Diffusion Model

Results from a mathematical analysis inspired from game theory and statistical physics.
(1) Coordination game...

- Both receive payoff \( q \).
- Both receive payoff \( 1 - q > q \).
- Both receive nothing.
(1)...on a network.
Threshold Model

State of agent $i$ is represented by

$\sum_{j \sim i} X_j \geq qd_i$
Model for the network?

Statistical physics: bootstrap percolation
Random graphs with given degree sequence introduced by Molloy and Reed (1995).

Examples:
- Erdös-Rényi graphs, $G(n, \lambda/n)$.
- Graphs with power law degree distribution.

We are interested in large population asymptotics.

Average degree is $\lambda$. 

$\lambda$
q = relative threshold
λ = average degree
$\lambda$
Some experiments: Seed = one node, $\lambda=3$ and $q=0.24$ (source: the Technoverse blog)
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Seed = one node, $\lambda = 3$ and $1/q > 4$

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Some experiments: Seed = one node, $\lambda = 3$ and $q=0.24$ (or $1/q > 4$) (source: the Technoverse blog)
In accordance with (Watts 2002)
A new Phase Transition
Pivotal players

- Giant component of players requiring only one neighbor to switch.

Tipping point: Diffusion like standard epidemic

Chasm: Pivotal players = Early adopters
Monotone dynamic $\rightarrow$
Minimal size of the seed, $q > \frac{1}{4}$

Chasm:

Connectivity hurts

Tipping point:

Connectivity helps
Conclusion

• Simple tractable model:
  – Threshold rule introduces local dependencies
  – Random network: heterogeneity of population

• 2 regimes:
  – Low connectivity: tipping point
  – High connectivity: chasm

• More results in the paper:
  – Heterogeneity of thresholds, active/inactive links,
  – Equilibria of the game and coexistence.
Thanks!

- Diffusion and Cascading Behavior in Random Networks.

Available at http://www.di.ens.fr/~lelarge