Distributed demand control in power grids
and ODEs for Markov decision processes

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Challenges

Challenges of renewable power generation

Impact of wind and solar on net-load at CAISO

Ramp limitations cause price-spikes

Price spike due to high net-load ramping need when solar production ramped out

Negative prices due to high mid-day solar production

Load and Net-load

Renewable Generation

$/MWh

GW

GW

Ramps

Jan 01 Jan 02 Jan 03 Jan 04 Jan 05 Jan 06

Wind generation in BPA, Jan 2015

PCI
Challenges of renewable power generation

Impact of wind and solar on net-load at CAISO

- Price spike due to high net-load ramping need when solar production ramped out
- Negative prices due to high mid-day solar production

Ramp limitations cause price-spikes

\[ G_W(t) = \text{Wind generation in BPA, Jan 2015} \]
Challenges of renewable power generation

Balancing control loop

- wind and solar volatility seen as disturbance
- grid level measurements: scalar function of time (ACE) a linear combination of frequency deviation and the tie-line error (power mismatch between the scheduled and actual power out of the balancing region)
- compensation $G_c$ designed by a balancing authority
- In many cases control loops are based on standard PI (proportional-integral) control design.

\[
\Delta P_{\text{delivered}} = H_a + H_b + \cdots + Y(t)\pi(t)
\]
Challenges of renewable power generation

Increasing needs for **ancillary services**

In the past, provided by the generators - **high costs!**
Challenges

Tracking Grid Signal with Residential Loads

Tracking objective:

Prior work

- Deterministic centralized control:
  Sanandaji et al. 2014 [HICSS], Biegel et al. 2013 [IEEE TSG]

- Randomized control:
  Mathieu, Koch, Callaway 2013 [IEEE TPS] (decisions at the BA)
  Meyn, Barooah, B., Chen, Ehren 2015 [IEEE TAC]
  (local decisions, restricted load models)
Tracking Grid Signal with Residential Loads

Example: 20 pools, 20 kW max load

Each pool consumes 1 kW when operating
12 hour cleaning cycle each 24 hours

Power Deviation:

![Power Deviation Graph](image)

Nearly Perfect Service from Pools
Meyn, Barooah, B., Chen, Ehren 2015 [IEEE TAC]
using an extension/reinterpretation of Todorov 2007 [NIPS] (linearly solvable MDPs)
Tracking Grid Signal with Residential Loads

Example: 300,000 pools, 300 MW max load

Each pool consumes 1kW when operating
12 hour cleaning cycle each 24 hours

Power Deviation:

![Graph showing power deviation with 300,000 pools and 300 MW max load.]

Nearly Perfect Service from Pools
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What About Other Loads?
Control Goals and Architecture

Macro control

High-level control layer: BA or a load aggregator.

The balancing challenges are of many different categories and time-scales:

- Automatic Generation Control (AGC); time scales of seconds to 20 minutes.
- Balancing reserves. In the Bonneville Power Authority, the balancing reserves include both AGC and balancing on timescales of many hours. Balancing on a slower time-scale is achieved through real time markets in some other regions of the U.S.
- Contingencies (e.g., a generator outage)
- Peak shaving
- Smoothing ramps from solar or wind generation
Control Goals and Architecture

Local Control: decision rules designed to respect needs of load and grid

Demand Dispatch: Power consumption from loads varies automatically to provide service to the grid, without impacting QoS to the consumer.

- Min. communication: each load monitors its state and a regulation signal from the grid.
- Aggregate must be controllable: randomized policies for finite-state loads.
Load Model

Controlled Markovian Dynamics

- Discrete time: \( i \)th load \( X^i(t) \) evolves on finite state space \( X \)
- Each load is subject to *common* controlled Markovian dynamics.

\[
\text{Signal } \zeta = \{\zeta_t\} \text{ is broadcast to all loads}
\]

- Controlled transition matrix \( \{P_\zeta : \zeta \in \mathbb{R}\} \):

\[
P\{X^i_{t+1} = x' \mid X^i_t = x, \zeta_t = \zeta\} = P_\zeta(x, x')
\]

Questions

- How to analyze aggregate of similar loads?  
- Local control design?
Aggregate model
How to analyze aggregate?

Mean field model

$N$ loads running independently, each under the command $\zeta$.

