A Stochastic Geometry Framework for Modeling of Wireless Communication Networks

Bartłomiej Błaszczyszyn

INRIA-ENS / University of Wroław

X KzP Będlewo 2008

X Konferencja z probabilistyki

Będlewo, 19-23 maja 2008

OUTLINE OF THE TALK

I WIRELESS NETWORKS — a crash course in wireless communications

II STOCHASTIC GEOMETRY (SG) — classic models in a new context

III SINR COVERAGE MODEL

IV SOME MORE (APPLIED) WORKS

V CONCLUSIONS & PERSPECTIVES

WIRELESS NETWORKS

what we build on

Networking:

coexistence and cooperation in time and space of many communications.

WIRELESS NETWORKS

what we build on

Networking:

coexistence and cooperation in time and space of many communications.

wireless networks $\Leftarrow$ SG

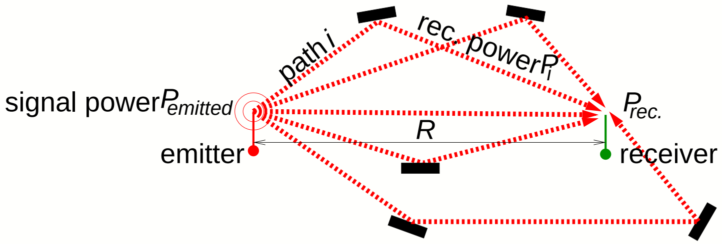
$\Uparrow$

signal detection models

$\Uparrow$

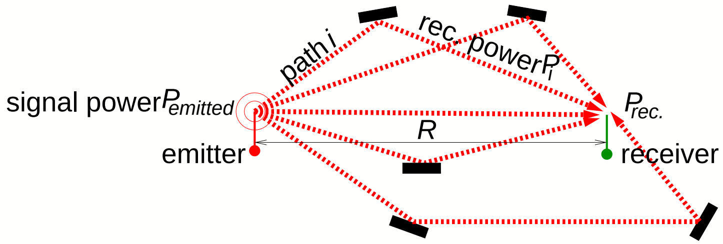
radio wave propagation models

radio wave propagation models



detailed Maxwell's electromagnetic field equations — too complex and not needed

radio wave propagation models

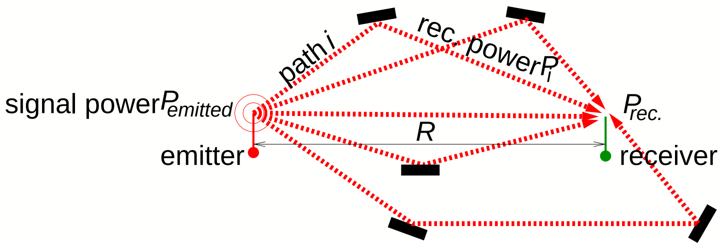


detailed Maxwell's electromagnetic field equations — too complex and not needed

(D) Deterministic model: $P_{rec.} \approx P_{emitted} \cdot l(R)$,

where $l(R) \sim R^{-\beta}$ is deterministic attenuation function,

radio wave propagation models



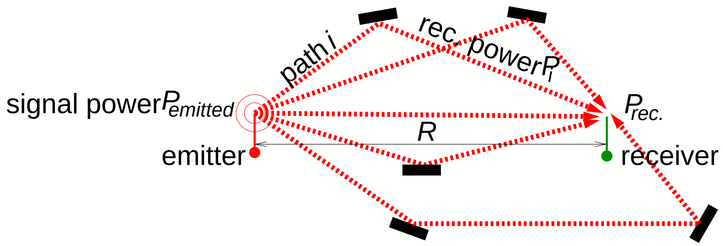
detailed Maxwell's electromagnetic field equations — too complex and not needed

(D) Deterministic model: $P_{rec.} \approx P_{emitted} \cdot l(R)$,

(G) Stochastic ("far-field") model: $P_{rec.} = \sum_i P_i \overset{CLT}{\approx} P_{emitted} \cdot F \cdot l(R)$,

where $l(R) \sim R^{-\beta}$ is deterministic attenuation function, F is random fading variable.

radio wave propagation models



detailed Maxwell's electromagnetic field equations — too complex and not needed

(D) Deterministic model: $P_{rec.} \approx P_{emitted} \cdot l(R)$,

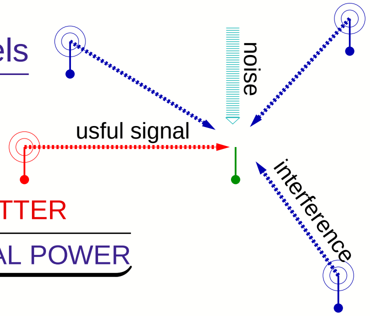
(G) Stochastic ("far-field") model: $P_{rec.} = \sum_i P_i \overset{CLT}{\approx} P_{emitted} \cdot F \cdot l(R)$,

where $l(R) \sim R^{-\beta}$ is deterministic attenuation function, F is random fading variable.

