An Aloha Protocol for Multihop Mobile Wireless Networks

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***Abstract*— We define an Aloha type access control mechanism for large mobile, multihop, wireless networks. The access scheme is designed for the multihop context, where it is important to find a compromise between the spatial density of communications and the range of each transmission. More precisely, we optimize the product of the number of simultaneously successful transmissions per unit of space (spatial reuse) by the average range of each transmission. The optimization is obtained via an averaging over all Poisson configurations for the location of interfering mobiles in a model where an exact evaluation of signal over noise ratio is used. The main mathematical tools stem from stochastic geometry and are spatial versions of the so called additive and max shot noise processes. The resulting MAC protocol can be implemented in a decentralized way provided some local geographic informations are available to the mobiles. This MAC protocol shows very interesting properties. First its transport capacity is proportional to the square root of the density of mobiles and its stability can be shown under mobility conditions discussed in this paper. The delay provided by this protocol for transporting information from one node to any another node is proportional to the distance between them and to the square root of the network density. Furthermore this protocol is self adapting to the network density; more precisely to the best of the authors knowledge, it is the first protocol to reach the Gupta and Kumar bound that does not require prior knowledge of the node density.**

***Index Terms*— Network design, stochastic process, point process, stochastic geometry, queuing theory, optimization, transport capacity, signal to interference ratio, interference, collision, multiple access protocol, MAC layer**

I. Introduction

This paper concentrates on the medium access control (MAC) of wireless networks with several mobile emitters and receivers sharing a common Hertzian medium, like in e.g. certain classes of mobile ad hoc networks or sensor networks. One of the main difficulties for tuning MAC within this context stems from the mobility and the resulting unpredictability of the geometrical properties of the emission patterns. Mobility may in particular lead to random spatial clustering rendering some sets of simultaneous transmissions impossible due to high interferences.

Within this context, the MAC protocols aim at defining policies where mobiles access the shared medium in such a way that spatial or temporal clustering does not happen or only rarely happens. This is done by some exclusion mechanism that prevents mobiles that are close to some emitting mobile (and also receiving mobile in the case of IEEE 802.11 with the RTS-CTS option) from emitting at the same time. In wired networks, the MAC algorithm is supposed to prevent simultaneous transmissions from happening, as often as possible, since such transmissions are bound to produce collisions. Another and contradictory requirement is that MAC protocols should nevertheless allow as many simultaneous and successful transmissions as possible in different parts of the network. This ability is known as *spatial reuse*.

Aloha [1] with TDMA (Time Division Multiple Access) was one of the first protocol used in radio network. The work presented in the present paper is an adaptation and an optimization of Aloha in the context of a multihop network of mobile nodes. This optimization of Aloha uses various mathematical tools mainly borrowed from stochastic geometry. This paper complements previous studies in [2]. In particular, we relax the mathematical assumption of exponential emitted powers and discuss stability issues in the case without mobility. We develop also the implementation issues.

The paper is organized as follows. Section II introduces related works and positions the contributions of this paper among these works. Section III introduces the mathematical model. Section IV focuses on SR-Aloha (for spatial reuse Aloha), namely on the optimization of the Medium Access Probability (MAP) p when each station expects to make a hop of length R, or on the best hop length R when p is fixed. As we shall see, this simple optimization fails to determining the optimal MAC setting. In § IV-C, we also compare SR-Aloha with the CSMA (Carrier Sense Multiple Access) technique which is the basis of the MAC protocol for the WLANs standards IEEE 802.11 [3] and Hiperlan type 1 [4]. The main result of the paper is introduced in Section V where the optimization of MSR-Aloha (for multihop spatial reuse Aloha) is addressed. In Section VI we discuss capacity and stability issues for MSR-Aloha. We consider the time dynamic of the protocol and suppose that each mobile initiates a stationary flow of packets of intensity τ to be transported to some random destination. We give arguments showing under some assumptions concerning the mobility of users, that if τ is smaller than a threshold that is given in closed form, MSR-Aloha should be dynamically stable and the delay to transmit a packet between any o-d pair should be proportional to the distance between origin and destination. We will comment also on the particular case where nodes have no mobility at all. In this case, the previous dynamic stability result does not hold anymore. However, under mild assumptions the protocol nevertheless provides a positive throughput to any node of the network and is still optimal in some sense. Implementation issues are briefly discussed in Section VII, with a particular emphasis on a decentralized implementation of the protocol. Appendix gathers proofs of two technical results concerning the Poisson shot-noise process used in the main stream.

II. State of the art and related works

Aloha is a widely deployed and studied access protocol. The initial paper presenting Aloha has been published in 1970 [1] and Aloha is now used in most cellular networks to request access. A lot of both theoretical and practical studies have been carried out to improve Aloha. Initial studies [5], [6] sought methods to stabilize the protocol. The first paper studying Aloha in a multihop context is [7]. In this work Nelson and Kleinrock computed the probability of successful transmission

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in a random planar Aloha packet radio network with a simple model where interferences only propagate two hops away. In 1988 Ghez, Verdu and Schwartz introduced a model for slotted Aloha with multipacket reception capability in a widely referenced paper [8]. To the best of the authors knowledge, this paper introduced a widely accepted model for Aloha in a network with spatial reuse.

The present article revisits the spatial reuse Aloha MAC mechanism in the context of multihop mobile wireless networks. Compared to [8], our main contributions are

- an exact representation of the signal to interference ratio for each transmission and hence of the collisions of the Aloha scheme, taking into account all interferers;
- various optimizations of the Aloha protocol: SR-Aloha, which concerns the case where some predefined range of transmission is set; MSR-Aloha, which is meant for the multihop context.

The routing protocol that will be considered here is close to MFR (most forward with radius). In this greedy routing protocol introduced by Takagi and Kleinrock [9], a node selects the neighbor with the shortest projected distance to the receiver. The main new step in the present paper is the merging of the geometric routing notion of ”most forwarding” with the MAC notion of ”transmission success” into a unique geometric function to optimize.

For this, we introduce an abstract geometric model allowing one to address the key concerns outlined above concerning MAC, namely spatial reuse and range of transmission can be simultaneously addressed as properties of simple random geometrical objects. To simplify the considerations we have assumed a slotted Aloha model. All interferences are taken into account in an exact way and the success of some transmission will be decided in function of the Signal to Interference Ratio (SIR) at the receiver, as it would be the case under the classical Gaussian channel model of information theory.

Keeping the “random access” spirit of the Aloha protocol, numerous works tried to design more efficient protocols. Two main ways have been investigated; the first one consists in taking advantage of the history of the channel in order to adopt a better retransmission strategy than the blind Aloha re-emission strategy. The second one consists in improving the control of the channel by carrier sensing: that is the CSMA (Carrier Sense Multiple Access) technique. In [10] it is shown that CSMA actually outperforms Aloha in wired networks. However Tobagi points out in [11] that CSMA protocol may suffer from hidden collisions and numerous papers mostly in the 90s proposed dedicated protocols to cope with this problem [12], [13], [14], [15]. Actually these protocols can be seen as enhanced CSMA protocols where the carrier sense effect is also used around the receiver to protect its reception. As a byproduct of the proposed analytical model, the present article offers a tentative comparison between Aloha and CSMA in a multihop network under the general SIR model. This is a pertinent comparison sine one may question the benefit of CSMA in adhoc network which suffers from hidden nodes.

In 2000 Gupta and Kumar published a now widely referenced article [16] in which it is shown how the throughput of multihop adhoc networks scales with the node density. If one uses the material produced by Gupta and Kumar in the part dedicated to the derivation of the lower bound, it is very easy to show that an ad hoc network using a CSMA scheme with a fixed carrier sense range will not be able to scale its throughput in $O(\frac{\sqrt{\lambda}}{\sqrt{log(\lambda)}})$ (the maximum achievable scaling law for the throughput of a random ad hoc network as found by [16]) but rather in $O(1)$. Thus for a CSMA ad hoc network to reach its maximum throughput scaling, an adaptation of the carrier sense range to the node density is required. Gupta and Kumar give access and routing protocols allowing the network to reach the lower bound throughput proposed in their paper; however, the proposed solution does not allow one to derive implementable protocols to reach this bound. The present paper proposes an access scheme reaching the Gupta and Kumar bound which does not require prior knowledge of the network density. It shows that MSR-Aloha gives a density of progress (notion related to Gupta and Kumar’s transport capacity) of the form $K(p)\sqrt{\lambda}$; we give a closed form for $K(p)$, allowing for an optimization with respect to p, which is one of the main results of the paper.

As we will see, although it allows the transmission of packets over time from any node to any other node, MSR Aloha does not require connectivity at any given time. This explains why the scaling that it provides is in $O(\sqrt{\lambda})$ instead of the expected $O(\frac{\sqrt{\lambda}}{\sqrt{log(\lambda)}})$. In ad hoc networks, connectivity is usually enforced via complex neighborhood management algorithms which lead to significant overhead (see e.g. [17], [18], [19], [20]). The fact that connectivity is not required by MSR Aloha can thus be thought of as an important argument in favor of this access protocol.

III. A STOCHASTIC GEOMETRY MODEL

We consider an infinite planar network. Let $\Phi = \{(X_i, (e_i, S_i, T_i))\}$ be a marked Poisson point process with intensity λ on the plane $\mathbb{R}^2$, where

- $\Phi = \{X_i\}$ denotes the locations of stations,
- $\{e_i\}_i$ the medium access indicator of station i; $e_i = 1$ for the stations which is allowed to emit and $e_i = 0$ means the station is (a potential) receiver. Here, the random variables e_i are independent, with $\mathbf{P}(e_i = 1) = p$.
- $\{S_i\}$ denote powers emitted by stations (stations for which $e_i = 1$); the random variables $\{S_i\}$ will always be assumed independent and identically distributed with mean $1/\mu$. If not otherwise specified, the S_is have a general distribution. Under this general distribution assumption, we will be able to prove our qualitative results. An important special case, in which a quantitative analysis is possible, is that with exponential powers.
- $\{T_i\}$ are the SINR thresholds corresponding to some channel bit rates or bit error rates; here, for simplicity, we will take $T_i \equiv T$ constant.

