

# Dynamic matching models

*Sujet de stage M2*

## Context

We consider a dynamic matching model with random arrivals – a dynamic version of the bipartite matching model. As in the static setting, it is based on a bipartite graph. In the discrete-time dynamic model there are arrivals of units of ‘supply’ and ‘demand’ that can wait in queues located at the nodes in the network. A control policy determines which items are matched at each time.

The theory of matching has a long history in economics, mathematics, and graph theory [5, 7], with applications found in many other areas such as chemistry and information theory. Most of the work is in a static setting. The dynamic model has received recent attention in [4, 3, 6]. The most compelling application is organ donation: United Network for Organ Sharing (UNOS) offers kidney paired donation (KPD). This is a transplant option for candidates who have a living donor who is medically able, but cannot donate a kidney to their intended candidate because they are incompatible (i.e., poorly matched) [1]. In this application, or application to resource allocation (such as in scheduling in a power grid) [2, 8], data arrives sequentially and randomly, so that matching decisions must be made in real-time, taking into account the uncertainty of future requirements for supply or demand, or the uncertainty of the sequence of classification tasks to be undertaken. The choice of matching decisions can be cast as an optimal control problem for a dynamic matching model.

## Objectives

The objective of this internship is to study the performance (the number of unmatched customers/servers in the system) of this model under various matching policies including: ML (Match the Longest), MS (Match the Shortest), FIFO (match the oldest), priorities. There are several possible directions for the internship: comparing different policies (numerically and/or analytically); searching for heuristics to minimize the total number of unmatched customers/servers in the system; extending the model to include impatience (unmatched items disappear after some delay), or the matching preferences of the customers.

**Prerequisites:** Basic knowledge of random structures and algorithms. Some prior experience with programming (C, C++ or Matlab) is strongly recommended.

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## References

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