Network Security: an Economic Perspective

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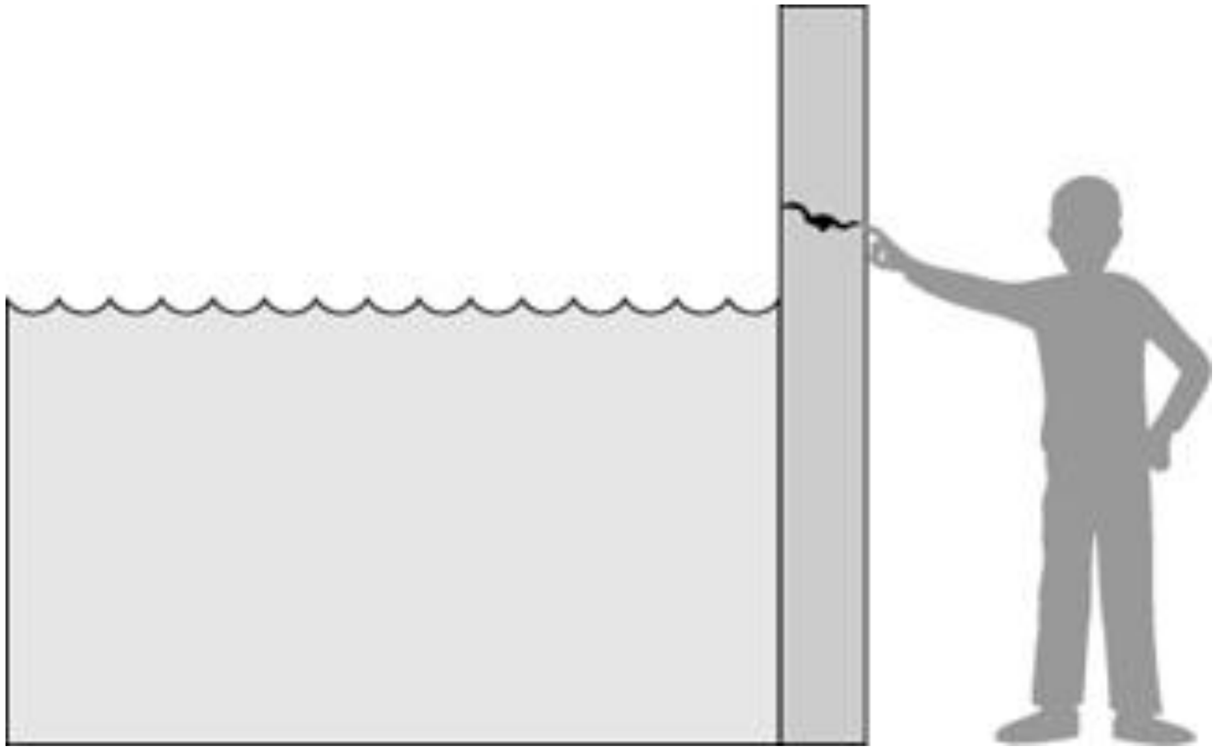
Threats and Vulnerabilities



Attacks are exogenous

Contribution

(1) Optimal security investment for a single agent

- Gordon and Loeb model, 1/e rule
- Monotone comparative statics

(2) Optimal security investment for an interconnected agent

- Network externalities

(3) Equilibrium analysis of the security game

- Free-rider problem, Critical mass, PoA

(1) Single agent

- Two parameters:
  - Potential monetary loss: ℓ
  - Probability of security breach without additional security: v
- Agent can invest x to reduce the probability of loss to: $p(x, v) \leq v$
- Optimal investment:

$$\phi(v, \ell) = \arg\min\{\ell p(x, v) + x,\ x \geq 0\}$$

(1) Gordon and Loeb

- Class of security breach probability functions:

