3^{rd} Millennium Computing

1. Binary Values

   Number, Text, Sound, Image, Measure, Command, Packet ...

2. Discrete Time

- Asynchronous: Neurons, Internet, Captors...
- Synchronous: Circuits, Airplanes, GPS...

3. Time Scales

- 1 **s** = 10^3 **ms** = 10^6 **μs** = 10^9 **ns** = 10^{12} **ps** = 10^{15} **fs** = 10^{18} **as**
- Heart -> Nerve -> Neuron -> μProc -> Circuit -> Molecule->Photon
- 1 *attosecond* =~ time for light to travel 3 hydrogen atoms
- 100 *attoseconds* =~ shortest time measured as of 2004=~ 16 decimal digits

	s/hour	s/day	s/week	s/year	ns/s	ns/day	ns/month	ns/year	ns/century
Log_{10} scale	4	5	6	7	9	14	15	16	18

Synchronous Circuits

1. Digital Algebra
   - Infinite synchronous binary sequences
   - Hacking Science
2. Boolean Normal Forms
   - Decision Diagrams
   - Integer Operations
3. Regular Normal Forms
   - Regular dimension
   - Minimal NDXA
4. Circuit Compiler
   - From C to NetList
   - From NetList to ASIC

Cookie: Moore's Law

Binary Integers

n	B^n_0	B^n_1	B^n_2	B^n_3	$B^n_{4\ldots}$	$[n]$	$\{k\varepsilon n\}$
0	0	0	0	0	0^ω	0^ω	{}
1	1	0	0	0	0^ω	10^ω	{0}
2	0	1	0	0	0^ω	010^ω	{1}
3	1	1	0	0	0^ω	110^ω	{0,1}
4	0	0	1	0	0^ω	0010^ω	{2}
5	1	0	1	0	0^ω	1010^ω	{0,2}
6	0	1	1	0	0^ω	0110^ω	{1,2}
7	1	1	1	0	0^ω	1110^ω	{0,1,2}
8	0	0	0	1	0^ω	00010^ω	{3}

n	B^n_0	B^n_1	B^n_2	B^n_3	$B^n_{4\ldots}$
-1	1	1	1	1	1^ω
-2	0	1	1	1	1^ω
-3	1	0	1	1	1^ω
-4	0	0	1	1	1^ω
-5	1	1	0	1	1^ω
-6	0	1	0	1	1^ω
-7	1	0	0	1	1^ω
-8	0	0	0	1	1^ω
-9	1	1	1	0	1^ω

$$\boxed{k \in n \Leftrightarrow 1 = B^n_k}$$

$$\boxed{n = \sum_k B^n_k 2^k = \sum_{k \in n} 2^k}$$

$$n \in \mathbb{N} \quad \Leftrightarrow \quad \nu(n) = \sum_{k \in \mathbb{N}} B^n_k = \left|\{k \in n\}\right| < \infty$$

$$\neg 0 = 1 + 2 + 4 + 8 + 16 + \cdots = -1$$

$$\boxed{\neg n = -(n+1)}$$

$$n \in \mathbb{Z} \quad \Leftrightarrow \quad \exists l, \forall k : k > l \Rightarrow B^n_k = B^n_l \quad \Leftrightarrow \quad \text{utimately constant.}$$

Magic Masks

B^n_0	B^n_1	B^n_2
1	1	1
0	1	1
1	0	1
0	0	1
1	1	0
-1/3	-1/5	-1/17

$$n = \sum_k B_k^n 2^k = \sum_{k \in n} 2^k$$

$$\begin{array}{lcl} \mu^0 & : & 10101010\cdots \\ +2\mu^0 & : & 01010101\cdots \\ =3\mu^0 & : & 11111111\cdots \\ 3\mu^0 & = & -1 \end{array}$$

$$\mu_0 = 1 + 4 + 16 + \cdots = -\frac{1}{3}$$

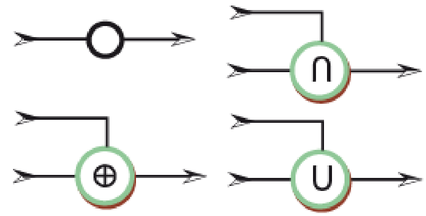
$$\boxed{\mu^k = \sum_n B_k^{\neg n} 2^n = \sum_{n \notin k} 2^n}$$

$$\boxed{\mu^k = (1^{2^k} 0^{2^k})^\infty}$$

$$\boxed{\mu^k = \frac{-1}{1 + 2^{2^k}}}$$

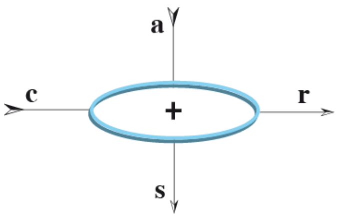
Synchronous *Gates*

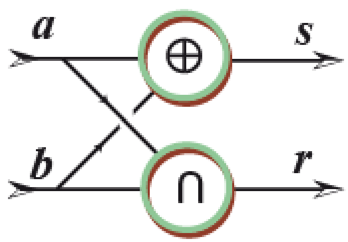
Boolean Gates



VDD

$$-1 = 1^{\omega} = [\mathbb{N} \to 1]$$

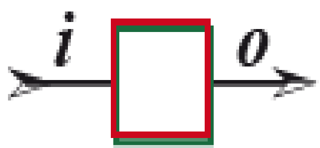




HalfAdder

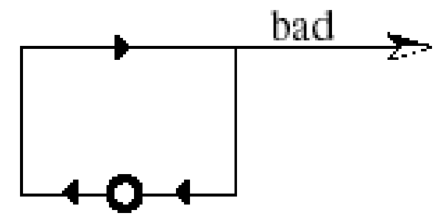
$$s_t = a_t \oplus b_t = a_t + b_t - 2a_t b_t$$
$$r_t = a_t \cap b_t = a_t b_t$$

