Wireless communications: from simple stochastic geometry models to practice III Capacity

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Workshop on Probabilistic Methods in Telecommunication

WIAS Berlin, November 14–16, 2016

Outline

- COVERAGE
- CONNECTIVITY
- **CAPACITY**

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- Ability to serve simultaneously many users. How many? Quality of service in function to the number of served users.

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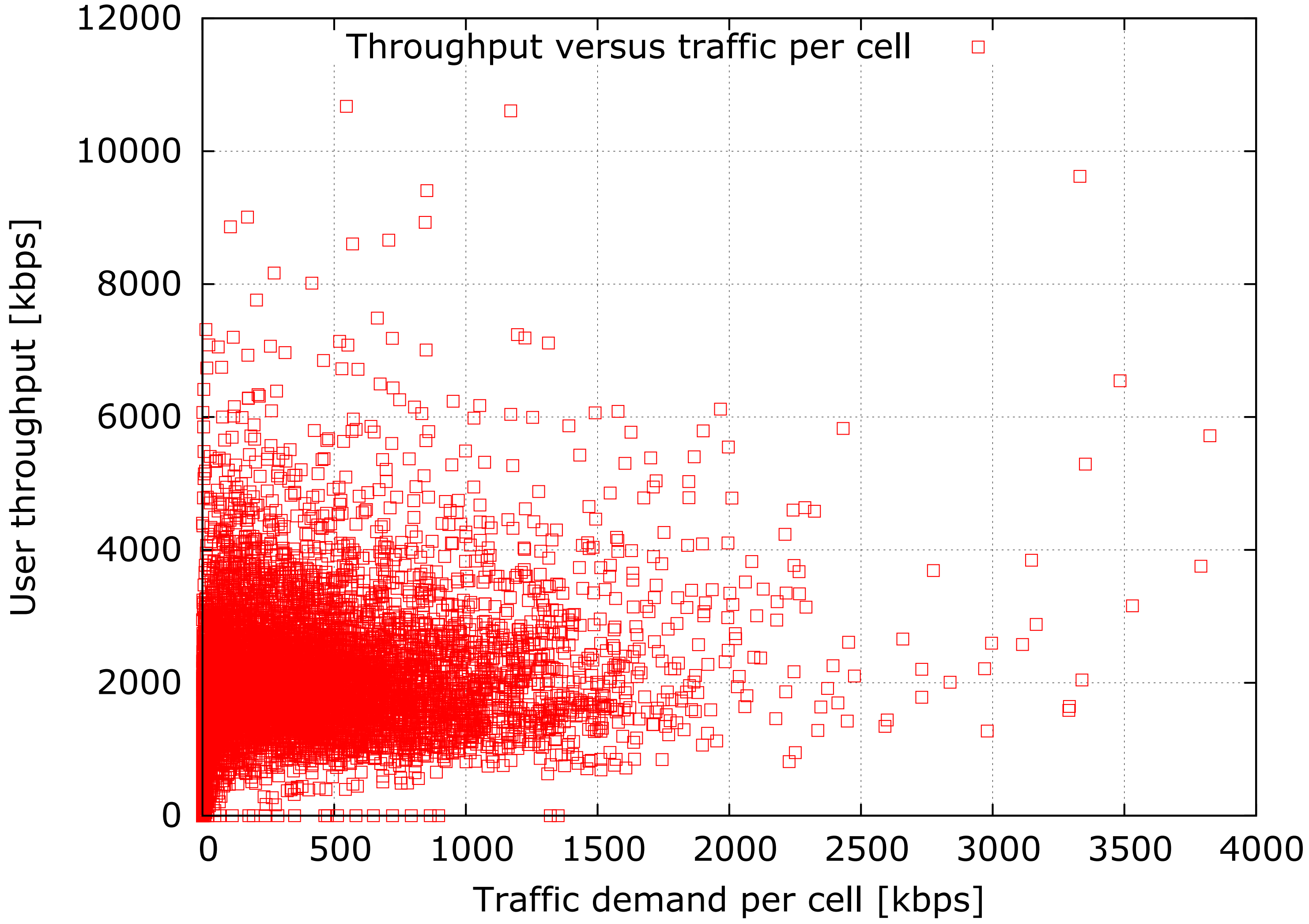
- Ability to serve simultaneously many users. How many? Quality of service in function to the number of served users.
- Queueing theory in association with stochastic geometry.
- Space-time models. Simulations required for quantitative results.
- We shall present some model capturing the dependence between the traffic demand and the quality of service in large cellular networks, validated w.r.t. some real data.
- Fruit of long-standing collaboration with M.K. Karray from Orange.

Motivation

Real data



Cellular network deployed in a big city. 9288 measurements (387 stations during 24 hours) of some given day.

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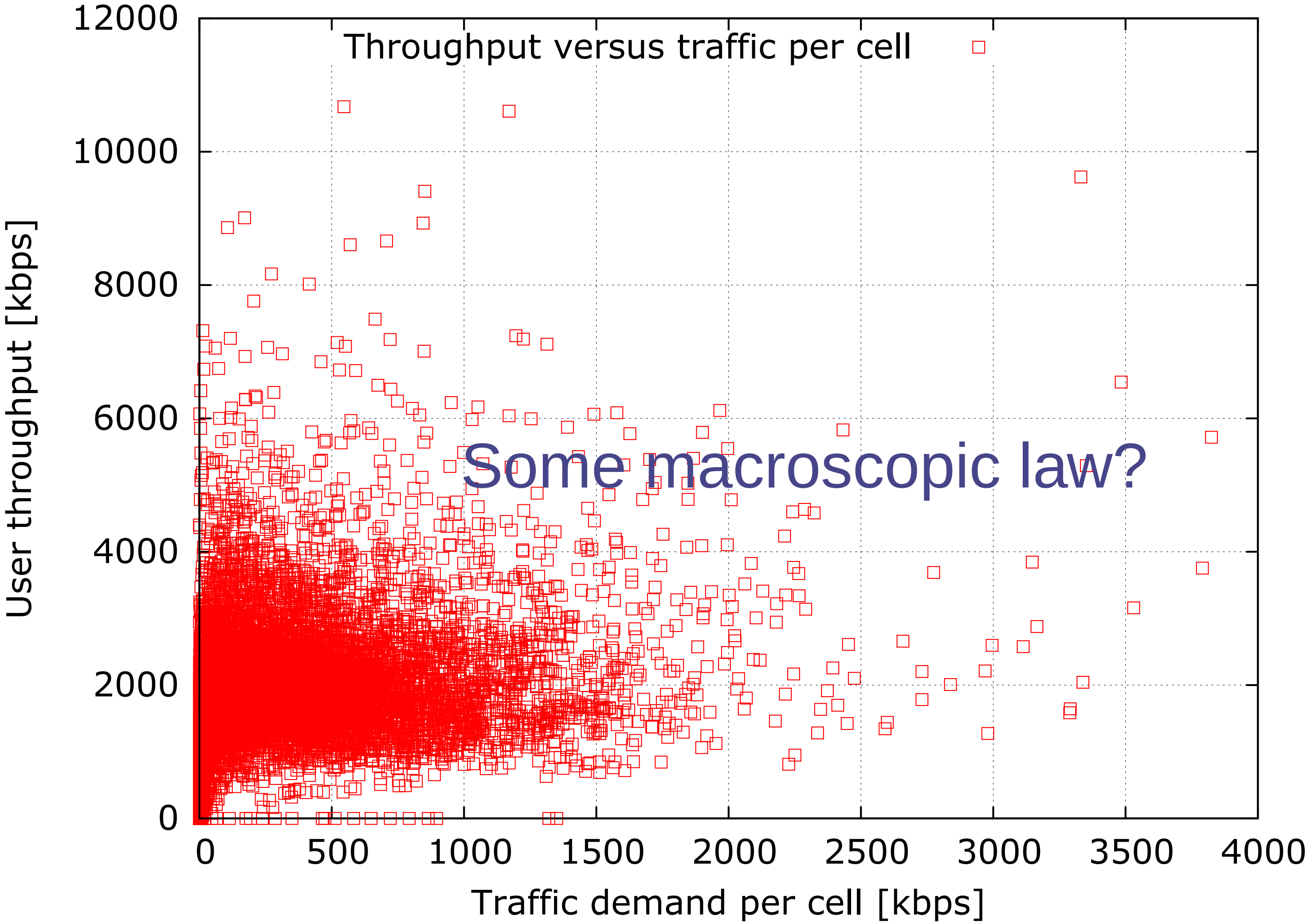

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- Mean user throughput: average “speed” [bits/second] of data transfer during a typical data connection.
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- User-centric QoS metric.
- Network heterogeneous in space and time. Appropriate temporal and spatial averaging required.

Various levels of averaging

- Information theory (over bits processed in time)
- Queueing theory (over users/calls served in time)
- Stochastic geometry (over geometric patterns of cells and users)

We are interested in the radio part of the problem.

Outline

- HOMOGENEOUS NETWORKS
- TECHNOLOGY HETEROGENEOUS NETWORKS; micro/macro cells
- SPATIALLY INHOMOGENEOUS NETWORKS; varying density of BS deployment, frequency dimensioning problem.

