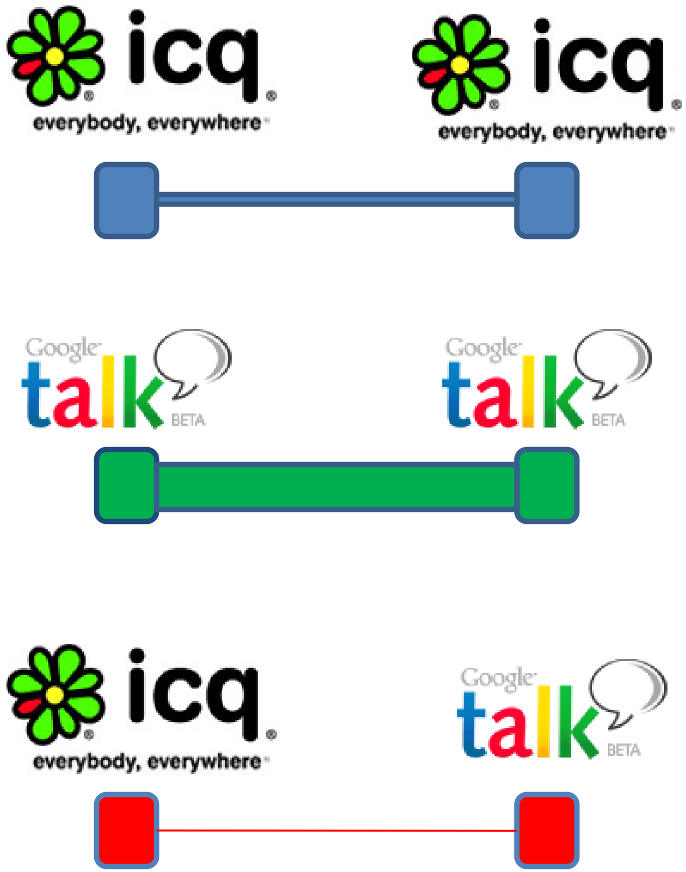
Crossing the Chasm
(Moore 1991)



(1) Diffusion Model

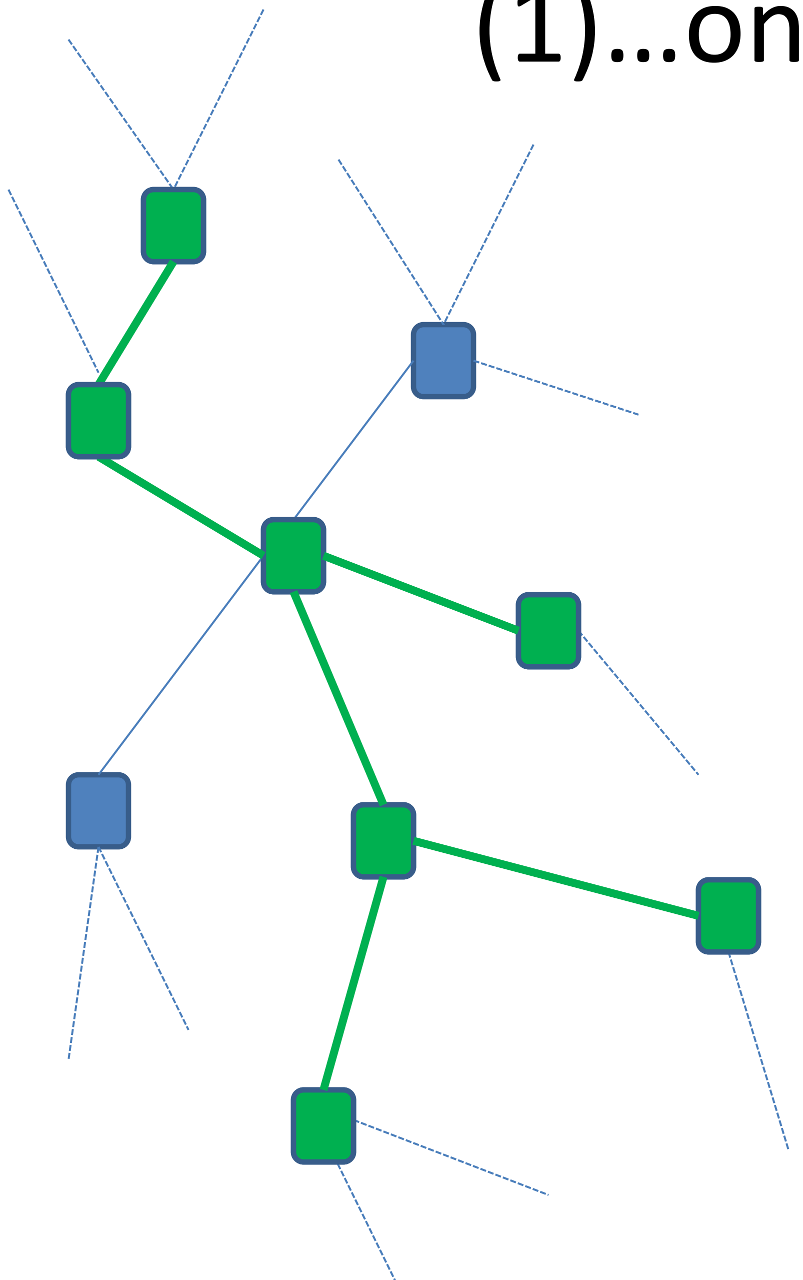
(2) Results

(1) Coordination game…



- Both receive payoff q.
- Both receive payoff 1-q>q.
- Both receive nothing.

