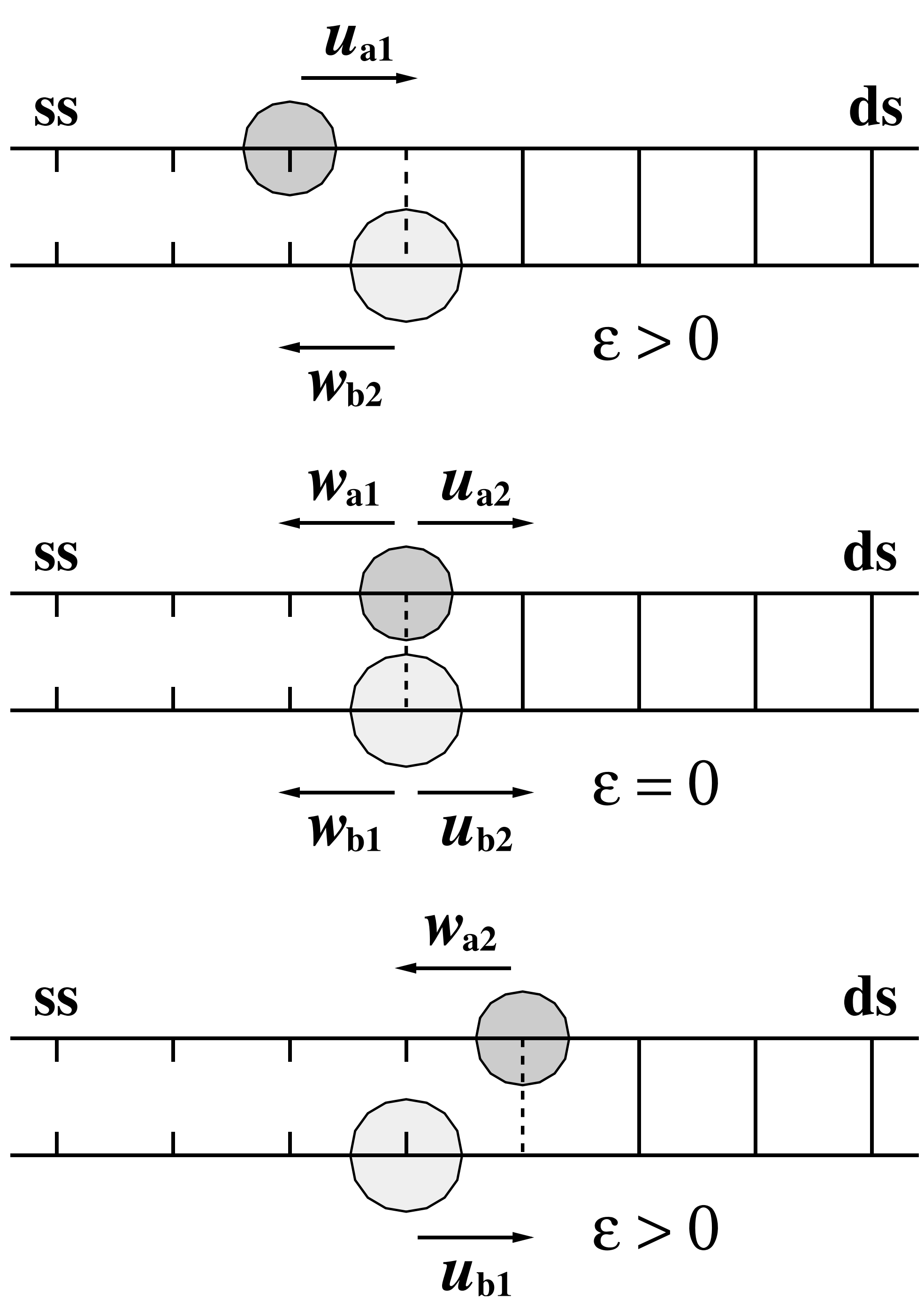
the Stukalin example (DNA walker)

1. a tale of two engines
   - ring assembly (combat small rings)
   - symmetric motors
   - directional
   - challenge: mean velocity > individual one

