STOCHASTIC ANALYSIS OF REAL AND VIRTUAL STORAGE IN THE SMART GRID

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joint work with
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Outline

1. Introduction and motivation
2. Managing Storage
3. Impact of Storage
4. Impact of Demand Response
Wind and solar energy make the grid less predictable

Mean error: 1–2%

Mean error: 20%
Storage can mitigate volatility

- Batteries, Pump-hydro
- Demand Response = Virtual Storage

Switzerland (mountains)

Voltalis Bluepod switches off thermal load for 60 mn

Projects: artificial islands (north sea)

- Belgium
- Copenhagen

A Manmade Island to Store Wind Energy

Belgium has plans for an artificial "energy astolf" to store excess wind power in the North Sea.
Questions addressed in this talk

1. How to manage one piece of storage
2. Impact of storage on market and prices
3. Impact of demand response on market and prices
2. MANAGING STORAGE

Storage

- Stationary batteries, pump hydro

Cycle efficiency
≈ 70 – 85%
1a. Forecast load $D^f_t (t + n)$ and renewable supply $W^f_t (t + n)$

1b. Schedule dispatchable production $P^f_t (t + n)$

2. Compensate deviations from forecast by charging / discharging $\Delta$ from storage
Full compensation of fluctuations by storage may not be possible due to power / energy capacity constraints

- Fast ramping energy source \((CO_2\) rich) is used when storage is not enough to compensate fluctuation

- Energy may be wasted when
  - Storage is full
  - Unnecessary storage (cycling efficiency < 100%)

- Control problem: compute dispatched power schedule \(P_t^f (t + n)\) to minimize energy waste and use of fast ramping
Example: The Fixed Reserve Policy

- Set $P^f_t (t + n)$ to $D^f_t (t + n) - W^f_t (t + n) + r^*$ where $r^*$ is fixed (positive or negative)

- Metric: Fast-ramping energy used (x-axis)
  Lost energy (y-axis) = wind spill + storage inefficiencies

Aggregate data from UK (BMRA data archive https://www.elexonportal.co.uk/)
scaled wind production to 20% (max 26GW)

$B_{\text{max}} = 100\text{GWh}$, $C_{\text{max}} = D_{\text{max}} = 2\text{GW}$

Efficiency $\eta = 0.8$

Efficiency $\eta = 1$
**Theorem.** Assume that the error \( e(t+n) = W(t+n) - W_t^f(t+n) \) conditioned to \( \mathcal{F}_t \) is distributed as \( \mathcal{E} \). Then for any control policy:

\[
\begin{align*}
(i) \quad \bar{G} & \geq \mathbb{E}[(\varepsilon + \bar{u})^-] - \text{ramp}(\bar{u}) \\
\bar{L} & \geq \mathbb{E}[(\varepsilon + \bar{u})^+] - \text{ramp}(\bar{u}) \\
\text{where} \quad \text{ramp}(\bar{u}) := \mathbb{E}[\min(\eta(\varepsilon + \bar{u})^+, \eta C_{\max}, (\varepsilon + \bar{u})^-, D_{\max})]
\end{align*}
\]

(ii) The lower bound is achieved by the Fixed Reserve when storage capacity is infinite.

- Assumption valid if prediction is best possible
Lower bound is attained for $B_{\text{max}} = 100 \text{GWh}$

- $C_{\text{max}} = D_{\text{max}} = 2 \text{GW}$
- $C_{\text{max}} = D_{\text{max}} = 6 \text{GW}$

Efficiency $\eta = 0.8$

Efficiency $\eta = 1$
Concrete Policies

BGK policy [Bejan et al, 2012] = targets fixed storage level

Dynamic Policy (Gast, Tomozei, L. 2014) minimizes average anticipated cost using policy iteration

Small storage

Large storage

Efficiency $\eta = 0.8$

What this suggests about Storage

- **A lower bound exists for any type of policy**
  - **Tight** for large capacity (>50GWh)
  - Open issue: bridge gap for small capacity

- **(BGK policy: )** Maintain storage at **fixed level: not optimal**
  - Worse for low capacity
  - There exist better heuristics, which use error statistics

- Can be used for sizing
  - UK 2020: 50GWh and 6GW is enough for 26GW of wind
3.