In addition to this marked point process, the model is based on a function $l(x, y)$ that gives the attenuation (path-loss) from y to x in $\mathbb{R}^2$. We will assume that the path-loss depends only on the distance; i.e, with a slight abuse of notation $l(x, y) = l(|x-y|)$; As an important special case of the *simplified* attenuation function we will take

$$l(u) = (Au)^{-\beta} \quad \text{for } A > 0 \text{ and } \beta > 2. \tag{3.1}$$

Note that such $l(u)$ explodes at $u = 0$, and thus in particular *is not* correct for a small distance r and large intensities λ.

We also consider an independent of Φ, external noise (e.g. thermal) and denote it (at a given location) by W.

Note first that Φ can be represented as a pair of independent Poisson p.p. representing emitters $\Phi^1 = \{X_i : e_i = 1\}$, and receivers $\Phi^0 = \{X_i : e_i = 0\}$, with intensities, respectively, λp and $\lambda(1-p)$.

Suppose there is a station located at x that emits with power S and requires SINR T. Suppose there is a user located at $y \in \mathbb{R}^2$. The station can establish a channel to this user with a given bit-rate (which will be taken as the unit throughput in what follows) iff

$$\frac{Sl(|x-y|)}{W + I_{\Phi^1}(y)} \geq T\,, \tag{3.2}$$

where I_{Φ^1} is the shot-noise process of Φ^1: $I_{\Phi^1}(y) = \sum_{X_i \in \Phi^1} S_i l(|y - X_i|)$. Denote by $\delta(x, y, \Phi^1)$ the indicator that (3.2) holds. Note that by stationarity of Φ^1, the probability $\mathbf{E}[\delta(x, y, \Phi^1)]$ depends only on the distance $x - y$ and *not* on the specific locations of (x, y); so we can use the notation $p_{|x-y|}(\lambda p) = \mathbf{E}[\delta(x, y, \Phi^1)]$, where λp is the intensity of the emitters Φ^1. The following lemma is the basis of our quantitative analysis of the model.

Lemma 3.1: For exponential S with mean $1/\mu$,

$$\begin{aligned} p_R(\lambda) &= \exp\Big\{ -2\pi\lambda \int_0^\infty \frac{u}{1 + l(R)/(Tl(u))}\,\mathrm{d}u \Big\} \\ &\quad \times \psi_W(\mu T/l(R))\,, \end{aligned} \tag{3.3}$$

where $\psi_W(\cdot)$ is the Laplace transform of W.

Proof: Note by (3.2) that

$$\begin{aligned} p_R(\lambda) &= \mathbf{P}(S \geq T(W + I_\Phi)/l(R)) \\ &= \int_0^\infty e^{-\mu s T/l(R)}\,\mathrm{d}\Pr(W + I_\Phi \leq s) \\ &= \psi_{I_\phi}(\mu T/l(R))\,\psi_W(\mu T/l(R))\,, \end{aligned}$$

where $\psi_{I_\Phi}(\cdot)$ is the Laplace transform of $I_\Phi = I_\Phi(0)$. Note that ψ_W does not depend on λ, whereas it is known that for a general Poisson shot-noise

$$\psi_{I_\Phi}(\xi) = \exp\Big\{ -\lambda \int_{\mathbb{R}^2} 1 - \mathbf{E}\Big[e^{-\xi S l(|x|)}\Big]\,\mathrm{d}x \Big\}. \tag{3.4}$$

Since S is exponential with mean $1/\mu$

$$\begin{aligned} \psi_{I_\Phi}(\xi) &= \exp\Big\{ -\lambda \int_{\mathbb{R}^2} 1 - \frac{\mu}{\mu + \xi l(|x|)}\,\mathrm{d}x \Big\} \\ &= \exp\Big\{ -2\pi\lambda \int_0^\infty \frac{u}{1 + \mu/(\xi l(u))}\,\mathrm{d}u \Big\} \end{aligned}$$

that concludes the proof. ∎

Corollary 3.2: For exponential S, $W \equiv 0$ and the simplified attenuation function (3.1) we have $p_R(\lambda) = e^{-\lambda R^2 T^{2/\beta} C}$, where

$$C = C(\beta) = \Big(2\pi\Gamma(2/\beta)\Gamma(1 - 2/\beta)\Big)/\beta \tag{3.5}$$

and $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\,\mathrm{d}t$ is the Gamma function.

For general power distribution, we do not know any explicit form of $p_R(\lambda)$. However, the following scaling result will be useful.Denote by $\bar{p}_r$ the value of $p_r(1)$ calculated for the model with the simplified attenuation function (3.1), $T \equiv 1, W \equiv 0$ and normalized emitted powers $\bar{S}_i = \mu S_i$.

Lemma 3.3: For general power distribution and the simplified attenuation function (3.1), when the external noise $W \equiv 0$, we have $p_R(\lambda) = \bar{p}_{RT^{1/\beta}\lambda^{1/2}}$.

Note that $\bar{p}_r$ does not depend on any parameter of the model other than the distribution of the normalized emitted power $\bar{S}$.

Proof: The Poisson point process Φ with intensity $\lambda > 0$ can be represented as $\{X_i'/\sqrt{\lambda}\}$, where $\Phi' = \{X_i'\}$ is Poisson with intensity 1. Due to this the Poisson shot-noise with simplified attenuation function admits the following representation: $I_\Phi = \lambda^{\beta/2} I_{\Phi'}$. Thus for $W = 0$

$$\begin{aligned} p_R(\lambda) &= \Pr(S \geq T(AR)^\beta I_\Phi) \\ &= \Pr\Big(\mu S \geq \mu(ART^{1/\beta}\lambda^{1/2})^\beta I_{\Phi'}\Big) = \bar{p}_{RT^{1/\beta}\lambda^{1/2}}\,. \end{aligned}$$

∎

IV. Spatial Density of Successful Transmissions

In this section we suppose that each mobile $X_i \in \Phi$ attempts to transmit to one receiver Y_i located at a distance $R = |X_i - Y_i|$ to it via a channel based on the principle (3.2).

A. SR-Aloha: Best MAP Given Some Range

The first question that we investigate assumes that the range of all transmissions is given and looks for the value of MAP p that maximizes the mean number of emitters (and thus emitter-receiver pairs) that can successfully transmit, per unit area. The main result is that there exists an optimal MAP and thus a way to optimize Aloha once transmission range is fixed. The associated protocol will be referred to as SR-Aloha in what follows.

In fact, we don't ask here whether there is a receiver Y_i located at a distance R as we will do in the next section. This is why there is actually only one point process of intensity $\lambda^1 = \lambda p$ in the model of this section, and the optimization in p can actually be seen as that in λ^1. In order to simplify notation, we will drop the upper index 1 in this section and call here Φ the point process of emitters with intensity λ and look for the optimal λ.

We have the following simple formula for the *spatial density of successful transmissions* in the network.

Proposition 4.1: The mean number of emitters per unit area that can successfully transmit at range R is $\lambda p_R(\lambda)$.

Proof: The respective mean number of emitters per unit area set $B \subset \mathbb{R}^2$ is equal to

$$\begin{aligned} &\mathbf{E}\Big[\sum_{X_i \in \Phi} \mathbb{1}(X_i \in B)\delta(X_i, Y_i, \Phi)\Big] \\ &= \lambda \int_{\mathbb{R}^2} \mathbb{1}(x \in B) p_R(\lambda)\,\mathrm{d}x = \lambda p_R(\lambda)\,. \end{aligned}$$

∎

Now we look for $\lambda_{\text{max}} = \text{maxarg}_{\lambda \geq 0}\{\lambda p_R(\lambda)\}$ that maximizes the *spatial density of successful transmissions* in the network. We will see that for S exponential λ_{max} is well defined. For general S, we have the following technical result in this matter.

Lemma 4.2: If $\Pr(S > 0) > 0$ then $p_r(\lambda)$ is continuous in λ. If moreover $\mathbf{E}[S^{3/\beta}] < \infty$ then $\lim_{\lambda\to\infty} \lambda p_r(\lambda) = 0$ for any $r > 0$.

The proof is forwarded to the Appendix. As a consequence of the above lemma, the function $\lambda \mapsto \lambda p_r(\lambda)$ attains it maximum in $(0, \infty)$. Define λ_{max} to be the smallest λ for which the spatial density of successful transmissions is maximal. Then $0 < \lambda_{\text{max}} < \infty$.

Proposition 4.3: For a general S, the simplified attenuation function (3.1) and $W \equiv 0$ we have $\lambda_{\text{max}} = c_1/(R^2 T^{2/\beta})$, and $\lambda_{\text{max}} p_R(\lambda_{\text{max}}) = c_2/(R^2 T^{2/\beta})$, where the constants c_1, c_2 do not depend on R, T, μ, provided λ_{max} is well defined. For exponential S, $c_1 = 1/C$ and $c_2 = 1/(eC)$ with C defined in (3.5).