(1)…on a network.



- Everybody start with icq
- Total payoff = sum of the payoffs with each neighbor.
- A seed of nodes switches to Google talk

(Morris 2000)

(1) Threshold Model

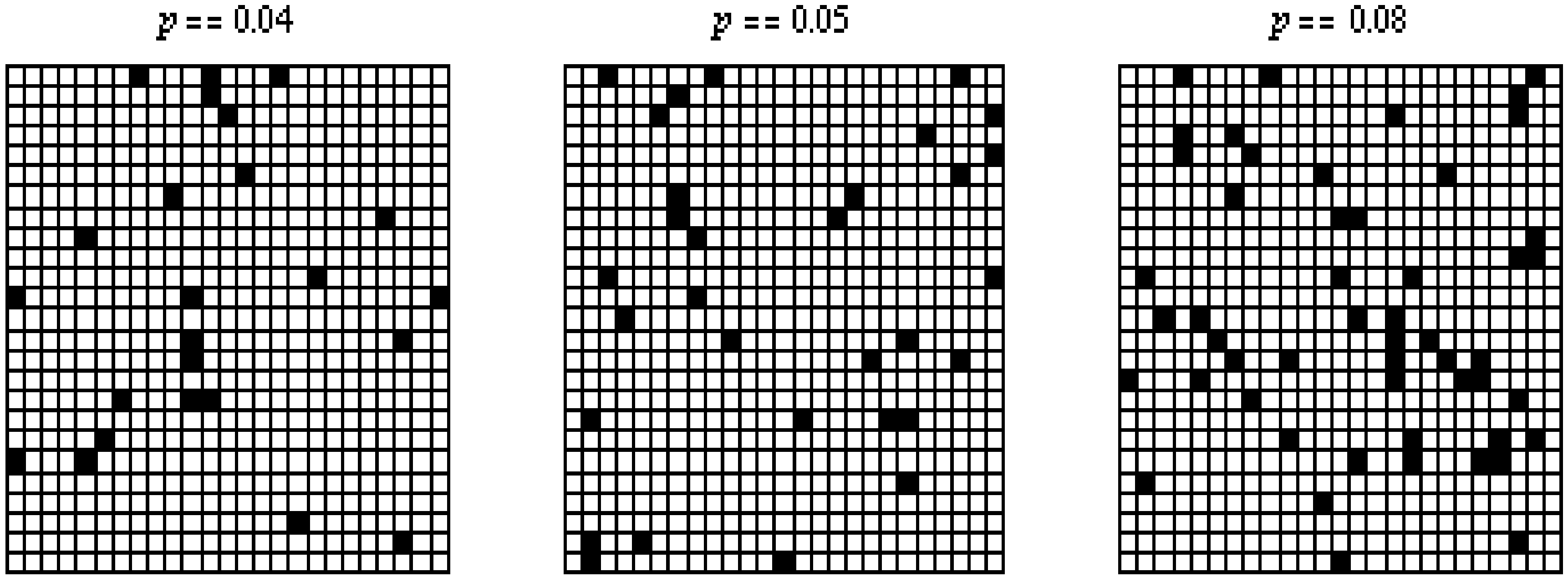
- State of agent i is represented by

$$X_i = \begin{cases} 0 & \text{if icq} \\ 1 & \text{if Google talk} \end{cases}$$

- Switch from icq to Google talk if:

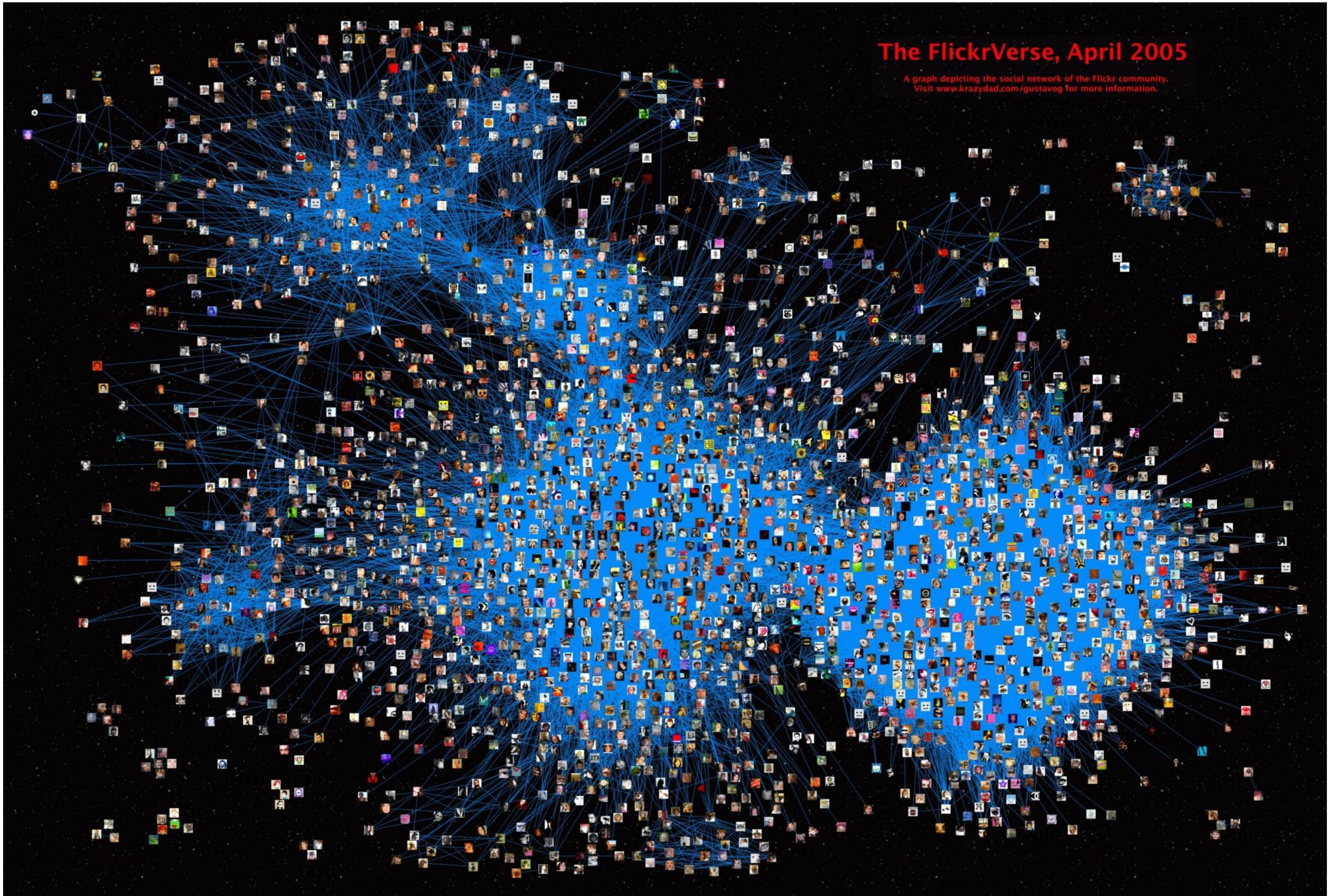
$$\sum_{j \sim i} X_j \geq q d_i$$

(1) Model for the network?



Statistical physics: bootstrap percolation.

(1) Model for the network?



(1) Random Graphs

- Random graphs with given degree sequence introduced by Molloy and Reed (1995).
- Examples:
  - Erdös-Réyni graphs, G(n,λ/n).
  - Graphs with power law degree distribution.
- We are interested in large population asymptotics.
- Average degree is λ.