2. interested in replacing/scaling micro-CTMC to SDEs ...
3. detailed balance
4. translational invariance

PRL 2005 – Stukalin et al.



$$
\begin{tikzcd}
 & x, x+1 & \\
x-1, x & x, x \arrow[l, "w_{a1}/u_{a1}"'] \arrow[r, "u_{a2}/w_{a2}"] \arrow[u, "u_{b2}/w_{b2}"] \arrow[d, "w_{b1}/u_{b1}"] & x+1, x \\
 & x, x-1 &
\end{tikzcd}
$$

$$
\begin{tikzcd}
x, x+1 \arrow[r, "u_{a1}"] & x+1, x+1 \arrow[d, "w_{b1}"] \\
x, x \arrow[u, "u_{b2}"] & x+1, x \arrow[l, "w_{a2}"]
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
x, x+1 \arrow[d, "w_{b2}"'] & x+1, x+1 \arrow[l, "w_{a1}"'] \\
x, x \arrow[r, "u_{a2}"'] & x+1, x \arrow[u, "u_{b1}"']
\end{tikzcd}
$$

$$
\log(w_{b2}/u_{b2}) + \log(w_{a1}/u_{a1}) \quad = \quad \log(w_{a2}/u_{a2}) + \log(w_{b1}/u_{b1})
$$

$$\begin{aligned} u_{i1}/w_{i1} &= (u_i/w_i)\,\gamma \\ u_{i2}/w_{i2} &= (u_i/w_i)\,\gamma^{-1} \end{aligned}$$

$$\gamma = e^{\beta\epsilon}$$

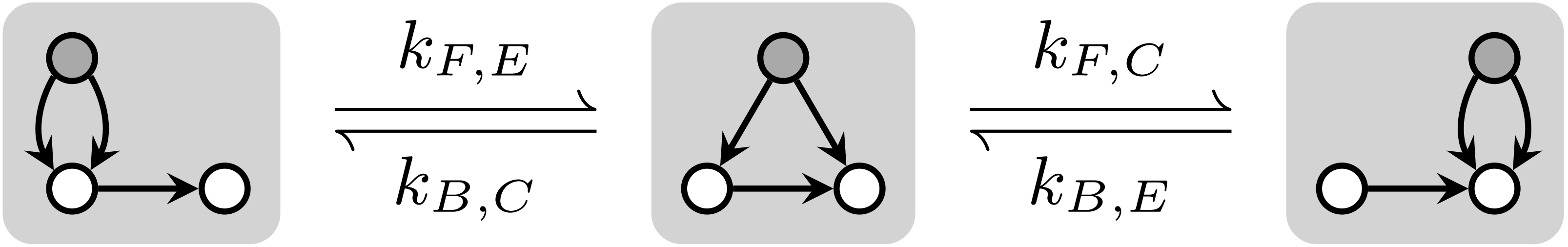
$$\begin{aligned} &\log(u_{a1}/w_{a1}) = \beta\epsilon + \log(u_a/w_a) \\ &\log(w_{a2}/u_{a2}) = \beta\epsilon - \log(u_a/w_a) \\ &\log(u_{b1}/w_{b1}) = \beta\epsilon + \log(u_b/w_b) \\ &\log(w_{b2}/u_{b2}) = \beta\epsilon - \log(u_b/w_b) \end{aligned}$$

$$\beta\epsilon - \log(u_b/w_b) - \beta\epsilon - \log(u_a/w_a) = \beta\epsilon - \log(u_a/w_a) - \beta\epsilon - \log(u_b/w_b)$$