Proof: Suppose $\lambda_{\text{max}} = \text{maxarg}_{\lambda \geq 0}\{\lambda p_R(\lambda)\}$ is well defined. By Lemma 3.3 $c_1 = \text{maxarg}_{\lambda \geq 0}\{\lambda \bar{p}_{\lambda^{1/2}}\}$ and $c_2 = \max_{\lambda \geq 0}\{\lambda \bar{p}_{\lambda^{1/2}}\}$, which completes the proof for general S. For exponential S, general attenuation function and general distribution of $W \geq 0$, by Lemma 3.1, and the differentiation of the function $\lambda p_R(\lambda)$ w.r.t. λ, it is easy to see that its unique maximum is attained at

$$\lambda_{\text{max}} = \left(2\pi \int_0^\infty \frac{u}{1 + l(R)/(Tl(u))}\, \mathrm{d}u \right)^{-1}$$

and the maximal value is given by $\lambda_{\text{max}} p_R(\lambda_{\text{max}}) = e^{-1}\lambda_{\text{max}}\, \psi_W(\mu T/l(R))$. Plugging in the simplified attenuation function (3.1) and $W \equiv 0$ and evaluating the integral

$$2\pi \int_0^\infty \frac{u}{1 + l(R)/(Tl(u))}\, \mathrm{d}u = \frac{2\pi R^2 T^{2/\beta}}{\beta} \Gamma(2/\beta)\Gamma(1-2/\beta)\,,$$

we get the result. ■

Note that under this optimal choice of λ, the mean distance progressed by transmissions per unit space, for exponential S and simplified attenuation function, is $R\lambda_{\text{max}} p_R(\lambda_{\text{max}}) = c_2/(RT^{2/\beta})$, which is maximal for $R = 0$. The apparent conclusion is here that the smallest the range, the best the network operates. We will come back to this in § V.

B. Spatial Reuse

We can also interpret the last results in terms of the so-called *spatial reuse factor* defined as the distance to the receiver R divided by the (mean) distance between adjacent emitters. For this last quantity, we take the mean distance between neighboring points in Poisson-Voronoi tessellation (more precisely, the mean edge length of the typical triangle in the Poisson-Delaunay triangulation; see e.g. [21]), which is $32/(9\pi\sqrt{\lambda_{\text{max}}})$. Thus we get, for exponential S *Spatial reuse*$=T^{-1/\beta}\frac{9\pi}{32\sqrt{C}}$. For the network based on the perfect triangular mesh spatial reuse is analyzed in [22] and is given by the formula $1/2\, T^{-1/\beta}\sqrt{3}/(6\zeta(\beta-1))^{1/\beta}$, where $\zeta(s) = \sum_{n=1}^{\infty} 1/n^s$ is the Riemann zeta function. Figure 1 compares the values of spatial reuse in these two cases for $T = 10$dB and different β. Note that in FDMA hexagonal networks with super-hexagonal frequency reuse, spatial reuse would range from $1/6 = 0.133$ to $\sqrt{3}/12 = 0.144$ (depending on whether we take the receiver to be located in the middle of hexagonal cell edge or at its end.

C. Tentative comparison of SR-Aloha and CSMA

The aim of this section is a tentative comparison between SR-Aloha and a generic CSMA protocol. By this latter we understand the model where each communication with a targeted transmission range R is protected by the exclusion disc centered at the transmitter with radius $R_{cs} > R$, within which other transmissions are inhibited. Note that our SR-Aloha can be seen as model with *random* exclusion areas.

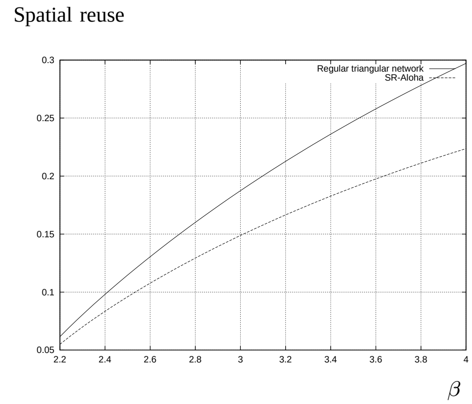


Fig. 1. Comparison of the spatial reuse factor for Poisson (lower curve) and perfect triangular network (upper curve) for $T = 10$dB and different β. In hexagonal TDMA networks, with super-hexagonal frequency reuse, this parameter is between 0.133 and 0.144 regardless of β.

Throughout the section, we assume a random Poisson network, the simplified attenuation function (3.1) and $W = 0$. We suppose that the radius of the carrier sense range R_{cs} is set at $R_{cs} = RT^{1/\beta}\frac{2(6\zeta(\beta-1))^{1/\beta}}{\sqrt{3}}$, where R denotes the targeted transmission range. According to [22] we are sure that with this value, there will be no collision for a receiver in a radius of range R if the transmitters in the network are on a triangular regular network with neighboring transmitters separated by R_{cs}. It is in a triangular regular network that the density of nodes being at least at R_{cs} away is maximum. Thus, whatever the pattern of simultaneous emitters respecting the CSMA rule with R_{cs}, a transmission to a receiver within radius R will always be collision free.

In order to compare SR-Aloha to CSMA protocol, we have to compute the intensity of an extracted point process satisfying the CSMA exclusion rule. Of course the intensity of this process will depend on the selection algorithm. An intuitive algorithm consists in picking nodes randomly and adding them to the CSMA transmission set if they are not in the carrier sense range of an already selected node. This algorithm is close to the effective behavior of a simple CSMA system. However this model does not seem to be easily tractable mathematically. Another selection algorithm is that based in the Matern hard core process [23], [21]. This process is a thinning of the initial Poisson point process in which points are selected according to random marks. A point of the process is selected if its mark is larger than all marks in a radius of range R_{cs}. It is easy to check that the selected points follow the CSMA rule. The spatial intensity $\lambda_{R_{cs}}$ of the Matern hard-core process can be obtained in function of the spatial intensity of the initial Poisson point process by the formula: $\lambda_{R_{cs}} = (1 - e^{-\pi\lambda R_{cs}^2})/(\pi R_{cs}^2)$; see [21].

Simulations show that the intensity of this process is smaller than the intensity obtained through the random pick algorithm alluded to above, while giving results of the same order of magnitude. We can notice that the Matern hard core process is a natural model for the access scheme of HiPERLAN type 1. The MAC of HiPERLAN type 1 actually uses an advanced version of CSMA. A signaling burst is sent before the packet; the (random) length of this elimination burst will be the mark which allows one to derive the Matern process.

Since we know R_{cs}, it is then easy to compute the trans-

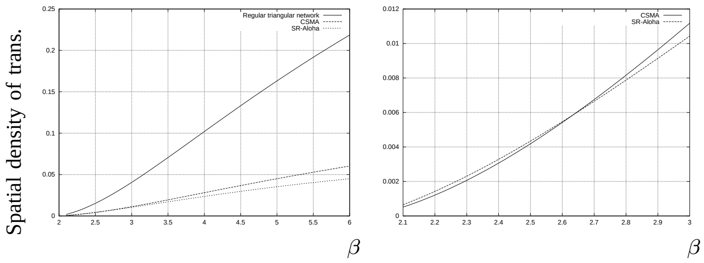


Fig. 2. Left: spatial intensity of successful transmissions for CSMA (Matern selection model) and for SR-Aloha scheme in function of β, $T = 10$dB. The top curve gives the throughput of a regular triangular network. Right: Zoom of the comparison CSMA-SR-Aloha for β between 2 and 3.

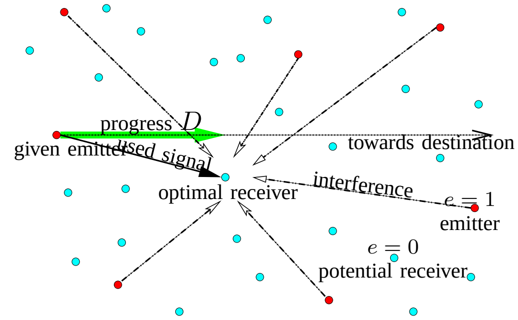


Fig. 3. Progress.

mission density for a CSMA scheme and to compare it with the spatial density of successful transmission of our SR-Aloha scheme given by Proposition 4.3.

This comparison is given in Figure 2. Figure 2 (top) compares the spatial intensity of CSMA (selection of active nodes as in a Matern hard core process) and the spatial density of successful transmission of SR-Aloha scheme in function of β, for $T = 10$dB. The curve on the top gives the spatial intensity of CSMA in a regular triangular network. On the bottom we have a zoom for β between 2 and 3. We see that, near 2, the optimized Aloha scheme actually outperforms the CSMA scheme.

Figure 2 shows that under these assumptions, the performance of SR-Aloha is very close to that of the CSMA scheme. This observation is consistent with [7] ,where a similar result reports that Aloha and CSMA have close performance. However the study in [7] uses a simplified transmission model (interference is only considered to propagate two hops away) and the carrier sense range and transmission range are supposed to be the same. In [24] a convenient tuning of the carrier sense range is shown to be important for the global performance of the network.

As a result of this tentative comparison we can conclude that SR-Aloha and a generic CSMA algorithm will have comparable result, a better framework and further studies will be necessary to precise this comparison.

D. Best Range Given Some MAP

Assuming some intensity λ of emitters given, we will use the following notation and definition:

$$r_{\text{max}}(\lambda) = \underset{r\geq 0}{\text{maxarg}}\{rp_r(\lambda)\} \quad (4.1)$$

$$\rho(\lambda) = \max_{r\geq 0}\{rp_r(\lambda)\}. \quad (4.2)$$

We call $r_{\text{max}}(\lambda)$ *the best range attempt* for λ and $\rho(\lambda)$ the *best mean range*. For exponential S, by Lemma 3.1, $0 < r_{\text{max}} < \infty, 0 < \rho < \infty$. For a general S we have the following technical result.