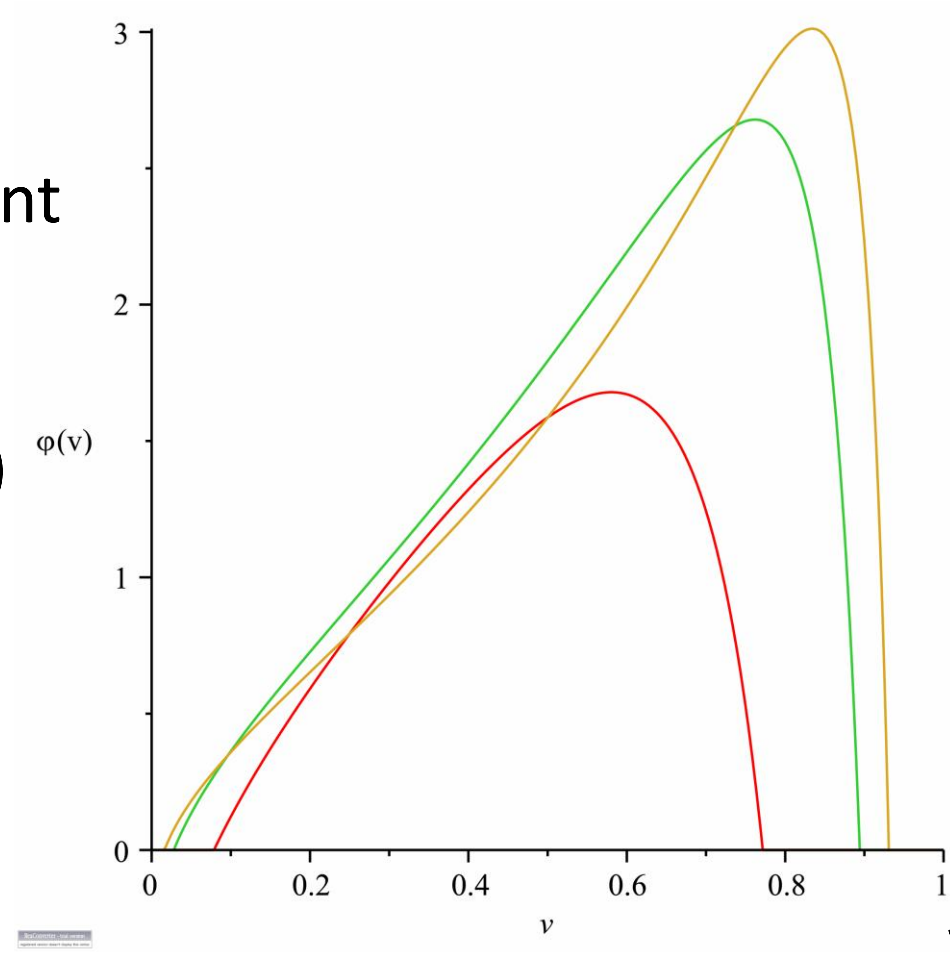
$$p_{GL}(x, v) = v^{\alpha x + 1} \text{ for } \alpha > 0$$

- α measure of the productivity of security.

Gordon and Loeb (2002)