Register



$$o_0 = 0, \; o_{t+1} = i_t$$

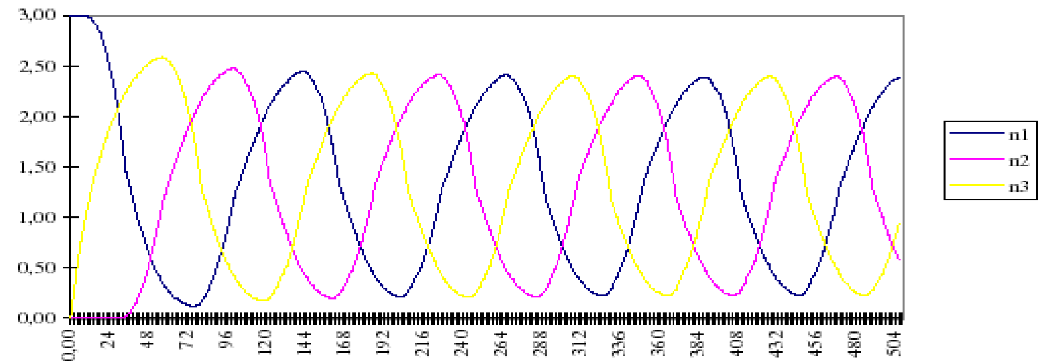
Feedback



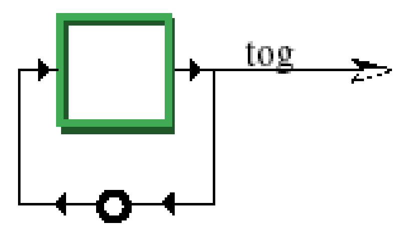
Problem: bad = ¬bad

$$bad_0 = 1 - bad_0$$

$$bad_0 = \frac{1}{2} \notin \{0,1\}$$



No problem: tog = REG(¬tog)



$$tog_0 = 0 \qquad tog_{2N} = 0$$

$$tog_{N+1} = 1 - tog_N \qquad tog_{2N+1} = 1$$

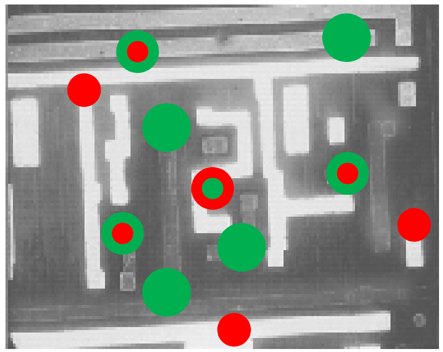
$$\boxed{\neg tog = \mu^0 = (10)^\infty = -1/3}$$

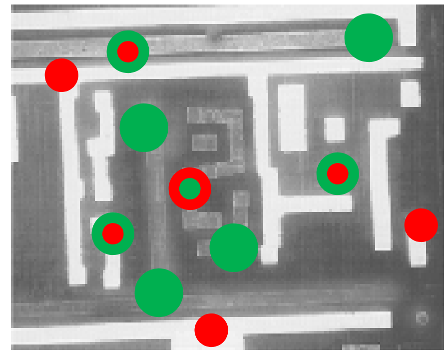
Solution: ***No*** combinational feedback !

Pulse Code modulation

Time=0 or even... **Time=1 or odd...**

EBM Images: 1GHz Circuit





10 µm

=> 4 Binary Transitions

$0 \rightarrow 0$ $\quad pcm(t) = 0$

$0 \rightarrow 1$ $\quad t < 1 \Rightarrow pcm(t) = 0$

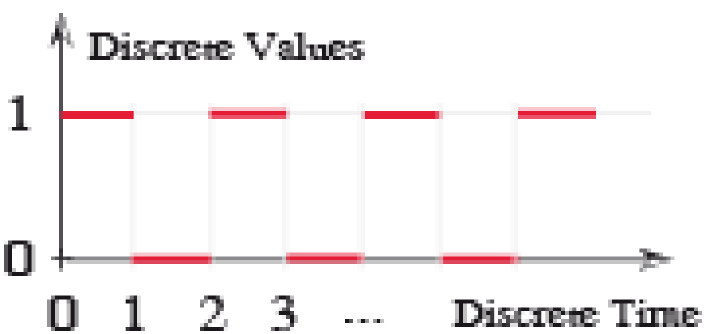
$1 = pcm(t) + pcm(t+1)$

$1 \rightarrow 1$ $\quad pcm(t) = 1$

$1 \rightarrow 0$ $\quad t < 1 \Rightarrow pcm(t) = 1$

$1 = pcm(t) + pcm(t+1)$

$pcm(t) = \lfloor t \rfloor \cap 1$



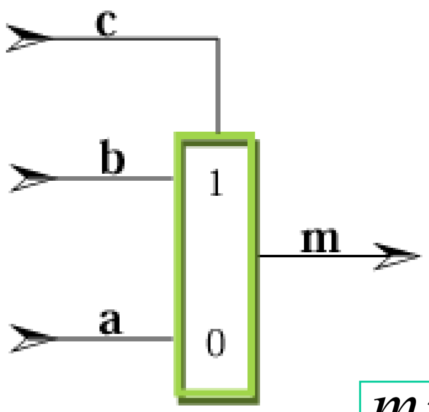
$$dirac(s)(t) = \sum_{n \in \mathbb{N}} \partial(n) s_n$$

$$pcm(s)(t) = \int_{\lfloor t \rfloor}^{\lfloor t \rfloor + 1} dirac(s)(x)dx$$

Digital Synchronous Circuit

- Syntax
  - Compose *<Vdd, And, Xor, Reg>*
  - Feed-forward Boolean Gates
  - Hierarchical Synthesis
- Semantics
  - Binary Values
    - Zero delay: *<In, Not, And, Xor, Reg>*
    - All variables have binary values at all times!
  - Discrete Time
    - All *Registers* have the very same unit delay.
    - Constant or variable time period.