Lemma 4.4: If $\Pr(S > 0) > 0$ then $p_r(\lambda)$ is continuous in r. If moreover $\mathbf{E}[S^{2/\beta}] < \infty$ then $\lim_{r\to\infty} rp_r(\lambda) = 0$ for any $\lambda > 0$.

The proof is forward to the Appendix. As a consequence of the above lemma the function $r \mapsto rp_r(\lambda)$ attains it maximum in $(0,\infty)$ and if we take r_{max} to be, for instance, the largest r for which the spatial density of successful transmissions is maximal then $0 < r_{\text{max}} < \infty$.

By Lemmas 3.1, and 3.3 we have the following result.

Proposition 4.5: For a general S, the simplified attenuation function (3.1) and $W \equiv 0$ we have $r_{\text{max}}(\lambda) = c_3/(T^{1/\beta}\sqrt{\lambda})$ and $\rho(\lambda) = c_4/(T^{1/\beta}\sqrt{\lambda})$, where the constants c_3, c_4 do not depend on R, T, μ, provided λ_{max} is well defined. For exponential S, $c_3 = 1/\sqrt{2C}$ and $c_4 = 1/\sqrt{2eC}$.

Here again, trying to maximize the cumulated mean range of all transmissions initiated per unit of space w.r.t. λ, namely trying to maximize $\lambda\rho(\lambda)$ in λ, leads to a degenerate answer since the maximum is for $\lambda = \infty$ which again gives $R = 0$.

V. Multihop Networks

A. Progress

We now return to the model of Section III with emitters Φ^1 and receivers Φ^0 and focus on the multihop context. Suppose an emitter, say X_0, located at the origin $X_0 = 0$ has to send information in some given direction (say along the x axis) to some destination located far from it (say at infinity – see Figure 3). Since the destination is too far from the source to be able to receive the signal in one hop, the source tries to find a non-emitting station in Φ^0 such that the hop to this station maximizes the distance traversed towards the destination, among those which are able to receive the signal. This station will later forward the data to the destination or next intermediary station. In this model, the "effective" distance traversed in one hop, which we will call the progress, is equal to

$$D = \max_{X_j\in\Phi^0}\left(\delta(0, X_j, \Phi^1)|X_j|\Big(\cos(\arg(X_j))\Big)^+\right), \quad (5.1)$$

where $\arg(y)$ is the argument of the vector $y \in \mathbb{R}^2$ ($-\pi < \arg(y) \leq \pi$) and $\delta(x, y, \Phi^1)$ the indicator that (3.2) holds. We are interested in the expectation $d(\lambda, p) = \mathbf{E}[D]$ that only depends on λ and on the MAP p, once given the parameters concerning emission and reception, Note that similarly to Proposition 4.1, we have the following formula for the *(spatial) density of progress*:

Proposition 5.1: The mean total distance traversed in one hop by all transmissions initialized in some unit area (density of progress) is equal to $\lambda p d(\lambda, p)$.

B. MSR-Aloha and Optimal Progress

Note that for given λ, there is the following trade-off in p between the spatial density of communications and the range of each transmission. For a small p, there are few emitters only per unit area, but they can likely reach very remote receiver as a consequence of little I_{Φ^1}. On the other hand, a large p means many emitters per unit area that create interference and thus prevent each other from reaching a remote receiver. Another feature associated with large p is the paucity of receivers, which makes the chances of a jump in the right direction smaller. In the sequel we try to quantify this tradeoff and find p that maximizes the density of progress. Since this optimization is adapted to the multihop context, the corresponding MAC protocol will be referred to as MSR-Aloha.

For mathematical convenience and also for the reasons that will be discussed in Section VII we will not study $d(\lambda, p)$ directly but rather a lower bound of this quantity which we now introduce. Let

$$\tilde{D} = \max_{X_j \in \Phi^0} \left(p_{|X_j|}(\lambda p)|X_j| \Big(\cos(\arg(X_j))\Big)^+ \right) \qquad (5.2)$$

and let $\tilde{d}(\lambda, p) = \mathbf{E}[\tilde{D}]$. We have an obvious bound $\tilde{D} \leq \rho(\lambda p)$.

Proposition 5.2: For all λ, p, $d(\lambda, p) \geq \tilde{d}(\lambda, p)$.

Proof: Let $\mathbf{E}^1, \mathbf{E}^0$ denote expectation w.r.t Φ^1 and Φ^0, respectively. Note that $\mathbf{E}[\tilde{D}] = \mathbf{E}^1\mathbf{E}^0[\tilde{D}]$ due to the independence between Φ^1 and Φ^0. The result now follows from Jensen's inequality, since the functional $\varphi(f) = \mathbf{E}^0[\max_{X_j \in \Phi^0}(f(X_j)|X_j|(\cos(\arg(X_j)))^+)]$ is convex on the space of real functions $f : \mathbb{R}^2 \to \mathbb{R}$. ■

The aim of the remaining part of this section is to determine the value of MAP p that optimizes $\lambda p \tilde{d}(\lambda, p)$.

We will use the notation (cf §IV-D) $r_{\text{max}} = r_{\text{max}}(\lambda p) = \text{maxarg}_{r \geq 0}\{r p_r(\lambda p)\}$ and $\rho = \rho(\lambda p) = \max_{r \geq 0}\{r p_r(\lambda p)\}$. For $z \in [0, 1]$, let

$$G(z) = \frac{2}{r_{\text{max}}^2} \int_{\{r \geq 0: \rho z/(r p_r) < 1\}} r \arccos\Big(\frac{\rho z}{r p_r}\Big) \, \mathrm{d}r \, . \qquad (5.3)$$

Remark 5.3: Note that if we assume simplified attenuation function (3.1) and $W = 0$, then Proposition 4.5 shows that $G(z)$ does not depend on the model parameters λ, p, T, μ. Indeed, in this case

$$G(z) = \frac{2}{c_3} \int_{\{r \geq 0: c_4 z/(r \bar{p}_r) < 1\}} r \arccos(\frac{c_4 z}{r \bar{p}_r}) \, \mathrm{d}r.$$

In particular, for exponential S, we have

$$G(z) = 2 \int_{\{t: e^t/\sqrt{2et} \leq 1/z\}} \arccos\Big(\frac{z e^t}{\sqrt{2et}}\Big) \, \mathrm{d}t \, . \qquad (5.4)$$

We now study the distribution function of $\tilde{D}$.

Proposition 5.4: We have

$$F_{\tilde{D}}(z) = \mathbf{P}(\tilde{D} \leq z) = e^{-\lambda(1-p)(r_{\text{max}}(\lambda p))^2 G(z/\rho(\lambda p))} \, .$$

Proof: Note in (5.2) that $\tilde{D}$ has the form of the so-called *extremal shot-noise* $\max_{X_i \in \Phi^0} g(X_i)$ with the response function $g(x) = |x| p_{|x|}(\cos(\arg(x)))^+$. Its distribution function can be expressed by the Laplace transform of the (additive) shot noise

$$\mathbf{P}(\max_{X_i \in \Phi^0} g(X_i) \leq z) = \mathbf{E}\left[\exp\left[\sum_{X_i \in \Phi^0} \ln(\mathbb{1}(g(X_i) \leq z))\right]\right]$$

and thus, by (3.4), for Poisson p.p. Φ^0 with intensity $\lambda(1-p)$

$$\mathbf{P}(\tilde{D} \leq z) = \exp\left[-\lambda(1-p) \int_{\mathbb{R}^2} \mathbb{1}(g(x) > z) \, \mathrm{d}x\right] .$$

Passing to polar coordinates in the integral $\int_{\mathbb{R}^2} \ldots \mathrm{d}x$, we get

$$\int_{\mathbb{R}^2} \mathbb{1}(g(x) > z) \, \mathrm{d}x \quad = \quad r_{\text{max}}^2 G(z/\rho) \, ,$$

which completes the proof. ■

Immediately from the Proposition 5.4:

Proposition 5.5: The expectation of $\tilde{D}$ is equal to

$$\tilde{d}(\lambda, p) = \mathbf{E}[\tilde{D}] = \rho(\lambda p) \int_0^1 1 - e^{-\lambda(1-p)(r_{\text{max}}(\lambda p))^2 G(z)} \, \mathrm{d}z \, .$$

Corollary 5.6: For the model with the simplified attenuation function (3.1) and $W = 0$, the expected modified progress is equal to

$$\tilde{d}(\lambda, p) = \frac{c_4}{T^{1/\beta}\sqrt{\lambda p}} H(p, T) \, , \qquad (5.5)$$

and the spatial density of modified progress is

$$\lambda p \tilde{d}(\lambda, p) = \frac{c_4\sqrt{\lambda p}}{T^{1/\beta}} H(p, T) \, , \qquad (5.6)$$

where

$$H(p, T) = \int_0^1 1 - \exp\Big[\Big(1 - \frac{1}{p}\Big)\frac{G(z)}{c_3 T^{2/\beta}}\Big] \, \mathrm{d}z \qquad (5.7)$$

Thus the maximal density of progress is attained for MAP $p^* = p^*(T)$ satisfying $H(p^*, T) = \sup_{0 \leq p \leq 1} H(p, T)$. For exponential S this is equivalent to

$$\int_0^1 \Big(1 + \frac{G(z)}{p^* T^{2/\beta} C}\Big) \exp\Big[\Big(1 - \frac{1}{p^*}\Big)\frac{G(z)}{2T^{2/\beta} C}\Big] \, \mathrm{d}z = 1 \, . \quad (5.8)$$

Note that p^* does not depend on λ and μ.