Empirical Distributions:

$$\mu^N_t(x) = \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}\{X^i(t) = x\}, \quad x \in X$$

$U(x)$ power consumption in state $x$,

$$y^N_t = \frac{1}{N} \sum_{i=1}^{N} U(X^i_t) = \sum_x \mu^N_t(x) U(x)$$

Mean-field model:

via Law of Large Numbers for martingales

$$\mu_{t+1} = \mu_t P_{\zeta_t}, \quad y_t = \langle \mu_t, U \rangle$$

$$\zeta_t = f_t(y_0, \ldots, y_t) \quad \text{by design}$$
Local Control Design
Local Control Design

**Goal:** Construct a family of transition matrices \( \{ P_\zeta : \zeta \in \mathbb{R} \} \)

---

**Myopic Design:**

\[
P_\zeta^{myop}(x, x') := P_0(x, x') \exp(\zeta U(x') - \Lambda_\zeta(x))
\]

with \( \Lambda_\zeta(x) := \log \left( \sum_{x'} P_0(x, x') \exp(\zeta U(x')) \right) \) the normalizing constant.
Local Design

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**Exponential family design:**

\[
P_\zeta(x, x') := P_0(x, x') \exp(h_\zeta(x, x') - \Lambda_{h_\zeta}(x))
\]

with

\[
h_\zeta(x, x') = \zeta H_0(x, x').
\]

The choice of \( H_0 \) will typically correspond to the linearization of a more advanced design around the value \( \zeta = 0 \) (or some other fixed value of \( \zeta \)).
Local Design

**Goal:** Construct a family of transition matrices \( \{P_\zeta : \zeta \in \mathbb{R}\} \)

**Individual Perspective Design**

Consider a finite-time-horizon optimization problem: For a given terminal time \( T \), let \( p_0 \) denote the pmf on strings of length \( T \),

\[
p_0(x_1, \ldots, x_T) = \prod_{i=0}^{T-1} P_0(x_i, x_{i+1}),
\]

where \( x_0 \in X \) is assumed to be given. The scalar \( \zeta \in \mathbb{R} \) is interpreted as a weighting parameter in the following definition of total welfare. For any pmf \( p \),

\[
\mathcal{W}_T(p) = \zeta \mathbb{E}_p \left[ \sum_{t=1}^{T} U(X_t) \right] - D(p||p_0)
\]

where the expectation is with respect to \( p \), and \( D \) denotes relative entropy:

\[
D(p||p_0) := \sum_{x_1, \ldots, x_T} \log \left( \frac{p(x_1, \ldots, x_T)}{p_0(x_1, \ldots, x_T)} \right) p(x_1, \ldots, x_T)
\]
Local Design

**Goal:** Construct a family of transition matrices \( \{ P_\zeta : \zeta \in \mathbb{R} \} \)

It is easy to check that the myopic design is an optimizer for the horizon \( T = 1 \),

\[
P^{myop}_\zeta(x_0, \cdot) \in \arg \max_p \mathcal{W}_1(p).
\]

The infinite-horizon mean welfare is denoted,

\[
\eta^*_\zeta = \lim_{T \to \infty} \frac{1}{T} \mathcal{W}_T(p_T^*).
\]

Explicit construction via eigenvector problem:

\[
P_\zeta(x, y) = \frac{1}{\lambda} \frac{v(y)}{v(x)} \hat{P}_\zeta(x, y), \quad x, y \in X,
\]

where \( \hat{P}_\zeta v = \lambda v \), \( \hat{P}_\zeta(x, y) = \exp(\zeta \mathcal{U}(x))P_0(x, y) \)

Extension/reinterpretation of [Todorov 2007] + [Kontoyiannis & Meyn 200X]
Example: pool pumps
How Pools Can Help Regulate The Grid

Needs of a single pool

- Filtration system circulates and cleans: Average pool pump uses 1.3kW and runs 6-12 hours per day, 7 days per week
- Pool owners are oblivious, until they see frogs and algae
- Pool owners do not trust anyone: Privacy is a big concern

Single pool dynamics: $X = \{(m, j) : m \in \{0, 1\}, j \in \{1, 2, \ldots, I\}\}$. 

On

Off
Local Control Design

Tracking Grid Signal with Residential Loads

Example: 20 pools, 20 kW max load

Each pool consumes 1kW when operating
12 hour cleaning cycle each 24 hours

Power Deviation:

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Power Deviation:

Nearly Perfect Service from Pools
Meyn et al. 2013 [CDC], Meyn et al. 2015 [IEEE TAC]
Range of services provided by pools
Example: 10,000 pools, 10 MW max load

12 hr/day cycle

Power Deviation
Reference

MW

MW

t/hour
Local Design

Extending local control design to include exogenous disturbances

State space for a load model: $X = X_u \times X_n$.

