Wireless Cellular Networks with Shadowing

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NetGCooP

Paris, 12–14 October 2011.

Shadowing

In wireless communications, shadowing is deviation of the power of the received electromagnetic signal from an average value.

Caused by obstacles affecting the wave propagation.

May vary with geographical position and/or radio frequency.

Usually modelled as a random process.

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In particular, we will show that the call blocking probability in cellular networks is <u>not</u> always increasing with the shadowing variance.

Outline of the Remaining Part of the Talk

- Model Description,
- Numerical Results
- Some Mathematical Results
- Concluding Remarks and Questions

MODEL DESCRIPTION

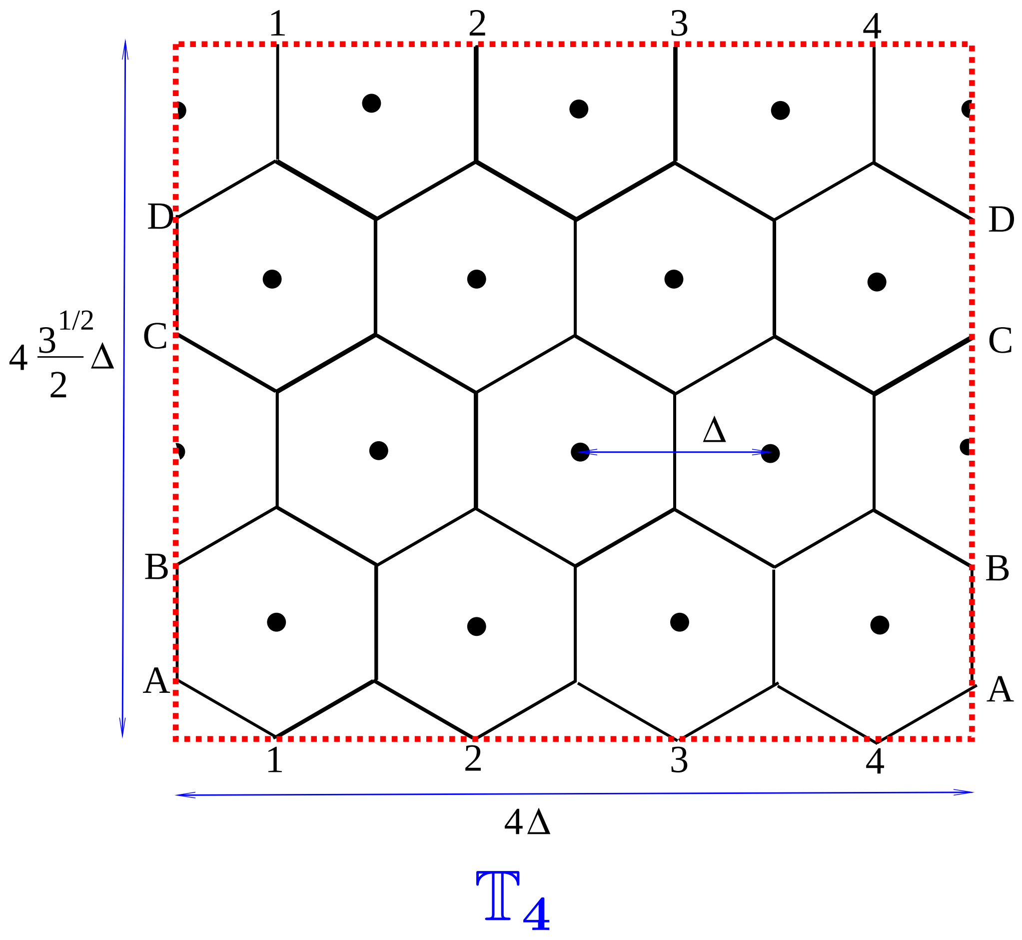
Geometry of BS's

Hexagonal network Φ_H

Δ distance between adjacent vertexes, surface of the cell $\sqrt{3}\Delta^2/2$, thus the intensity $\lambda_H = 2/(\sqrt{3}\Delta^2)$ BS/km^2.

Uniformly shifted to make it stationary.

Infinite or finite considered on torus to neglect boundary effects.

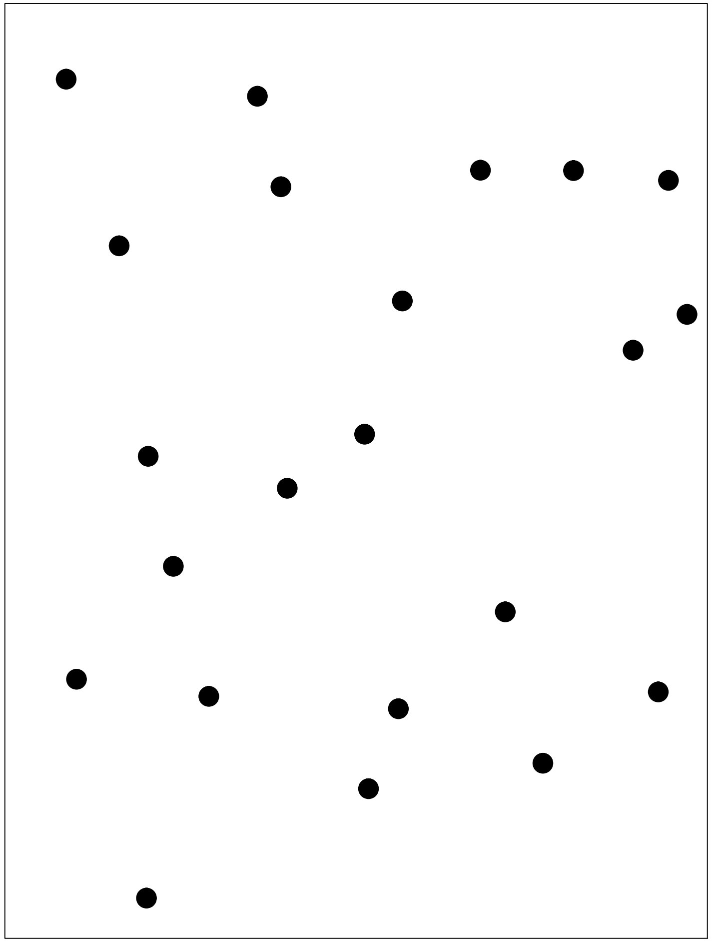


Geometry of BS's; cont'd

Poisson network Φ_P

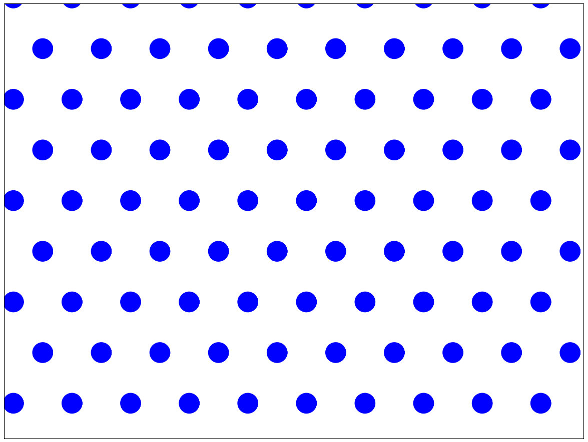
- number of points of Φ_p in any set A, $\Phi_P(A)$, is Poisson random variable with mean λ_P times the surface of A,
- numbers of points $\Phi_P(A_i)$ of Φ_p in disjoint sets A_i are independent random variables.