(1) Diffusion Model

q = relative threshold

λ = average degree

(2) Results

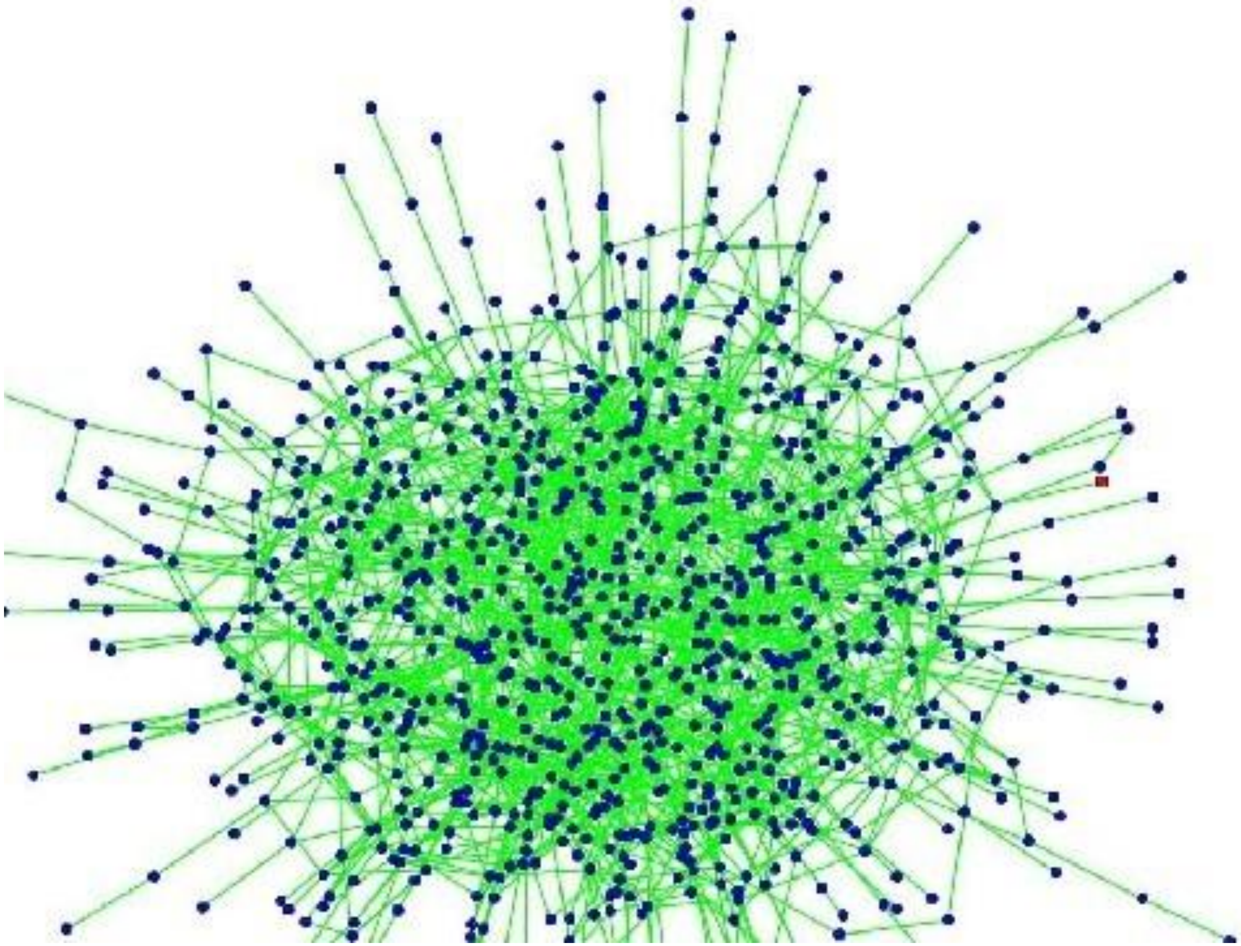
(1) Diffusion Model

q = relative threshold

λ = average degree

