Second Homework Exercise

January 14, 2022

Please type your answers using the Latex. Please hand in your assignment (by email or by hard copy) before the class of January 20, 2022.

Please feel free to contact me if you have questions.

Exercise 1

Given $G = (V, E)$, the *max cut* problem asks for a partition of V into S and $\overline{S} = V \backslash S$, so that the number of the edges connecting a vertex in S and a vertex is $\overline{S}$ is maximized. The following is an algorithm:

> Choose arbitrary two nodes u and v and create two sets $A = \{u\}$ and $B = \{v\}$. Next consider the rest of the nodes one by one. If the node w has more neighbors in A than in B, we add it into B; otherwise, we add w into A.

Prove that this is a 1/2-approximation algorithm.

Exercise 2

Consider the following machine scheduling problem. Given a single machine, suppose that there are n jobs, each with processing time p_j, weight w_j, and a due date d_j. We schedule these jobs one by one on the machine and we gain the weight w_j of job j if its finishing time is at most d_j. We want to maximize the total weight gained.

This problem is NP-hard. But we will design an FPTAS for it in this exercise. (Do not get intimidated by this problem. You can look carefully again over how we have designed an FPTAS for knapsack problem).

Here are some hints.

1. Observe that there is an optimal schedule in which all on-time jobs finish before all late jobs.
2. Next observe we can also assume that in this optimal schedule, the on-time jobs complete in an earliest due date order.

3. Using these two observations, you should be able to solve the problem in $O(nW)$ time, where $W = \sum w_j$, using dynamic programming.

4. And then you should be able to turn this pseudo-polynomial time algorithm into an FPTAS. What is the running time?