Intensity λ_P of BS/km^2.
Infinite or finite considered on torus to neglect boundary effects.



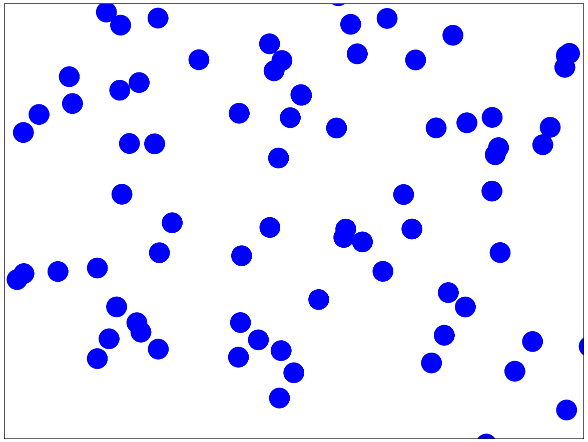
Geometry of BS's; cont'd

Hexagonal



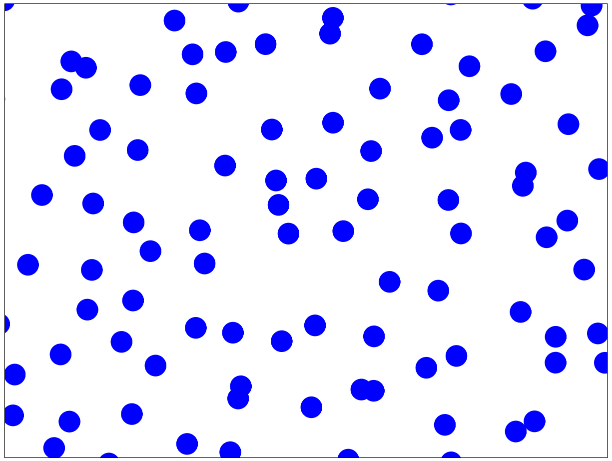
"ideal" model

Poisson



"ad-hoc" deployed network

A general point process Φ?



(Signal-Power) Path-Loss with Shadowing

Given BS $\boldsymbol{X} \in \boldsymbol{\Phi}$ ($\boldsymbol{\Phi} = \boldsymbol{\Phi_P}$ or $\boldsymbol{\Phi_H}$) and location $\boldsymbol{y} \in \mathbb{R}^{\mathbf{2}}$ on the plane, denote by $\mathbf{1}/\boldsymbol{L_X}(\boldsymbol{y})$ the power of the signal received at $\boldsymbol{y}$ from $\boldsymbol{X}$.

(Signal-Power) Path-Loss with Shadowing

Given BS $X \in \Phi$ ($\Phi = \Phi_P$ or Φ_H) and location $y \in \mathbb{R}^2$ on the plane, denote by $1/L_X(y)$ the power of the signal received at y from X.

Assume:

(1)
$$L_X(y) = \frac{L\left(|X - y|\right)}{S_X(y)}$$

where

- $L(\cdot)$ is a non-decreasing, deterministic path-loss function,
- $S_X(\cdot)$ is a random shadowing field related to the BS X.

Deterministic Path-Loss Function

If not otherwise specified

(2) $$L(r) = (Kr)^{\beta}$$

where $K > 0$ and $\beta > 2$ (path-loss exponent) are some constants.

Shadowing Distribution

We will always assume that:

- $\{S_X(y)\}$ are i.i.d. across BS's $X \in \Phi$ and $y \in \mathbb{R}^2$ non-negative r.v's.
- normalized to $\mathsf{E}[S_X(y)] = 1$.
- mean path-loss is finite, i.e., $\mathsf{E}[1/S] < \infty$.

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Of special interest is:

- $S_X(y) \equiv 1$ — no shadowing,
- $S_X(y) = S$ is log-normal with mean 1; $S = e^{-\sigma^2/2+\sigma N}$, where N is standard $(0, 1)$ Gaussian random variable. Call $v = \sigma 10/\log 10$ logarithmic standard deviation (log-SD) of the shadowing; this is the SD of the path-loss $L_X(y)$ expressed in dB.

Spatial Service Policy

A given user at location $\boldsymbol{y} \in \mathbb{R}^{\mathbf{2}}$ is served by the BS $\boldsymbol{X}^*_{\boldsymbol{y}} \in \boldsymbol{\Phi}$ from which it receives the strongest signal (i.e., the weakest path-loss)

(3) $$\boldsymbol{L}_{\boldsymbol{X}^*_{\boldsymbol{y}}}(\boldsymbol{y}) \leq \boldsymbol{L}_{\boldsymbol{X}}(\boldsymbol{y}) \qquad \text{for all } \boldsymbol{X} \in \boldsymbol{\Phi}\,,$$

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with any tie-breaking rule.

- In the case of no shadowing ($\boldsymbol{S}_{\boldsymbol{X}}(\boldsymbol{y}) \equiv \mathbf{1}$) and strictly increasing path-loss function $\boldsymbol{L}(\cdot)$ the above policy corresponds to the geographically closest BS.
- For infinite network models with random shadowing, one has to prove that the minimum of the path-loss is achieved for some BS.

QoS “pre-metric” 1: Path-loss Factor

For a given location $y \in \mathbb{R}^2$ we define the path-loss factor as

$$l(y) := L_{X_y^*}(y) = \frac{1}{\max_{X\in\Phi} \frac{S_X(y)}{L(|X-y|)}}; \tag{4}$$

i.e., as the path-loss experienced at y with respect to the serving BS (not to be confused with the path-loss exponent β).

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We will be interested in the mean path-loss factor

$$\mathsf{E}[l(y)] = \mathsf{E}[l\big(y, \Phi, S_X(y), X \in \Phi\big)].$$

By stationarity (of toroidal or infinite network model) $\mathsf{E}[l] := \mathsf{E}[l(y)] = \mathsf{E}[l(0)]$.

QoS “pre-metric” 2: Interference Factor

For a given location $y \in \mathbb{R}^2$ we define the interference factor as

(5) $$f(y) := \sum_{X \in \Phi, X \neq X_y^*} \frac{L_{X_y^*}(y)}{L_X(y)} = \frac{\sum_{X \in \Phi} \frac{S_X(y)}{L(|X-y|)}}{\max_{X \in \Phi} \frac{S_X(y)}{L(|X-y|)}} - 1\,.$$

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$$\mathsf{E}[f(y)] = \mathsf{E}[f\big(y, \Phi, S_X(y), X \in \Phi\big)]\,.$$

By stationarity (of toroidal or infinite network model) $\mathsf{E}[f] := \mathsf{E}[f(y)] = \mathsf{E}[f(0)]$.

Why $\mathbf{E}[l]$ and $\mathbf{E}[f]$?

The two QoS "pre-metrics": mean path-loss factor $\mathsf{E}[l]$ and mean interference factor $\mathsf{E}[f]$ are rather elementary characteristics, which give insight into more involved QoS metrics, as e.g. the blocking probability.