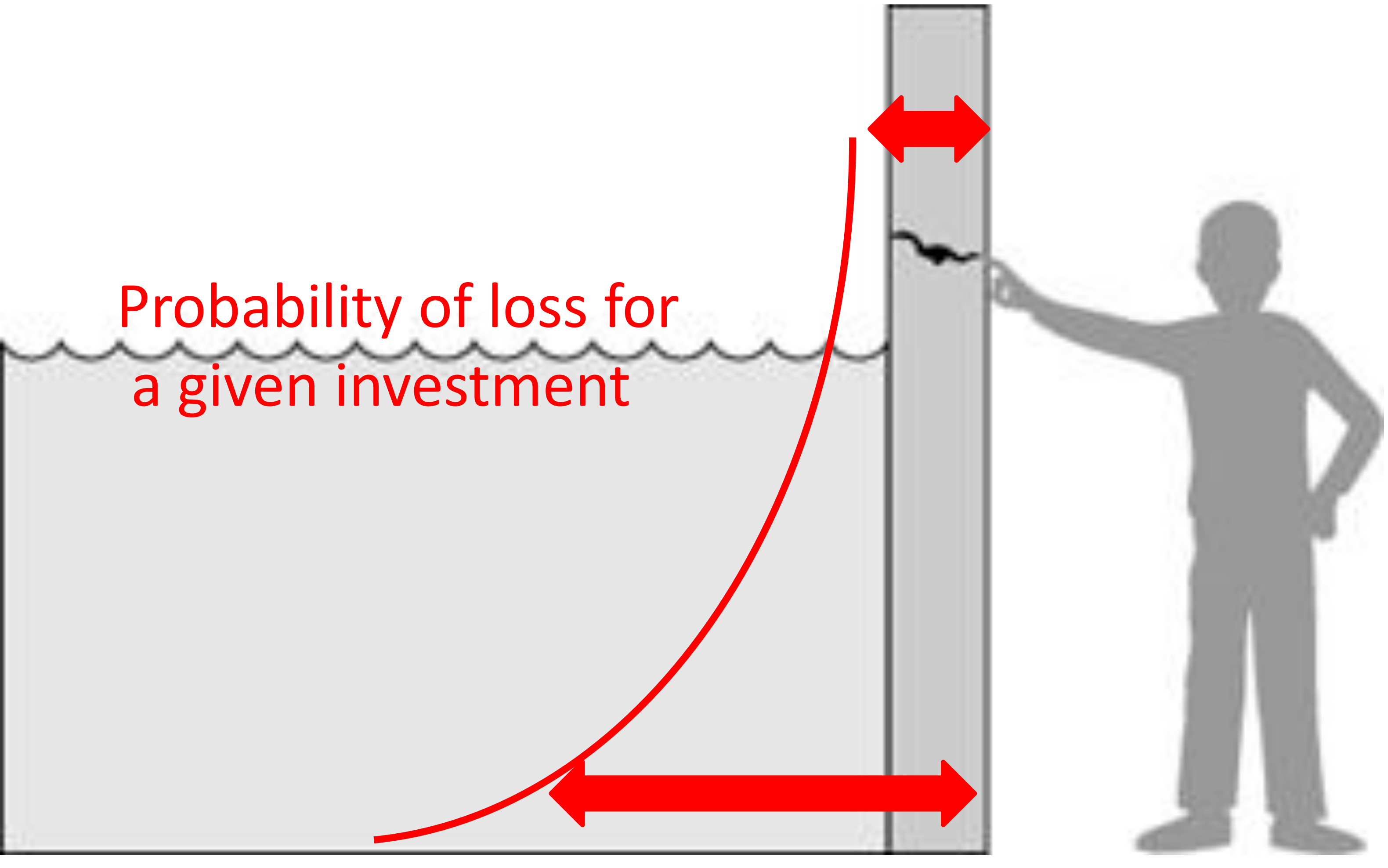
(1) Gordon and Loeb (cont.)

Optimal
investment
(size of
potential
loss fixed)