HOMOGENEOUS NETWORKS

Queuing theory

for one cell

Little's law

Consider a service system in its steady state. (Here one network cell during a given hour). Denote

N — mean (stationary, time average) number of users (calls) served at a given time

λ — average number of call arrivals per unit of time [second]

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Applies in to a very general system of service, production, communication... No probabilistic assumptions regarding the distribution of the arrivals, service times. Not related to a particular service policy. Just stationarity!

Mean user throughput via Little's law

Denote:

$\frac{1}{\mu}$ — average data volume [bits] transmitted during one call

$\rho := \frac{1}{\mu} \times \lambda$ mean traffic demand [bits/second]

$r := \frac{1}{\mu}/T$ mean user throughput [bits/second]

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From Little's law

$$N = \lambda T \quad \Rightarrow \quad \frac{1}{T} = \frac{\lambda}{N} \quad \Rightarrow \quad \frac{1}{\mu T} = \frac{\lambda}{\mu N}$$

$$r = \frac{\rho}{N} = \frac{\text{mean traffic demand}}{\text{average number of users served at a given time}}$$

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But N depends on ρ. What is the relation between N and ρ?

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Number of users in the system has a geometric distribution with the mean

$$N = \begin{cases} \frac{\theta}{1-\theta} & \text{when } \theta < 1 \\ \infty & \text{when } \theta \geq 1 \end{cases} \qquad (*)$$

Spatial processor sharing queue

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Denote: $\boldsymbol{V}$ — the service zone (cell)

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How to model service rates $\boldsymbol{R(y)}$?

Information theory

for the link quality

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E.g. the Shannons law for the Gaussian channel says

$$R(y) = aW \log(1 + \text{SNR}(y)),$$

where

W — channel bandwidth [Hertz]

SNR — signal-to-noise ratio

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More specific expressions for MMSE, MMSE-SIC, MIMO, etc...

Concluding for one cell

Throughput r v/s traffic demand ρ

Putting together previously explained relations for one cell V $r = \rho/N$, $N = \theta/(1-\theta)$, $\theta = \rho/R$ one obtains

$$\boxed{r = (\rho^c - \rho)^+,}$$

where

$$\rho^c := R = \frac{|V|}{\int_V 1/R(\text{SINR}(y))\,dy}$$

can be interpreted as the critical traffic demand for cell V and $R(\text{SINR}(y))$ are location dependent user peak service rates, which depend on the SINR experienced at y.

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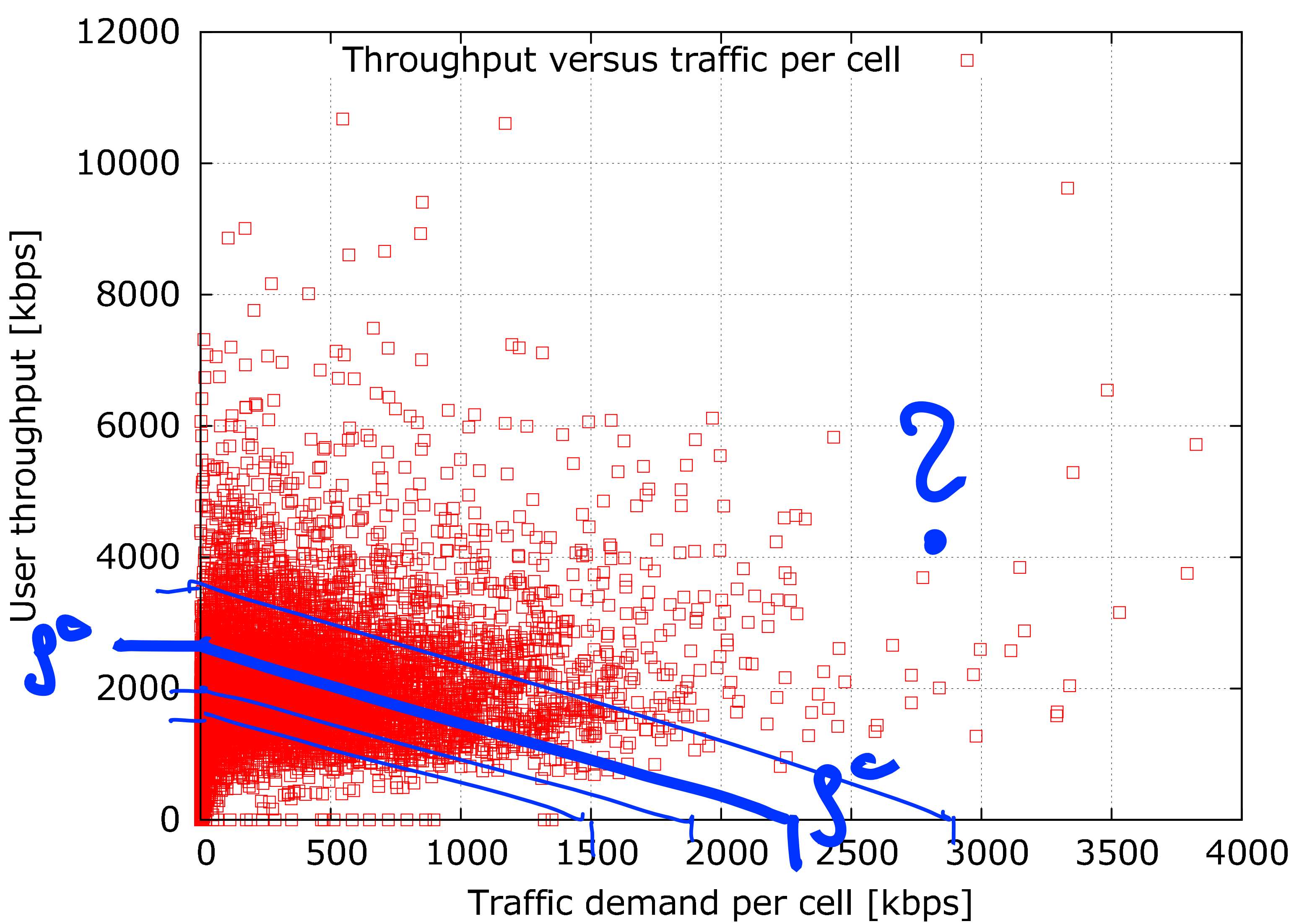
Adequate model for the spatial distribution of the SINR in cellular networks is required!

$r = (\rho^c - \rho)^+$ cell by cell?



Stochastic geometry

for a large multi-cell network

Network of interacting cells

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However, $\mathrm{SINR}_i(y)$ depends on the extra-cell interference.
Study of such dependent PS-queues is impossible!

Decoupling of cells

Simplifying idea:
"decoupling cells in time", keeping only spatial dependence

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Come up with a model in which stochastic processes describing the evolution of PS-queues at different cells are conditionally independent, given locations of network BS, which will be assumed (random) point process.

Cell load equations

Assume that the cell loads θ_i $i = i, \ldots$ satisfy the system of fixed-point equations

$$\theta_i = \rho \int_{V_i} \frac{1}{R\left(\frac{P/l(|y-X_i|)}{N+P\sum_{j\neq i}\min(\theta_j,1)/l(|y-X_j|)}\right)} dy$$

where X_i is the location of the BS i, $l(\cdot)$ is the path loss function, N external noise, P BS transmit power.

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Existence of a solution! We assume uniqueness; partially supported by [Siomina&Yuan, "Analysis of cell load coupling for LTE network ..." IEEE TWC 2012].

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There is no one global network stability condition.
Recall: for a given traffic demand ρ per unit of surface, cell i is stable provided $\rho_i = \rho|V_i| < \rho_i^c$.

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Denote:
$\mathcal{S} = \bigcup_{i:\rho_i<\rho_i^c} V_i$ — union of all stable cells
$\pi_{\mathcal{S}}$ — fraction of the surface covered by $\mathcal{S}$; equivalently: probability that the typical user is covered by a stable cell.

Mean user throughput in large network

Define the mean user throughput in the network as the ratio

$$r = \frac{\text{average number of bits per data request}}{\text{average duration of the data transfer in the stable part } \mathcal{S}}$$

in the stable part $\mathcal{S}$ of the network;
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We have

$$\boxed{r := \frac{\rho\,\pi_{\mathcal{S}}}{\lambda_{\mathrm{BS}} N^0}}$$

where
λ_{BS} is the density of BS deployment (stationary, ergodic)
$N^0 := 1/n \sum_{i:\rho_i<\rho_i^c}^{n} N_i$ is the spatial average of the (steady-state mean) number of users per stable cell;
(typical (stable) network cell interpretation).

“Mean cell” approach

Mean cell load: constant $\bar{\theta}$ satisfying

$$\bar{\theta} = \frac{\rho}{\lambda_{BS}} \mathbf{E}\left[1/R\left(\frac{P/l\left(|X^*|\right)}{N + P\sum_{X_j \neq X^*} \bar{\theta}/l\left(|y - Z|\right)}\right)\right]$$

, where

X^* BS serving the typical user (located at 0).