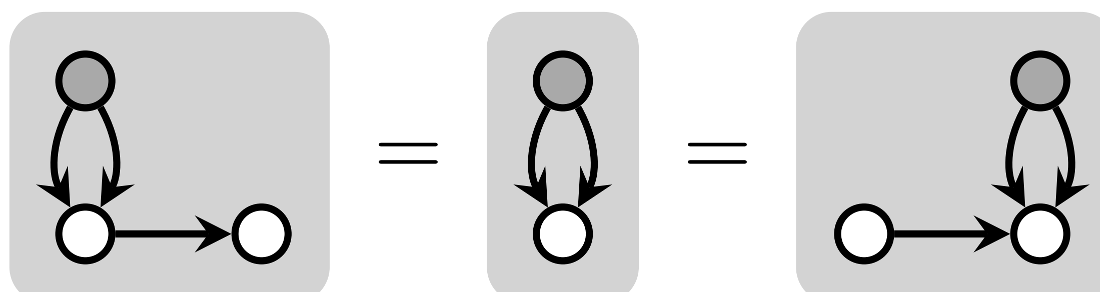
$$
\begin{tikzcd}
 & \beta\epsilon - \log(u_b/w_b) & \\
\beta\epsilon + \log(u_a/w_a) & \bullet \arrow[l] \arrow[r] \arrow[u] \arrow[d] & \beta\epsilon - \log(u_a/w_a) \\
 & \beta\epsilon + \log(u_b/w_b) &
\end{tikzcd}
$$

$$\tau := \log(u_b/w_b) + \log(u_a/w_a) = 0$$

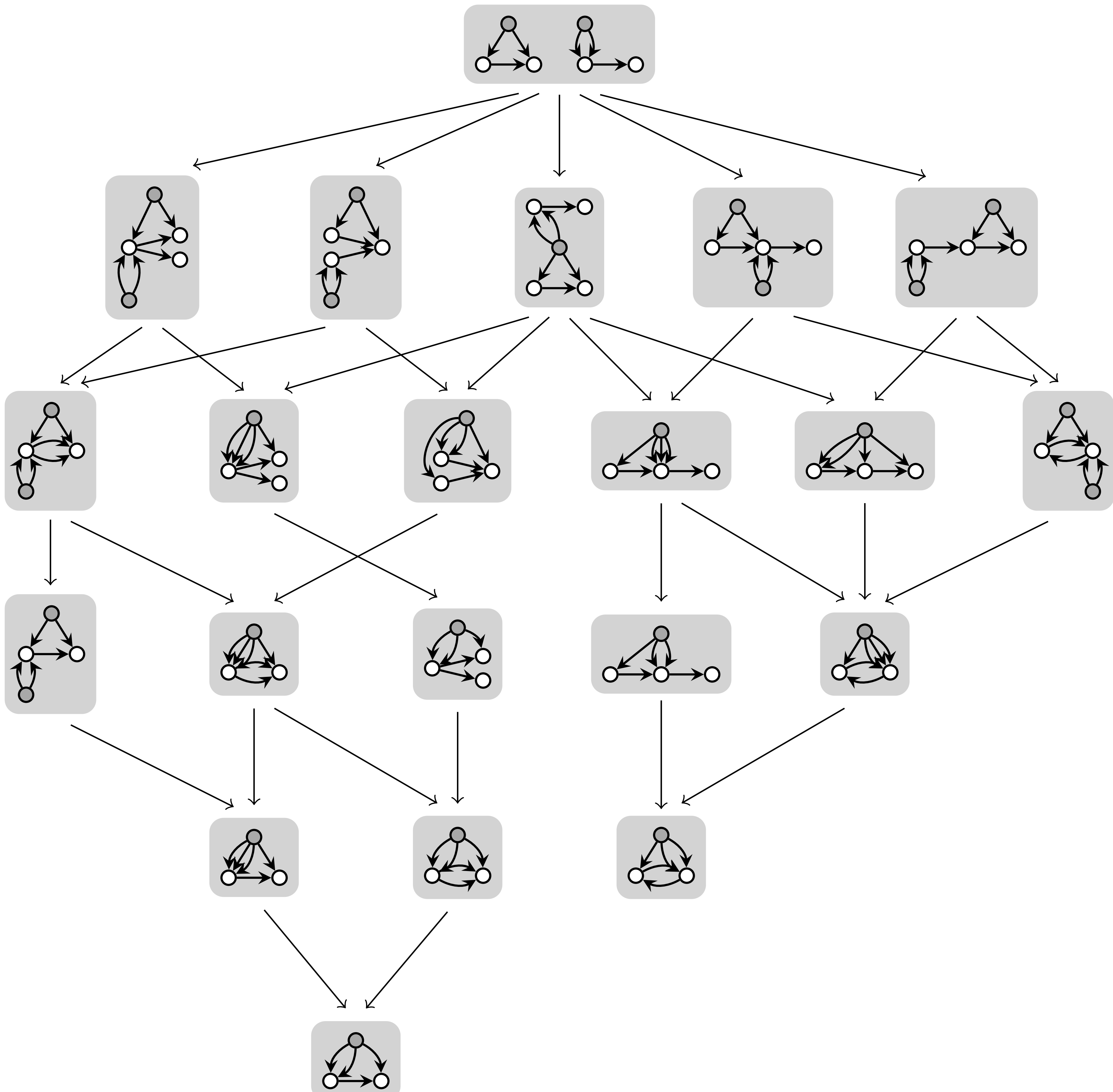
PRL 2005 – Stukalin et al.



