Third Homework Exercise

January 26, 2022

Please type your answers using the Latex. Please hand in your assignment (by email or by hard copy) before the class of January 27, 2022.

Please feel free to contact me if you have questions.

Exercise 1

In class we presented a PTAS for the problem of makespan minimisation in the identical machine setting. Suppose that now the number of the machines is constant. Design an FPTAS. (Remember, the running time has to be a polynomial of n *and* $1/\epsilon$).

Exercise 2

Given a graph $G = (V, E)$, with edge cost $c : E \to \mathbb{R}_+$, you choose a subset $X \subset V$ where $|X| \leq k$ with the objective of minimize

$$\max_{v \in V} \min_{x \in X} dist(v, x).$$

(Recall that $dist(v, x)$ means the shortest distance between v and x according to edge cost c). Here you can imagine that X is the places that you want to open an Carrefour center. You have only money to open k of them and you want to minimize the travelling distance of any client (to the closest opened Carrefour).

2(a)

Let S be a "maximal" independent set in the graph H derived from G. Here H has the same vertex set as G and it has an edge (u, v) if and only if $dist(u, v) \leq 2R$.

Prove that in the original graph G, if we can only open $|S|-1$ vertices as the Carrefour centers, then at least one client needs to travel a distance of *strictly larger* than R to find an open Carrefour.

2(b)

Use the previous observation to design a 2-approximation algorithm.