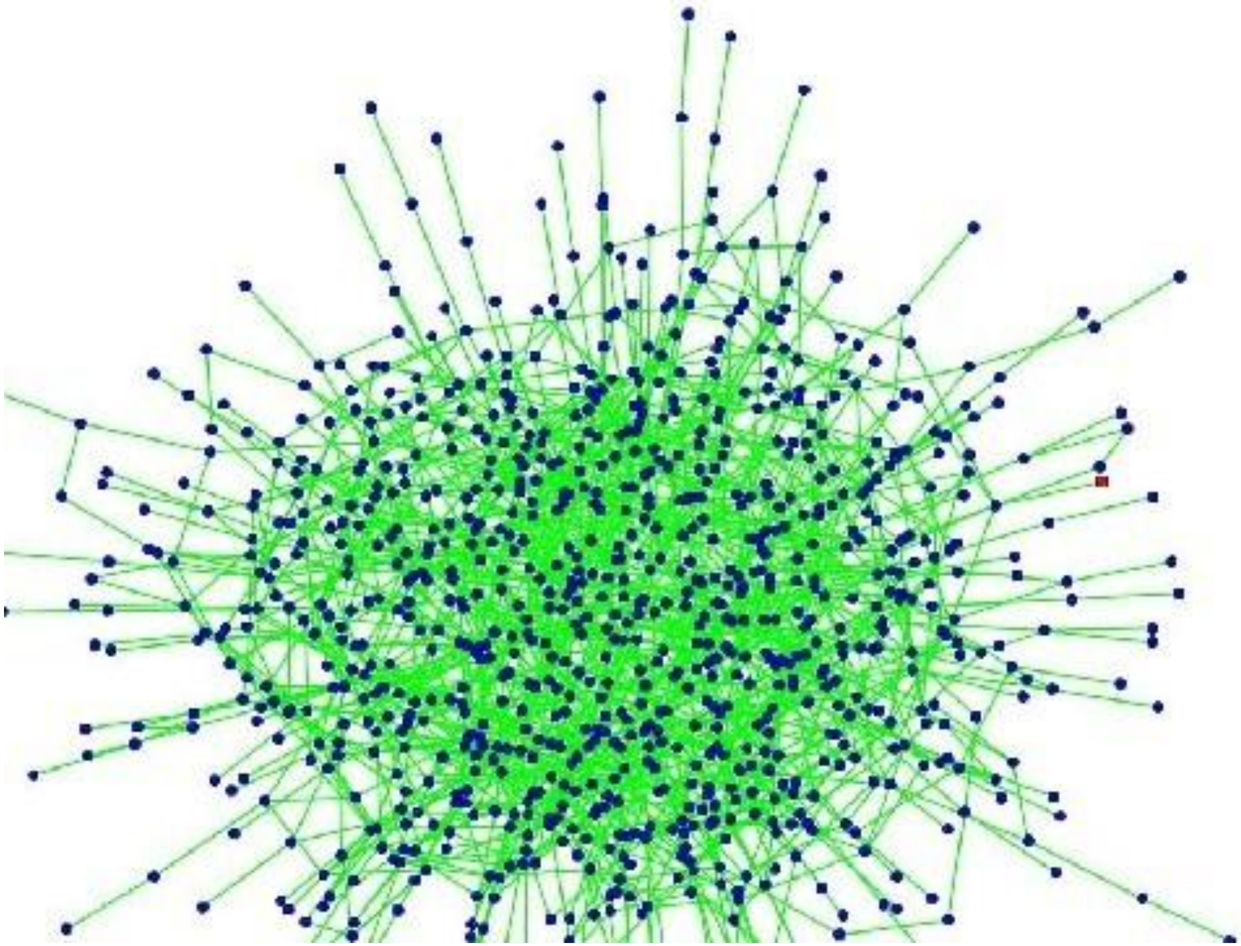
(2) Results

(2) Some experiments



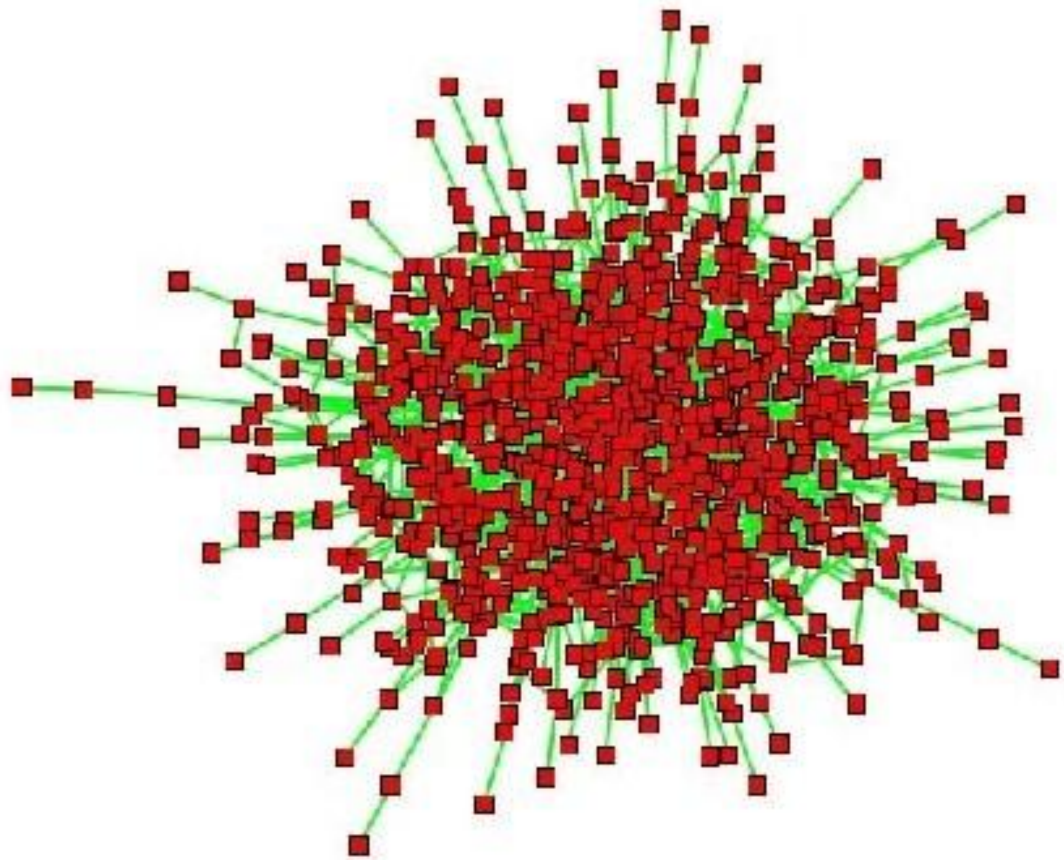
Seed = one node, λ=3 and q=0.24
(source: the Technoverse blog)