C. Numerical Examples and Discussion

Now, we evaluate numerically the optimal MAP in the exponential case, and discuss the issue of the distance to the receiver that realizes the maximum in (5.2). This distance should not be large when one wants to implement the algorithm. We will show, that at the optimal MAP p^*, the receiver that realizes the maximum in (5.2), is very likely in the vicinity of the emitter. However replacing the optimal receiver in (5.2) by the nearest one in some angle towards destination gives essentially suboptimal density of progress.

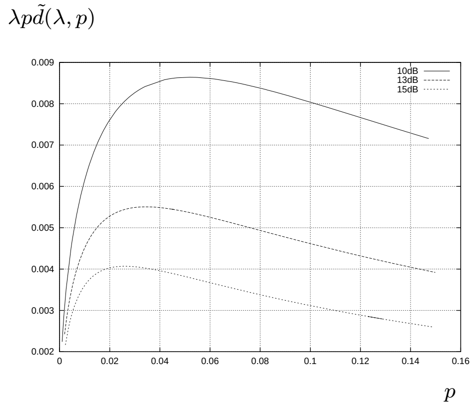
1) Numerical approximations of p^*: The successful numerical calculation of $\tilde{d}$ and of the solution p of (5.8) maximizing the density of progress requires an efficient way of calculating the function G given by (5.4). Bellow we show some properties of G that involve the so called *Lambert W* functions LW^0 and LW^1. These functions can be seen as the inverses of the function te^t in the domains $(-1, \infty)$ and $(-\infty, -1)$ respectively; i.e., for $s \geq -1/e$, $\text{LW}^0(s)$ is the unique solution of $\text{LW}^0(s)e^{\text{LW}^0(s)} = s$ satisfying $\text{LW}^0(s) \geq -1$, whereas for $0 > s \geq -1/e$, $\text{LW}^1(s)$ is the unique solution of $\text{LW}^1(s)e^{\text{LW}^1(s)} = s$ satisfying $\text{LW}^1(s) \leq -1$. Let $L^0(s) = -\frac{1}{2}\text{LW}^0(-s^{-2}e^{-1})$ and $L^1(s) = -\frac{1}{2}\text{LW}^1(-s^{-2}e^{-1})$. The following representation of G is equivalent to that in (5.4):

$$G(z) \quad = \quad 2 \int_{\arcsin(z)}^{\pi/2} \Big(L^1(\frac{\sin s}{z}) - L^0(\frac{\sin s}{z})\Big) \, \mathrm{d}s \, .$$

Moreover, the following function

$$G_\sim(z) = \pi(1 - z) - 2\ln(z)\arccos(z)$$

approximates G very well on the whole interval $0 < z < 1$. Figure 4 shows the density of progress calculated by means of $G_\sim$ for $\beta = 3$, $\lambda = 1$ and three values of the SIR threshold $T = \{10, 13, 15\}$dB. On the plot, we can identify the MAP p^* that maximizes the density of progress for a given T.



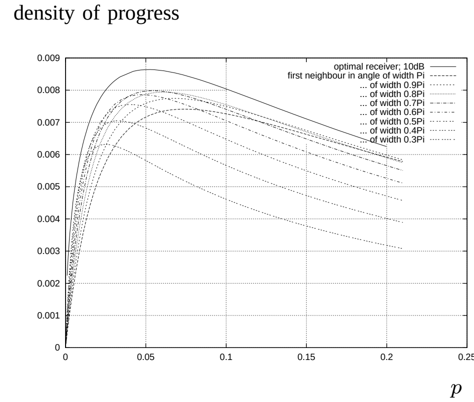


Fig. 4. Density of progress for the model with exponential S and simplified attenuation function with $\beta = 3$, $\lambda = 1$ and $W = 0$, with $T = \{10, 13, 15\}$dB (curves from top to bottom). The optimal values $(\mathrm{maxarg}, \max)$ are respectively $\{(0.052, 0.0086), (0.034, 0.0055), (0.026, 0.0040)\}$.

Fig. 5. Density of progress for the model with exponential S and simplified attenuation function with $\beta = 3$, $\lambda = 1$ and $W = 0$, with $T = 10$dB for "optimal receiver" and "nearest neighbor" case.

2) Location of the Optimal Receiver: First we will show that the optimal density of progress can be approached in a model with a reasonably restricted domain of reception. By this we mean that we exclude in the definition of D and $\tilde{D}$ the receivers laying outside some disk with given radius R. Note first that we have the following straightforward generalization of our previous results.

Proposition 5.7: The Propositions 5.1, 5.2, 5.4 and 5.5 remain true if we take $\max_{X_i\in\Phi^0,|X_j|\le R}(\dots)$ the definitions (5.1) and (5.2). In this case the function G has to be modified by taking the integral in (5.3) over the region $\{0 \le r \le R : \rho z/(rp_r) < 1\}$.
The case considered above will be referred to as the restricted range model in what follows.

We look for a reception radius R such that for a given p the density of progress in the restricted range model is close enough to that of the unrestricted range model. It is convenient to relate the reception radius R with the intensity λ of emitters. As we will see later on, its even more convenient to take $R = Kr_{\max}$ for some constant $K \ge 0$ (recall, that $r_{\max} = r_{\max}(\lambda p)$ is the distance at which the mean range $rp_r(\lambda p)$ is maximal). Denote by G_K the function defined by (5.3) with the integral taken over $\{0 \le r \le Kr_{\max} : \rho z/(rp_r) < 1\}$.

We will continue with the simplified attenuation function (3.1) and $W = 0$. In this case

$$G(z) = \frac{2}{c_3}\int_{\{0\le<r\le c_3K : c_4 z/(r\bar{p}_r)<1\}} r\arccos(\frac{c_4 z}{r\bar{p}_r})\,\mathrm{d}r.$$

We can now prove the following continuity result.

Proposition 5.8: For the simplified attenuation function and $W = 0$

$$0 \le \tilde{d} - \tilde{d}_K \le \rho z_K\,, \tag{5.9}$$

for some function $K \mapsto z_K$, such that $\lim_{K\to\infty} z_K = 0$. For exponential S, we can take $z_K = Ke^{1/2-K^2/2}$ for $K \ge 1$.

Proof: Since $r\bar{p}_r \to 0$ when $r \to \infty$ (see Appendix), for each $K \ge 0$, there exists z_K such that $G_K(z) = G(z)$ for $z \ge z_K$. Moreover $z_K \to 0$ when $K \to \infty$. Thus (5.9) follows from Propositions 5.5 and 5.7. ■

Take for example, $p = 0.035$, which for $T = 13$dB gives in exponential S case the mean progress in the unbounded model (about) $\tilde{d} = 0.0055/0.035 = 0.157$ (cf. Figure 4), the best mean range is attained for the range attempt $r_{\max} = 0.506$ and is equal to $\rho = 0.307$. In order to have a relative difference $\epsilon = 0.01$ we find the minimal $K \ge 1$ such that $Ke^{1/2-K^2/2} \le 0.01 \cdot 0.157/0.307 = 0.00513$, which is $K = 3.768$. This means that in the model with reception radius $R = Kr_{\max} = 3.768 \cdot 0.506 = 1.905$, the mean progress is 1% close to the optimal one, obtained in the unrestricted model.

On the other hand, it is easy to calculate the progress in the model when the emitter chooses always the nearest (to him) receiver in the cone of a given angle α towards the destination. Formally let $\tilde{D}^{\text{near}}_\alpha = p_{|X_*|}(\lambda p)|X_*|\cos(\arg(X_*))$, where $X_* = X_*(\alpha)$ is such that $|X_*| = \min\{|X_j| : X_j \in \Phi^0, |\arg(X_j)| \le \alpha/2\}$. Since the distribution function of $|X_*|$ for the Poisson process Φ^0 with intensity $\lambda(1-p)$ is known to be $\Pr(|X_*| \ge r) = \Pr(\text{the Poisson p.p. has no points in the resp. cone}) = \exp[-\lambda(1-p)\alpha/2\pi]$ and $\arg(X_*)$ is independent of $|X_*|$, uniformly distributed on $(-\alpha/2, \alpha/2)$, we easily get the following result on the mean progress in this scenario.

Proposition 5.9: For the simplified attenuation function, exponential S and $W = 0$ we have

$$\mathbf{E}[D^{\text{near}}_\alpha] = \frac{\Gamma(3/2)}{\sqrt{\lambda}}\frac{\sin(\alpha/2)(1-p)}{\Big((1-p)\alpha/2 + pT^{2/\beta}C\Big)^{3/2}}\,.$$

Figure 5 compares the density of progress $p\tilde{d}(1,p)$ in the "optimal receiver" case to the density of progress $p\mathbf{E}[D^{\text{near}}_\alpha]$ in the "nearest neighbor" case, for various values of α when $T = 10$dB and S is exponential. The optimal choice of α is about 0.72π, for which the optimal MAP p is about $p = 0.056$ which gives $p\mathbf{E}[D^{\text{near}}_\alpha] = 0.0080$, to be compared to $p^*\tilde{d}(1,p^*) = 0.0086$ for $p^* = 0.052$.

Finally, note on Figure 6, that for T in the range 0dB–10dB the optimal value of the density of progress $p^*\tilde{d}(p^*,1)$ is linear in p^*, which means that the mean progress $\tilde{d}(p^*,1)$ does not depend much on T in this range. We see on Figure 7, that making T very small increases the optimal MAP p^*, rather than the mean progress $\tilde{d}(p^*,1)$.