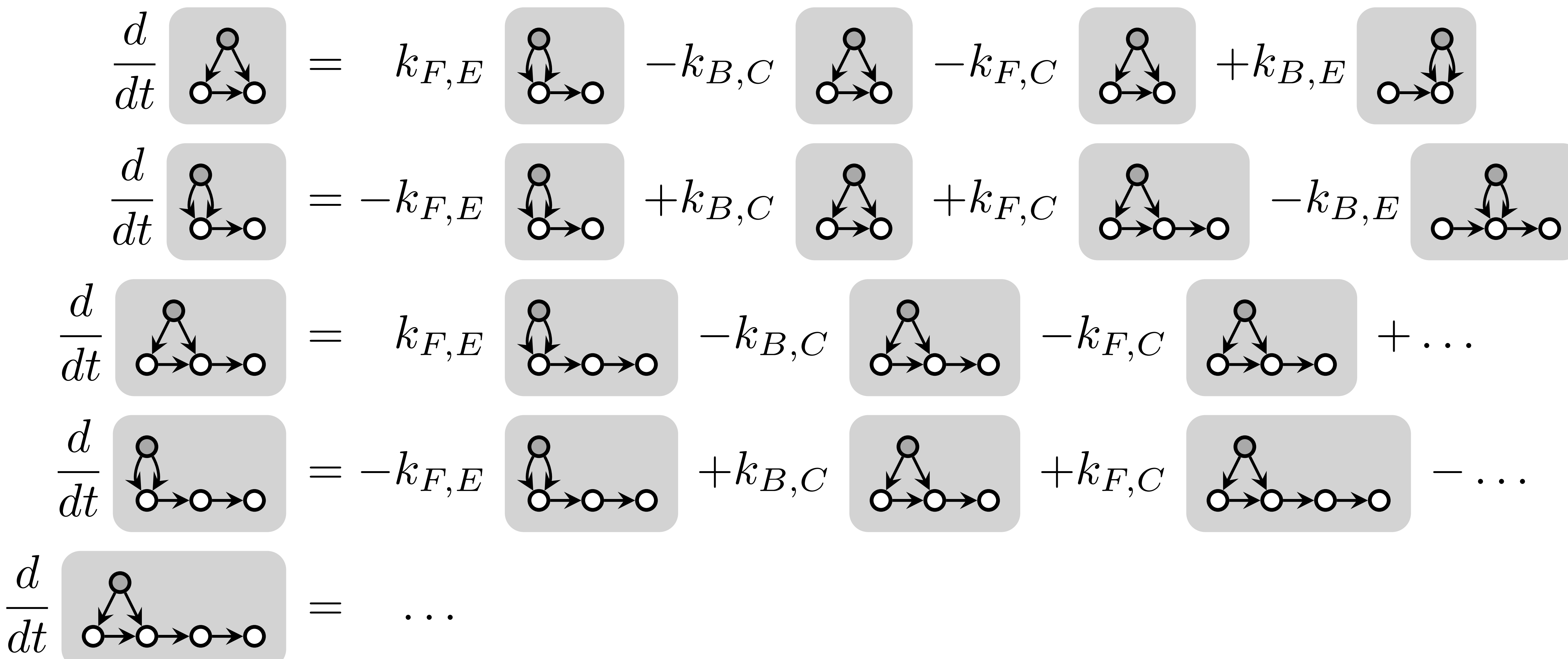
no- or cyclic-boundary invariants



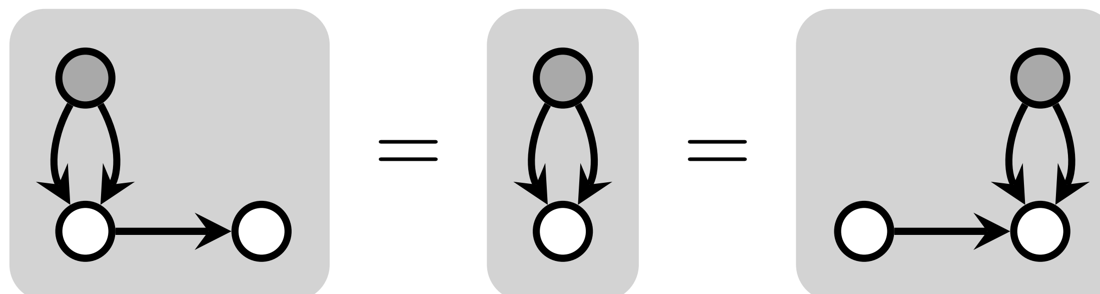
minimal gluings/minimal unions



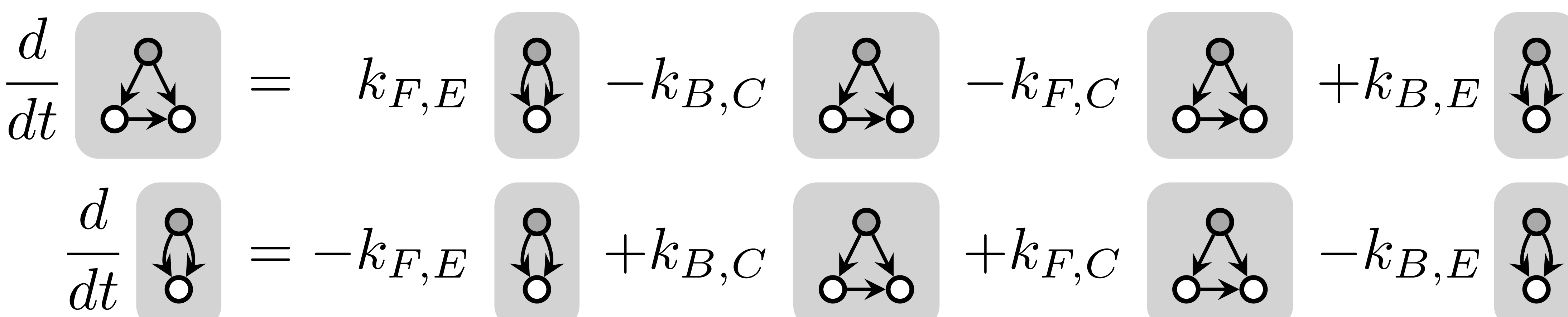