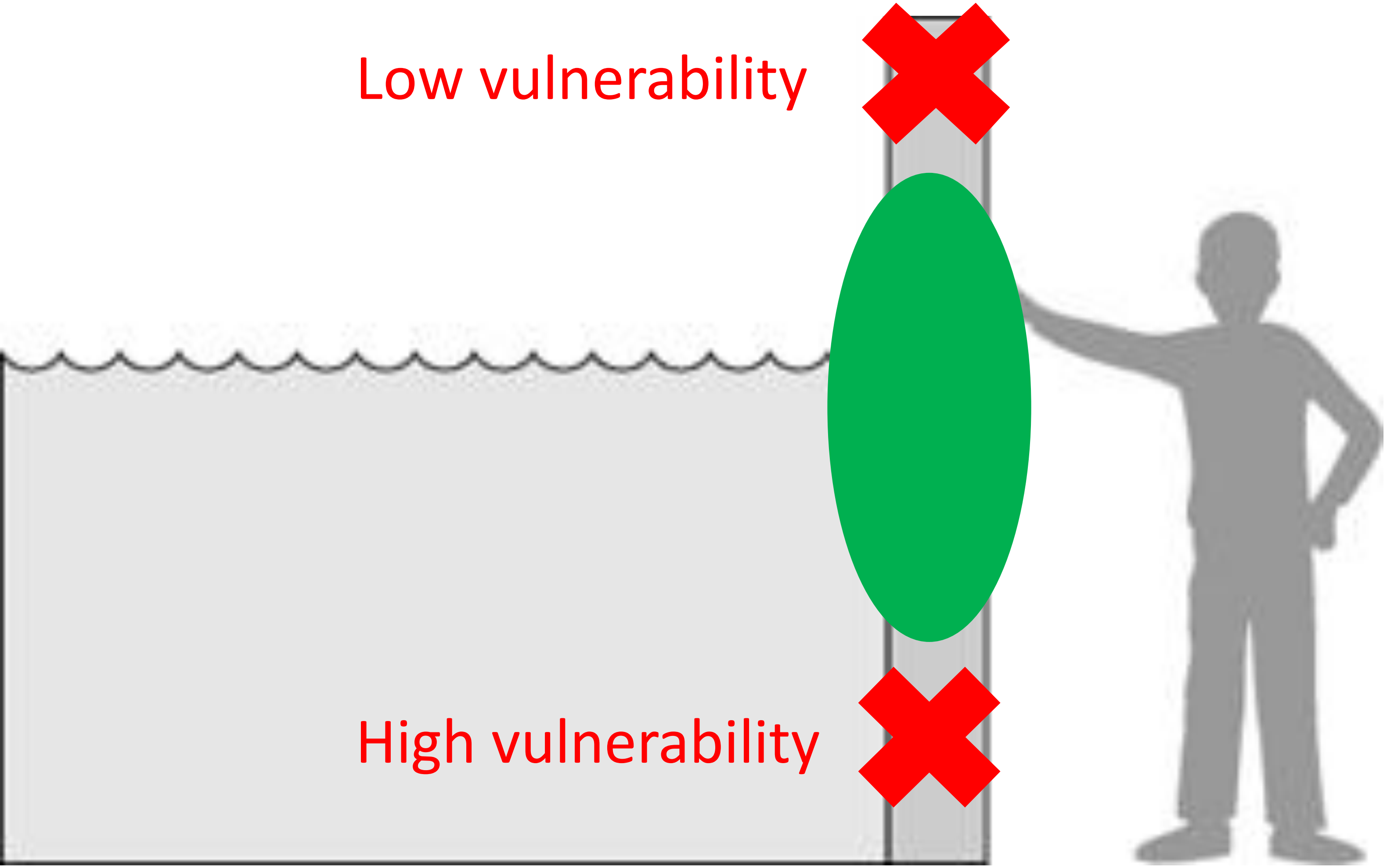


vulnerability

(1) Gordon and Loeb (cont.)



(1) Gordon and Loeb (cont.)



(1) Conditions for monotone investment

- If $\frac{\partial p}{\partial x}(x,v) \leq 0$ and $\frac{\partial^2 p}{\partial x \partial v}(x,v) \leq 0$

then $\phi(v,\ell)$ is non-decreasing

- Augmenting return of investment with vulnerability:

$$v^H > v^L \Rightarrow \left|\frac{\partial p}{\partial x}(x,v^H)\right| \geq \left|\frac{\partial p}{\partial x}(x,v^L)\right|$$

- Extension to submodular functions.

(1) The 1/e rule

- If the function $p(x, v)$ is log-convex in x then the optimal security investment is bounded by: $\frac{\ell v}{e}$,i.e

$\frac{1}{e} \approx 37\%$ of the expected loss

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(2) Effect of the network

- Agent faces an internal risk and an indirect risk.
- Information available to the agent: γ in a poset (partially ordered set).
- Optimal security investment:

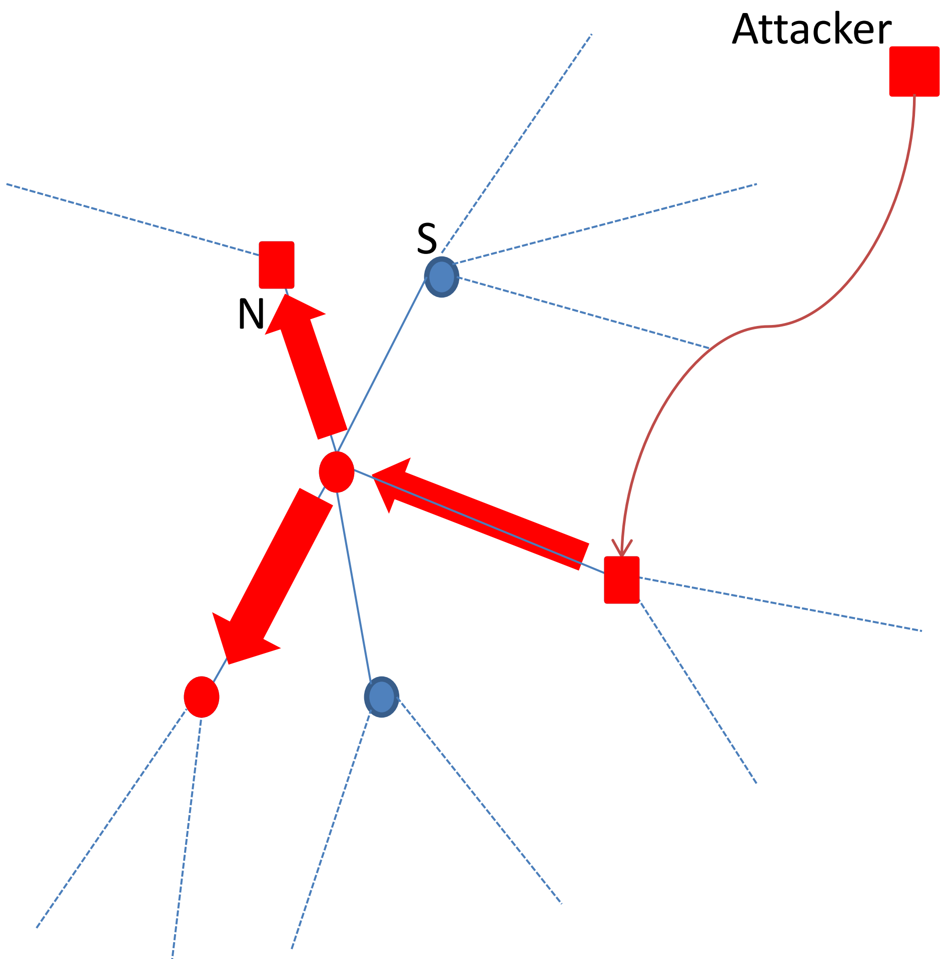
$$\phi(v,\ell,\gamma) = \arg\min\{\ell p(x,v,\gamma) + x,\ x \geq 0\}$$

(2) How to estimate the probability of loss?

- Epidemic risk model
- Binary choice for protection $x \in \{0, 1\}$
- Limited information on the network of contagion (physical or not): degree distribution.
  - Best guess: take a graph uniformly at random.

Galeotti et al. (2010)

(2) Epidemic Model



- Attacker directly infects an agent N with prob. p.
- Each neighbor is contaminated with prob. q if in S or $q^+ \geq q$ if in N.

(2) Monotone comparative statics

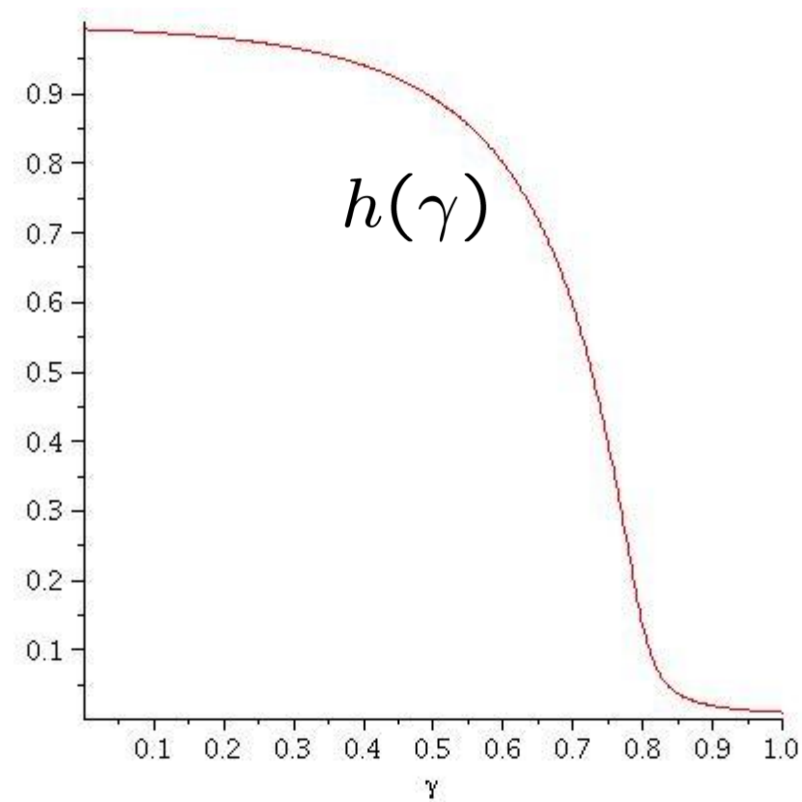
- If the function $p(x, v, \gamma) - p(x, v', \gamma')$ is strictly decreasing in x for any $(v', \gamma') > (v, \gamma)$ then the optimal investment $\phi(v, \ell, \gamma)$ is non-decreasing.
- Equivalent to:

Network externalities function is decreasing:

$$h(\gamma) = p(0, \gamma) - p(1, \gamma)$$

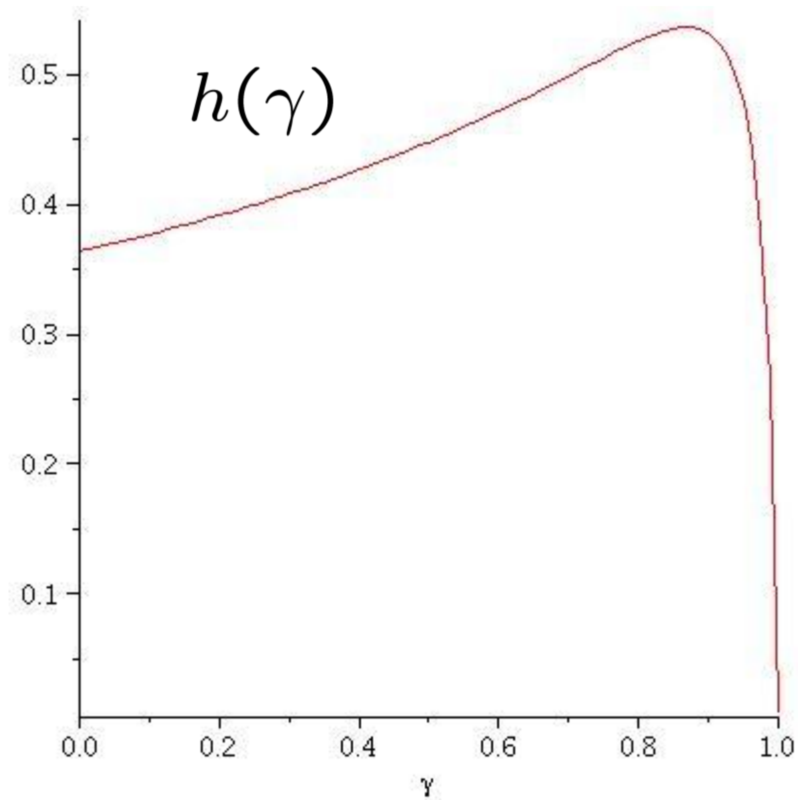
(2) Strong protection

- An agent investing in S cannot be harmed by the actions of others: $q = 0$ in previous equation.
- Decreasing network externalities function.



(2) Weak protection

- If $q > 0$, the network externalities function is:



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(3) Fulfilled expectations equilibrium

- Concept introduced by Katz & Shapiro (85)
- Willingness to pay for the agent of type ℓ_i :

$$(p(0,\gamma^e) - p(1,\gamma^e))\ell_i = h(\gamma^e)\ell_i$$

multiplicative specification of network externalities, Economides & Himmelberg (95).

- C.d.f of types: % with $\ell_i \leq x = F(x)$
- Willingness to pay for the 'last' agent:

$$w(\gamma,\gamma^e) = h(\gamma^e)F^{-1}(1-\gamma)$$

(3) Fulfilled expectations equilibrium

- In equilibrium, expectation are fulfilled:

$$\gamma = \gamma^e$$

- The willingness to pay is:

$$w(\gamma) = h(\gamma)F^{-1}(1-\gamma)$$

- Extension of Interdependent Security

2 players game introduced by Kunreuther & Heal (03).