Multiplexer



$$\boxed{\begin{aligned} m &= mux(c,b,a) \\ c_N = 0 &\Rightarrow m_N = a_N \\ c_N = 1 &\Rightarrow m_N = b_N \end{aligned}}$$

$$m = (c \cap b) \cup (\neg c \cap a)$$
$$m_N = c_N b_N + (1 - c_N) a_N$$
$$m = a \oplus (c \cap (a \oplus b))$$

$$\boxed{mux \in < \neg, \cap, \cup >}$$

$$\boxed{mux \in < \cap, \oplus >}$$

$$\neg x = mux(x, 0, \neg 0)$$
$$x \cap y = mux(x, y, x)$$
$$x \cup y = mux(x, x, y)$$

$$\boxed{\neg, \cap, \cup \in < 0, \neg 0, \mathrm{mux} >}$$

$$\boxed{f \in < 0, \overline{0}, x_1, \ldots, x_i, \mathrm{mux} >}$$

Shannon Decomposition

$$\boxed{\begin{aligned} f(x_1 x_2 \cdots x_i) &= (\neg x_1 \cap f(0 x_2 \cdots x_i)) \cup (x_1 \cap f(1 x_2 \cdots x_i)) \\ &= mux(x_1, f(1 x_2 \cdots x_i), f(0 x_2 \cdots x_i)) \\ &= (1 - x_1) f(0 x_2 \cdots x_i) + x_1 f(1 x_2 \cdots x_i) \end{aligned}}$$

Boolean Bases

$$< op_1,\ldots,op_k > \text{ is a Boolean Base } =: \quad \forall \mathrm{f} \in \mathbb{B}^n \to \mathbb{B} : \quad f(x_1,\ldots x_n) \in < x_1,\ldots,x_n,op_1,\ldots,op_k >$$

$$< 0, \neg 0, \mathrm{mux} >$$

$$< 0, \neg, \cap, \cup, \oplus >$$

$$< \neg, \cap, \cup >$$

$$0 = x \oplus x$$

$$a \oplus b = (a \cap \neg b) \cup (\neg a \cap b)$$

$$< x, \neg mux >$$

$$-1 \notin < \oplus, \cap, \cup > = < \oplus, \cap >$$

$$-1 = \neg 0 = {}_2 1^\infty = \frac{1}{1-z} = \sum_{n \in \mathbb{N}} 2^n$$

$$< -1, \cap, \oplus > = < -1, halfAdd >$$

$$\neg x = \neg 0 \oplus x$$

$$a \cup b = a \oplus b \oplus (a \cap b)$$

Technology Bases

$$\neg a = a \overline{\cap} a$$

$$a \cup b = (\neg a) \overline{\cap} (\neg b)$$

$$a \cap b = \neg(a \overline{\cap} b)$$

Nand $$a \overline{\cap} b = \neg(a \cap b)$$

Universal Gate $$\forall f : f(x_1, \ldots, x_n) < x_1, \ldots, x_n, \overline{\cap} >$$

Wolfram's Axiom $$c = ((a \overline{\cap} b) \overline{\cap} c) \overline{\cap} (a \overline{\cap} ((a \overline{\cap} c) \overline{\cap} a))$$

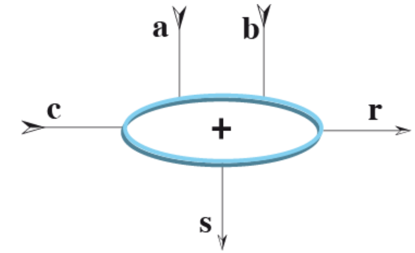
Nor $$a \overline{\cup} b = \neg(a \cup b)$$

ASIC Gates

Feedback Transitor networks: 4 states

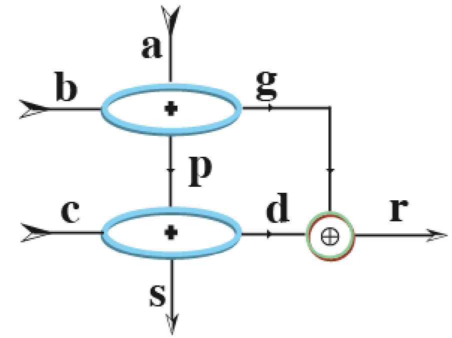
Full Adder

Icon



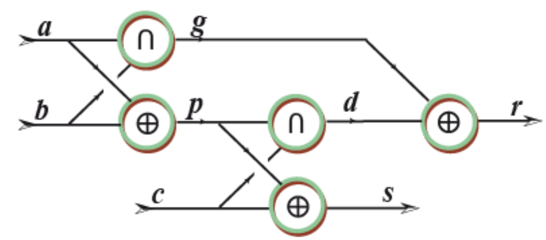
$$fullAdd(a,b,c)=(s,r)$$
$$\{\quad (p,g)=halfAdd(a,b)$$
$$(s,d)=halfAdd(c,g)$$
$$r=g\oplus d\quad \}$$

Definition



$$p=a\oplus b \quad g=a\cap b$$
$$s=c\oplus p \quad d=c\cap p$$
$$r=g\oplus d$$

Netlist



Invariant

$$\boxed{\begin{aligned} \forall t: S_t &= a_t+b_t+c_t=s_t+2r_t \\ S_t {\cdot}|{\cdot} 2 &= s_t=a_t\oplus b_t\oplus c_t \\ S_t \div 2 &= r_t=a_tb_t\oplus b_tc_t\oplus c_ta_t \end{aligned}}$$

$$\boxed{a+b+c=s+2r}$$

$$a+b=p+2g$$
$$c+p=s+2d$$
$$r=g+d$$

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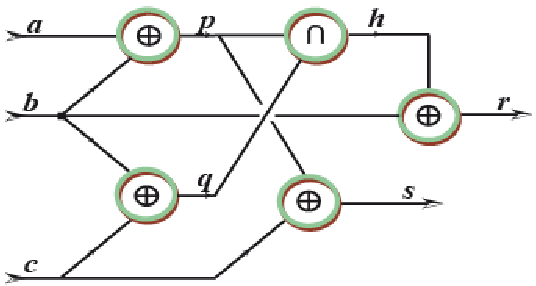
Circuit Simulations



$$p = a \oplus b \quad q = b \oplus c$$

$$s = c \oplus p \quad h = p \cap q$$

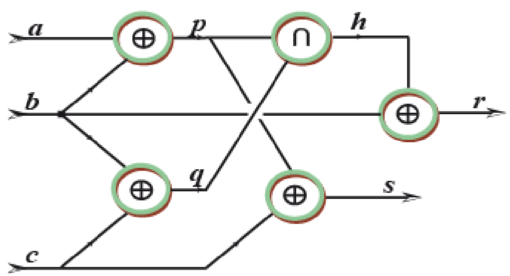
$$r = b \oplus h$$

a	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
b	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
c	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
p	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	1	1
q	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
s	0	0	0	1	1	1	1	0	0	0	0	0	1	1	1	1	1	0	0
h	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
r	0	0	0	0	0	0	0	0	1	1	1	0	0	1	1	1	1	1	1