Blocking Probability in a Spatial Erlang's Loss Model

Finite network (on torus), required space-time scenario;

- (Poisson) arrival process (in time) of calls to the network,
- calls require predefined (exponential) service times,
- calls require predefined, fixed bit-rates (CBR),
- blocking of arrivals when not enough resources (power, frequency etc),
- fraction of blocked arrivals in the long run of the system is called the blocking probability (bp).

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- fraction of blocked arrivals in the long run of the system is called the blocking probability (bp).
- Erlang's loss formula: bp it is equal to the conditional probability that in the stationary configuration of the (non-blocked) arrival process the system cannot admit a new user, given all users in the current configuration can be served.

Blocking Probability; basic facts cont'd

- Multi-Erlang (ME) form of call admission condition:

(6)
$$\sum_{y: X^*_y = X} \varphi\big(l(y), f(y)\big) \le 1\,,$$

where the summation is over all users (including a new arrival) to be served by the BS X and $\varphi(\cdot,\cdot)$ is some function of the path-loss and interference factorr (can be more general) of user y.

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- If ME admission condition then the Erlang's loss formula can be practically evaluated, e.g. discretizing the geometry and using the Kaufman-Roberts algorithm.

Multi-Erlang; downlink CDMA

$$\varphi(l, f) = \frac{\xi}{1+\alpha\xi}\frac{1}{1-\epsilon}\left(\frac{Nl}{\tilde{P}} + \alpha + f\right);$$

- $\tilde{P}$ is the maximal BS power,
- ϵ is the fraction of this maximal power used in common (pilot) channels,
- α is the intra-cell orthogonality factor,
- N external noise power
- $\xi = \psi^{-1}(r/W)$ is the SINR threshold corresponding to the required bit-rate r of user given the link performance function ψ and the system bandwidth W,
- ψ is the link performance function (e.g. for AWGN channel with maximal capacity coding $\psi(\xi) = \log_2(1+\xi)$ is the Shannon's formula).

Multi-Erlang; downlink OFDMA

$$\varphi(l, f) = \frac{r}{W\psi\big((1-\epsilon)/((Nl/\tilde{P}) + \alpha + f)\big)},$$

with the notation as for CDMA.

RESULTS

Blocking Probability

We consider Hexagonal BS models on the torus $\mathbb{T}_{\boldsymbol{N}}$ with power-law path-loss function (2) and log-normal shadowing.

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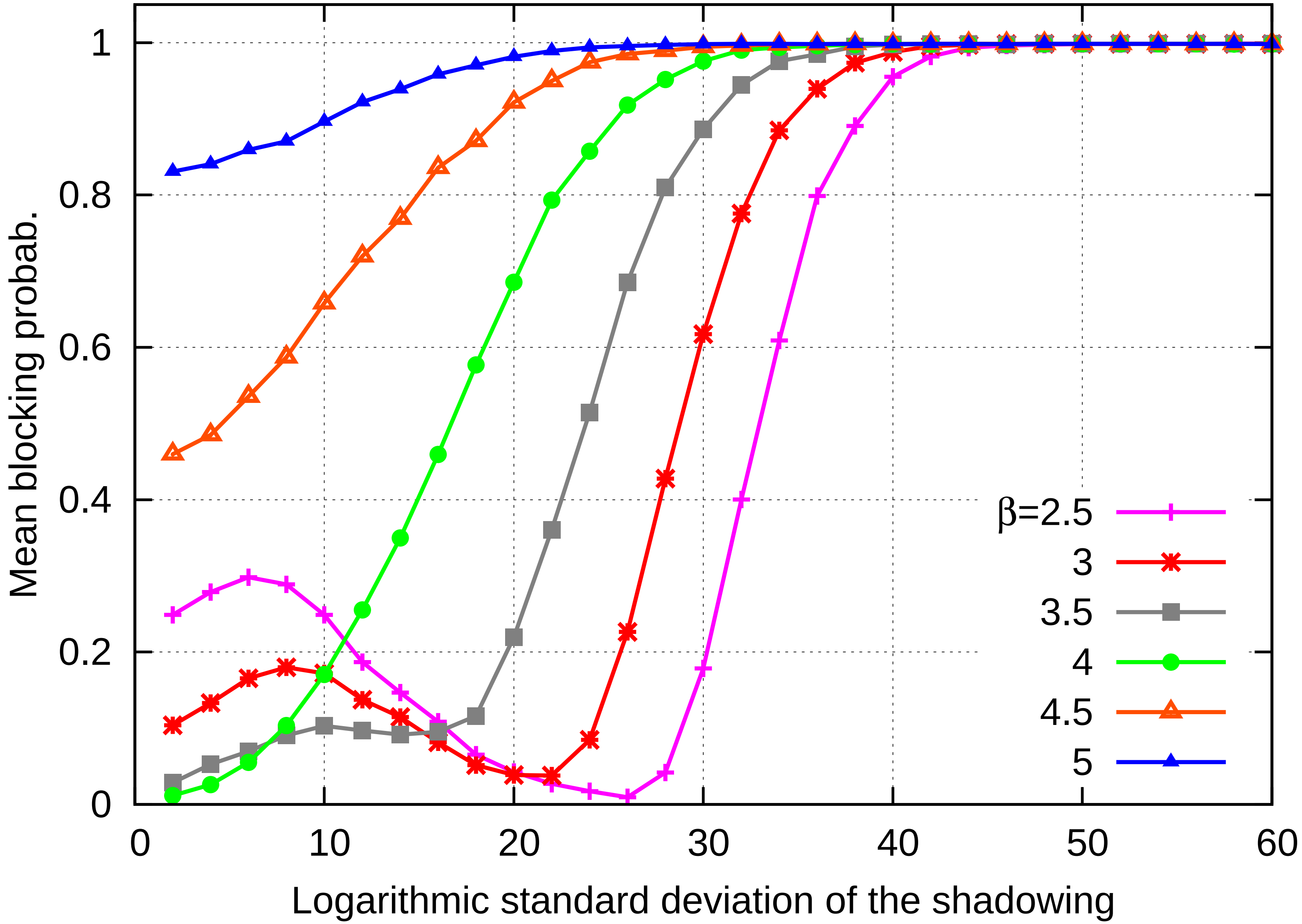
For a given realization of the shadowing field (in a suitably discretized geometry), we calculate the blocking probability via the Kaufman-Roberts algorithm.

Next, we average over many realizations of the shadowing field.