VI. Capacity and Stability

A. Transport Capacity

The spatial density of progress introduced above is related to Gupta and Kumar's [16] notion of *transport capacity*. In [16], it is shown how to construct a spatial and temporal scheme for scheduling transmissions in a bounded region such that the

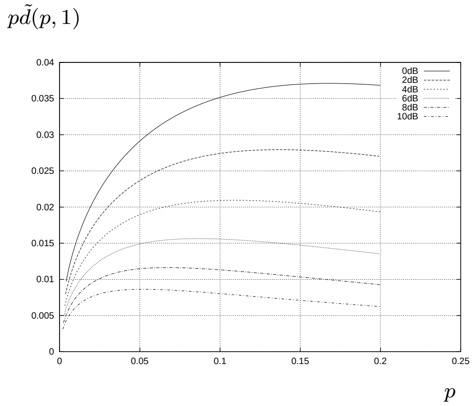


Fig. 6. Density of progress for the model with exponential S and simplified attenuation function with $\beta = 3$ and $W = 0$ for moderate values of T.

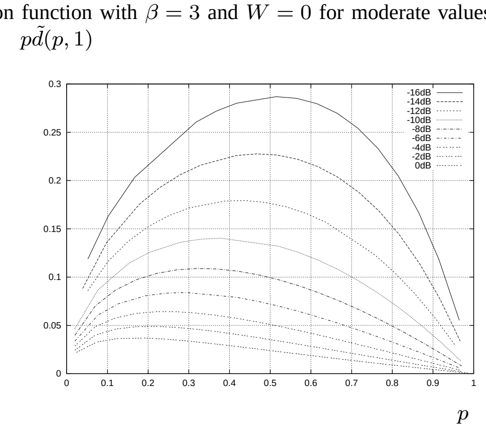


Fig. 7. Density of progress for the model with exponential S and simplified attenuation function with $\beta = 3$ and $W = 0$ for small values of T.

number of bit meters pumped by the network every second is of the order of $O\left(\frac{\sqrt{\lambda}}{\sqrt{\log(\lambda)}}\right)$ when the intensity $\lambda \to \infty$.

Our MSR-Aloha protocol also pumps a certain number of bit meters every second. If the bit rate corresponding to the threshold T is b, then the density of progress is $\lambda p d(\lambda, p)$ and MSR-Aloha pumps $C_t = b\lambda p d(\lambda, p)$ bit meters per second and per unit area. From the Proposition 5.2 and formula (5.6), we can lower-bound this transport capacity per unit area by

$$C_t = b\left(\frac{\sqrt{p^*}}{T^{1/\beta}\sqrt{2eC}} H(p^*, T)\right)\sqrt{\lambda} = O\left(\sqrt{\lambda}\right), \qquad (6.1)$$

where $H(p, T)$ is given by (5.7), letting p^* to maximize $H(p, T)$. So, we conclude that MSR-Aloha, achieves the optimal transport capacity of Gupta and Kumar. The reason for which it is $O(\sqrt{\lambda})$ and not $O(\frac{\sqrt{\lambda}}{\sqrt{log(\lambda)}})$ is there are no extra connectivity requirements. In Section VII, we will describe some conditions under which MSR-Aloha can be implemented in a purely decentralized way. Under these conditions MSR-Aloha can then be seen as a way of achieving optimal transport capacity in a decentralized way. To our best knowledge it is the first protocol to achieve this property.

B. Stability of MSR Aloha

Up to now, we analyzed spatial properties of the MSR-Aloha mechanism. We cannot really address stability issues unless we define temporal evolution of the model.

Suppose each mobile has the following transmission dynamics: it has a queue of packets to be transmitted at the bit rate specified by the SIR threshold T. This queue is fed by *packets* which are either *fresh packets* originating from this mobile or arriving from another mobile and to be *relayed*. Each mobile tries to transmit the packet head of the line according to the MSR-Aloha scheme, namely tries to transmit this packet with probability p and either succeeds or keeps this packet head of line in case of collision (to be identified with the instant progress $D = 0$).

Each packet transport consists in several transmission hops between a random source and a random destination. We assume that the set of such packet transports is homogeneous (for instance forming a random segment process with uniform orientation and mean length L). Then, assuming MSR-Aloha mechanism, the transport of each new packet requires an average of $L/d(\lambda, p)$ individual transmissions. Let τ denote the mean number of fresh packets initiated per time slot and per mobile. Thanks to the homogeneity assumptions, the average number of transmissions that are created by the network per slot and per unit of space is therefore $\lambda\tau L/d(\lambda, p)$.

We also know that when all stations have packets to transmit, the mean number of packets that are allowed to transmit per unit area and per slot is λp.

These two observations lead to the following conclusion: if the time intensity τ of fresh packets per station is larger than $pd(\lambda, p)/L$, then there is no way for the protocol to cope with the traffic load during periods where most stations have to transmit in some area. Thus the quantity $C_d = \frac{pd(\lambda,p)}{L}$ is an upper bound on the mean number of fresh packets per station and per slot that MSR-Aloha can handle at a given MAP p.

The question whether any time intensity of communications smaller than C_d per mobile leads to a stable dynamics for a network controlled by MSR-Aloha is quite natural by analogy with what we know of Aloha or Ethernet.

We show below that under simple independence and non-degeneracy assumptions on mobility, this dynamic stability can be conjectured.

1) Stability under Mobility: The slotted mobility model is close to the way point model: mobiles are numbered in some way (e.g. using the distance to the origin at slot 0). Mobile i, which is located at X_i^n in slot n, has a random and independent motion m_i^n during this slot, so that its position at slot $n+1$ is $X_i^{n+1} = X_i^n + m_i^n$. If the $\{m_i^n\}$ sequence is made of independent and identically distributed (i.i.d.) random variables in n and i, then $\{X_i^n\}$ is a Poisson point process at all time n if it is at time 0. The law of m_i^n is assumed to be non-degenerate (i.e. the norm of m_i^n is assumed to be positive with a positive probability). This implies that the sequence of configurations seen by mobile i over time (by configuration seen by mobile i at time n, we understand the family of points $\{X_j^n - X_i^n\}_j$) is stationary *and* ergodic (see [25] for these definitions). The distribution each such configurations is the Palm distribution of a planar Poisson p.p of intensity λ.

Given the ergodicity of the configurations seen by mobile i over time and the homogeneity assumptions, it makes sense to assume that the (time) point process of packets (fresh or to be relayed) arriving into the queue of station i is stationary and ergodic, with a time intensity τ' equal to $\tau' = \tau L/d(\lambda, p)$. The ergodicity assumption would not be justified in case of a 0 motion as mobile i might then be a bottleneck having to relay a larger number of packets or experiencing a larger collision rate due to its particular location in configuration 0. The worst case scenario for mobile i (or equivalently an upper bound to the content of its queue) is obtained when considering the case where all other queues are always full (which is the analogue of the situation where all stations are backlogged in standard Aloha). In contrast with what happens in standard Aloha, where the probability of success in an infinite population model is 0 when all stations are blocked, in our model, thanks to spatial reuse, the probability for mobile i to transmit is still positive, equal to p, even when all mobiles have infinite backlogs The sequence of successful transmission times that mobile i would experience if backloged is also a stationary and ergodic since it is based on the stationary and ergodic sequence of configurations seen by i over time and the i.i.d. sequences of transmission coin tosses in all mobiles. It makes sense to assume that the sequence of successful transmission of mobile i when backloged and the arrival process in the queue of station i, are *jointly* stationary and ergodic since they are both functionals of the same sequence of configurations seen by station i. Loynes' theorem [25] can then be invoked to show that under assumption $\tau' < p$, that is equivalent to $\tau < C_d$, mobile i (and hence any mobile) has a queue size that is upper bounded by a finite stationary and ergodic process, which is a satisfactory definition of dynamic stability.

Of course, the above argument does not extend to the case with no mobility at all, where one can fear a bad behavior of the plain MSR Aloha protocol in some parts of the plane due to long lasting bottleneck local situations. However, we will show in the second part of this section, that under mild assumptions the MSR-Aloha protocol nevertheless provides a positive throughput and a positive progress to any node of the network and is still optimal in some sense.

Remark 6.1: Connectivity in mobile ad hoc networks is most often addressed as a *static* percolation question. One typically considers a snapshot of such a network and one says that two nodes are connected if a successful transmission is possible between them within this snapshot. One then defines connectivity either as the property that all nodes belong to the same connected component (e.g. [16]) or as the existence of an infinite connected component (e.g. [26]). Note that this snapshot connectivity condition is one of the requirements in Gupta and Kumar's transport capacity estimate [16]. The setting of this section can actually be viewed as a *dynamic* framework for addressing connectivity: within the framework described above, the network has to transport an infinite flow of fresh packets originating from all nodes, each with its own destination. The existence of a sequence of successful transmissions over time allowing the network to transport each fresh packet of this infinite flow in a finite number of slots is a new and quite natural definition of connectivity.

Let us now look at the average end to end delays. We concentrate on the case where $\tilde{d}$ is used and on the simplified attenuation model. When all queues are stable and reach a stationary regime as alluded to above, this new definition of connectivity is satisfied and in steady state, the mean delay for transporting a packet from origin to destination ought to be proportional to $L/\tilde{d}(\lambda, p^*)$. The multiplicative constant is the average steady state queuing delay through one relay. Each relay is a slotted queue with arrival rate $\tau L/\tilde{d}(\lambda, p^*)$ and service rate p^* per slot. Assume now that λ varies in a range where L is large compared to $1/\sqrt{\lambda}$, which is required for the multihop model of the last sections to make sense. Also assume that τ is chosen in such a way that the load factor $\tau L/(p^* \tilde{d}(\lambda, p^*))$ of each such queue is equal to some positive $\theta < 1$ when λ varies, which is required for dynamic stability. This last assumption can be rephrased by stating that we adapt τ to the density of nodes according to the formula $\tau = \theta p^* \tilde{d}(\lambda, p^*)/L = O\left(\frac{1}{\sqrt{\lambda}}\right)$. Then it makes sense to assume that the average stationary delay through one relay is approximately constant in λ. Since $\tilde{d}(\lambda, p) = \kappa/\sqrt{\lambda}$ for some κ (see (5.5)), we conclude that under the assumptions made above, this average stationary origin to destination delay ought to be proportional to $L\sqrt{\lambda}$.