(2) Some experiments



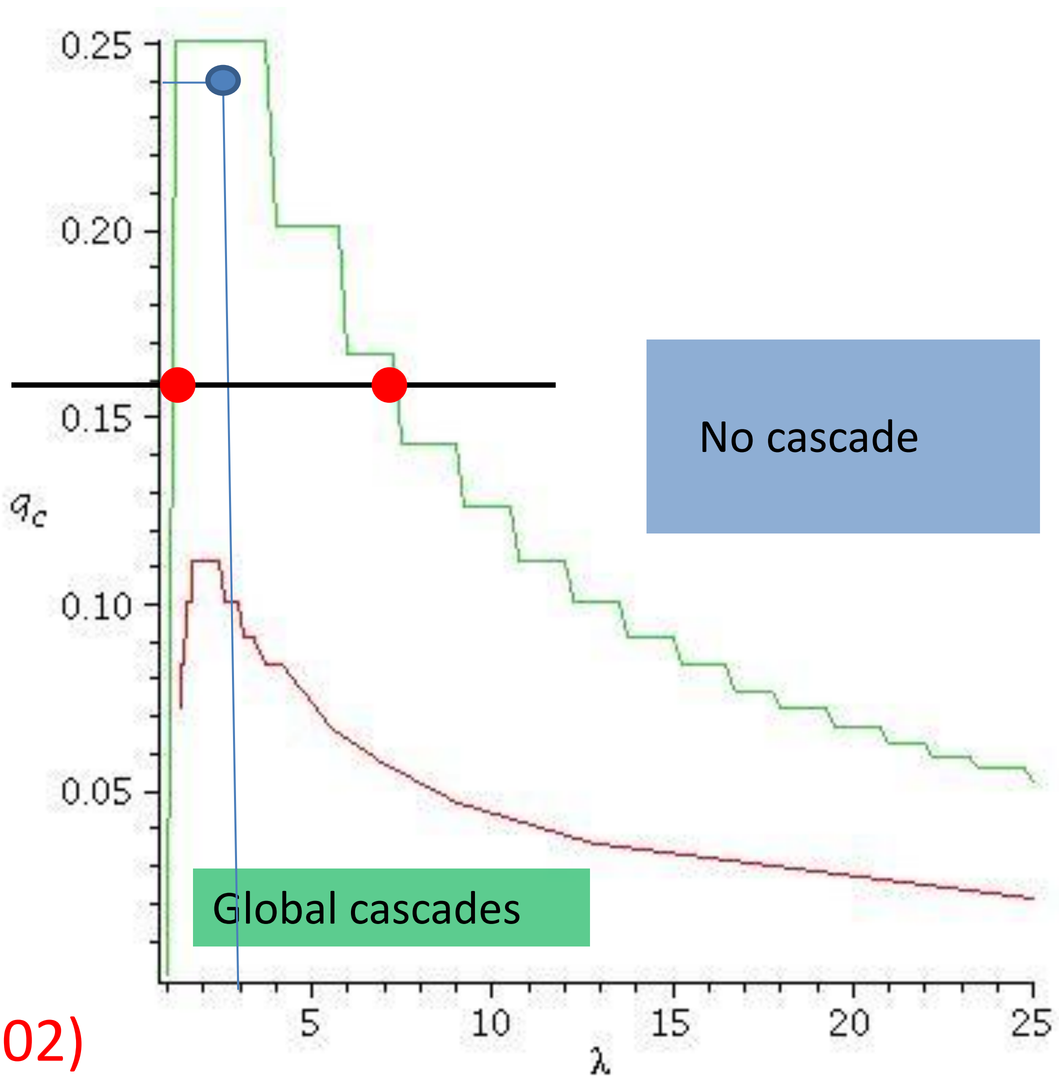
Seed = one node, λ=3 and 1/q>4
(source: the Technoverse blog)