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Other “mean cell” characteristics calculated from $\bar{\theta}$ and $\bar{\rho}$ as in the case of a single (isolated) cell:

$\bar{N} = \dfrac{\bar{\theta}}{1 - \bar{\theta}}$ — mean number of users

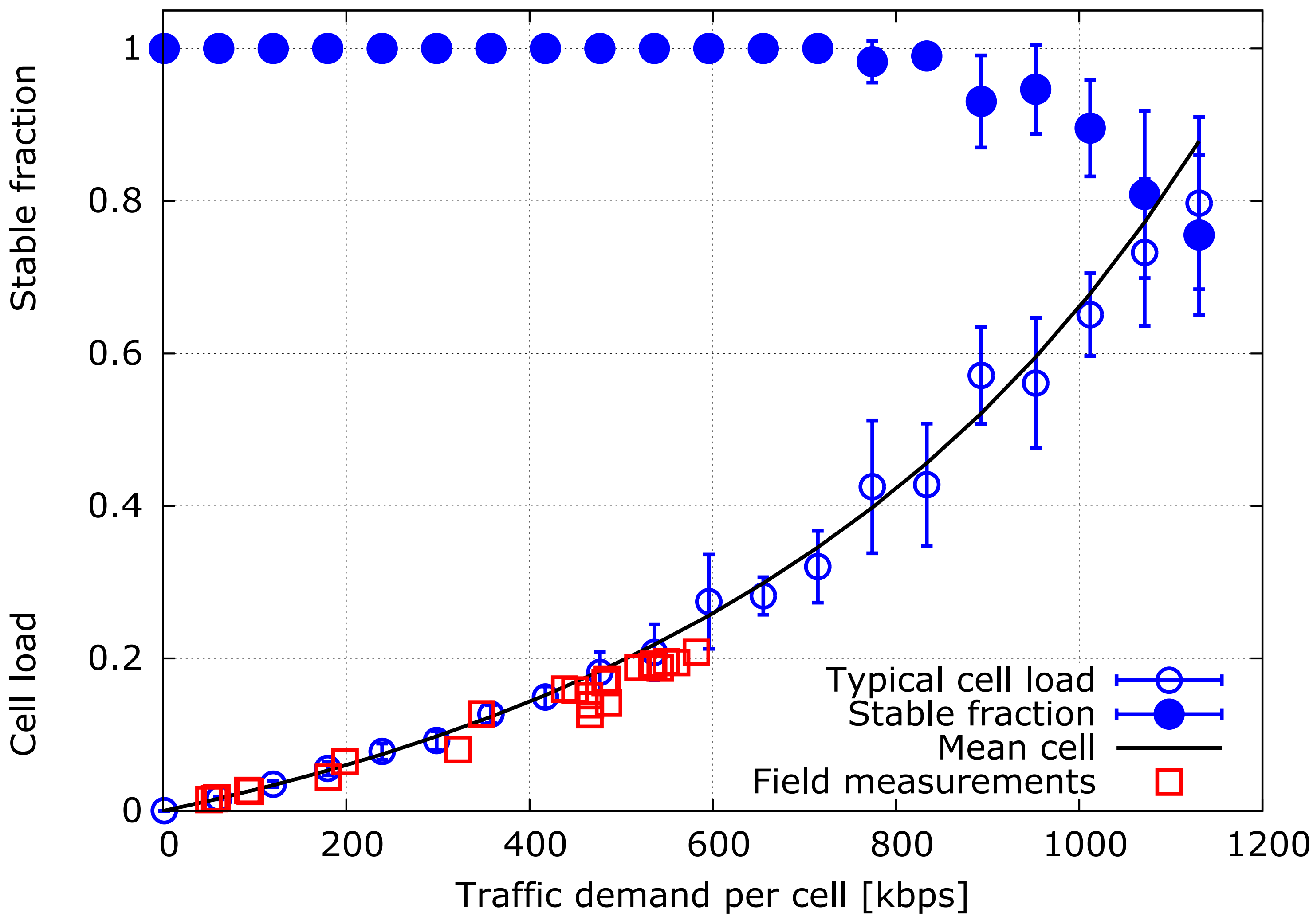
$\bar{r} = \bar{\rho}(1/\bar{\theta} - 1)$ — mean user throughput.

Numerical results

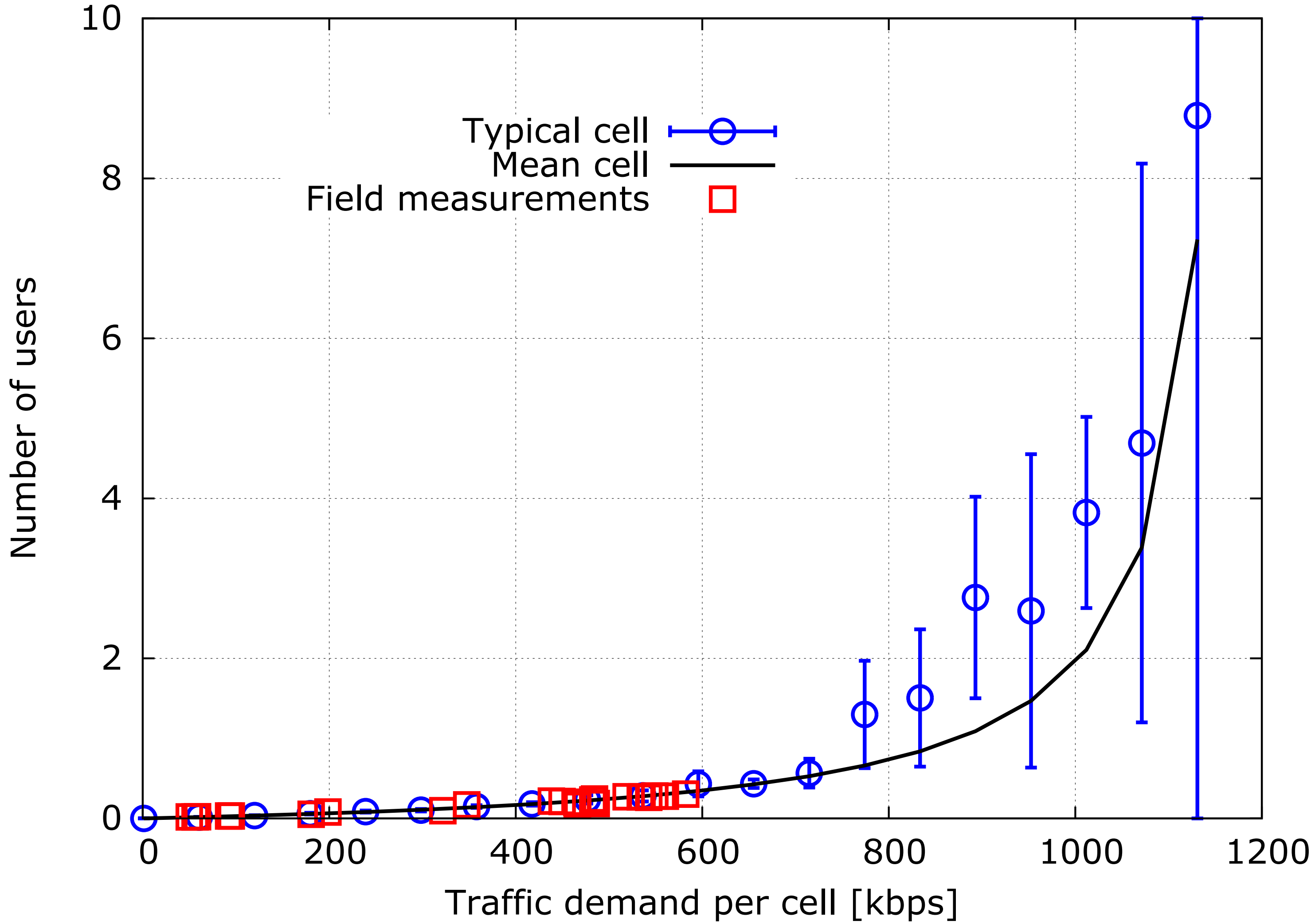
for some real network

(a homogeneous BS deployment region)