2) The Case without Mobility: The aim of this part of the section is to study the behavior of MSR-Aloha as defined in §V-B in the particular case where nodes have no mobility at all. In this case, the dynamic stability result discussed in the first part of this section does not hold anymore. We show below that the under mild assumptions the protocol nevertheless provides a positive throughput and a positive progress to any node of the network and is still optimal in a sense defined below.

The setting is as follows:

- $\Phi = \{X_i\}$ denotes the locations of nodes; we still assume an infinite number of nodes in the plane with locations that remain fixed for all time slots;
- the medium access sequence of station i is an independent sequence of i.i.d. r.v's $\{e_n^i\}$, taking value 1 with probability p in slot n if the station is allowed to emit in this slot, and 0 else;
- the potential powers of node i is also an independent sequence of i.i.d. r.v's $\{S_n^i\}$.

Each time n when node i is allowed to emit, the interference for the signal emitted by node i at receiver j is $I_\Phi(n, i, j) = \sum_{k \neq i} S_n^k e_n^k l(|X_j - X_k|)$. This sequence is i.i.d.

Our only assumption is that the series $J_\Phi(n, x) = \sum_{k \neq i} S_n^k l(|X_j - x|)$ is almost surely (a.s.) convergent for all x. A simple example where this assumption is satisfied is that where the locations $\{X_i\}$ are a realization of an homogeneous Poisson point process and the $\{S_n^i\}$ have finite mean. It then follows from shot noise theory that the series itself is a.s. convergent indeed.

Let us show that for all positive real numbers η the probability that the random variable $J_\Phi(n, x)$ is less than η is positive: since the series $J_\Phi(n, x)$ is convergent, for all $\eta > 0$, there exists a finite subset of the indices F such that the sum of all its terms over the indices that do not belong to F is less than η. Hence, the probability that $J_\Phi(n, x)$ is less than η is is larger than the probability that $e_n^i = 0$ for all $i \in F$, which is positive since F is finite. Using this and the fact that S_n^i is independent of $I_\Phi(n, i, j)$, it is easy to check that the success of a transmission from i to j in slot n, namely the event $\{S_n^i l(|X_j - X_i|) > T I_\Phi(n, i, j)\}$ is of positive probability, which in turns implies that the progress from node i toward any final destination has a positive expectation.

Hence, MSR-Aloha provides a positive throughput to any node of any infinite network provided the interference created by all nodes in this network is finite at any point of the plane.

VII. Implementation Issues

This section addresses the design issues of a MSR-Aloha MAC protocol based on the notion of progress. As described in the model, MSR-Aloha is a slotted protocol. The slots can be obtained via the timing information of a positionning

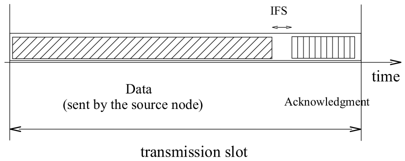


Fig. 8. Slot structure in MSR-Aloha

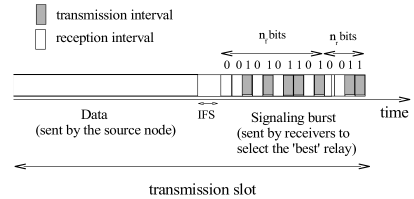


Fig. 9. Active signaling technique. When a burst is detected in a reception interval, the node quits the selection process. The "best relay" will be the receiver having used the greatest binary sequence for its signaling burst.

system as GPS or local atomic clocks (cesium-beam, rubidium clocks or hydrogen maser clocks) can provide nodes with such a synchronisation. MSR-Aloha being a random access MAC protocol, we also have to cope with collisions. Of course MAC collisions can be handled above the MAC layer but it can be easily shown that this leads to inefficient communication systems. This is why a good implementation of MSR-Aloha should use MAC acknowledgments for point to point packets as it is done in MAC protocols used for WLANs standards [3], [4]. We have assumed that MSR-Aloha is slotted. The slot can be divided in two parts: a data part (the main part) used by the emitter to send the packet and an acknowledgment part used by the receiver to indicate that it has correctly received the packet, (see Figure 8). There is an issue concerning the correct reception of the acknowledgment since the global geometry of the transmissions of acknowledgments is different from that of the transmission of data packets. This issue can be solved by using CDMA codes to send the acknowledgments. Each data packet mentions the CDMA code with which the recipients have to reply. As we will see later, all the receivers of a given packet will use this one code, and if the number of available CDMA codes is large enough, a random selection amongst available CDMA codes will make collision in codes of neighboring packet transmissions very unlikely. Since gains of more than 10dB are very easy to build, the correct reception of acknowledgments is very likely. In case a packet is not correctly acknowledged, MSR-Aloha will just have to send the packet again still using p (p^* in the optimized case) as the transmission probability.

Actually the MAC transmission policy of MSR-Aloha is extremly simple; whenever a MSR-Aloha node has a packet to send or to retransmit, it must send it using p (p^* in the optimized case) as transmission probability on each slot. The reception of an acknowledgement packet is used to qualify the correct transmission of a packet. The computation of p^* can be done a priori since it only requires to know the capture threshold T. Thus no special channel monitoring is needed.

Since MSR-Aloha is optimized for multihop networks, it must be closely related to a routing protocol. It is beyond the scope of this article to describe routing algorithms or to fully study how routing algorithms could work with MSR-Aloha. Most existing routing protocols do not use the geographical locations of nodes to compute routes, but research has shown that geographical location information can improve routing performance in mobile multihop networks [27], [28]. In the following we give a few hints concerning the use of MSR-Aloha with geographical position information.

We can imagine two techniques for MSR-Aloha: the next hop towards the final destination is directly computed or it is the result of a real transmission.

A. Direct computation of the next hop

For this solution it will be assumed that each network node knows the locations of all network nodes including itself. Thus the emitter knows its location (say 0), the direction of the final destination and the locations X_i of the emitter's neighbors expressed in the referential centered in the emitter in 0 and such that the x axis points to the destination. It can hence evaluate the functions $p_{|X_i|}v(X_i)$ for all i, where $v(x) = |x|(\cos(\arg(x)))^+$ and determine which is the best neighbor to be the next hop towards the final destination. Notice that this algorithm can also be implemented by the receivers. For this solution we assume that each node knows its location and locations of all the other nodes. Although actually only the locations of the neighbor nodes and the destination node are to be known, we can not claim that this scheme is completely independent of the network density λ. The following solution will have this property.

B. Next hop selected in a real transmission

We are looking for a mechanism which can at the same time acknowledge the reception of the current transmission and select among the potential receivers the one which offers the largest progress towards the destination. Such a mechanism can be implemented using in receivers an active signaling scheme inspired from the scheme used in the Hiperlan type 1 [4] access technique called EY-NPMA (Elimination Yield Non Preemptive Multiple Access). Note that EY-NPMA can be precisely analyzed in a single hop context, see [29].

The transmission slot is divided in a main part used by the transmitter to send the data and a remaining part of fixed length at the end of the slot which is used by the potential receivers. In this remaining part of the slot, the potential receivers (i.e., these who have successfully received the packet, and one of whom will forward it as the best relayer) send a burst of active signaling used for the selection of the best receiver. This burst is composed of a sequence of intervals of the same length in which a given receiver can either transmit or listen (see Figure 9). During this active signaling phase, each receiver applies the following rule: if he senses a signal during any of his listening intervals, he quits the selection process (namely he stops transmitting during the whole remaining part of the active signaling phase). The reason for this stems from the construction of signaling bursts (described below): the sensing of a transmission during a listening interval implies that a better relay has also correctly received the data information sent in the first part of the slot.

1) Signaling burst: Let us now describe the way signaling bursts are built. Each such burst is best represented by a

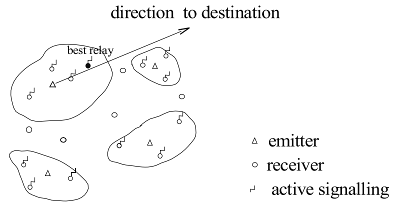


Fig. 10. Simultaneous transmissions with their receiving areas. The active signaling generate interference in other receiving areas.

binary sequence where 1 denotes a transmission interval and 0 denotes a listening one. This binary sequence is computed by each reception node as follows: the first n_f bits are computed by the receiver as a function of the progress the node offers as relay to the packet. Since we assume that the data packet includes the address of the source and the address of the final destination, a node can easily compute this progress it offers as relay to a received packet. For instance, we can assume that the first $n_f = 10$ bits give the progress offered by the relay coded in base 2. It is easy to check that if the progress offered by a receiver 1 is larger than that of receiver 2, then there exists an interval in which receiver 2 listens and receiver 1 emits, which is exactly the announced property. We then add n_r bits selected at random to discriminate between nodes offering the same progress. We will also assume that the sequence encompasses a last bit set to 1. This bit forces the receiver which remains active after the selection process to provide evidence of its activity. Thus if the emitter (the node who sent the data packet in the first part of the slot) cannot sense a signal in the last interval of the signaling burst, it infers that its packet has not been received or that the selection process between potential relays has failed and the data packet must be retransmitted according to the Aloha rule. To cope with interference between several selection processes taking place in different locations of the plane during the same signaling burst, we propose to use CDMA codes; the code to be used by all receivers of a given packet for acknowledging this packet and selecting the best relay can be provided in the packet.