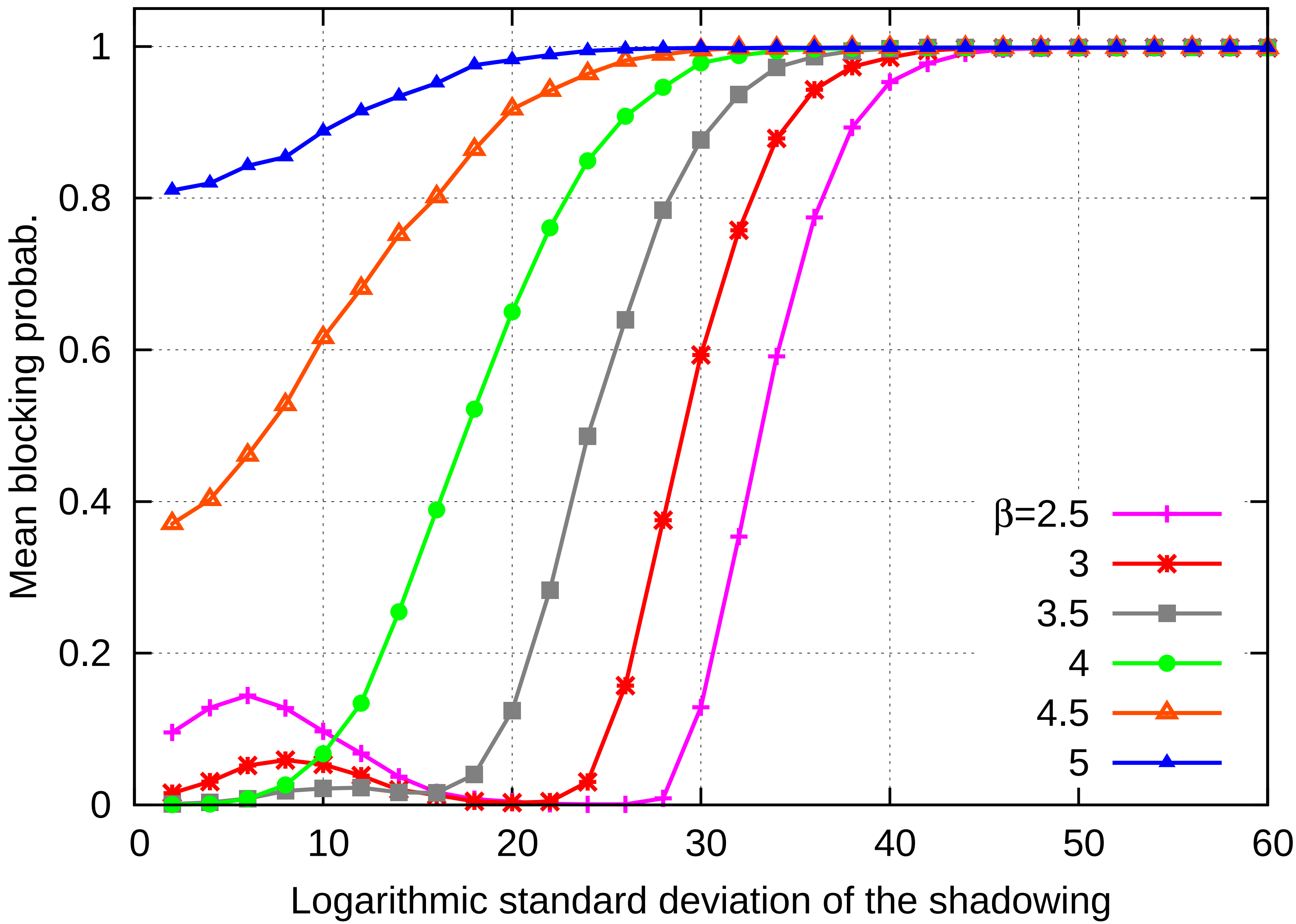
In practice, for a sufficiently large network ($\geq$ 36 BS in our setting) just one realization of the shadowing field is enough (spatial ergodicity).

Blocking probability v/s shadowing



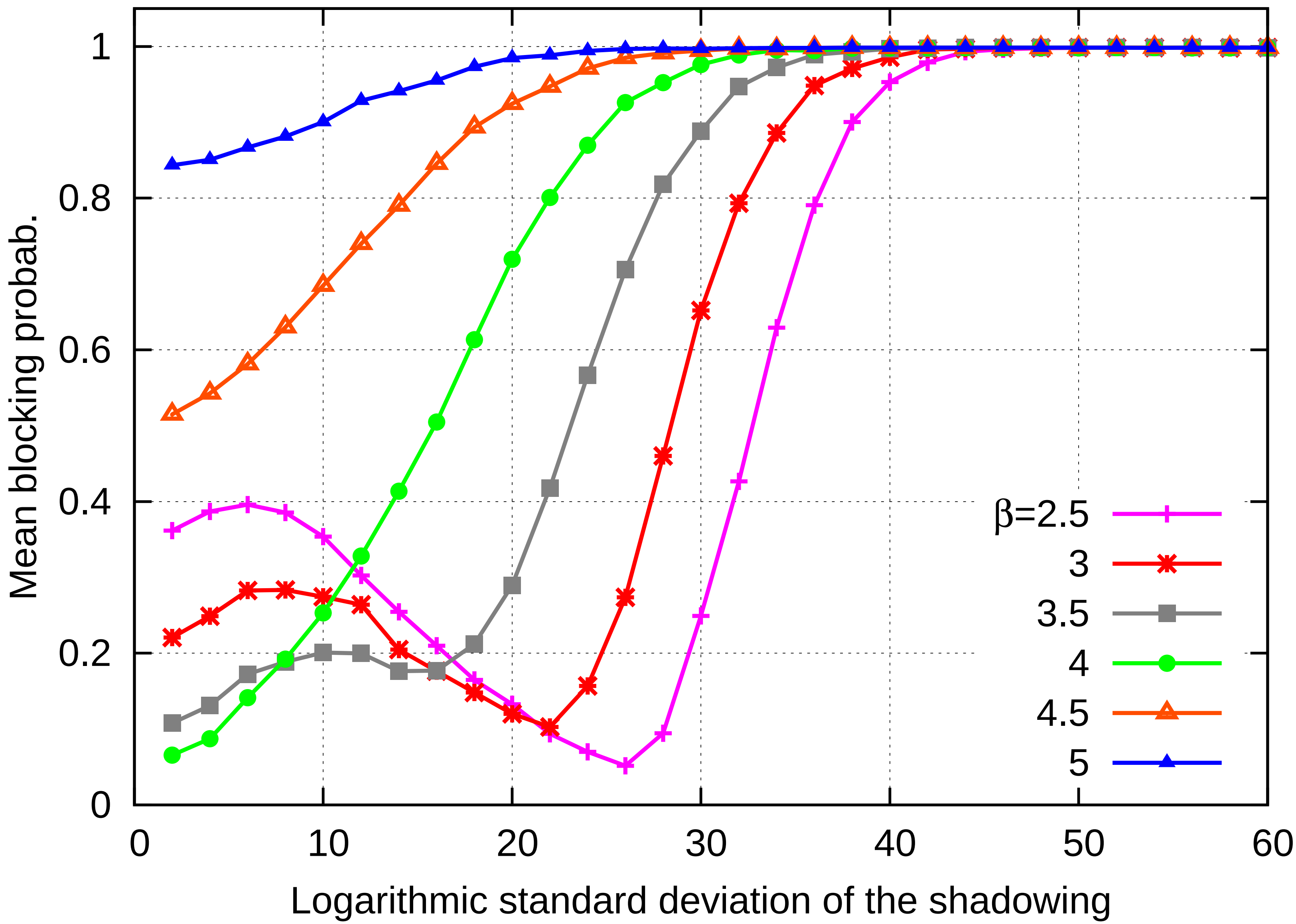
OFDMA, downlink, hexagonal network of 36 BS, cell radius 0.5 km.
log-normal shadowing, path-loss exponent β, traffic 34.6 Erlang per km^2.