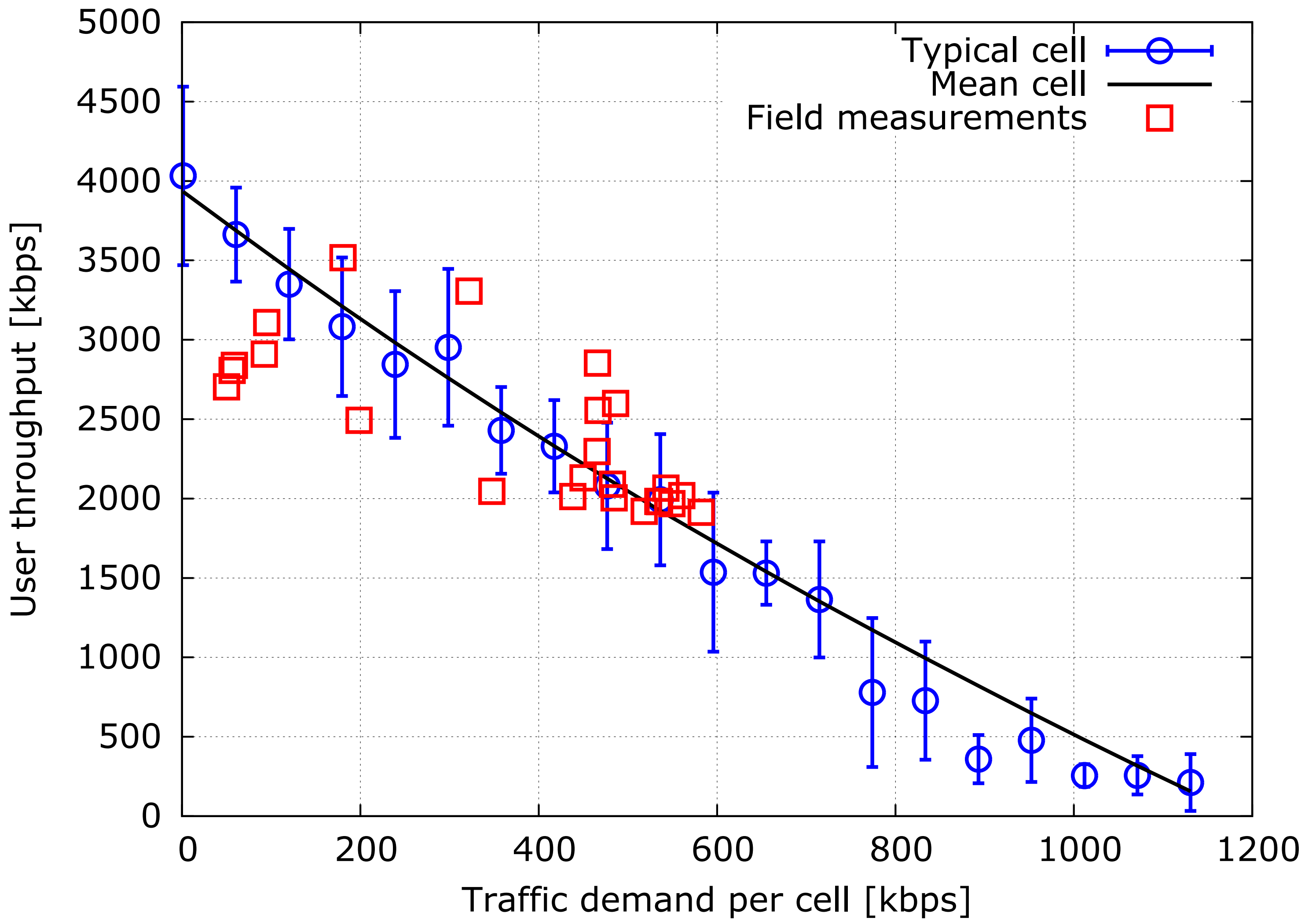
Mean cell load and the stable fraction



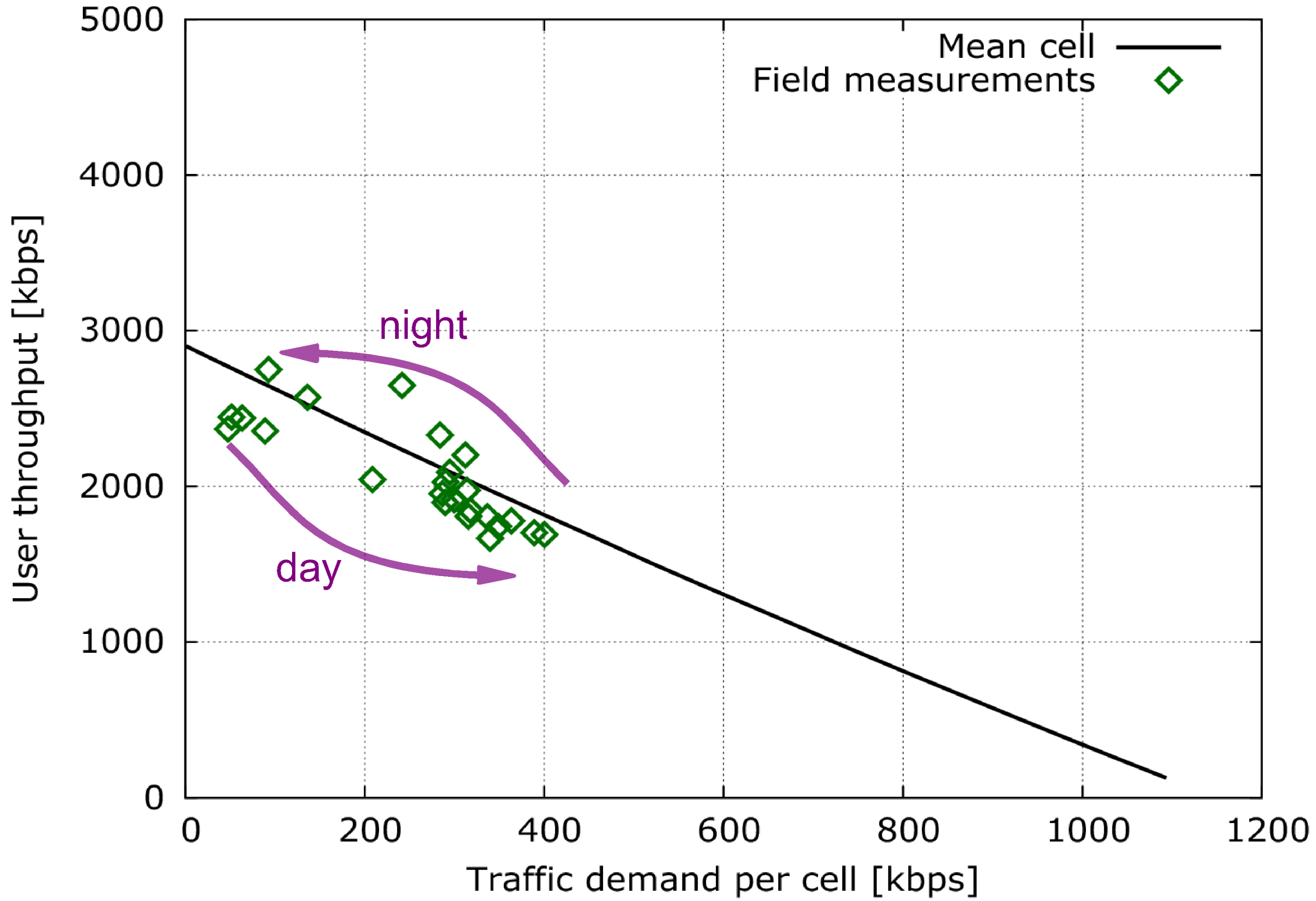
Mean number of users per cell



Mean user throughput



Mean user throughput, another example



Conclusions

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- QoS in large irregular multi-cellular networks using information theory (for link quality), processor sharing queues (traffic demand and service model, cell by cell), stochastic geometry (to handle a spatially distributed network).
- The mutual-dependence of the cells (due to the extra-cell interference) is captured via some system of cell-load equations accounting for the spatial distribution of the SINR.
- Identify macroscopic laws regarding network performance metrics involving averaging both over time and the network geometry.
- Validated against real field measurement in an operational network.

What next?

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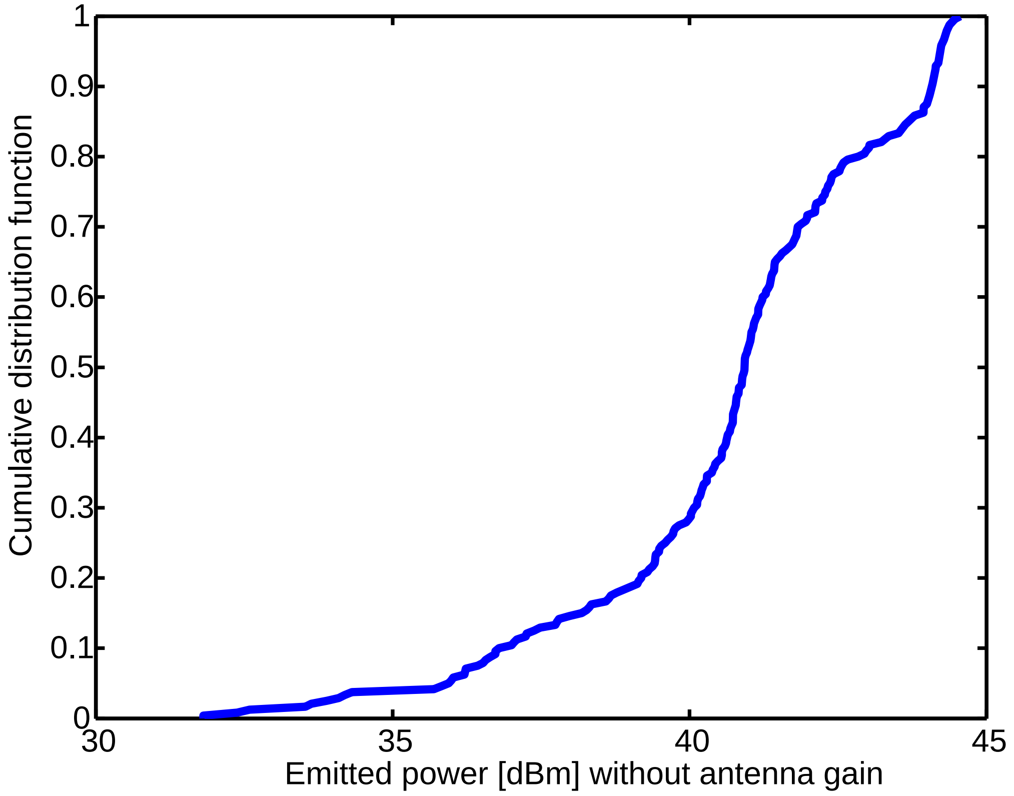
- Spatially inhomogeneous networks; varying density of BS deployment, as observed at the level of a whole country; useful for macroscopic network planning and dimensioning.