(M) Rayleigh fading: F is exponential ($= Z_I^2 + F_Q^2$ where Z_I, Z_Q i.i.d. N(0,1))

signal detection models

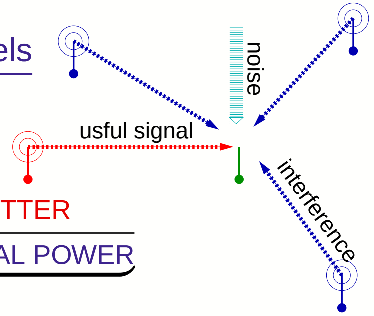
The principal (radio) channel characteristic:



$$\text{SINR} = \frac{\text{POWER RECEIVED FROM GIVEN EMITTER}}{\text{NOISE POWER} + \underbrace{\text{OTHER RECEIVED SIGNAL POWER}}_{\text{interference}}}$$

signal detection models

The principal (radio) channel characteristic:

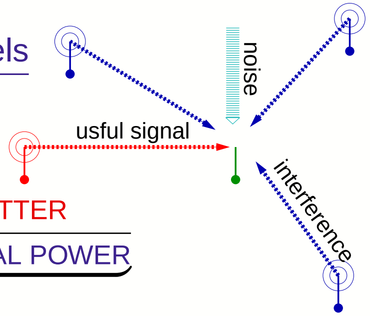


$$\text{SINR} = \frac{\text{POWER RECEIVED FROM GIVEN EMITTER}}{\text{NOISE POWER} + \underbrace{\text{OTHER RECEIVED SIGNAL POWER}}_{\text{interference}}}$$

depends very much on the geometry of the location of nodes!

signal detection models

The principal (radio) channel characteristic:



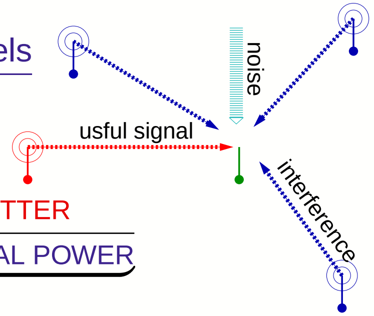
$$\text{SINR} = \frac{\text{POWER RECEIVED FROM GIVEN EMITTER}}{\text{NOISE POWER} + \underbrace{\text{OTHER RECEIVED SIGNAL POWER}}_{\text{interference}}}$$

depends very much on the geometry of the location of nodes!

Non-outage: $1(\text{SINR} > Const)$ — existence of connection

signal detection models

The principal (radio) channel characteristic:



$$\text{SINR} = \frac{\text{POWER RECEIVED FROM GIVEN EMITTER}}{\text{NOISE POWER} + \underbrace{\text{OTHER RECEIVED SIGNAL POWER}}_{\text{interference}}}$$

depends very much on the geometry of the location of nodes!

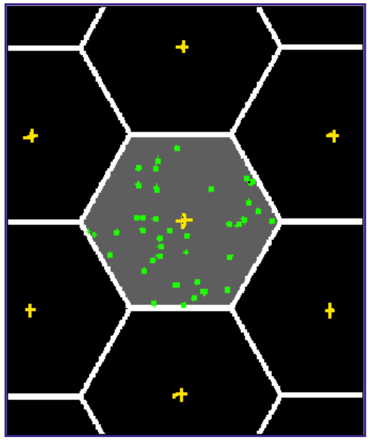
Non-outage: $1(\text{SINR} > Const)$ — existence of connection

Throughput: $f(\text{SINR})$ — # Bits/sec that can be send in the channel with a given bit-error probability; f depends on coding: linear for simple schemes, $f(x) \sim \log(1 + x)$ is max. theoretical bound (Shannon theorem).

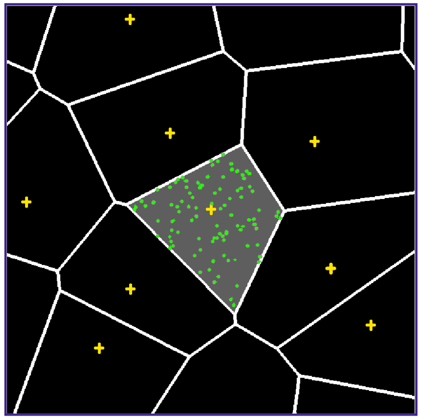
types of wireless networks

Cellular networks (GSM, UMTS, WiFi): Infrastructure of base stations or access points provided by an operator. Individual users talk to these stations and listen to them.

regular



irregular (mesh networks)



Key issues concerning cellular networks:

- How do the cells really look like?