- Stable Gates + Stable Composition yield Stable Circuit
- Delay Invariant Fixpoint – Binary Values
- Sufficient: Synchronous Clock Delay > Critical Path

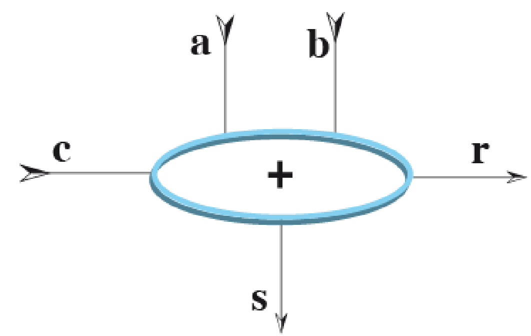
- *Gate Level Simulators: logical & electrical*
- *Event Driven Simulators: logical & electrical*

Computational Circuit Proofs

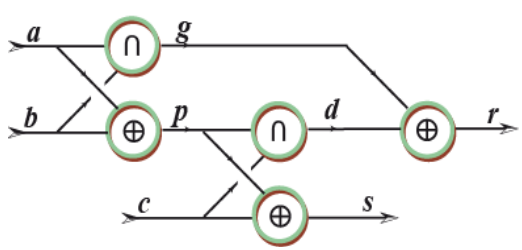


$$p = a \oplus b \quad q = b \oplus c$$
$$s = c \oplus p \quad h = p \cap q$$
$$r = b \oplus h$$

a	0	1	0	1	0	1	0	1
b	0	0	1	1	0	0	1	1
c	0	0	0	0	1	1	1	1
p	0	1	1	0	0	1	1	0
q	0	0	1	1	1	1	0	0
s	0	1	1	0	1	0	0	1
h	0	0	1	0	0	1	0	0
r	0	0	0	1	0	1	1	1
V	0	0	0	0	0	0	0	0



$$\boxed{a + b + c = s + 2r}$$



$$p = a \oplus b \quad g = a \cap b$$
$$s = c \oplus p \quad d = c \cap p$$
$$r = g \oplus d$$

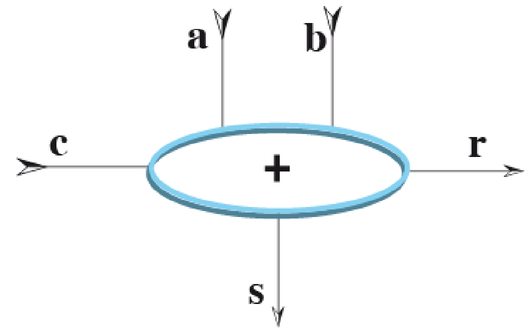
a	0	1	0	1	0	1	0	1
b	0	0	1	1	0	0	1	1
c	0	0	0	0	1	1	1	1
p	0	1	1	0	0	1	1	0
g	0	0	0	1	0	0	0	1
s	0	1	1	0	1	0	0	1
d	0	0	0	0	0	1	1	0
r	0	0	0	1	0	1	1	1
V	0	0	0	0	0	0	0	0

1. Topological Sort
2. Use word length 2^i computer
3. Assign magic masks to inputs
4. Evaluate gates in order - i.e.
5. Compute $g2^i$ Boolean results
6. Extract output truth-tables
7. Verify all invariants

Symbolic Circuit Proof

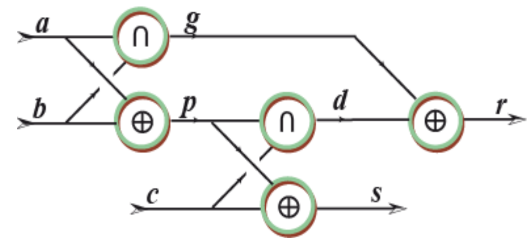
$$s = a \oplus b \oplus c$$
$$r = a \cap b \oplus b \cap c \oplus c \cap a$$

Specification



$$s = (a \oplus b) \oplus c$$
$$r = a \cap b \oplus (a \oplus b) \cap c$$

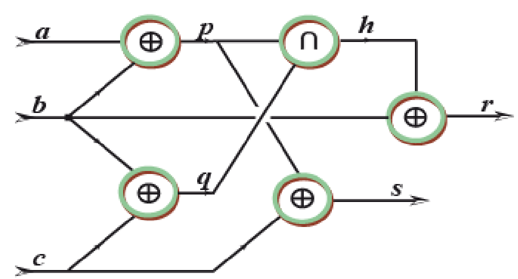
$$p = a \oplus b \quad g = a \cap b$$
$$s = c \oplus p \quad d = c \cap p$$
$$r = g \oplus d$$



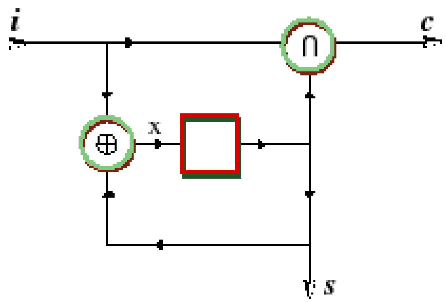
Reduce by sharing all equal variables

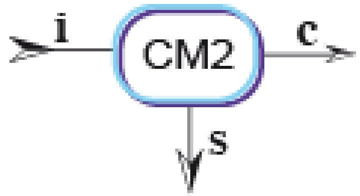
$$s = (a \oplus b) \oplus c$$
$$r = b \oplus (a \oplus b) \cap (b \oplus c)$$

$$p = a \oplus b \quad q = b \oplus c$$
$$s = c \oplus p \quad h = p \cap q$$
$$r = b \oplus h$$