TECHNOLOGY HETEROGENEOUS NETWORKS

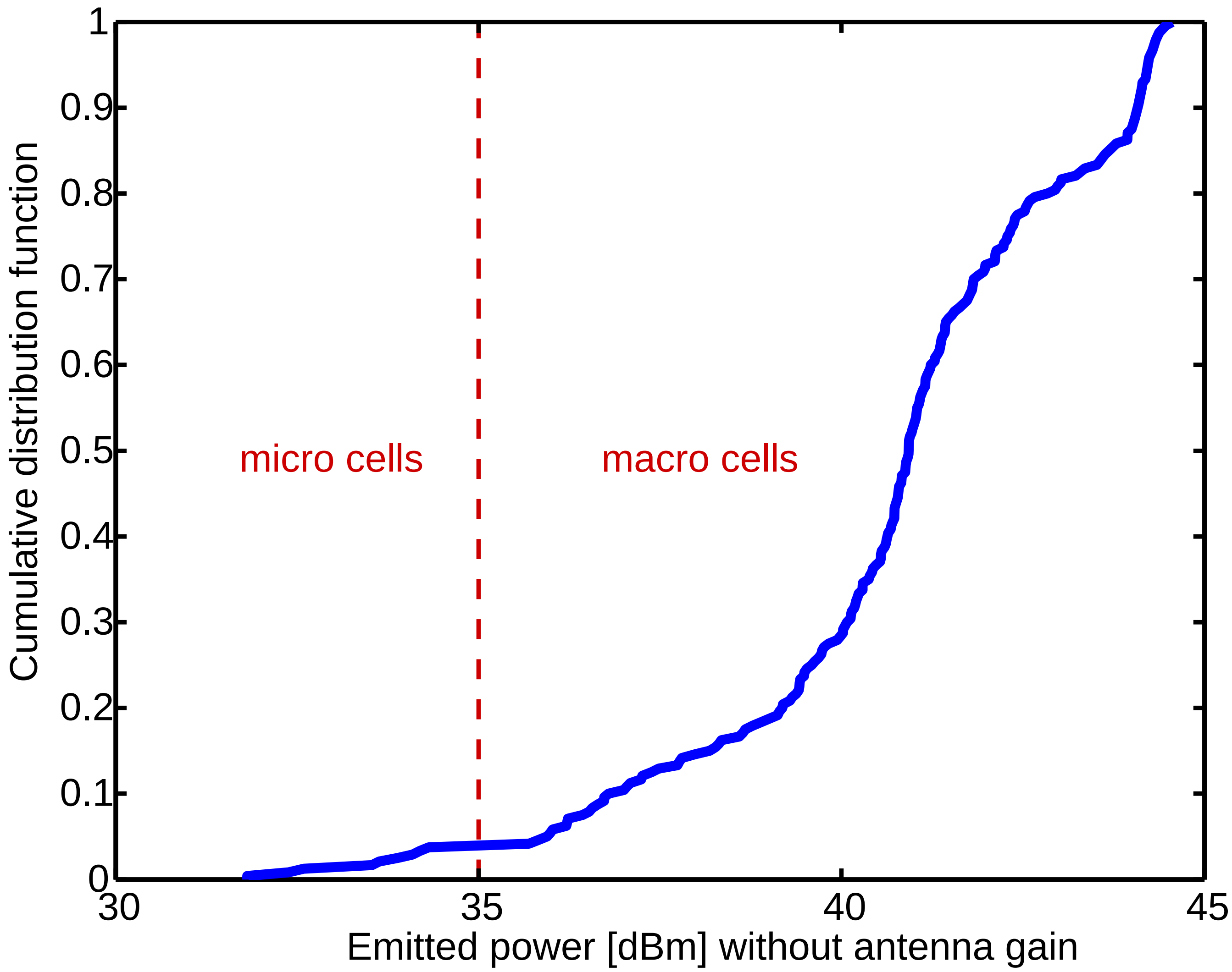
Motivation

Variable BS transmission power



Data from a commercial network in a big European city.

Multi-tier network approach



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Cell loads θ_i mutually dependent $\theta_i = \theta_i(\{\theta_j : j \neq i\})$ via the extra-cell interference (cell load equations).

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- Cell V_i of BS $X_i \in \Phi$ is the strongest received signal zone

$$V_i = \{y : P_{X_n}\,(y) \geq P_Y\,(y) \text{ for all } Y \in \Phi\}.$$

Cell load equations

Cell loads $\{\theta_i : i = i, \ldots\}$ is the (minimal) solution of the system of fixed-point equations

$$\theta_i = \rho \int_{V_i} \frac{1}{R\left(\frac{P_{X_i}(y)}{N + \sum_{j \neq i} \min(\theta_j, 1) P_{X_j}(y)}\right)} dy \quad \text{for all } i,$$

where N is the external noise power.

Multi-tier network, basic facts

Consider J types (tiers) of BS characterized by different (constant) transmitting powers P_j, $j = 1, \ldots, J$, modeled by independent homogeneous Poisson point processes Φ_j of intensity λ_j.

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FACT 2: Probability that the cell covering the typical user is of type j is equal to a_j/a, where $a = \sum_j a_j$ and

$$a_j := \frac{\pi \mathsf{E}\left[S^{2/\beta}\right]}{K^2} \lambda_j P_j^{2/\beta}.$$

Network equivalence

FACT 3: The distribution of the signal powers received by the typical user in the multi-tier network is the same as in the homogeneous network with all emitted powers equal to

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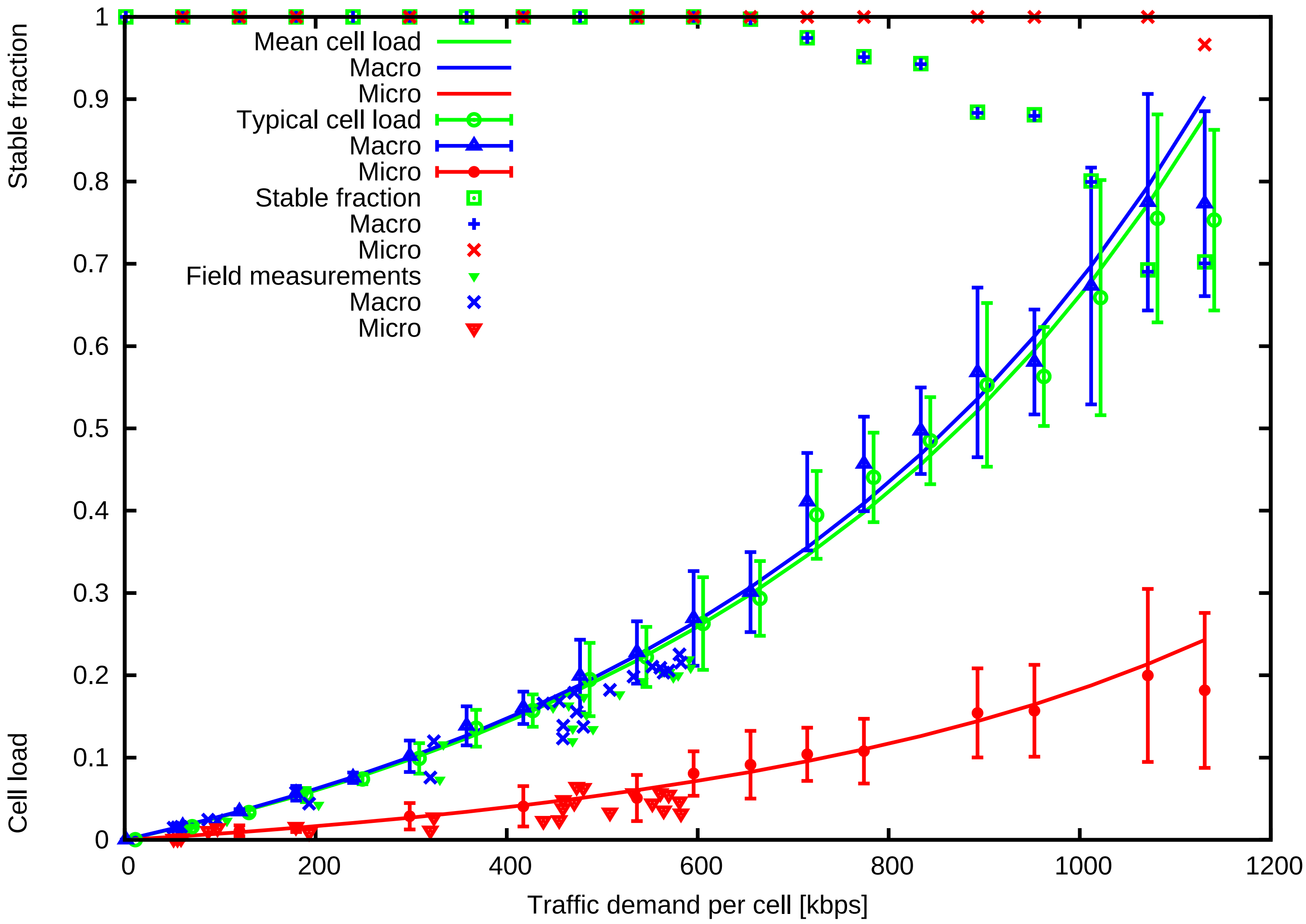
FACT 4: The mean load of the typical cell of type j is

$$\bar{\theta}_j = \bar{\theta} \frac{\lambda a_j}{\lambda_j a} = \bar{\theta} \frac{P_j^{2/\beta}}{P^{2/\beta}} ,$$

where $\bar{\theta}$ is the load of the typical cell in the equivalent homogeneous network.