IMPACT OF STORAGE ON MARKETS AND PRICES

We focus on the real-time market

Most electricity markets are organized in two stages

- **Day-ahead market**
  - Planned production \( g_{da}(t) \)
  - Forecast demand

- **Real-time market**
  - Real-time reserve
    \[ R(t) = G^a(t) - D^a(t) \]
  - Actual production \( G^a(t) \)
  - Actual demand \( D^a(t) \)

Compensate for **deviations from forecast**

Inelastic demand satisfied using:
- Thermal generation (ramping constraints)
- Storage (capacity constraints)
Real-time Market exhibit highly volatile prices

Efficiency or Market manipulation?
The first welfare theorem

- Impact of volatility on prices in real time market is studied by Meyn and co-authors: price volatility is expected

<table>
<thead>
<tr>
<th>Theorem (Cho and Meyn 2010). When generation constraints (ramping capabilities) are taken into account:</th>
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<tbody>
<tr>
<td>• Markets are efficient</td>
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<td>• Prices are never equal to marginal production costs.</td>
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What happens when we add storage to the picture?

- Does the market work, i.e. does the invisible hand of the market control storage in the socially optimal way?

A Macroscopic Model of Real-time generation and Storage

Randomness (forecast errors)

Assumption: \((D - \Gamma) \sim \text{Brownian motion}\)

Controllable generation

Ramping Constraint

\(-\xi \leq G(t) - G(t - 1) \leq \zeta\)

Supply

\[G^a(t) = g^{da}(t) + G(t) + \Gamma(t)\]

Demand

\[D^a(t) = d^{da}(t) + D(t)\]

Day-ahead

Storage cycle efficiency

(E.g. \(\eta = 0.8\))

Limited capacity

Macroscopic model

- At each time: generation = consumption

\[G^a(t) + u(t) = D^a(t)\]
A Macroscopic Model of Real-time generation and Storage

We consider 3 scenarios for storage ownership:
1. Storage ∈ Supplier (this slide)
2. Storage ∈ Consumer
3. Independent storage

(ownership does mostly not affect the results)

\[ P(t) = \text{stochastic price process on real time market} \]

\[ W_D(t) = v \min(D^a(t), E(t) + g^{da}(t)) - c^{bo}(D^a(t) - G^{da}(t) - u(t))^+ - P(t)E(t) - p^{da}(t)g^{da}(t) \]

\[ W_S(t) = P(t)E(t) + p^{da}(t)g^{da}(t) - cG(t) - c^{da}g^{da}(t) \]
Definition of a competitive equilibrium

Assumption: agents are price takers

\( P(t) \) does not depend on players’ actions

Both users want to maximize their average expected payoff:

- Consumer: find \( E \) such that
  \[
  E \in \text{argmax}_E \mathbb{E} \left[ \int W_D(t)e^{-\gamma t} \, dt \right]
  \]

- Supplier: find \( E, G, u \) such that
  \[
  G \text{ and } u \text{ satisfy generation constraints and}
  E, G, u \in \text{argmax}_E \mathbb{E} \left[ \int W_S(t)e^{-\gamma t} \, dt \right]
  \]

Question: does there exists a price process \( P \) such that consumer and supplier agree on the production?

\((P,E,G,u)\) is called a dynamic competitive equilibrium
**Dynamic Competitive Equilibria**

**Theorem.** Dynamic competitive equilibria exist and are essentially independent of who is storage owner [Gast et al, 2013]

For all 3 scenarios, the price and the use of generation and storage is the same.

Prices ≈ marginal value of storage
- Concentrate on marginal production cost when $\eta = 1$
- Oscillate for $\eta < 1$

**Parameters based on UK data:**
1 u.e. = 360 MWh, 1 u.p. = 600 MW, $\sigma^2 = 0.6$ GW2/h, $\zeta = 2$ GW/h, $C_{max} = D_{max} = 3$ u.p.
The social planner problem

The social planner wants to find $G$ and $u$ to maximize total expected discounted payoff

$$\max_{G,u} \mathbb{E} \int (W_S(t) + W_D(t)) e^{-\nu t} dt$$

- **satisfied demand**
  $$\nu \min(D^a(t), E(t) + g^{da}(t)) - c^{bo}(D^a(t) - G^{da}(t) - -u(t))^+$$
- **Frustrated demand**
  $$-cG(t) - c^{da} g^{da}(t)$$
- **Cost of generation**

The solution does not depend on storage owner, and depends on the relation between the reserve $R(t)$ and the storage level $B(t)$

(Where reserve = generation - demand: $R(t) := G^a(t) + u(t) - D^a(t)$)

**Theorem [Gast et al 2013]** The optimal control is s.t.:
- if $R(t) < \Phi(B(t))$ increase $G(t)$
- if $R(t) > \Phi(B(t))$ decrease $G(t)$
The Social Welfare Theorem
[Gast et al., 2013]

- Any dynamic competitive equilibrium for any of the three scenarios maximizes social welfare