Counter modulo 2





Icon

$$s = zx$$

$$x = i \oplus s$$

$$c = i \cap s$$

Netlist

$$cm2(i) = (s, c)$$

$$\{s = z(i \oplus s) \quad c = i \cap s\}$$

Specification

$$s_0 = 0 \quad s_{t+1} = x_t$$

$$x_t = i_t \oplus s_t$$

$$c_t = i_t s_t$$

Behaviour

$$x(z) = i(z) + zx(z) = \frac{i(z)}{1-z} \pmod 2$$

$$x_t = \sum_{k \le t} i_k \bmod 2$$

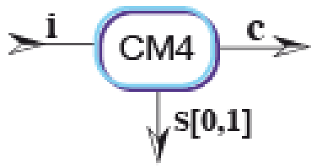
$$\forall t \in \mathbb{N} : \sum_{k<t} i_k = s_t + 2 \sum_{k<t} c_k$$

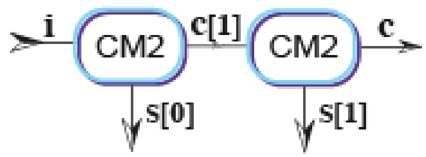
Invariants

s	0	1	0	1
i	0	0	1	1
x	0	1	1	0
c	0	0	0	1

State Transitions Truth Table

Counter modulo 4





Specification

Icon

$$\color{red}{cm4}(i)=(s[0,1],c)$$

$$\{(s[0],c[1])=\color{blue}{cm2}(i,s[0]) \quad (s[1],c)=\color{blue}{cm2}(c[1],s[1])\}$$

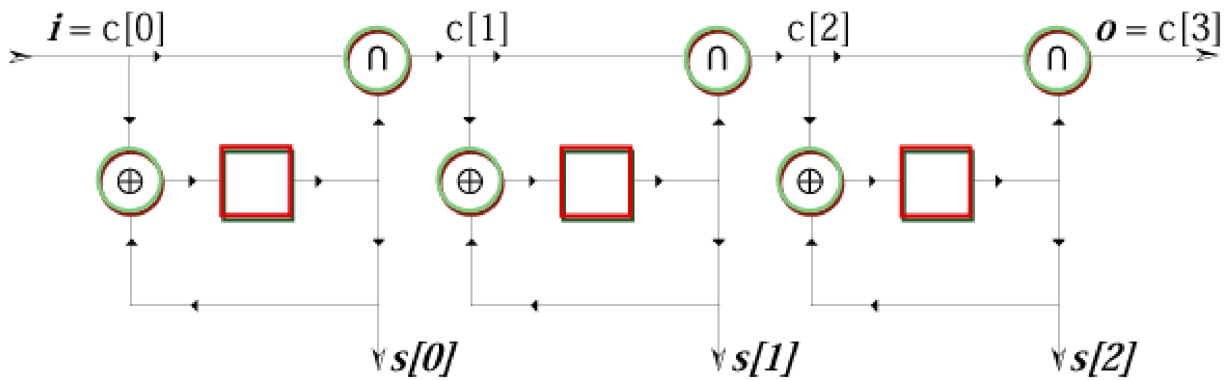
Netlist

$$\boxed{\begin{array}{lll} c[0]=i & s[0]=zx[0] & s[1]=zx[1] \\ & x[0]=c[0]\oplus s[0] & x[1]=c[1]\oplus s[1] \\ c=c[2] & c[1]=x[0]\cap s[0] & c[2]=x[1]\cap s[1] \end{array}}$$

Invariant

$$\forall t\in\mathbb{N}: \sum_{k<t} i_k = s_t[0]+2s_t[1]+4\sum_{k<t} c_k$$

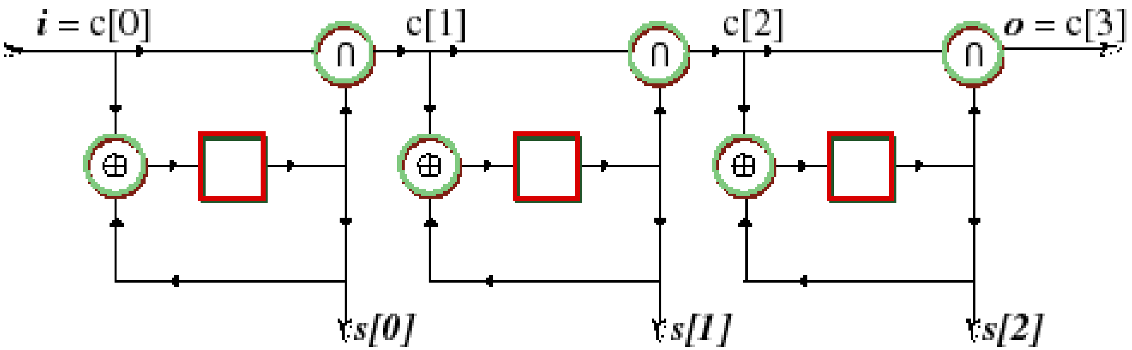
Binary n-bit Counter




```
// Binary n bit counter
Counter(n:int)(incr:net)= (s:net[n], ovfl:net)
{  c[0]=incr;   // carry in
   ovfl=c[n];   // carry out
   for (k<n)    (s[k] , c[k+1] ) = cm2(c[k]);  }
```