Blocking probability v/s shadowing, cont'd



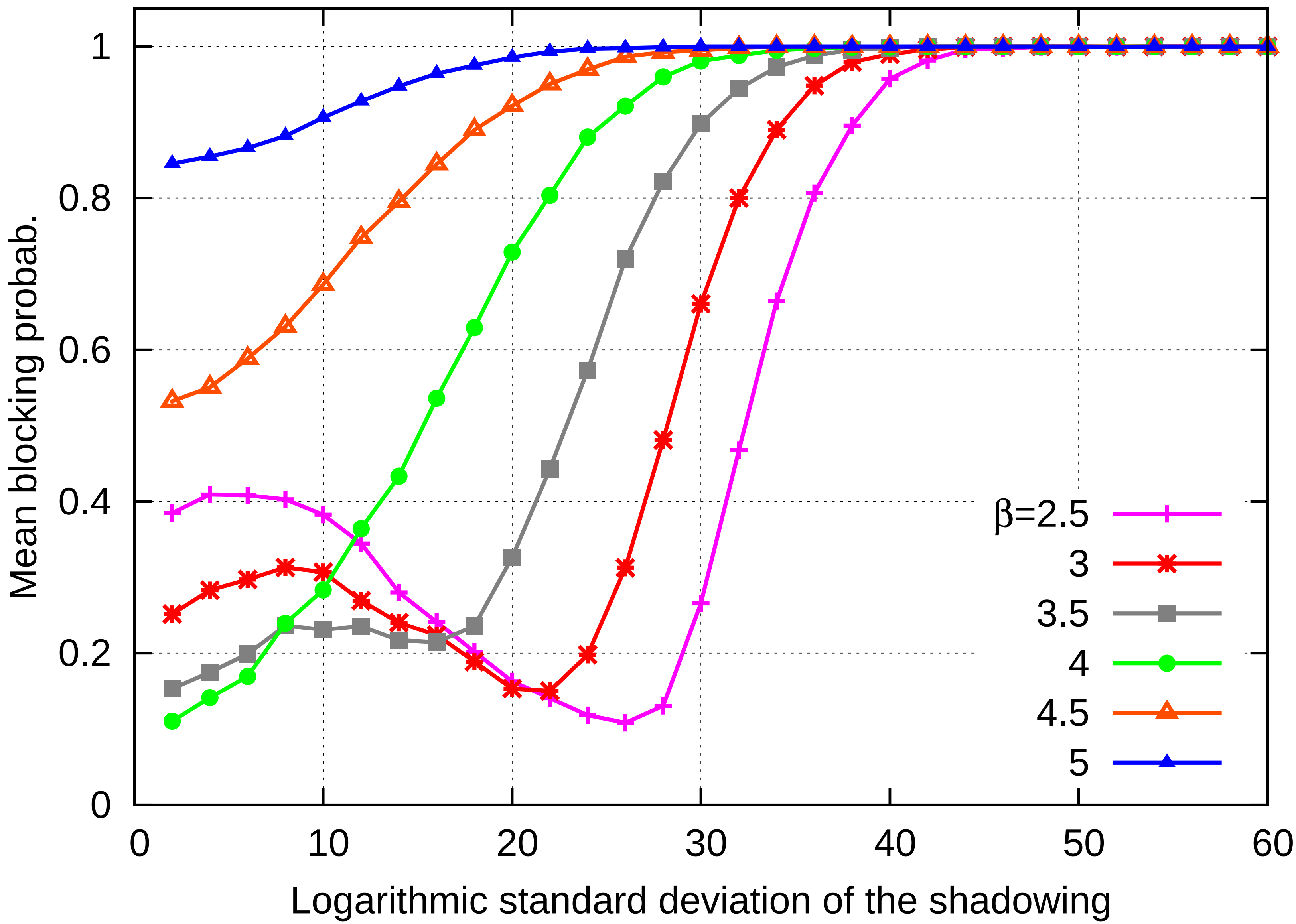
OFDMA, downlink, hexagonal network of 36 BS, cell radius 0.5 km.
log-normal shadowing, path-loss exponent β, traffic 23.1 Erlang per km^{2}.

Blocking probability v/s shadowing, cont’d



OFDMA, downlink, hexagonal network of 36 BS, cell radius 0.5 km.
log-normal shadowing, path-loss exponent β, traffic 46.2 Erlang per km^2.

Blocking probability v/s shadowing, cont'd

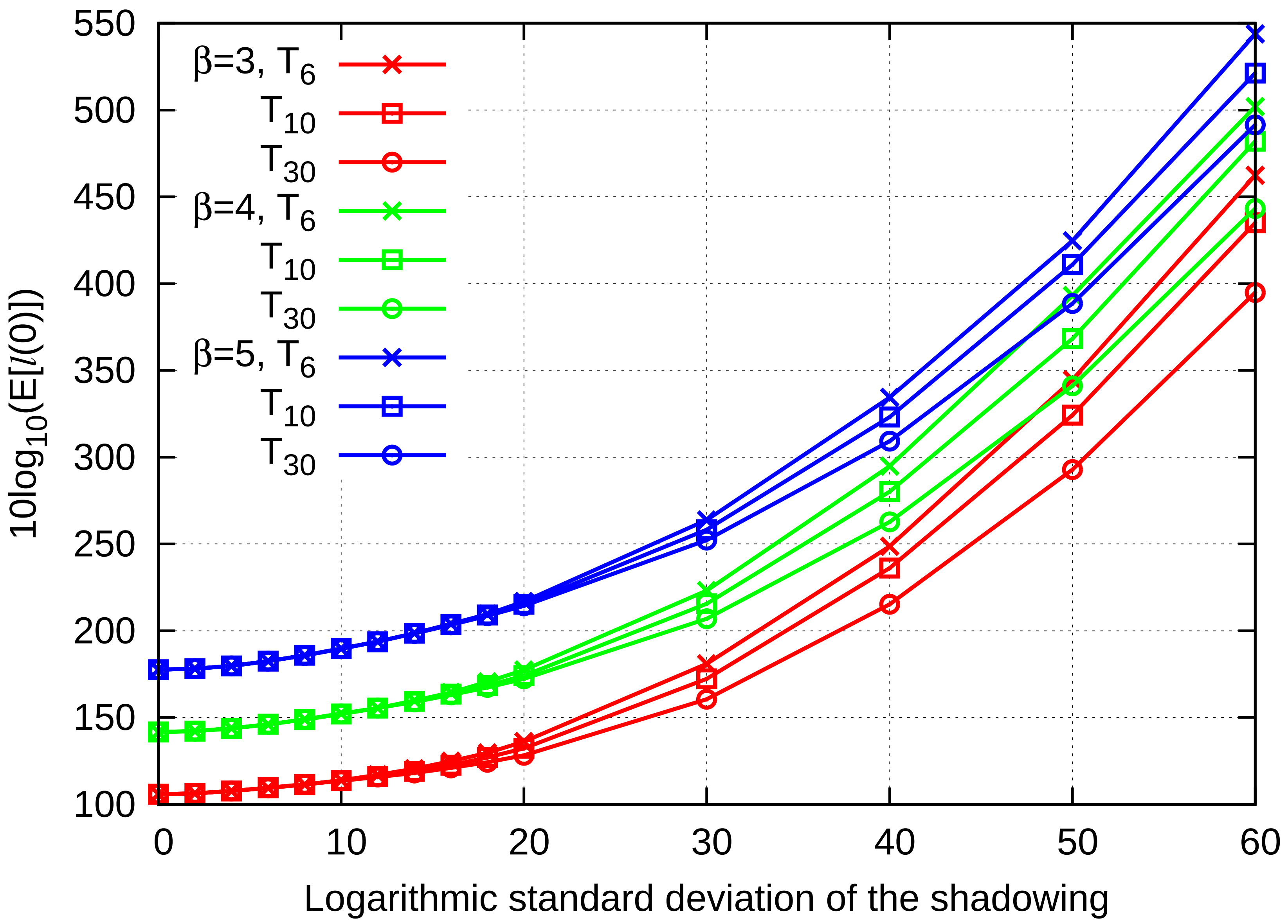


CDMA, downlink, hexagonal network of 36 BS, cell radius 0.5 km.
log-normal shadowing, path-loss exponent β, traffic 46.2 Erlang per km^2.