infinite ODE



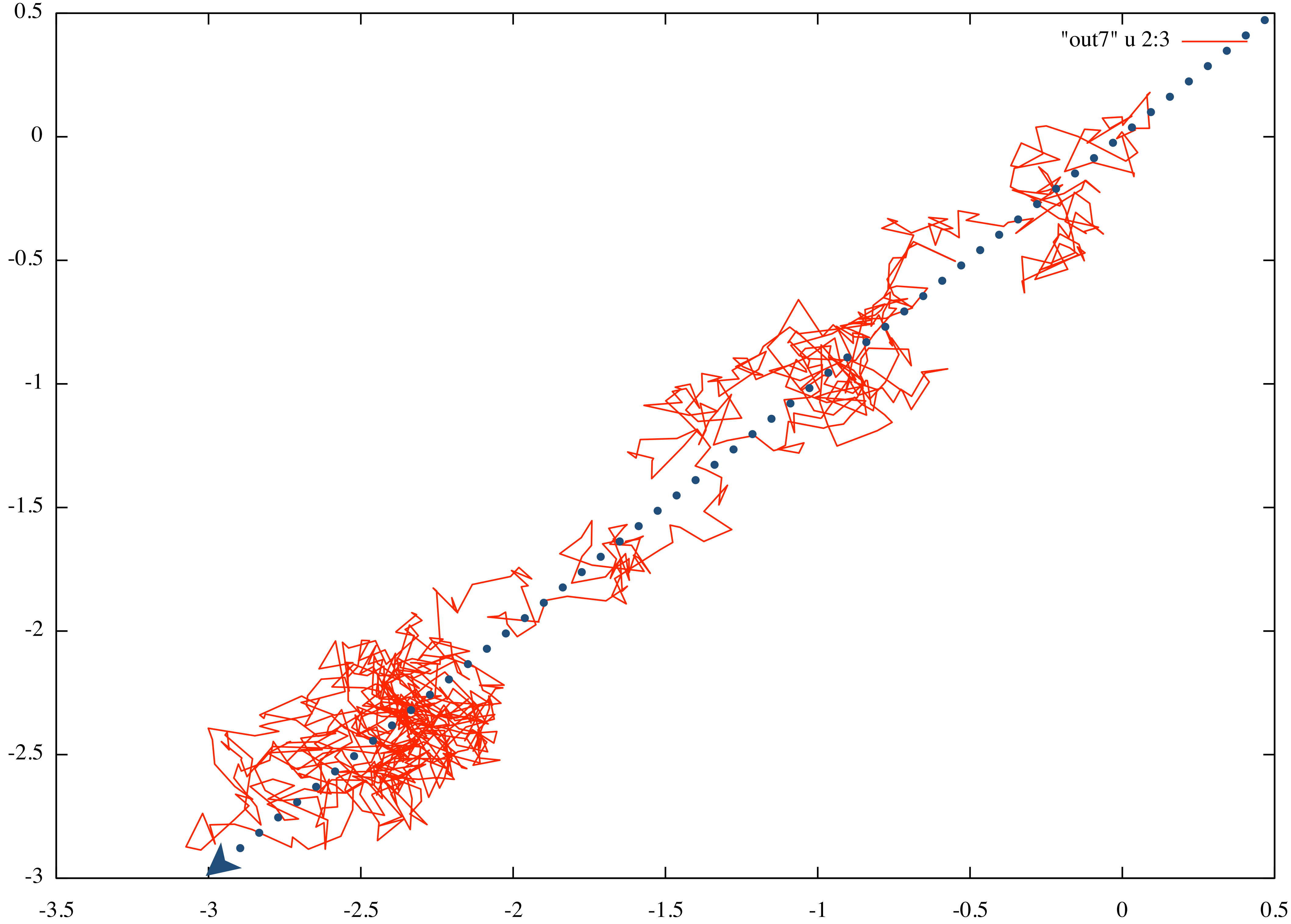
no- or cyclic-boundary invariants



The Stukalin MFA equations



same thing with SDEs?



diffusion limits – here a 2d OU

$$dX_t = -\theta(X_t - \mu)dt + \sigma\, dB_t$$

$$\begin{aligned} dX_1 &= -\theta_1(X_1 - \mu_1)dt + dB_t^1 \\ dX_2 &= -\theta_2(X_2 - \mu_2)dt + dB_t^2 \end{aligned}$$