Components $X_n$ are not subject to direct control (e.g. impact of the weather on the climate of a building).
Local Design
Extending local control design to include exogenous disturbances

State space for a load model: $X = X_u \times X_n$.
Components $X_n$ are not subject to direct control (e.g. impact of the weather on the climate of a building).

Conditional-independence structure of the local transition matrix

$$P(x, x') = R(x, x'_u)Q_0(x, x'_n), \quad x' = (x'_u, x'_n)$$

$Q_0$ models uncontroled load dynamics and exogenous disturbances.
Local Design

**Goal:** Construct a family of transition matrices \( \{P_{\zeta} : \zeta \in \mathbb{R}\} \)

**Nominal model**
A Markovian model for an individual load, based on its typical behavior.

- Finite state space \( X = \{x^1, \ldots, x^d\} \);
- Transition matrix \( P_0 \), with unique invariant pmf \( \pi_0 \).
Local Design

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Common structure for design
The family of transition matrices used for distributed control is of the form:

\[
P_\zeta(x, x') := P_0(x, x') \exp(h_\zeta(x, x') - \Lambda_{h_\zeta}(x))
\]

with \( h_\zeta \) continuously differentiable in \( \zeta \), and the normalizing constant

\[
\Lambda_{h_\zeta}(x) := \log\left(\sum_{x'} P_0(x, x') \exp(h_\zeta(x, x'))\right)
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\]

Assumption: for all \( x \in X, \ x' = (x'_u, x'_n) \in X, \ h_\zeta(x, x') = h_\zeta(x, x'_u) \).
Local Design

Goal: Construct a family of transition matrices \( \{P_\zeta : \zeta \in \mathbb{R}\} \)

Construction of the family of functions \( \{h_\zeta : \zeta \in \mathbb{R}\} \)

**Step 1:** The specification of a function \( \mathcal{H} \) that takes as input a transition matrix. \( H = \mathcal{H}(P) \) is a real-valued function on \( X \times X \).

**Step 2:** The families \( \{P_\zeta\} \) and \( \{h_\zeta\} \) are defined by the solution to the ODE:

\[
\frac{d}{d\zeta} h_\zeta = \mathcal{H}(P_\zeta), \quad \zeta \in \mathbb{R},
\]

in which \( P_\zeta \) is determined by \( h_\zeta \) through:

\[
P_\zeta(x, x') := P_0(x, x') \exp(h_\zeta(x, x') - \Lambda h_\zeta(x))
\]

The boundary condition: \( h_0 \equiv 0 \).
For any function \( H^\circ : X \to \mathbb{R} \), one can define

\[
H(x, x'_u) = \sum_{x'_n} Q_0(x, x'_n) H^\circ(x'_u, x'_n)
\]  

(1)

Then functions \( \{h_\zeta\} \) satisfy

\[
h_\zeta(x, x'_u) = \sum_{x'_n} Q_0(x, x'_n) h_\zeta^\circ(x'_u, x'_n),
\]

for some \( h_\zeta^\circ : X \to \mathbb{R} \). Moreover, these functions solve the \( d \)-dimensional ODE,

\[
\frac{d}{d\zeta} h_\zeta^\circ = \mathcal{H}^\circ(P_\zeta), \quad \zeta \in \mathbb{R},
\]

with boundary condition \( h_0^\circ \equiv 0 \).
Individual Perspective Design

- **Local welfare function:** \( \mathcal{W}_\zeta(x, P) = \zeta U(x) - D(P\|P_0) \),
  where \( D \) denotes relative entropy: \( D(P\|P_0) = \sum_{x'} P(x, x') \log \frac{P(x, x')}{P_0(x, x')} \).

- **Markov Decision Process:** \( \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E[\mathcal{W}_\zeta(X_t, P)] \)

Average reward optimization equation (AROE):

\[
\max_P \left\{ \mathcal{W}_\zeta(x, P) + \sum_{x'} P(x, x')h^*_\zeta(x') \right\} = h^*_\zeta(x) + \eta^*_\zeta
\]

where \( P(x, x') = R(x, x'_u)Q_0(x, x'_n) \), \( x' = (x'_u, x'_n) \)
ODE method for IPD design:

Family \( \{ P_\zeta \} : \quad P_\zeta(x, x') := P_0(x, x') \exp(h_\zeta(x, x') - \Lambda h_\zeta(x)) \)