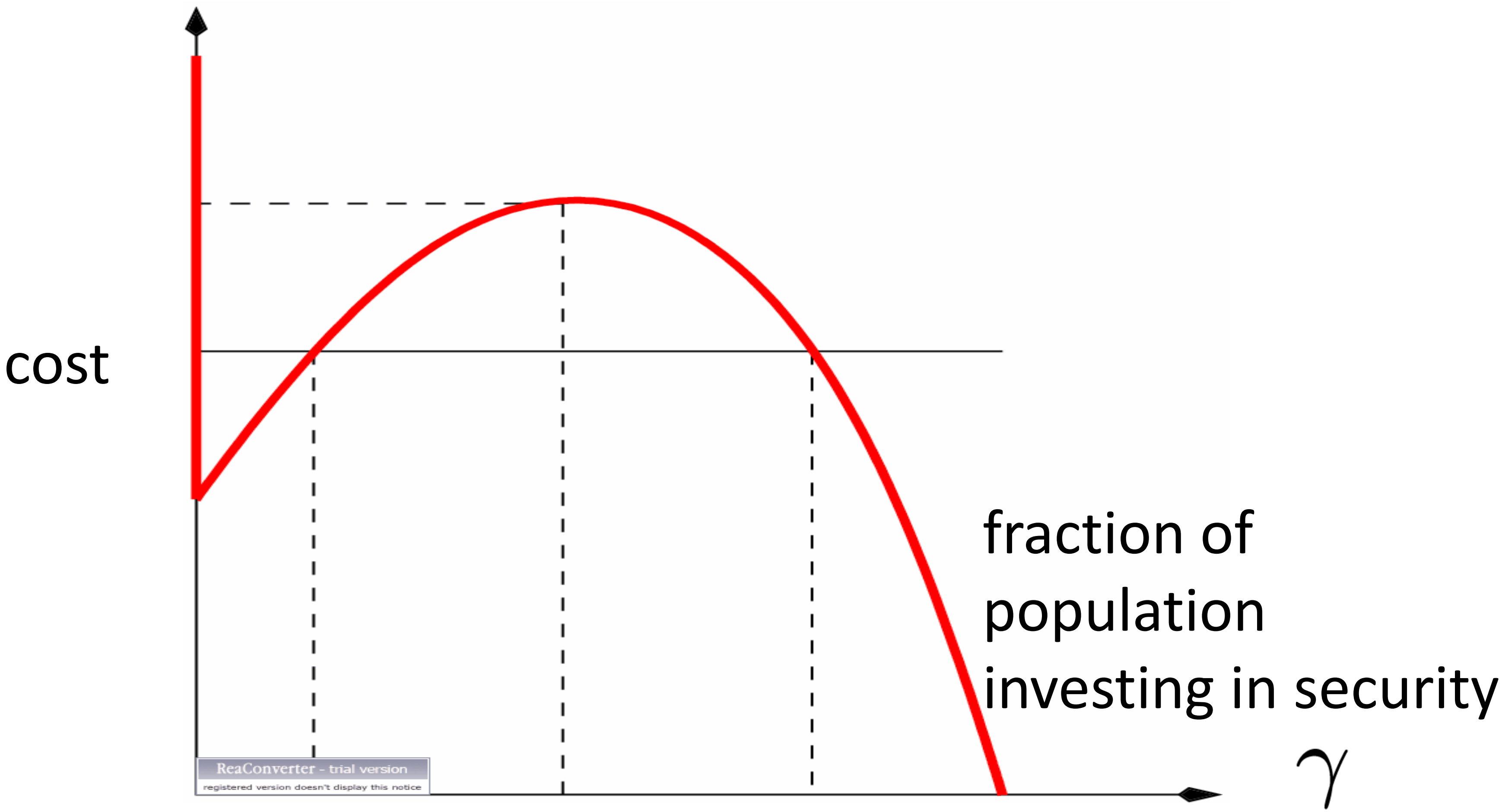
(3) Critical mass

- Equilibria given by the fixed point equation

$$c = w(\gamma) = h(\gamma)F^{-1}(1-\gamma)$$

(3) Critical mass (cont.)