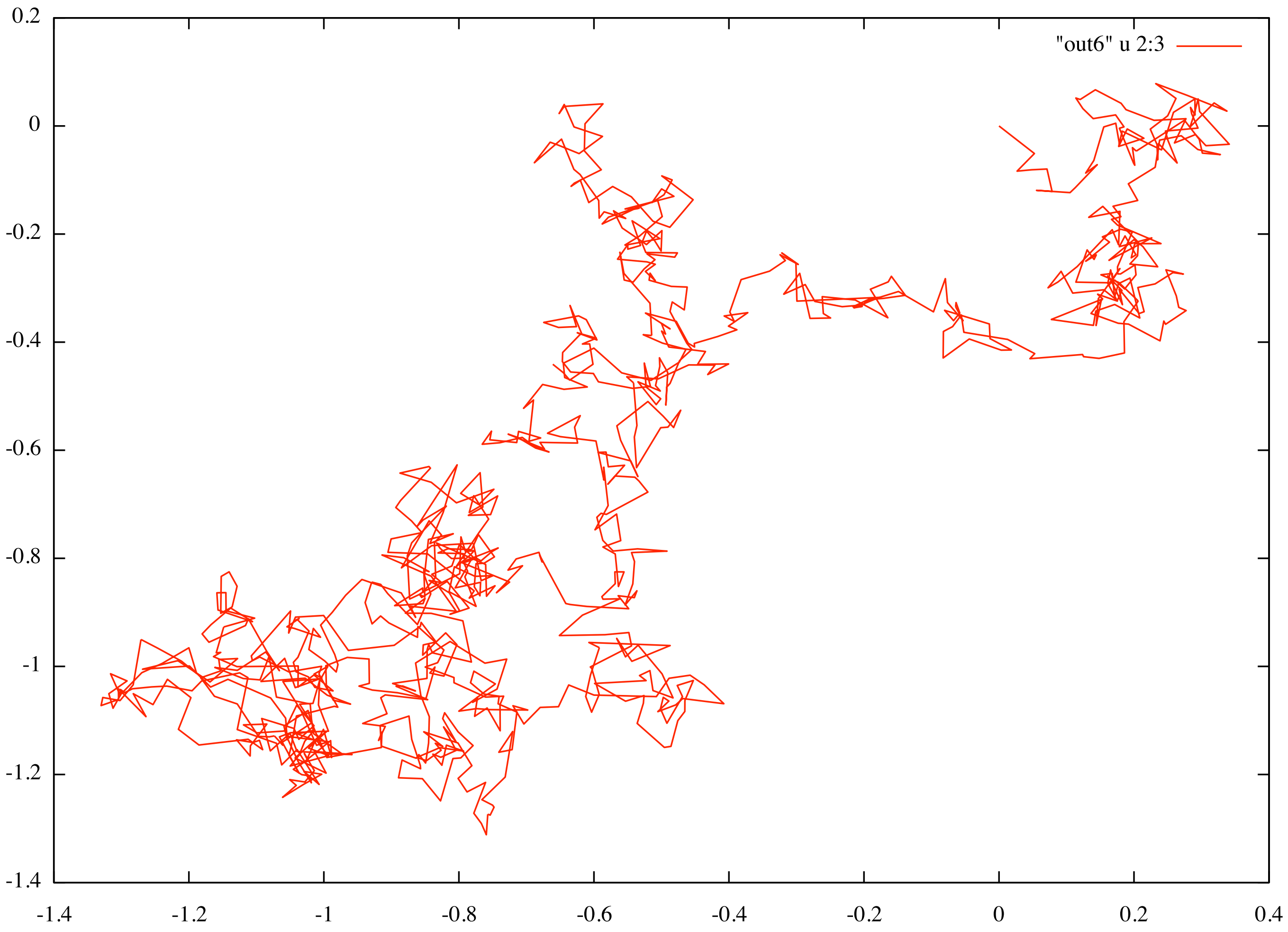


Figure 2: The OU SDE above, starting at 0, 0 and run with $\mu_1 = \mu_2 = 0$, $\theta_1 = \theta_2 = 1.0$, $\Sigma = I$, $e = 1000$, $dt = 0.01$

$$dX_t = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} dt + \begin{pmatrix} -\theta_1 & \theta_1 \\ \theta_2 & -\theta_2 \end{pmatrix} X_t \, dt + \Sigma \, dB_t$$