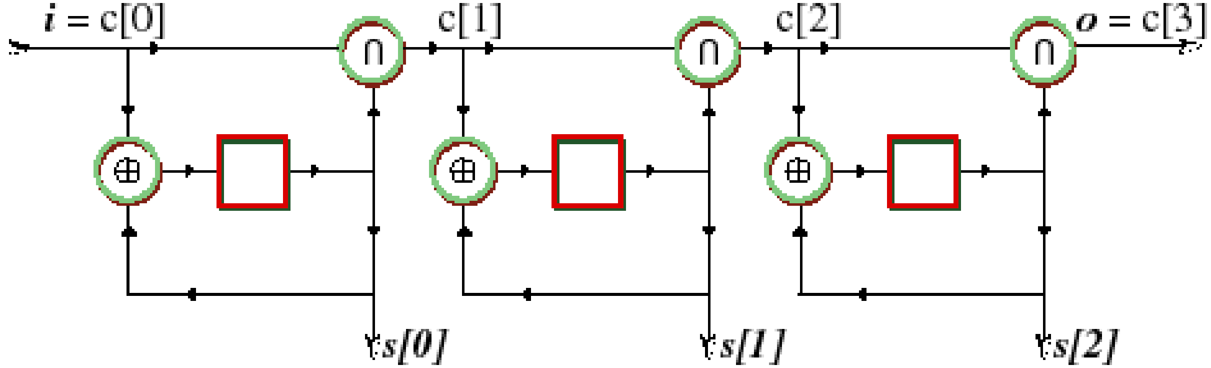
Invariant $\forall t \in \mathbb{N}: \sum_{k<t} i_k = \sum_{k<t} s_t[k]2^k + 2^n \sum_{k<t} c_k$

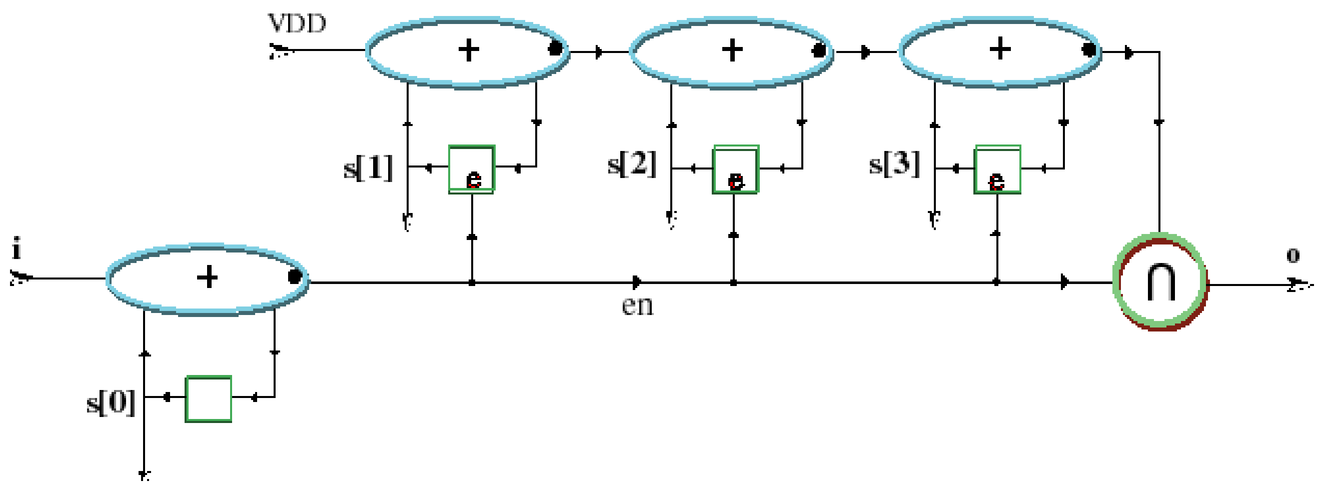
Binary Counter



cycle	i = c[0]	s[0]	s[1]	s[2]	c[1]	c[2]	c[3]
0	1	0	0	0	0	0	0
1	1	1	0	0	1	0	0
2	1	0	1	0	0	0	0
3	1	1	1	0	1	1	0
4	1	0	0	1	0	0	0
5	1	1	0	1	1	0	0
6	1	0	1	1	0	0	0
7	1	1	1	1	1	1	1
8	1	0	0	0	0	0	0

Fast Silicon Counter





Memory-less Circuits

- Boolean Gates
- No feedback
- Topological order
- Reduced

- Automatic Verification
- Automatic Synthesis
- Automatic Optimization
- All symbolic computations

Boolean Normal Forms:

$$mux(x,a,b) = x \cap a \oplus \neg x \cap a$$

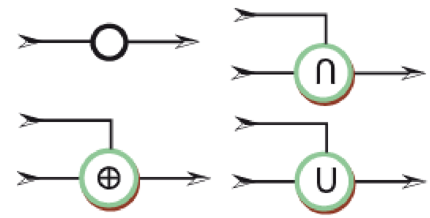
$BDD \quad < 0, \neg 0, mux >$ where x is an input variable.

Input order $x_1, \ldots, x_n$

$$lin(a,x,c) = a \oplus x \cap c$$

$BMD \quad < \neg 0, lin >$ where x is an input variable.

Logic Gates



$$\neg a = \left[t \to (1 - B_t^a) \right]$$
$$a \cap b = \left[t \to (B_t^a \times B_t^b) \right]$$
$$a \cup b = \left[t \to (B_t^a + B_t^b - B_t^a \times B_t^b) \right]$$
$$a \oplus b = \left[t \to (B_t^a + B_t^b - 2\, B_t^a \times B_t^b) \right]$$

Arithmetic Definition

$$\neg a = \{t \notin a\}$$
$$a \cap b = \{t \in a\} \cap \{t \in b\}$$
$$a \cup b = \{t \in a\} \cup \{t \in b\}$$
$$a \oplus b = \{t \notin a \cap b\} \cap (\{t \in a\} \cup \{t \in b\})$$

Set Definition

n	B^n_0	B^n_1	B^n_2	B^n_3	$B^n_{4\ldots}$	*w[n]*	*{kεn}*
0	0	0	0	0	0^ω	0^ω	{}
1	1	0	0	0	0^ω	ε	{0}
2	0	1	0	0	0^ω	0	{1}
3	1	1	0	0	0^ω	1	{0,1}
4	0	0	1	0	0^ω	00	{2}
5	1	0	1	0	0^ω	10	{0,2}
6	0	1	1	0	0^ω	01	{1,2}
7	1	1	1	0	0^ω	11	{0,1,2}
8	0	0	0	1	0^ω	000	{3}

$n \in \mathbb{D}$:

$n = [t \to B_t^n]$

$\{t \in n\} = \{t \in \mathbb{N}\text{: } 1 = B_t^n\}$

binary sequence $\mathbb{N} \to \mathbb{B}$

integer subset $2^{\mathbb{N}}$

$k \in a \quad \Leftrightarrow \quad B_k^a = 1$

$\min\{n\text{: } 5 \in n\} = ?$

$a \subseteq b \quad \Leftrightarrow \quad \{k \in a\} \subseteq \{k \in b\} \quad \Leftrightarrow \quad a = a \cap b$

$\forall a, b \in \mathbb{N} \qquad (a \subset b) \Rightarrow (a < b)$

subset order

Can we replace $\mathbb{N}$ by $\mathbb{Z}$, $\mathbb{P}$?