- the same price process controls optimally both the storage AND the production i.e. the invisible hand of the market works

\[ P^*(t) = \begin{cases} 0 & \text{storage compensates fluctuations} \\ \eta \frac{\partial V}{\partial b} (R^*(t), B^*(t)) + v + c_b, & \text{storage cannot store} \end{cases} \]

Overproduction that storage cannot store

Underproduction that storage cannot satisfy

Prices are dynamic
Lagrange multipliers

Figure 6: Steady-state distribution of prices for various storage energy capacities $B_{\max}$. For $B_{\max} = 10 \text{ u.e.}$, we zoom on $c=1$ to compare $\eta = 0.8$ and $\eta = 1$. 
The Invisible Hand of the Market may not be optimal

- Any dynamic competitive equilibrium for any of the three scenarios maximizes social welfare.
- However, this assumes a given storage capacity.
- Is there an incentive to install storage?
  - No, stand alone operators or consumers have no incentive to install the optimal storage

Can lead to market manipulation (undersize storage and generators)
Scaling laws and optimal storage sizing

- (steepness) being close to social welfare requires the optimal storage capacity

- optimal storage capacity scales like \( \frac{\sigma^4}{\zeta^3} \)

- increase volatility and ramp-up capacity by \( x \)
  = increase storage by \( x \)

\[ \text{proportional to installed renewable capacity} \]

\[ \zeta \text{ proportional to ramp-up capacity of traditional generators} \]
What this suggests about storage:

- With a free and honest market, storage can be operated by prices.
- However, there may not be enough incentive for storage operators to install the optimal storage size.
  - Perhaps preferential pricing should be directed towards storage as much as towards PV.
- Storage requirement scales linearly with the amount of renewables.
4.

IMPACT OF DEMAND-RESPONSE ON MARKETS AND PRICES

Demand Response

- distribution network operator may interrupt / modulate power

- virtual storage

- elastic loads support graceful degradation

- Thermal load (Voltalis), washing machines (Romande Energie «commande centralisée»)
e-cars

Voltalis Bluepod switches off boilers / heating for ≤60 mn
Issue with Demand Response: Non Observability

- Widespread demand response may make load hard to predict

**Intention**
- **renewables**

**Real**
- load with demand response
  - «natural» load
Our Problem Statement

Does it really work as virtual storage?

Side effect with load prediction?

To this end we add demand response to the previous model.
Our Problem Statement

Does it really work as virtual storage?
Side effect with load prediction

To analyze this we add demand response to the previous model

We consider 2,3 or 4 actors, involving
1. Demand
2. Flexible Loads
3. Production
4. Storage

Controllable generation
Ramping Constraint
\[ \xi \leq G(t) - G(t - 1) \leq \zeta \]

Supply
\[ G^a(t) = g^{da}(t) + G(t) + \Gamma(t) \]

Demand
\[ D^a(t) = d^{da}(t) + D(t) \]

Flexible Loads
\[ F^a(t) = f^{da}(t) + F(t) \]

Storage
\[ \frac{\partial B}{\partial t} = -u(t)(1_{u(t)>0} + \eta 1_{u(t)<0}) \]

Storage cycle efficiency
(E.g. \( \eta = 0.8 \))
Limited capacity
Model of Flexible Loads

- Population of $N$ On-Off appliances (fridges, buildings, pools)

- Without demand response, appliance switches on/off based on internal state (e.g. temperature) driven by a Markov chain

- Demand response action may force an off/off transition but mini-cycles are avoided

- Consumer game: anticipate or delay power consumption to reduce cost while avoiding undesirable states
Results of this model with Demand Response

- **Social welfare theorem** continues to hold, i.e. demand response can be controlled by price and this is socially optimal, given an installed base.

- We **numerically compute** the optimum using:
  - A mean field approximation for a homogeneous population of $N$ appliances
  - Branching trajectory model for renewable production [Pinson et al 2009]
  - ADMM for solution of the optimization problem
  - We assume all actors do not know the future but know the stochastic model

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The Benefit of demand-response is similar to perfect storage

Social Welfare

Non-Observability Significantly Reduces Benefit of Demand-Response

We assume that:
- The demand-response operator knows the state of its fridges
- The day-ahead forecast does not.
The Invisible Hand of the Market may not be optimal

Demand Response stabilizes prices more than storage
What this suggests about Demand Response:

- With a free and honest market, storage and demand response can be operated by prices.

- However, there may not be enough incentive for storage operators to install the optimal storage size / demand response infrastructure.

- Demand Response is similar to an ideal storage that would have close to perfect efficiency.

- However, it is essential to be able to estimate the state of loads subject to demand response (observability).
Thank You!

More details on smartgrid.epfl.ch