(2) Some experiments



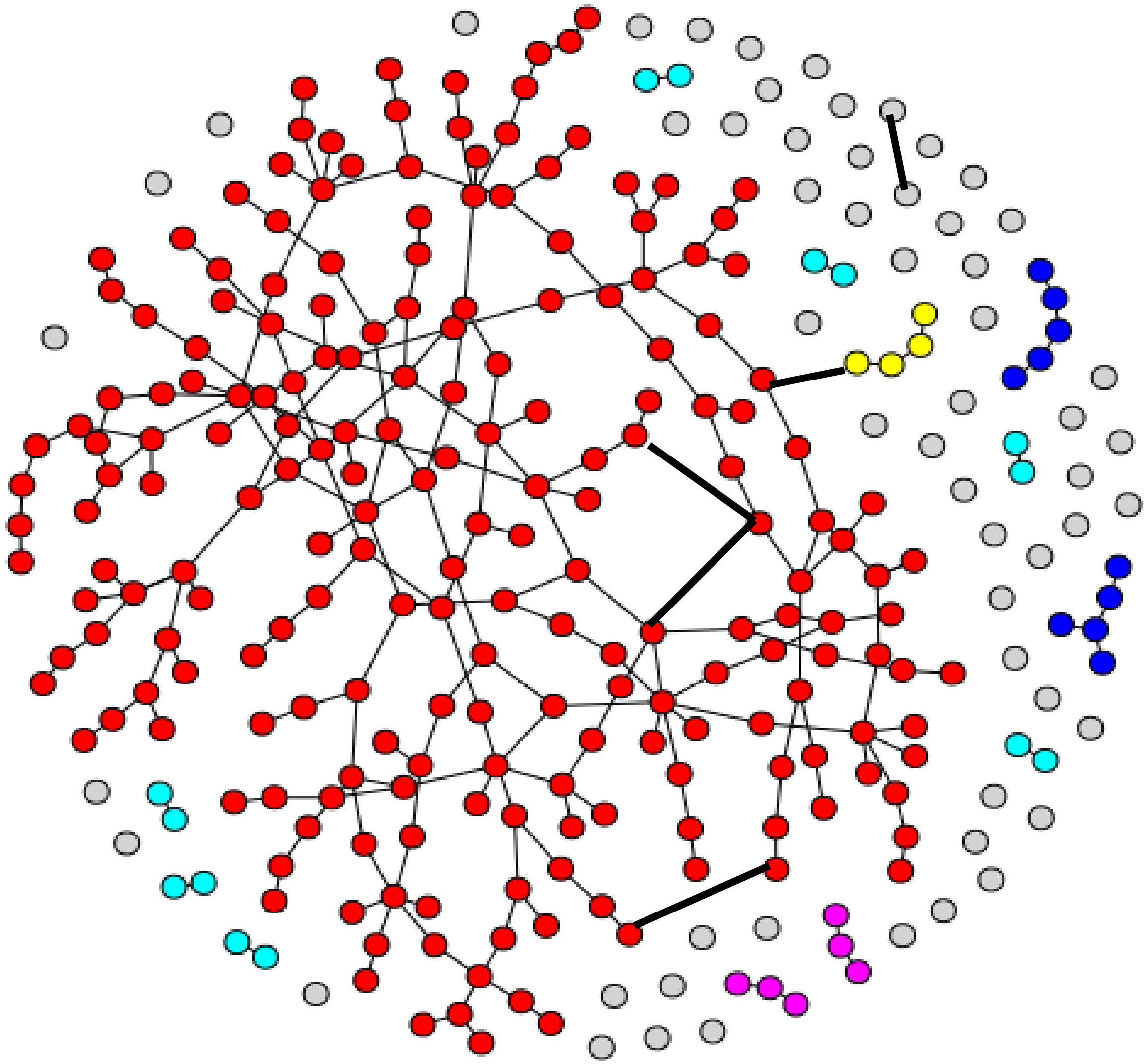
Seed = one node, λ=3 and q=0.24 (or 1/q>4)
(source: the Technoverse blog)

(2) Contagion threshold



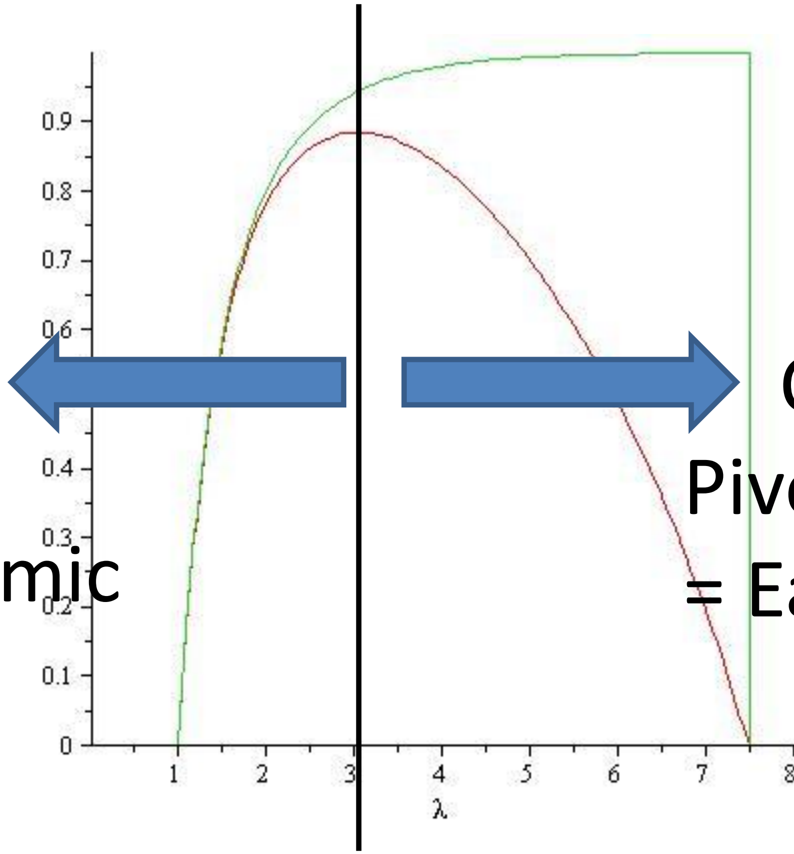
In accordance with (Watts 2002)

(2) A new Phase Transition



(2) Pivotal players

- Giant component of players requiring only one neighbor to switch.