Functions \( \{ h_\zeta \} : \quad h_\zeta(x, x'_u) = \sum_{x_n} Q_0(x, x'_n) h_\zeta^o(x'_u, x'_n), \)

for \( h_\zeta^o : X \to \mathbb{R} \) solutions of the \( d \)-dimensional ODE,

\[
\frac{d}{d\zeta} h_\zeta^o = H^o(P_\zeta), \quad \zeta \in \mathbb{R},
\]

with boundary condition \( h_0^o \equiv 0. \)

\[
H_\zeta^o(x) = \frac{d}{d\zeta} h_\zeta^o(x) = \sum_{x'} [Z_\zeta(x, x') - Z_\zeta(x^o, x')] U(x'), \quad x \in X,
\]

where \( Z = [I - P + 1 \otimes \pi]^{-1} = \sum_{n=0}^{\infty} [P_\zeta - 1 \otimes \pi]^n \) is the fundamental matrix.
Example: Thermostatically Controlled Loads

- refrigerators, water heaters, air-conditioning . . .
- TCLs are already equipped with primitive “local intelligence” based on a deadband (or hysteresis interval)
- The state process for a TCL at time $t$:

$$X(t) = (X_u(t), X_n(t)) = (m(t), \Theta(t)),$$

where $m(t) \in \{0, 1\}$ denotes the power mode (“1” indicating the unit is on), and $\Theta(t)$ the inside temperature of the load

Exogenous disturbances: ambient temperature, and usage
Example: Thermostatically Controlled Loads

The standard ODE model of a water heater is the first-order linear system,

$$\frac{d}{dt} \Theta(t) = -\lambda [\Theta(t) - \Theta^a(t)] + \gamma m(t) - \alpha [\Theta(t) - \Theta^{in}(t)] f(t),$$

- $\Theta(t)$ temperature of the water in the tank
- $\Theta^{in}(t)$ temperature of the cold water entering the tank
- $f(t)$ flow rate of hot water from the WH
- $m(t)$ power mode of the WH ("on" indicated by $m(t) = 1$).

Deterministic deadband control: $\Theta(t) \in [\Theta_-, \Theta_+]$

Nominal model for local control design: based on the specification of two CDFs for the temperature at which the load turns on or turns off
Example: Thermostatically Controlled Loads

Discrete-time control.

- At time instance $k$, if the water heater is on (i.e., $m(k) = 1$), then it turns off with probability,

$$p^\ominus(k + 1) = \frac{[F^\oplus(\Theta(k + 1)) - F^\oplus(\Theta(k))]}{1 - F^\oplus(\Theta(k))}$$

where $[x]_+ := \max(0, x)$ for $x \in \mathbb{R}$;

- Similarly, if the load is off, then it turns on with probability

$$p^\oplus(k + 1) = \frac{[F^\ominus(\Theta(k)) - F^\ominus(\Theta(k + 1))]}{F^\ominus(\Theta(k))}$$

The nominal behavior of the power mode can be expressed

$$P\{m(k) = 1 \mid \theta(k - 1), \theta(k), m(k - 1) = 0\} = p^\oplus(k)$$
$$P\{m(k) = 0 \mid \theta(k - 1), \theta(k), m(k - 1) = 1\} = p^\ominus(k)$$
Example: Thermostatically Controlled Loads

Myopic design - exponential tilting of these distributions:

\[ p_\zeta^\oplus(k) := \mathbb{P}\{m(k) = 1 \mid \theta(k-1), \theta(k), m(k-1) = 0, \zeta(k-1) = \zeta\} \]

\[ = \frac{p^\oplus(k)e^\zeta}{p^\oplus(k)e^\zeta + 1 - p^\oplus(k)} \]

\[ p_\zeta^\ominus(k) = \mathbb{P}\{m(k) = 0 \mid \theta(k-1), \theta(k), m(k-1) = 1, \zeta(k-1) = \zeta\} \]

\[ = \frac{p^\ominus(k)}{p^\ominus(k) + (1 - p^\ominus(k))e^\zeta} \]

If \( p_0^\oplus(k) > 0 \), then the probability \( p_\zeta^\oplus(k) \) is strictly increasing in \( \zeta \), approaching 1 as \( \zeta \to \infty \); it approaches 0 as \( \zeta \to -\infty \), if \( p_0^\oplus(k) < 1 \).
Example: Thermostatically Controlled Loads