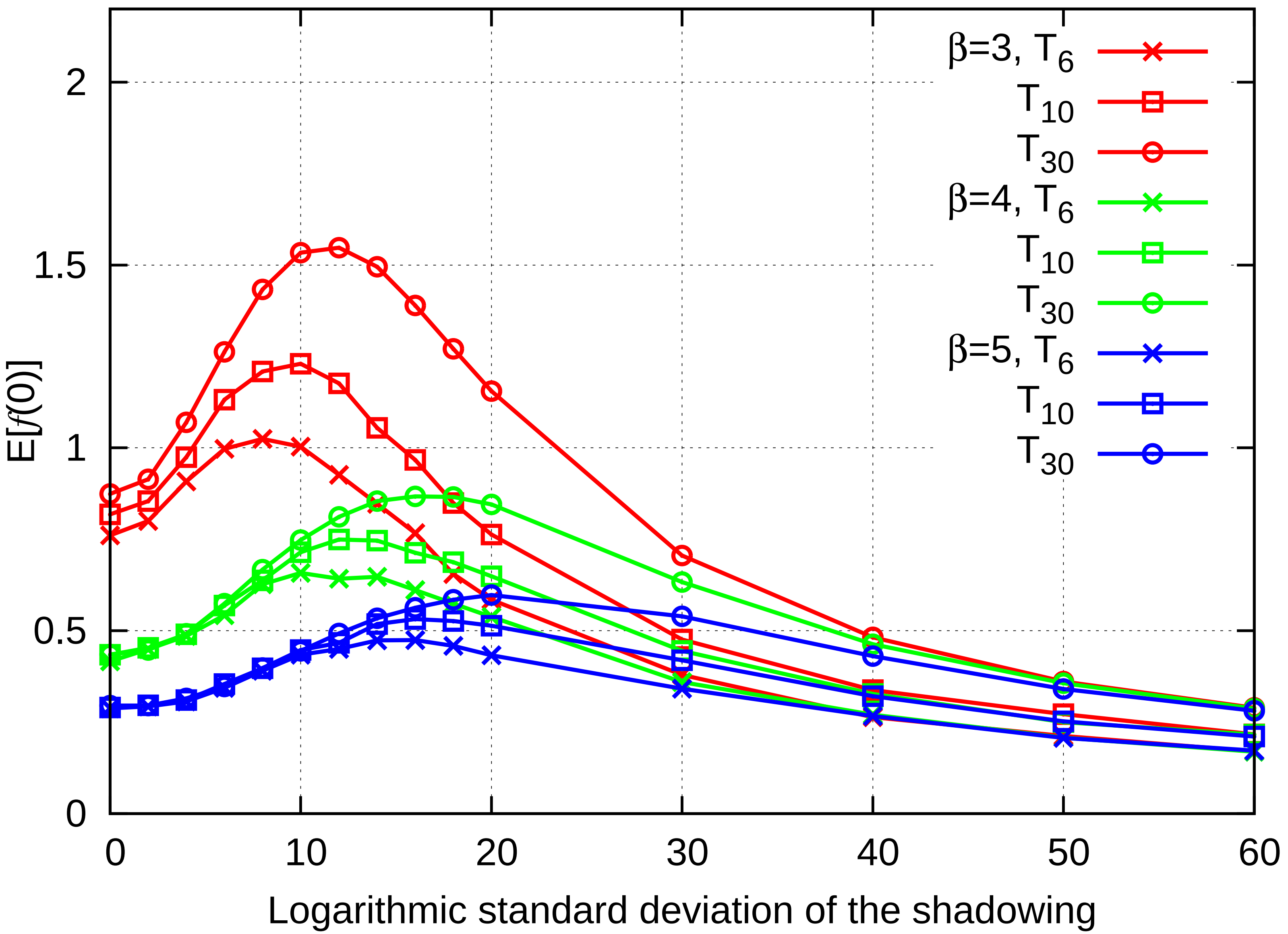
Back to Basics

Let's try to understand the impact of the model parameters (in particular the variance of the shadowing) on $\mathsf{E}[l]$ and $\mathsf{E}[f]$.