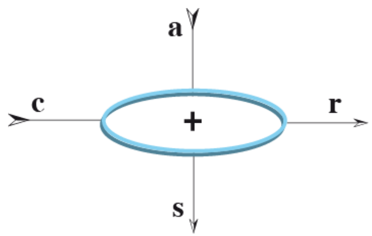
$\min\{n\text{: } k \in n\} = 2^k$

Binary Field

$$\mathbb{F}_2 = \langle 0 \quad 1 \quad \oplus \quad \cap \rangle$$

$\oplus$	0	1
0	0	1
1	1	0

$\cap$	0	1
0	0	0
1	0	1



$$a \cap b = a \times b$$
$$a \oplus b = a + b - 2ab$$

$$\neg a = \neg 0 \oplus a$$
$$a \cup b = a \oplus b \oplus a \cap b$$

	$\neg$
0	1
1	0

$\cup$	0	1
0	0	1
1	1	1

$\mathbb{F}_2$ is a Boolean $\begin{matrix} 0 = a \oplus a \\ a = a \cap a \end{matrix}$ Ring.

$\mathbb{F}_2$ *is* the one and only Boolean Ring which is a Field: 1=1/1.

A size 2 Boolean Algebra is isomorphic to $\mathbb{F}_2 = \langle 0 \quad 1 \quad \neg \quad \cap \quad \cup \rangle$.

Boolean Algebra

$n \in \mathbb{D}$:

$n = [t \to B_t^n]$

$\{t \in n\} = \{t \in \mathbb{N} : 1 = B_t^n\}$

binary sequence $\mathbb{N} \to \mathbb{B}$

integer subset $2^{\mathbb{N}}$

Boolean Algebra : $\langle \mathbb{D} \quad \neg \quad \cap \quad \cup \rangle$

De Morgan

$$\begin{array}{rcl} \neg(a \cup b) & = & \neg a \cap \neg b \\ \neg(a \cap b) & = & \neg a \cup \neg b \end{array}$$

$$\begin{array}{rcl} a \cap b & = & b \cap a \\ a \cap (b \cap c) & = & (a \cap b) \cap c \\ a \cap \neg 0 & = & a \\ a \cap \neg a & = & 0 \\ a \cap (b \cup c) & = & (a \cap b) \cup (a \cap c) \end{array}$$

$$\begin{array}{rcl} a \cup b & = & b \cup a \\ a \cup (b \cup c) & = & (a \cup b) \cup c \\ a \cup 0 & = & a \\ a \cup \neg a & = & \neg 0 \\ a \cup (b \cap c) & = & (a \cup b) \cap (a \cup c) \end{array}$$

Boolean Ring: $\langle \mathbb{D} \quad \oplus \quad \cap \rangle$

$$a \oplus b = (\neg a \cap b) \cup (a \cap \neg b)$$

$$a \cup b = a \oplus b \oplus (a \cap b)$$

$$\neg a = a \oplus -1$$

$$\begin{array}{rcl} a \cap b & = & b \cap a \\ a \cap (b \cap c) & = & (a \cap b) \cap c \\ a \cap \neg 0 & = & a \\ a \cap 0 & = & 0 \\ a \cap (b \oplus c) & = & (a \cap b) \oplus (a \cap c) \end{array}$$

$$\begin{array}{rcl} a \oplus b & = & b \oplus a \\ a \oplus (b \oplus c) & = & (a \oplus b) \oplus c \\ a \oplus 0 & = & a \\ a \oplus a & = & 0 \\ a \cap a & = & a \end{array}$$

Set Lattice

$$x \subseteq y \quad \Leftrightarrow \quad \forall N : B_N^x \leq B_N^y \quad \Leftrightarrow \quad \varnothing = x \cap \neg y$$

$$x \to y \quad \Leftrightarrow \quad x \subset y \quad \cap \quad \forall z : x \subset z \Rightarrow y \subseteq z$$