System identification

\[ \frac{d}{dt} \Theta(t) = -\lambda[\Theta(t) - \Theta^a(t)] + \gamma m(t) - \alpha[\Theta(t) - \Theta^{in}(t)]f(t), \]

- \( \Theta(t) \) temperature of the water in the tank
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- \( f(t) \) flow rate of hot water from the WH
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<table>
<thead>
<tr>
<th>Temp. Ranges</th>
<th>ODE Pars.</th>
<th>Loc. Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Theta_+ \in [118, 122] ) F</td>
<td>( \lambda \in [8, 12.5] \times 10^{-6} )</td>
<td>( T_s = 15 ) sec</td>
</tr>
<tr>
<td>( \Theta_- \in [108, 112] ) F</td>
<td>( \gamma \in [2.6, 2.8] \times 10^{-2} )</td>
<td>( \kappa = 4 )</td>
</tr>
<tr>
<td>( \Theta^a \in [68, 72] ) F</td>
<td>( \alpha \in [6.5, 6.7] \times 10^{-2} )</td>
<td>( \rho = 0.8 )</td>
</tr>
<tr>
<td>( \Theta^{in} \in [68, 72] ) F</td>
<td>( P_{on} = 4.5 ) kW</td>
<td>( \theta_0 = \Theta_- )</td>
</tr>
</tbody>
</table>

Heterogeneous population: 100 000 WHs simulated by uniform sampling of the values in the table

Usage data from Oakridge National Laboratory (35WHs over 50 days)
Local Control Design

Tracking performance and the controlled dynamics for an individual load

100,000 water-heaters
When on, individual load consumes 4.5 kW
With no usage, approx. 2% duty cycle, avg. power consumption 10MW.
Tracking performance
Potential for contingency reserves and ramping

Tracking two sawtooth waves with 100,000 water heaters: average power consumption 8MW

- Power deviation (MW)
- Time (hrs)

Reference Power Deviation

Power Deviation
Tracking performance
and the controlled dynamics for an individual load

Heterogeneous setting:

- 40,000 loads per experiment;
- 20 different load types in each case

Lower plots show the on/off state for a typical load
Unmodeled dynamics

Setting: 0.1% sampling, and

1. Heterogeneous population of loads
2. Load $i$ overrides when QoS is out of bounds

Closed-loop tracking

PI control: $\zeta_t = k_P e_t + k_I e_I^t$, $e_t = r_t - y_t$, $e_I^t = \sum_{s=0}^{t} e_s$
Control Architecture

Frequency Allocation for Demand Dispatch

10,000 pools

Bandwidth centered around its natural cycle

Reference (from Bonneville Power Authority) – Output deviation
Conclusions

Virtual storage from flexible loads

**Approach:** creating Virtual Energy Storage through direct control of flexible loads
- helping the grid while respecting user QoS
Conclusions
Virtual storage from flexible loads

**Approach:** creating Virtual Energy Storage through direct control of flexible loads
- helping the grid while respecting user QoS

**Challenges:**
- Stability properties for IPD and myopic design?
- Information Architecture: $\zeta_t = f(?)$
  Different needs for communication, state estimation and forecast.
- Capacity estimation (time varying)
- Network constraints
- Resource optimization & learning
  Integrating VES with traditional generation and batteries.
- Economic issues
  Contract design, aggregators, markets . . .
Conclusions

Thank You!
References: this talk


A. Bušić and S. Meyn. Passive dynamics in mean field control. *53rd IEEE Conf. on Decision and Control (CDC)* 2014.
References: related

Demand dispatch:


Markov processes:

Mean Field Model
Linearized Dynamics

**Mean-field model:**
\[ \mu_{t+1} = \mu_t P_{\zeta_t}, \quad y_t = \langle \mu_t, U \rangle \]
\[ \zeta_t = f_t(y_0, \ldots, y_t) \]

**Linear state space model:**
\[ \Phi_{t+1} = A \Phi_t + B \zeta_t \]
\[ \gamma_t = C \Phi_t \]

Interpretations: \(|\zeta_t|\) is small, and \(\pi\) denotes invariant measure for \(P_0\).
- \(\Phi_t \in \mathbb{R}^{|X|}\), a column vector with 
  \[ \Phi_t(x) \approx \mu_t(x) - \pi(x), \quad x \in X \]
- \(\gamma_t \approx y_t - y^0\); deviation from nominal steady-state
- \(A = P^T_0, \quad C = U^T\), and input dynamics linearized:
\[ B^T = \left. \frac{d}{d\zeta} \pi P_{\zeta} \right|_{\zeta=0} \]