E[l] in the Hexagonal Network; Simulation



$\mathbf{E}[f]$ in the Hexagonal Network; Simulation

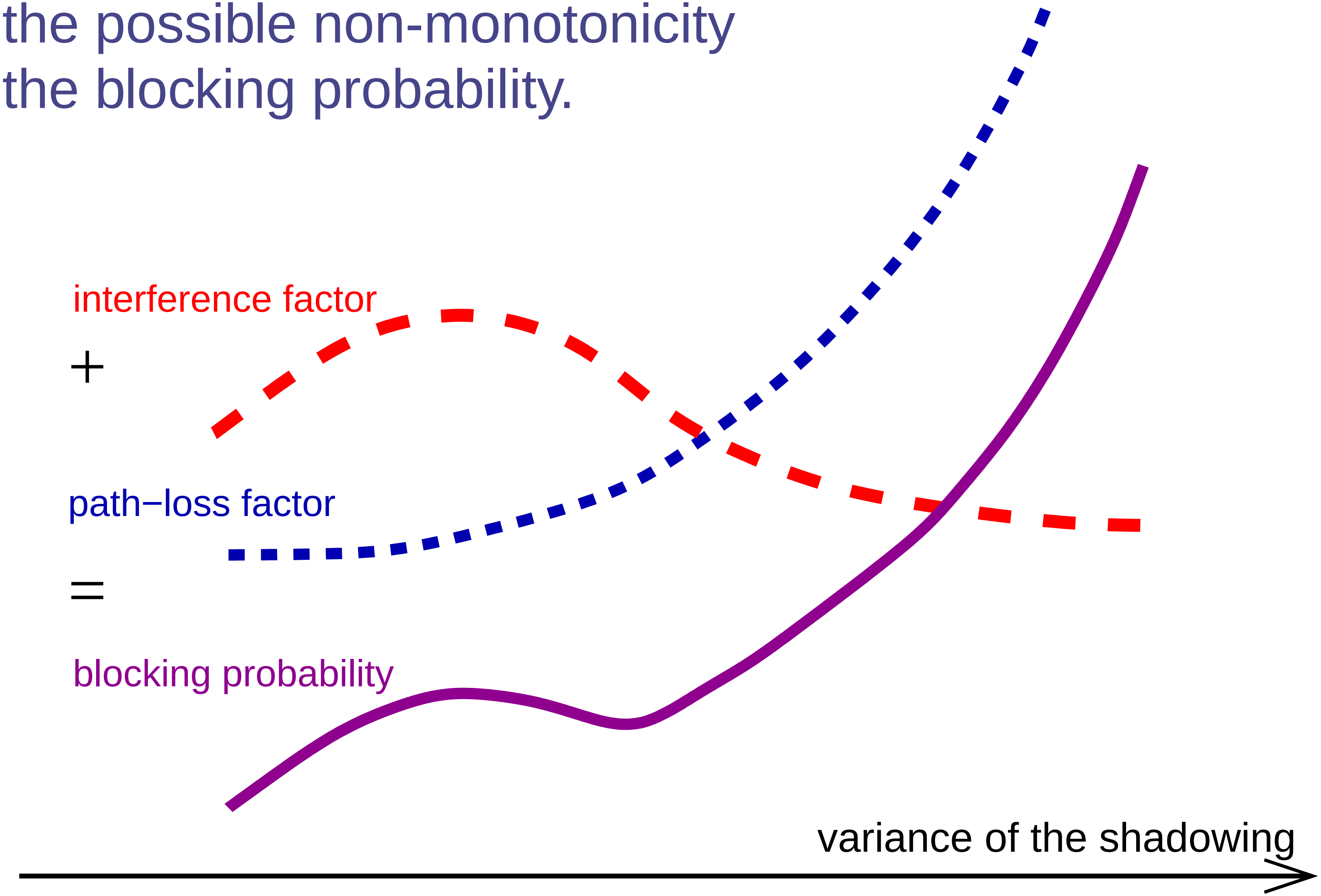


Observations for the Hexagonal Network

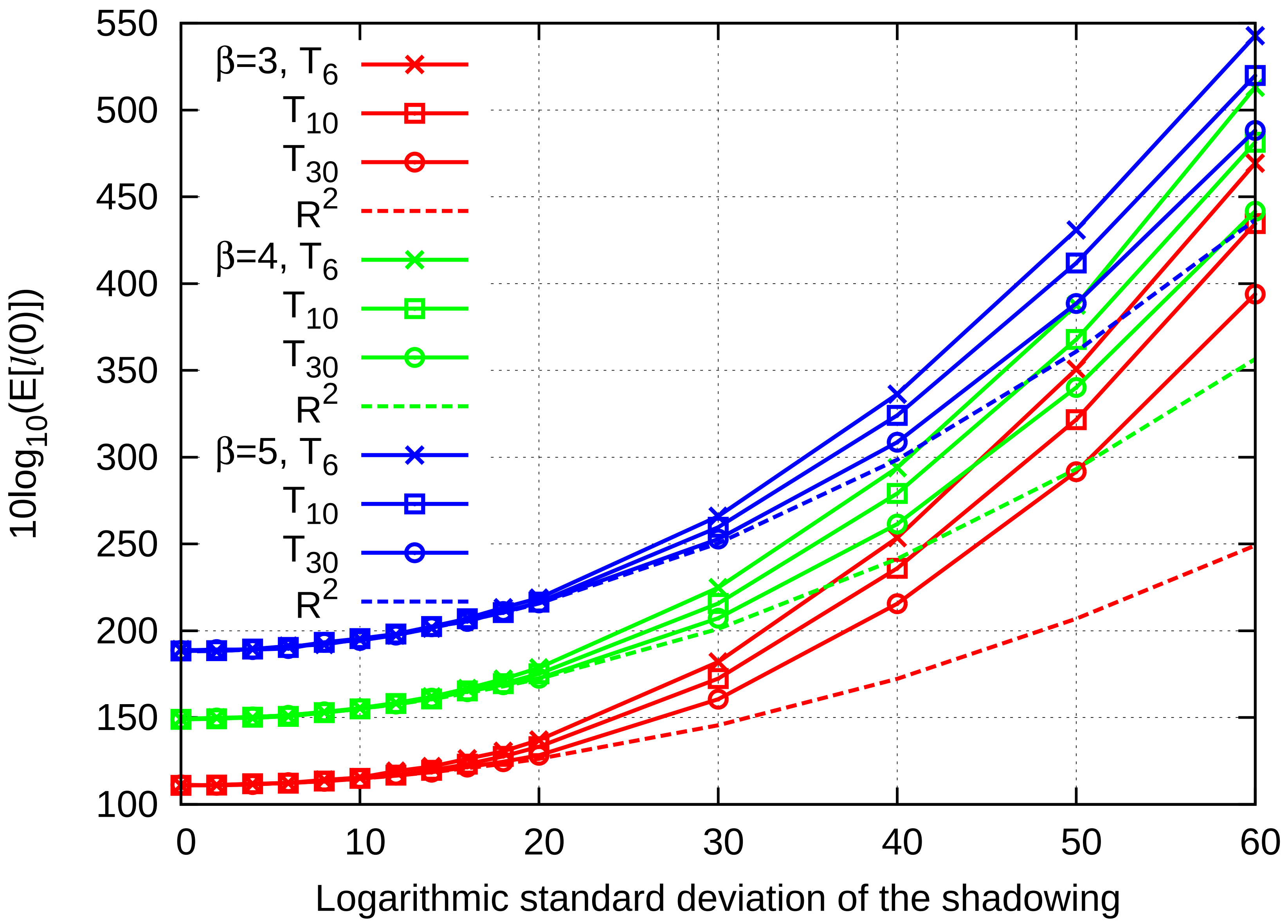
- Mean path-loss factor increases in (log-SD of) the shadowing, increases in path-loss exponent, but (slightly) decreases in the network size.
- Mean interference factor is not monotone in shadowing: first increases and then decreases (to 0, cf Prop. 1). It decreases in the path-loss exponent and increases in the network size.
- For more than 100 BS and reasonable path-loss model assumptions $\mathsf{E}[l(0)]$ and $\mathsf{E}[f(0)]$ correspond to these in the infinite model. However, for large values of log-SD of the shadowing $\mathsf{E}[f(0)]$ non-negligibly increases with the network size.