There remain two issues concerning the auto-selection-acknowledgment process.

2) Length of the signaling burst: First, the binary sequence of the active signaling used for the selection of the optimal receiver should be long enough to be able to discriminate between all the potential receivers. A brisk analysis shows that the expected number of successful receivers of a given packet is $O(1)$ when $\lambda \to \infty$ and thus it is possible to fix a length of this binary sequence that will be sufficient for all λ. Indeed, for the simplified attenuation function and $W \equiv 0$, by Lemma 3.3, this expected number is equal to

$$\mathbf{E}\Big[\sum_{X_i \in \Phi^0} \delta(0, X_i, \Phi^1)\Big] \quad = \quad \frac{1-p^*}{p^* T^{2/\beta}} \int \bar{p}_{|x|}\,\mathrm{d}x\,.$$

3) Interference in the signaling burst: The second issue concerns the interference created in the active signaling phase. In Figure 10, we have shown simultaneous transmissions with their related receiving nodes. The aim of the auto-selection-acknowledgment is to select the best 'relay' towards a given final destination. The signaling technique used to perform this selection generates interference.

It is beyond the scope of this paper to give a detailed stochastic-geometry analysis of this problem. Instead, we will briefly explain why it is possible to fix a CDMA code length that provides enough orthogonality to cope with interferences in this phase, for all $\lambda > 0$. Note that there are two possible misbehaviors due to the interference in this phase.

One is when a potential receiver, in one of his listening intervals, does not correctly received the signal coming from one of his competitors. It may in such case infer that there is energy from another transmission attempt as actually there is a signaling burst sent by a better relay for this very transmission.

Another is when a potential receiver (or even the emitter when its looks at the last interval of the signaling burst) takes the interference resulting from signaling burst of other auto-selection as a signaling burst for its own signaling process.

First Problem. Correct receptions must be validated on a SIR basis. The following approximation/bound of the probability of the correct reception can be considered: $p^{\text{ack}}_{R^{\text{ack}}} = \Pr\Big(S^{\text{ack}} \geq \gamma T^{\text{ack}} I^{\text{ack}} (AR^{\text{ack}})^{\beta}\Big)$, where S^{ack} is the power used in active signaling, T^{ack} is the SIR threshold, R^{ack} is the distance on which the right signal is attenuated, I^{ack} is the interference in this phase and γ is the orthogonality factor due to usage of the CDMA codes of a given length. Note that, due to our previous considerations, $R^{\text{ack}} = O(1/\sqrt{\lambda})$ and $I^{\text{ack}} \approx \sum_{X_i \in \Phi^1} N_i S^{\text{ack}}_i (A|X_i|)^{-\beta}$, where $N_i = O(1)$ $(\lambda \to \infty)$ is the number of potential receivers of the packet transmitted by the emitter X_i. Thus, for the simplified attenuation function and $W \equiv 0$, by Lemma 3.3,

$$p^{\text{ack}}_{R^{\text{ack}}} = \bar{p}^{\text{ack}}_{O(1/\sqrt{\lambda})\sqrt{\lambda p^*}(\gamma T^{\text{ack}})^{1/\beta}}\,,$$

where $\bar{p}^{\text{ack}}_R$ denotes the probability of success for the model with $\gamma = T^{\text{ack}} = \lambda = 1$. This shows that $p^{\text{ack}}_{R^{\text{ack}}} = O(1)$ when $\lambda \to \infty$ and moreover, $p^{\text{ack}}_{R^{\text{ack}}} \to 1$ when $\gamma \to 0$.

Second Problem. This problem cannot be validated on a SIR basis. We have to fix an *absolute threshold* for the power of the signal received in the auto-selection process, based on which the user will be able to distinguish between the burst of its own signaling process and a burst of a different auto-selection. In order to make the process decentralized and auto-adapting to the density λ, we let each receiver X_i fix this threshold as some fraction τ of the power $S(A|X_i|)^{-\beta}$ it received from the emitter in the data part of the transmission slot. The fraction τ should be set to a value such that the probability P_1 of the detection of the signaling process associated with its own emission is large, while the probability P_2 of the detection of a burst from a different auto-selection is small. These two probabilities can be approximated/bounded as follows:

$$\begin{aligned} P_1 &= \Pr\Big(S^{\text{ack}}(AR^{\text{ack}})^{-\beta} \geq \tau S(A|X_*|)^{-\beta}\Big)\,,\\ P_2 &= \Pr\Big(I^{\text{ack}} \geq \tau S(A|X_\dagger|)^{-\beta}\Big)\,, \end{aligned}$$

where we take X_* to be the user the nearest to the emitter (it determines the largest threshold) and $X_\dagger$ to be the most remote user in the cluster of receivers participating in the autos-election (it determines the smallest threshold). Assuming $S^{\text{ack}} \approx S$ and knowing that $R^{\text{ack}} = O(1/\sqrt{\lambda})$ we have

$$\begin{aligned} P_1 &= \Pr\Big(|X_*| \geq \tau^{1/\beta} R^{\text{ack}}\Big)\\ &= e^{-\lambda(1-p^*)\tau^{2/\beta}(R^{\text{ack}})^2} = e^{-(1-p^*)\tau^{2/\beta} O(1)} \end{aligned}$$

for $\lambda \to \infty$ and we can take $\tau > 0$ small enough to make P_1 close to 1. For the second probability, assuming $S^{\text{ack}} \approx S$ and knowing that $|X_\dagger| = O(1/\sqrt{\lambda})$, we have

$$P_2 = 1 - \bar{p}^{\text{ack}}_{|X_\dagger|\sqrt{\lambda p^*}(\gamma/\tau)^{1/\beta}}, = 1 - \bar{p}^{\text{ack}}_{\sqrt{p^*}(\gamma/\tau)^{1/\beta}O(1)}$$

and we can take $\gamma > 0$ large enough to make P_2 close to 0.

4) Summary: The receiver selection version of the MSR-Aloha protocol has the following interesting properties:

- for any given MAP p, it realizes a mean progress $d(\lambda, p)$;
- its throughput scales in at least $O(\sqrt{\lambda})$ (this follows from the results of §V-B);
- the protocol does not require that $p_r(\lambda p)$ and λ be known;
- there are no extra connectivity requirements (which explains why its throughput is in $O(\sqrt{\lambda})$ and not in $O(\frac{\sqrt{\lambda}}{\sqrt{log(\lambda)}})$) and hence no neighborhood management;
- it is fully decentralized and it scales to arbitrarily large configurations (as shown by the analysis considering an infinite number of nodes scattered through the whole plane with any given density).

APPENDIX

We give here sufficient conditions for

$$p_r(\lambda) = \Pr\left(r \le \frac{S^{1/\beta}}{A(T(W + I_\Phi))^{1/\beta}}\right) \qquad \text{(A.1)}$$

to be continuous function of r and λ and for $\lim_{\lambda\to\infty} \lambda p_r(\lambda) = \lim_{r\to\infty} r p_r(\lambda) = 0$.

Proposition A.1: If $\Pr(S > 0) > 0$ then the Poisson shot noise I_Φ is absolutely continuous w.r.t the Lebesgue measure (has a density), consequently the same is true for $\frac{S^{1/\beta}}{A(T(W+I_\Phi))^{1/\beta}}$ and hence $p_r(\lambda)$ is continuous in r and, by Lemma 3.3, in λ.

For the proof see Prop. A.2 in [30].

Proposition A.2: Suppose $\Pr(S > 0) > 0$. If $\mathbf{E}[S^{2/\beta}] < \infty$ then $\lim_{r\to\infty} r p_r(\lambda) = 0$. If $\mathbf{E}[S^{3/\beta}] < \infty$ then $\lim_{\lambda\to\infty} \lambda p_r(\lambda) = 0$.

Proof: Note by (A.1) that for $\lim_{r\to\infty} r p_r(\lambda) = 0$ it suffices to have $\mathbf{E}[\frac{S^{2/\beta}}{A(T(W+I_\Phi))^{2/\beta}}] < \infty$, whereas for $\lim_{r\to\infty} \lambda p_r(\lambda) = \lim_{r\to\infty} \lambda p_{r\sqrt{\lambda}}(1) = 0$ it suffices to have $\mathbf{E}[\frac{S^{3/\beta}}{A(T(W+I_\Phi))^{3/\beta}}] < \infty$. The result follows from independence of S, W, I_Φ and from the fact that if $\Pr(S > 0) > 0$ then $\mathbf{E}[I_\Phi^{-\gamma}] < \infty$ for any $\gamma > 0$. Indeed, take $\epsilon >$ such that $\Pr(S > \epsilon) > 0$ and observe that $I_\Phi \ge I'$ where $I' = \epsilon(A|X_*|)^{-\beta}$, where X_* is the point X_i of Φ which is the closest to 0 and such that $S_i > \epsilon$. The distribution function of $|X_*|$ is $\Pr(|X_*| \ge r) = e^{-\lambda \Pr(S>\epsilon)\pi r^2}$ and it is easy to see that $\mathbf{E}[I'^{-\gamma}] = \epsilon^{-\gamma} A^{\gamma\beta} \mathbf{E}[|X_*|^{\gamma\beta}] < \infty$ for any $\gamma > 0, \beta > 0$. ■

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