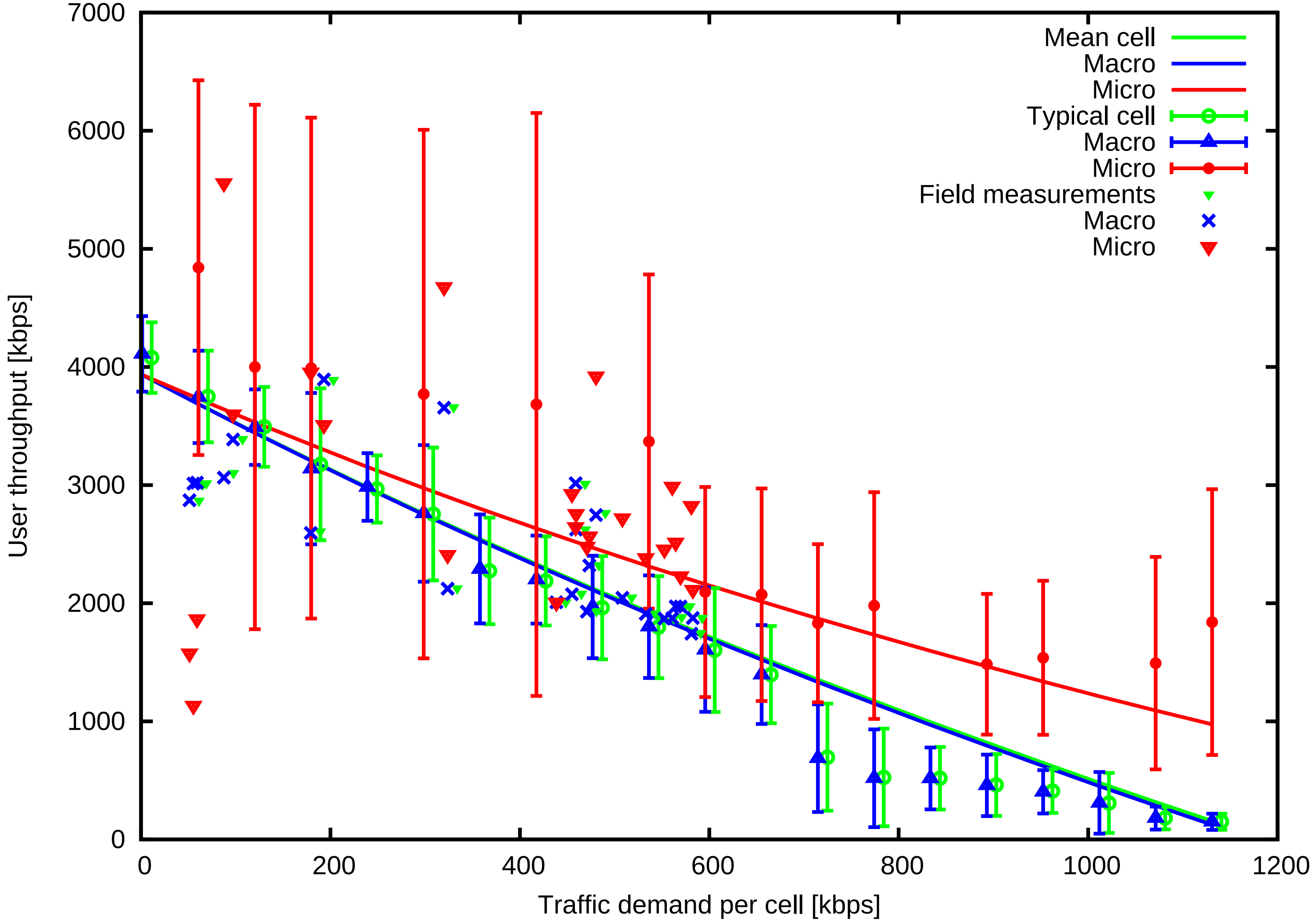
Cell load per cell type



Real data for micro/macro cells fit the analytical prediction.

(Data from a commercial network in a big European city.)

Mean user throughput prediction per cell typ



Real data for micro/macro cells (quite) fit the analytical prediction.

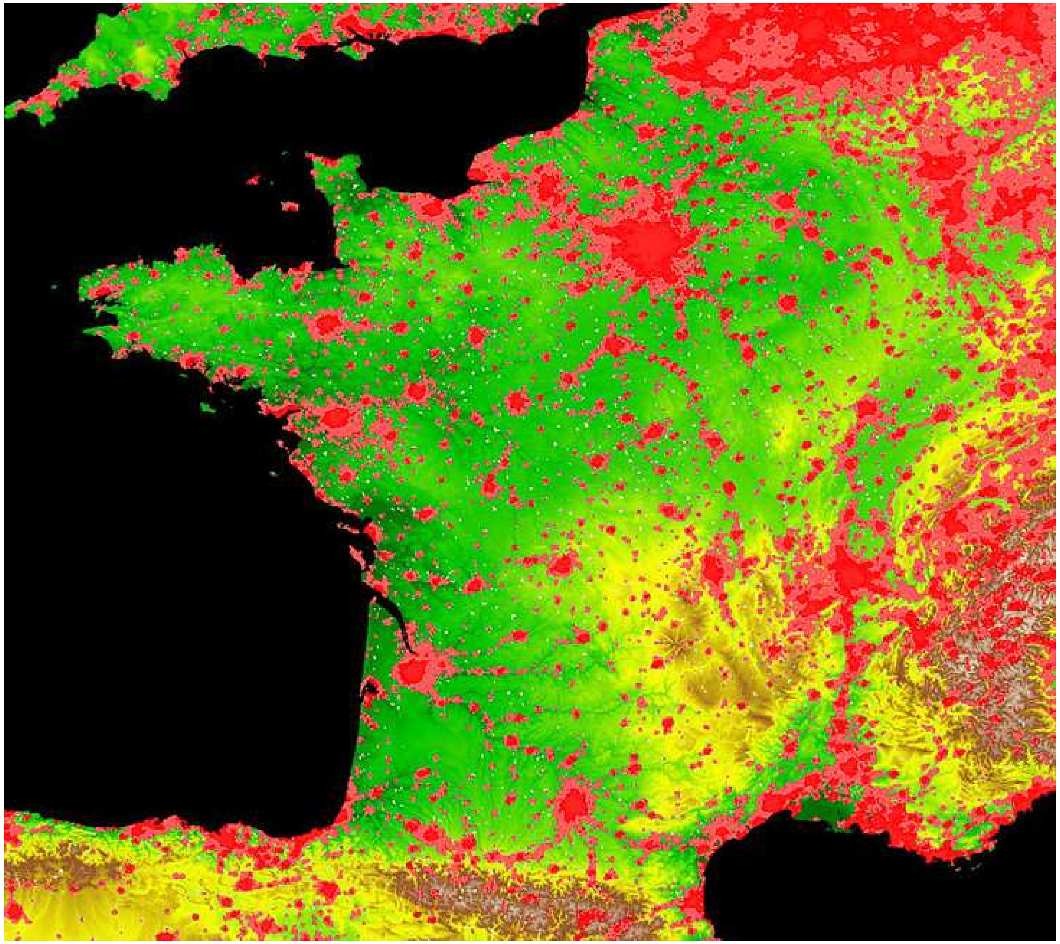
(Data from a commercial network in a big European city.)

SPATIALLY INHOMOGENEOUS NETWORKS

Motivation

Frequency dimensioning problem

What frequency bandwidth required for a network deployed across the whole country?



Scaling laws for homogeneous networks

- Assume path-loss function $l(x) = (K|x|)^{\beta}, \quad x \in \mathbb{R}^2$, where $K > 0$ and $\beta > 2$.
- For $\alpha > 0$ consider a network obtained from the original one by the following dilation:
  - base station locations $\Phi' = \{X' = \alpha X\}_{X \in \Phi}$
  - intensity of traffic demand $\rho' = \rho/\alpha^2$
  - distance coefficient $K' = K/\alpha$
  - shadowing processes $S_i'(y) = S_i\left(\frac{y}{\alpha}\right)$

  while preserving the original powers $P_i' = P_i$
- Consider the cells V_i' of the rescaled network and their respective characteristics their ρ_i', $\rho_i'^{c}$, r_i', N_i', θ_i'.

Scaling laws for homogeneous networks

FACT: For $\alpha > 0$ consider the homogeneous network scaling as above. We have $V_i' = \alpha V_i$. Moreover, the minimal solution of the system of load equations $\{\theta_i\}$ of the original network is the minimal solution of the scaled one

$$\theta_i' = \theta_i \quad \text{for all } i.$$

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Consequently $\rho_i'^{c} = \rho_i^{c}$, $r_i' = r_i$, $N_i' = N_i$.
And thus, the typical cell of the scaled networks has the same mean characteristics as the typical cell of the original network. In particular $\mathsf{E}^0[\theta_0'] = \mathsf{E}^0[\theta_0]$.