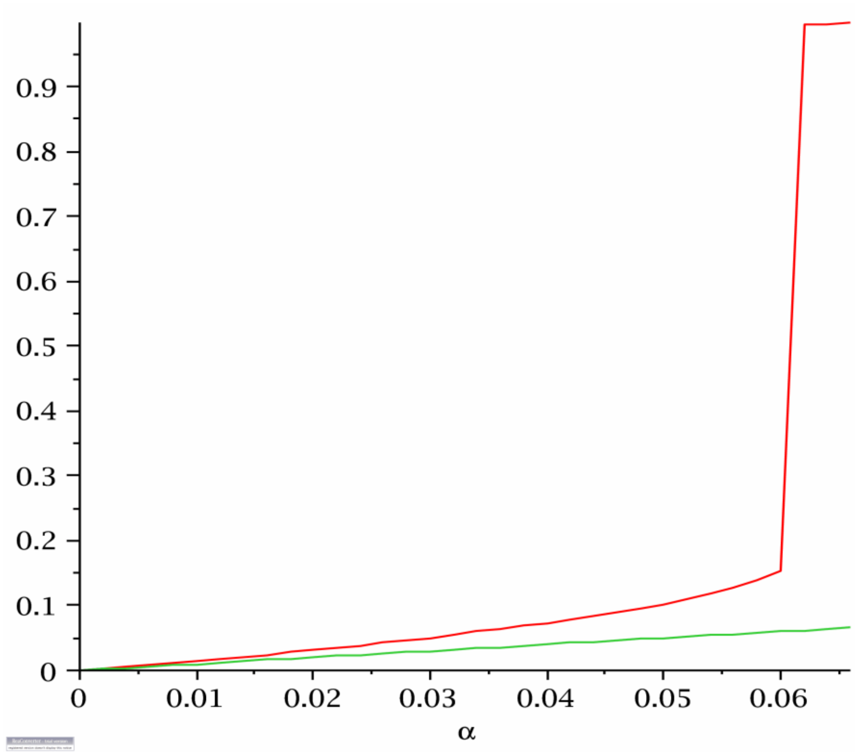


Tipping point:
Diffusion like standard epidemic

Chasm :
Pivotal players
= Early adopters

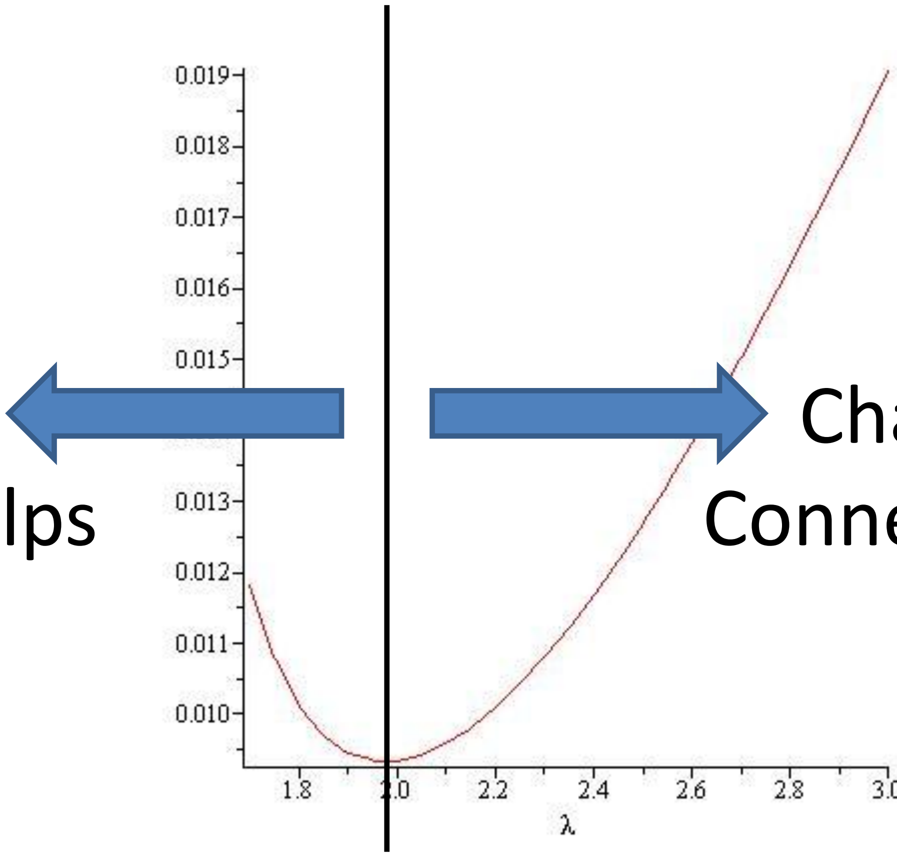
(2) q above contagion threshold

- New parameter: size of the seed as a fraction of the total population $0 < \alpha < 1$.
- Monotone dynamic → only one final state.



(2)Minimal size of the seed, q>1/4

Tipping point:
Connectivity helps



Chasm :
Connectivity hurts

Conclusion

- Simple tractable model:
  - Threshold rule introduces local dependencies
  - Random network : heterogeneity of population
- 2 regimes:
  - Low connectivity: tipping point
  - High connectivity: chasm
- More results in the paper:
  - heterogeneity of thresholds, active/inactive links, equilibria of the game and coexistence.

Thanks!

- Diffusion and Cascading Behavior in Random Networks. Available at http://www.di.ens.fr/~lelarge