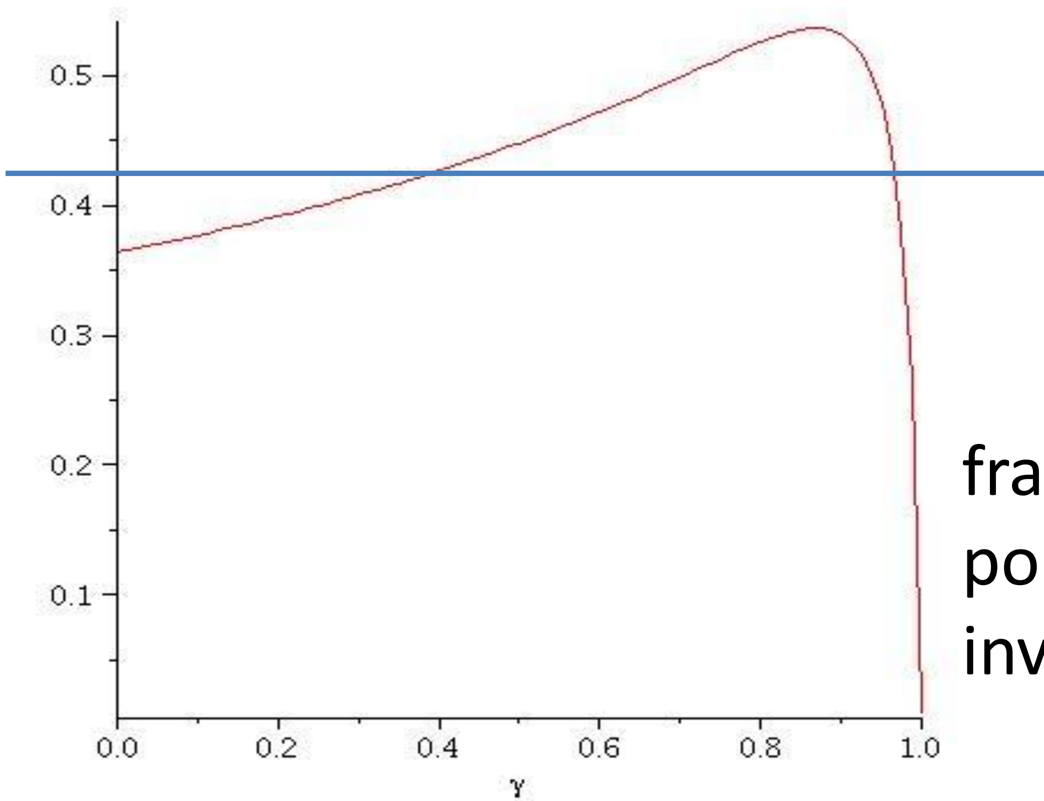
- Equilibria given by the fixed point equation



(3) Critical mass (cont.)

- If only one type: willingness to pay = network externalities function.

cost



fraction of population investing in security

γ

(3) Price of Anarchy

- The social welfare function:

$$W(\gamma) = \boxed{g(\gamma)\int_{\gamma}^{1} F^{-1}(1-u)du} + \boxed{(g(\gamma)+h(\gamma))\int_{0}^{\gamma} F^{-1}(1-u)du} - c\gamma,$$

where F is the c.d.f of types and:

$$\boxed{h(\gamma) = p^N(\gamma) - p^S(\gamma)}$$ Private externalities

$$\boxed{g(\gamma) = p^N(0) - p^N(\gamma).}$$ Public externalities

- Because of the public and private externalities, agent under-invest in security (in all cases).

Conclusion

- Simple single agent model: 1/e rule
  - General conditions for monotone investment
- Interconnected agents: network externalities function
  - General conditions to align incentives
- Equilibrium analysis of the security game
  - Critical mass, PoA
- Extensions: In this talk, agent is risk-neutral. What happens if risk-adverse? Insurance?

Thank you!

Feedbacks are welcome:

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