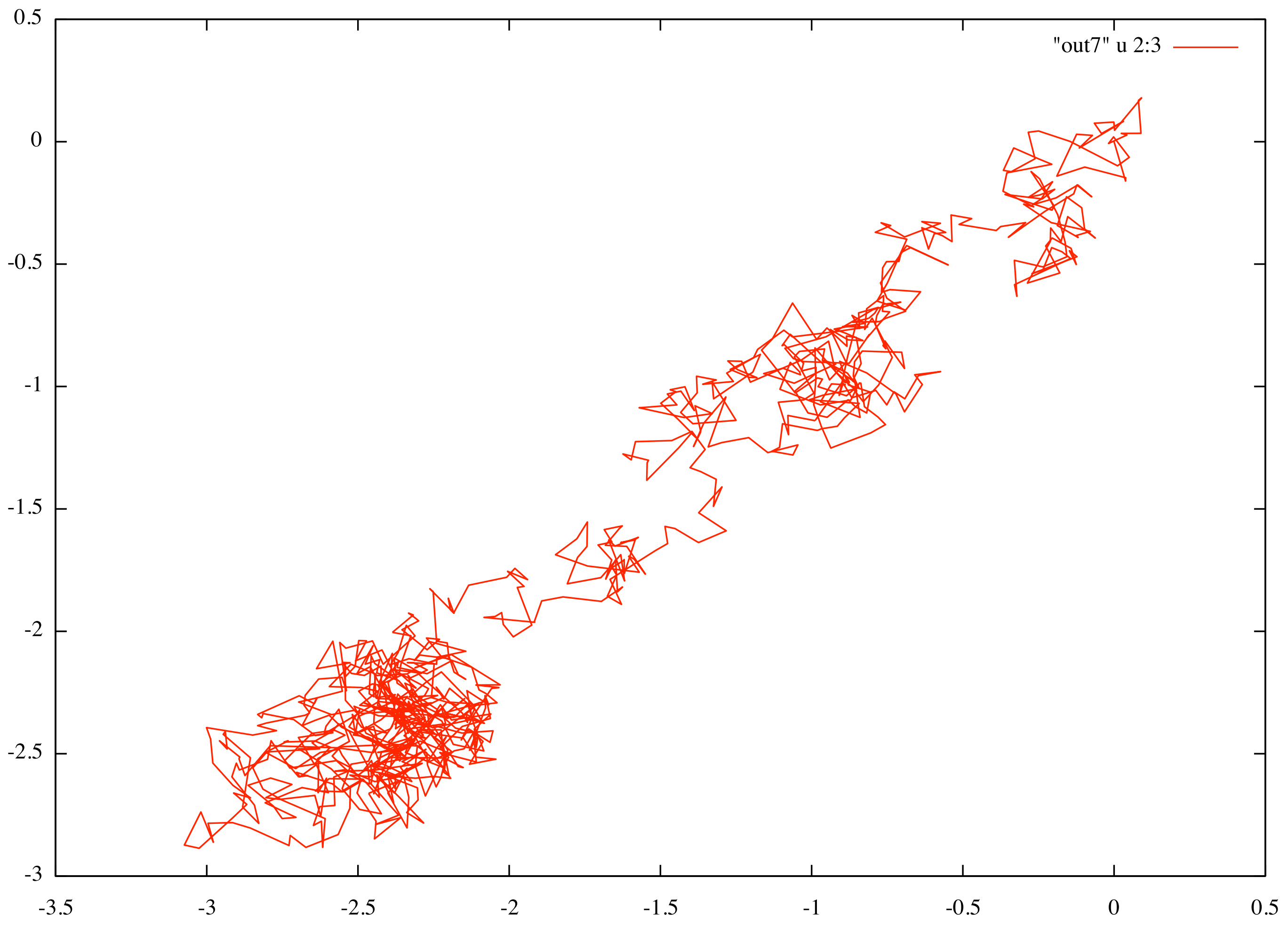


Figure 3: The OU SDE above, starting at 0, 0 and run with $\mu_1 = \mu_2 = 0$, $\theta_1 = \theta_2 = 1.0$, $\Sigma = I$, $e = 1000$, $dt = 0.05$; the concentration on $y = x$ is noticeable on this longer time.