Networks with homogeneous QoS response

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- The scaling laws: locally, in urban, suburban and rural areas, the same relations between the mean performance metrics and the (per-cell) traffic demand.
- One relation is enough to capture the key dependencies for heterogeneous network dimensioning!

Justifying assumptions

Assumption $K_i/\sqrt{\lambda_i} = \mathbf{const}$ means that the average distance D between neighbouring base stations is inversely proportional to the distance coefficient of the path-loss function: $D \times K = \mathbf{const}$.

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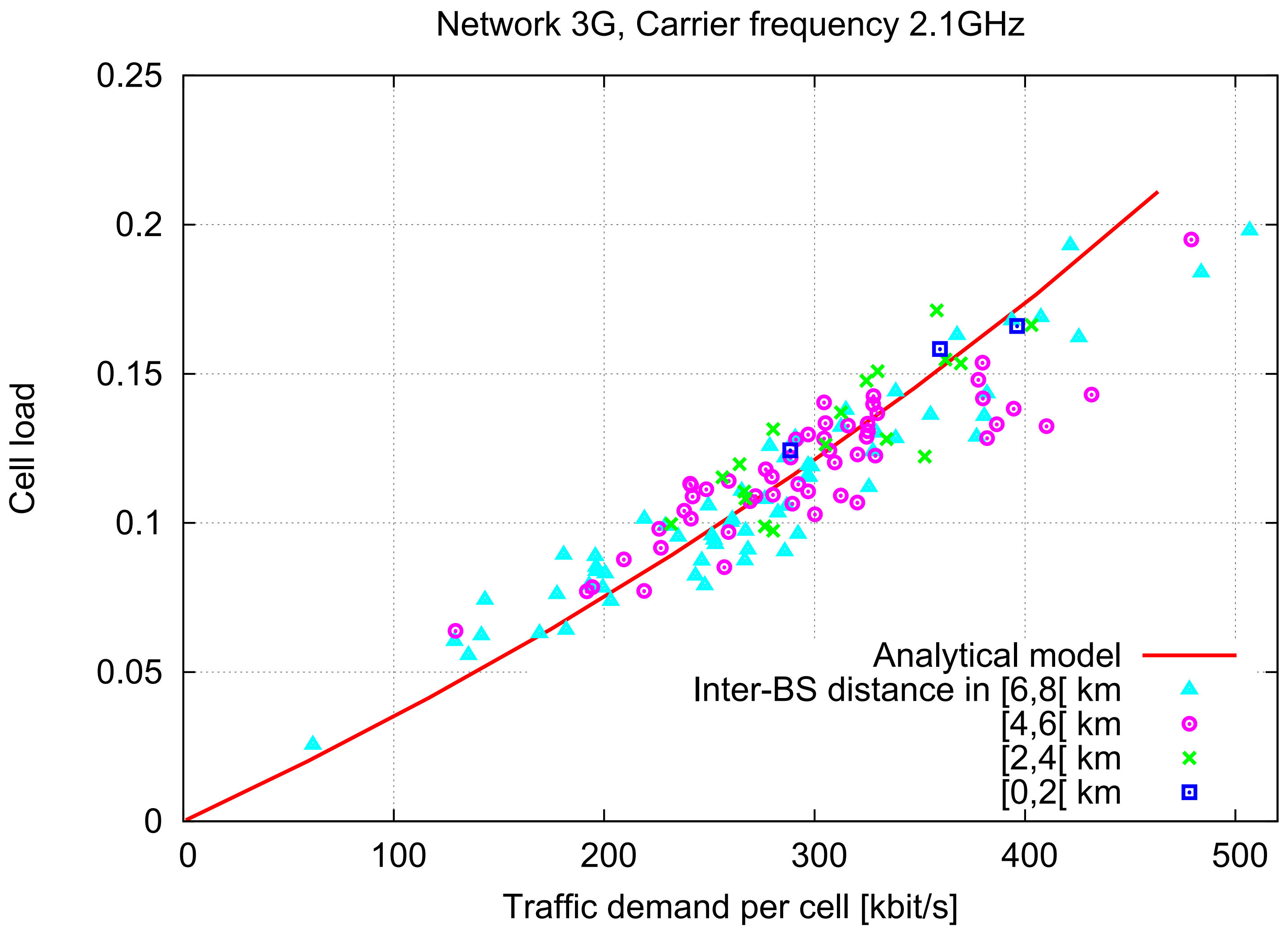
Example: propagation parameters for carrier frequency 1795MHz.

Environment	A	B	$K = 10^{A/B}$	K_{urban}/K
Urban	133.1	33.8	8667	1
Suburban	102.0	31.8	1612	5
Rural	97.0	31.8	1123	8

Suburban and rural BS distance D should be, respectively, 5 and 8 times larger than in the urban scenario. Realistic?

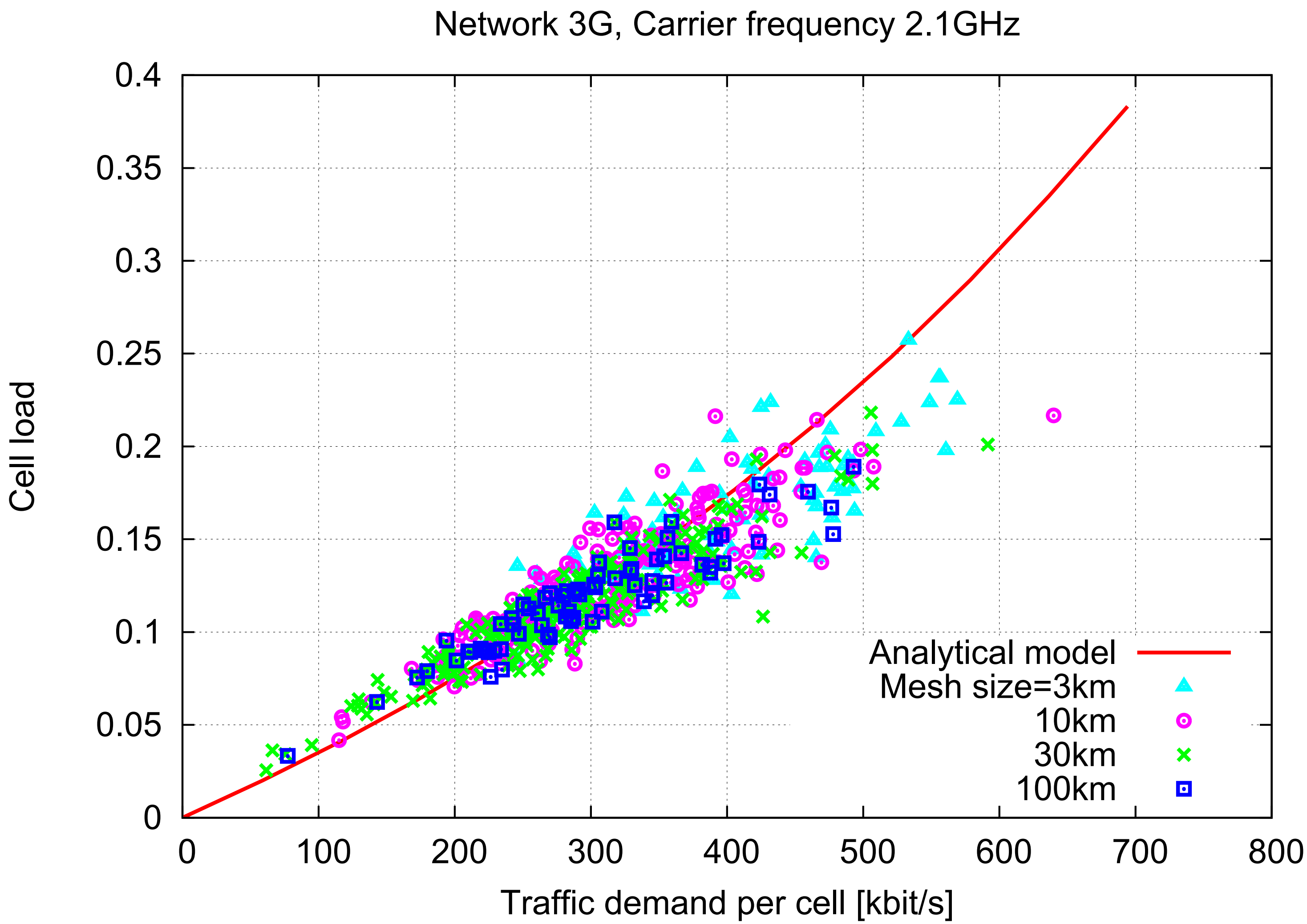
Cell load in different network-density zones



Real data in different zones fit the same analytical prediction.