Understanding the Blocking Probability

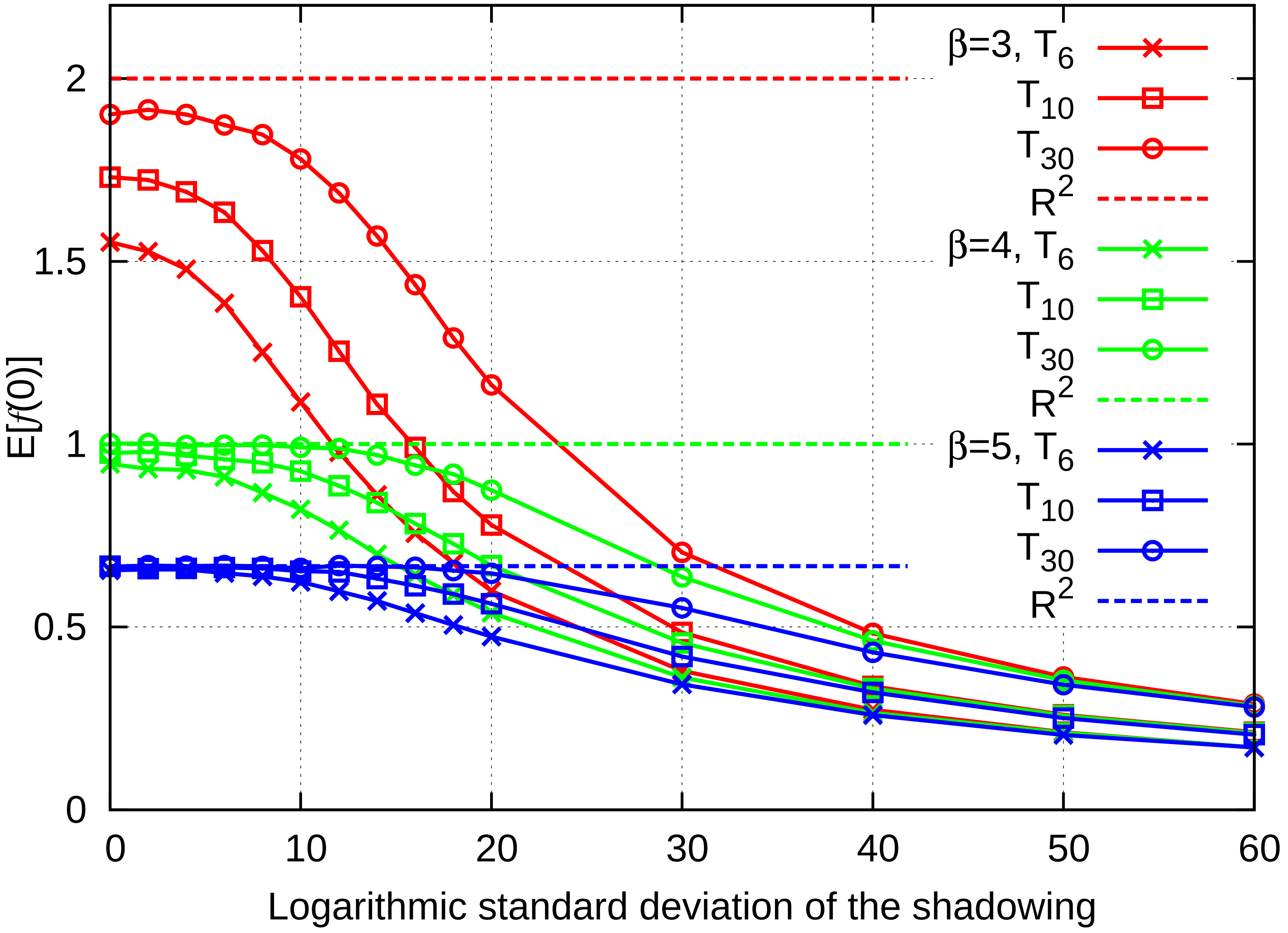
Graphical explanation
of the possible non-monotonicity
of the blocking probability.



Poisson Network, path-loss factor



Poisson Network, interference factor



Observations for Poisson network

- Mean path-loss factor increases in shadowing, increases in path-loss exponent, but decreases in the network size (same as for Hexagonal).
- Mean interference factor decreases in the shadowing (unlike in Hexagonal!). It decreases in the path-loss exponent and increases in the network size (Same as for Hexagonal).
- For infinite Poisson network with arbitrary distribution of shadowing, the QoS “pre-metrics” are known explicitly and do not depend on the shadowing!; cf. Prop 3.
- For large log-SD of the shadowing, the QoS “pre-metrics” of Hexagonal and Poisson network are very similar.

SO, WHAT CAN BE PROVED?
SOME MATHEMATICAL RESULTS.

Log-Normal Shadowing with mean 1

$S = e^{-\sigma^2/2+\sigma N}$ where N is the standard Gaussian random variable. For any fixed $\epsilon > 0$ we have

$$\mathsf{P}\{S \geq \epsilon\} \;=\; \mathsf{P}\{N \geq \sigma/2 + (\log \epsilon)/\sigma\} \stackrel{\sigma \to \infty}{\longrightarrow} 0\,,$$

which shows that the random variable S converges in probability to 0 (hence path-loss $1/S$ converges to ∞).

This shows that path-loss factor $l(y)$ converges in probability and in expectation to infinity.

Finite Networks, Increasing Shadowing

Proposition 1 *Assume an arbitrary, fixed, finite pattern* $\{X_1, X_2, \ldots, X_n\}$ *of BS locations. Consider any deterministic path-loss function* $0 < L(r) < \infty$ *and (independent) log-normal shadowing* $S_{X_i}(\cdot)$ *with the log-SD* v*. Then for any location* y *we have*

$$\lim_{v \to \infty} f(y) = 0 \quad \textit{in probability.}$$

Log-normal shadowing amplifies the ratio between the strongest signal and all other signals thus reducing the interference.

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Corollary 1 *The mean interference factor* $E[f(0)]$ *in the Poisson and hexagonal network on the torus* $\mathbb{T}_N$*, with log-normal shadowing converges in distribution and in expectation to 0 when log-SD of the shadowing goes to* ∞*.*

Finding Serving BS in Infinite Networks

Proposition 2 *Consider infinite Poisson* $\Phi = \Phi_P$ *or hexagonal* $\Phi = \Phi_H$ *model of BS, with shadowing whose marginal distribution has finite moment of order* $2/\beta$ *([a]). Then there exist almost surely the serving BS, i.e.,* $X_0^* \in \Phi$ *satisfying (3). Moreover, the interference factor calculated with respect to the restriction of* Φ *to* $\mathbb{T}_N$*, i.e.,* $f(0, \tilde{\Phi}^{\mathbb{T}_N})$*, converges almost surely and in expectation to* $f(0, \tilde{\Phi})$*.*

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[a] i.e., $\mathsf{E}[S^{2/\beta}] < \infty$. Note that $2/\beta < 1$ and thus the above assumption follows from our default assumption $\mathsf{E}[S] = 1 < \infty$.

Infinite Poisson Network

Proposition 3 *Assume infinite Poisson network with the deterministic path-loss function (2). The distribution of the path-loss factor $l(0) = l(0, \tilde{\Phi})$ and interference factor $f(0) = f(0, \tilde{\Phi})$ do not depend on the (marginal) distribution of the shadowing field $S = S_X(y)$ provided $E[S^{2/\beta}] < \infty$. Moreover, their expectations are given by*

$$
\begin{aligned}
E[f(0)] &= \frac{2}{\beta - 2}, \\
E[l(0)] &= \frac{K^{\beta}\Gamma(1 + \beta/2)}{(\pi\lambda E[S^{2/\beta}])^{\beta/2}},
\end{aligned}
$$

where $\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt$. Also, the distribution function of $l(0)$ admits a simple explicit expression.

CONCLUDING REMARKS & QUESTIONS

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- Studying two QoS “pre-metrics” (path-loss with respect to the serving BS and interference factor) one discovers a more subtle reality.
- As commonly expected: strong variance of the shadowing increases the mean path-loss with respect to the serving BS, which in consequence, degradates most of the QoS metrics.
- However, the mean interference is not monotonic in the variance of the shadowing. It first increases and then decreases (asymptotically to zero!), when the shadowing variance goes to infinity.

Concluding Remarks; cont'd

- For moderate shadowing, when the QoS is not yet compromised by the path-loss conditions, it may profit from the reduction of the interference. This is because increasing variance of the shadowing tends to "separate" the strongest (serving BS) signal from all other signals.

Concluding Remarks; cont'd

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- Such phenomenon can be compared to stochastic resonance phenomena observed for some other stochastic models.
- Might shed new light, in particular on the design of indoor communication scenarios.

QUESTIONS OR COMMENTS?

THANK YOU