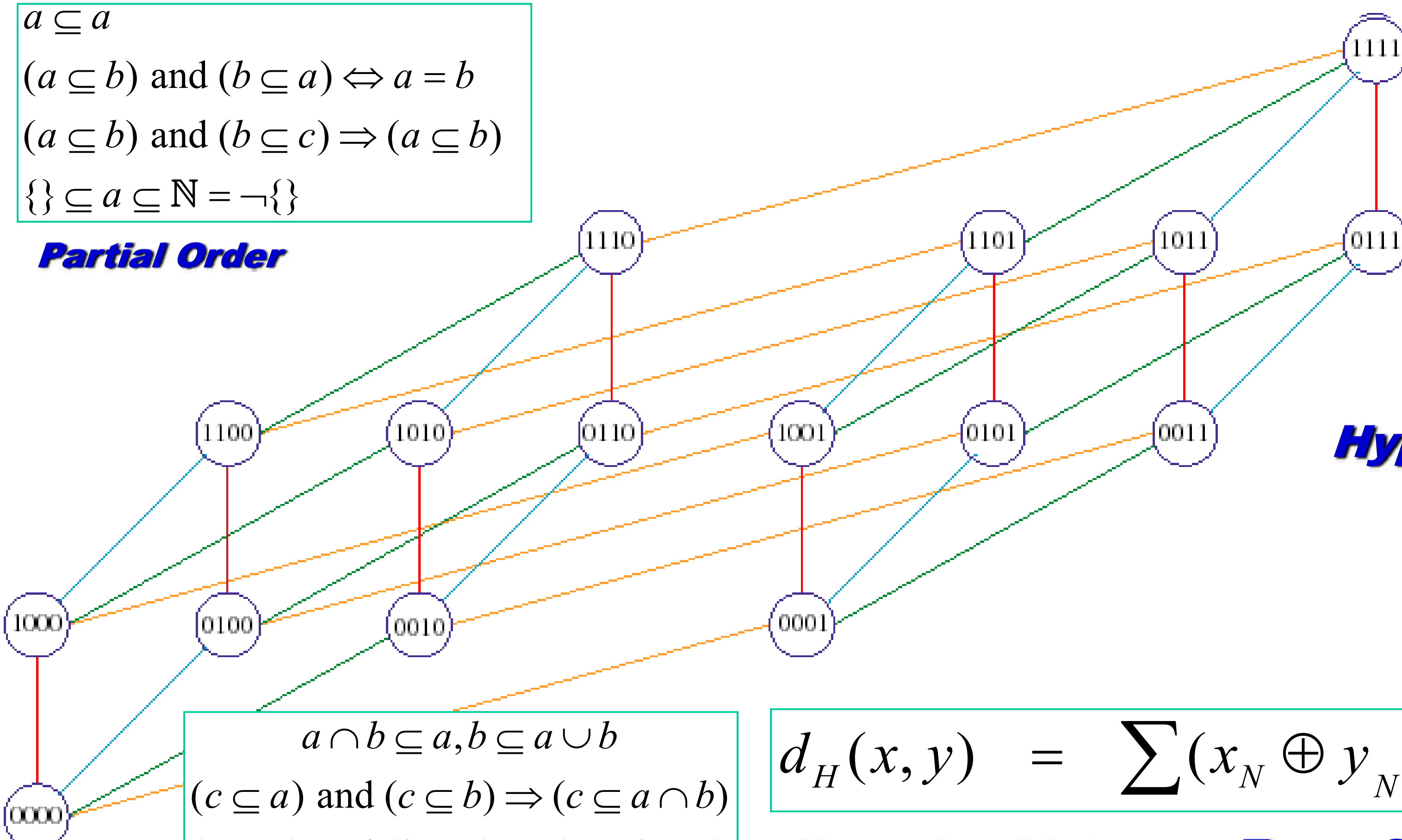
$a \subseteq a$

$(a \subseteq b)$ and $(b \subseteq a) \Leftrightarrow a = b$

$(a \subseteq b)$ and $(b \subseteq c) \Rightarrow (a \subseteq b)$

$\{\} \subseteq a \subseteq \mathbb{N} = \neg\{\}$

Partial Order



Hypercube H4

$a \cap b \subseteq a, b \subseteq a \cup b$

$(c \subseteq a)$ and $(c \subseteq b) \Rightarrow (c \subseteq a \cap b)$

$(a \subseteq c)$ and $(b \subseteq c) \Rightarrow (a \cup b \subseteq c)$

$$d_H(x, y) \quad = \quad \sum (x_N \oplus y_N)$$

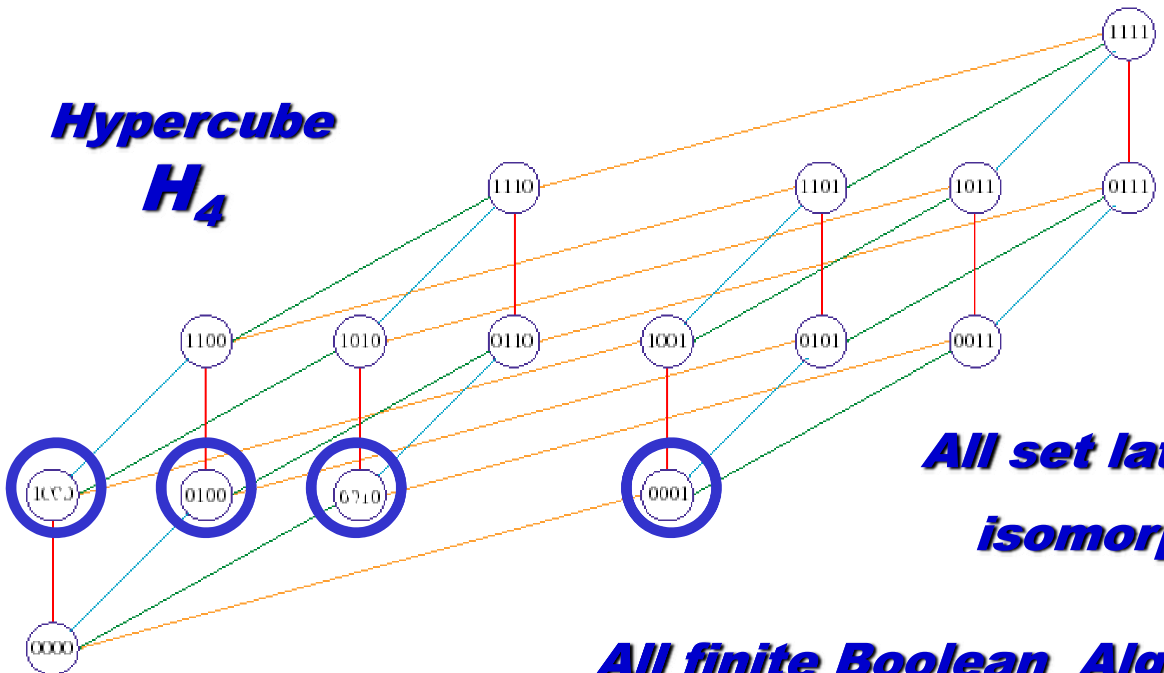
Hamming Distance ***Error Control***

Finite Set Lattices

$$a \cap b \subseteq a, b \subseteq a \cup b$$
$$(c \subseteq a) \text{ and } (c \subseteq b) \Rightarrow (c \subseteq a \cap b)$$
$$(a \subseteq c) \text{ and } (b \subseteq c) \Rightarrow (a \cup b \subseteq c)$$

Atoms $\forall x: \; x \subset a \quad \Leftrightarrow \quad x = 0$

Hypercube H_4



All set lattices with n atoms are isomorphic to Hypercube H_n

All finite Boolean Algebra have 2^n elements. Each (of these n!) is isomorphic to B^n.