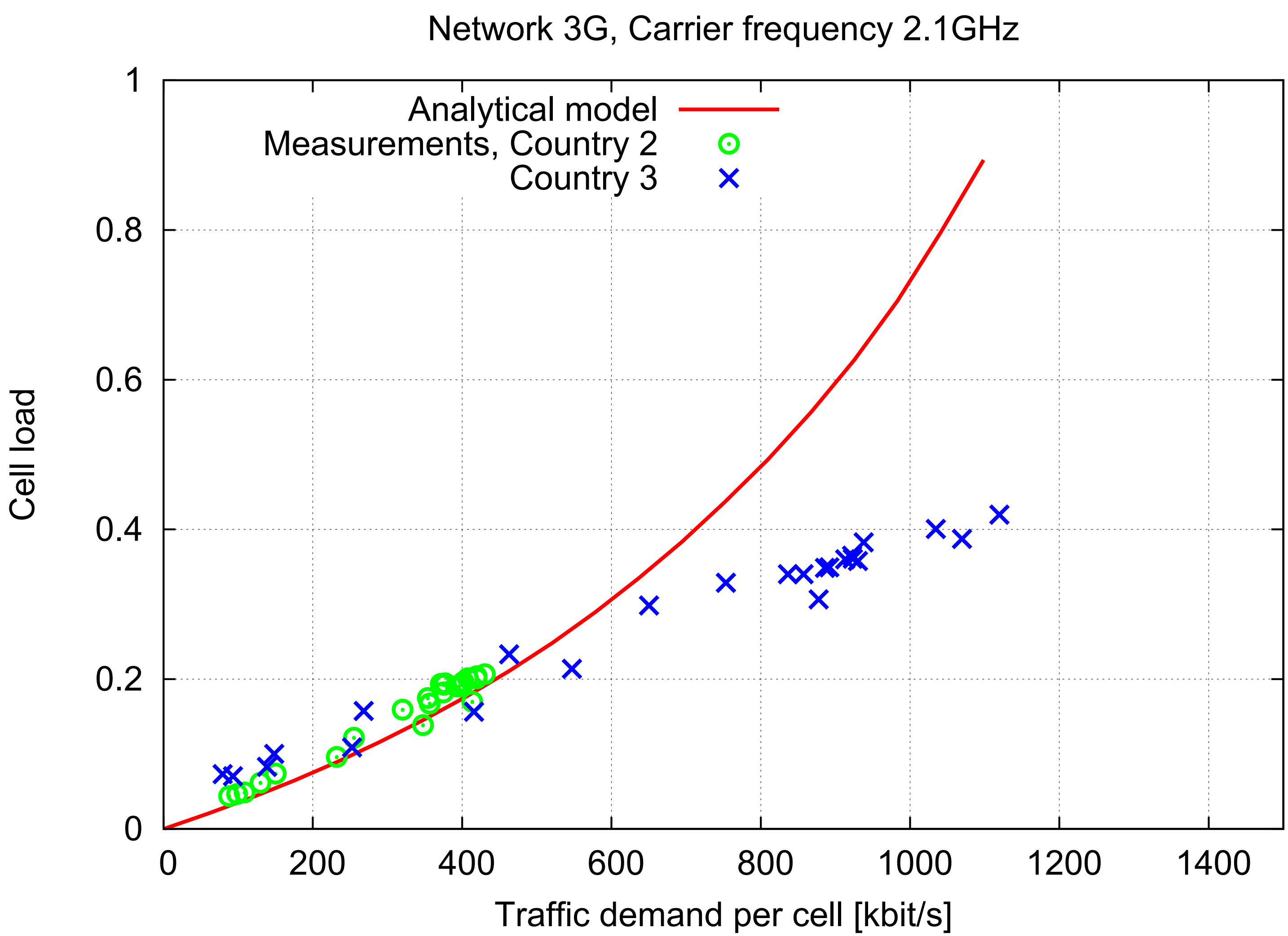
(Data from a commercial network of an international operator
in a big European country — a “reference network”)

Cell load with regular network decomposition



The analytical prediction fits the real data regardless of the network decomposition scale. The “reference network”.

Networks in two other countries



The analytical prediction fits the real data. In Country 2 (blue points), for the traffic $>$ 600 kbit/s per cell, an admission control is applied.

Bandwidth dimensioning

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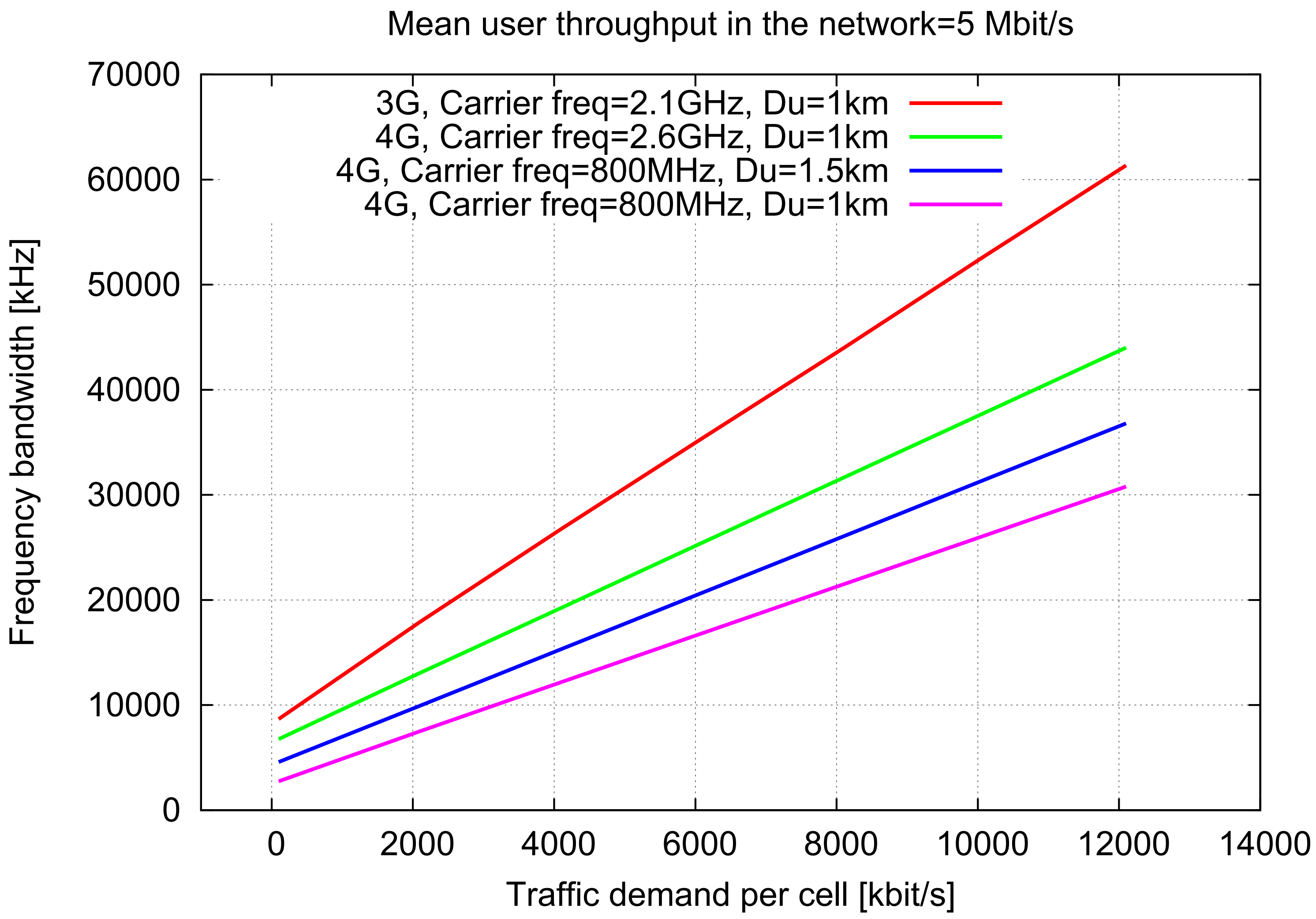
- What frequency bandwidth to run cellular network at a given QoS?
- Conjecturing the homogeneous QoS response network model focus on urban zones.
- Use the mean-cell model to predict the mean user throughput for increasing traffic demand given the network density and the frequency bandwidth.
- Find the minimal frequency bandwidth for which the prediction of the mean user throughput reaches a given target value.

Bandwidth dimensioning solution



Frequency bandwidth required to provide 5 MBit/s of mean user throughput; given the technology, carrier frequency and network density.

Conclusions

- We have presented macroscopic laws regarding network performance metrics involving averaging both over time and the network geometry.
- We are able to consider both
  - local network heterogeneity (e.g. micro/macro cells) and
  - spatially inhomogeneity of network deployment (varying density of BS)
- This latter extension is useful for macroscopic network planning and dimensioning.

More details in

- BB., Jovanovic, Karray, M. K. How user throughput depends on the traffic demand in large cellular networks. In Proc. of WiOpt/SpaSWiN 2014 (arxiv:1307.8409)
- Jovanovic, Karray, BB. QoS and network performance estimation in heterogeneous cellular networks validated by real-field measurements. In Proc. of ACM PM2HW2N 2014 (hal-01064472)
- BB, Jovanovic, Karray, Performance laws of large heterogeneous cellular networks In Proc. of WiOpt/SpaSWiN 2015 (arXiv:1411.7785)
- BB., Karray, What frequency bandwidth to run cellular network in a given country? - a downlink dimensioning problem. In Proc. of WiOpt/SpaSWiN 2015(arxiv:1410.0033)

Thank you!