First Homework Exercise

December 14, 2021

Please type your answers using the Latex. Please hand in your assignment (by email or by hard copy) before the class of January 13, 2022.

Please feel free to contact me if you have questions.

Exercise 1

Given a graph G which is a cycle on n vertices, numbered from 0 to $n-1$, in "clockwise" order. Some set R of requests are given, where each request (u, v) means that we need to send 1 unit from u to v either "clock-wise" or "counterclock-wise." The objective is to route the requests in such a way so that the maximum "load" is minimized. To be more precise, let L_i denote the number of requests that go through the edge $(i, i+1(\text{modulo } n))$. We want to minimize $\max_{i=0}^{i-1} L_i$.

Design a 2-approximation for this problem. Hint: Write an LP and go from there.

Exercise 2

Given n items whose sizes are $a_1, \cdots a_n \in (0.1]$, we want to put them into bins of size 1 (so a subset S of items can be fit into a bin if and only if $\sum_{i \in S} a_i \leq 1$), with the objective using as few bins as possible. This is an NP-hard problem. But there is a rather simple greedy algorithm. It works like this:

> You consider items one by one. If you can, put the item into the latest-opened bin (and only this one). If you cannot, open a new bin.

Prove that this algorithm gives a 2-approximation. Also give a tight (or as tightly as possible) example.

Exercise 3

Given a directed complete graph $G = (V, E)$ (so given every two vertices u and v, there is either an arc from u to v or the other way around), a feedback set $U \subseteq V$ is a set of vertices whose removal leaves the remaining graph acyclic (no more directed cycle).

The objective is to minimize the size of the feedback set. Our problem here is ask you to design a 3-approximation algorithm for this problem.

Hint: It suffices to kill directed cycles of length 3.