Key issues concerning cellular networks:

- How do the cells really look like?
- How many users a given infrastructure can reliably serve?

Key issues concerning cellular networks:

- How do the cells really look like?
- How many users a given infrastructure can reliably serve?
- Users send (uplink) and receive (downlink).

Key issues concerning cellular networks:

- How do the cells really look like?
- How many users a given infrastructure can reliably serve?
- Users send (uplink) and receive (downlink).
- Channels carry voice (fixed-birtate traffic) and/or data (elastic traffic).

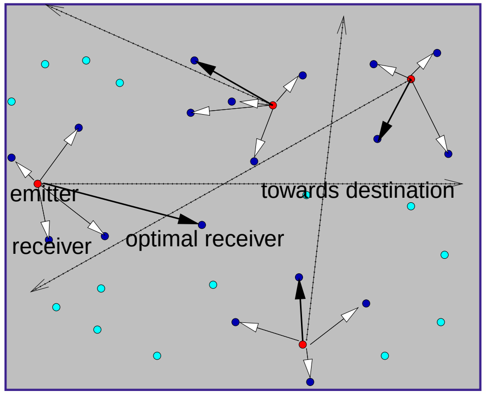
Key issues concerning cellular networks:

- How do the cells really look like?
- How many users a given infrastructure can reliably serve?
- Users send (uplink) and receive (downlink).
- Channels carry voice (fixed-birtate traffic) and/or data (elastic traffic).
- Users arrive to the network, move and depart.

Key issues concerning cellular networks:

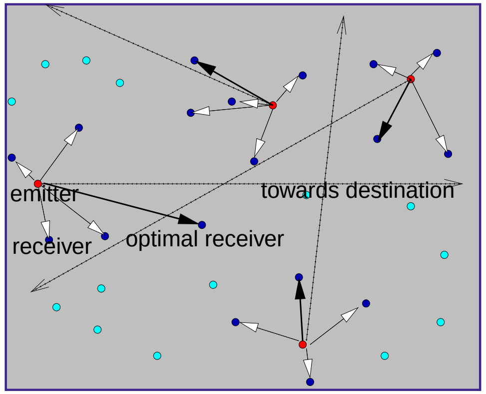
- How do the cells really look like?
- How many users a given infrastructure can reliably serve?
- Users send (uplink) and receive (downlink).
- Channels carry voice (fixed-birtate traffic) and/or data (elastic traffic).
- Users arrive to the network, move and depart.
- Evaluate Quality-of-Service characteristics of a “typical user” (e.g. call blocking probability).

Ad-hoc networks (IEEE 802.11 in mode ad hoc): No fixed infrastructure (no base stations, no access points, etc.)



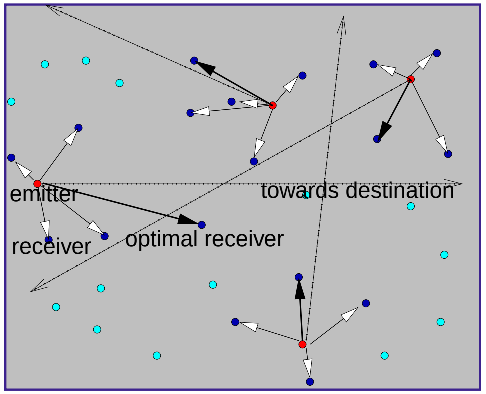
Ad-hoc networks (IEEE 802.11 in mode ad hoc): No fixed infrastructure (no base stations, no access points, etc.)

- A random set of users distributed in space and sharing a common Hertzian medium.



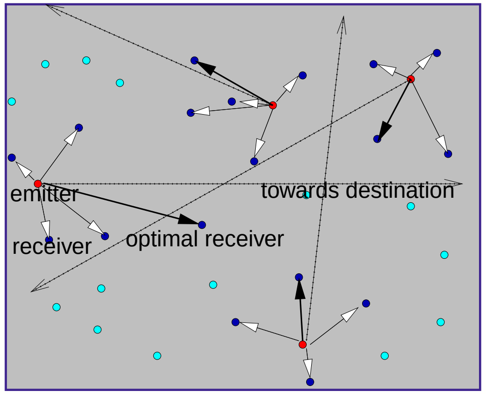
Ad-hoc networks (IEEE 802.11 in mode ad hoc): No fixed infrastructure (no base stations, no access points, etc.)

- A random set of users distributed in space and sharing a common Hertzian medium.
- Users constitute ad-hoc network that is in charge of transmitting information far away via several hops.



Ad-hoc networks (IEEE 802.11 in mode ad hoc): No fixed infrastructure (no base stations, no access points, etc.)

- A random set of users distributed in space and sharing a common Hertzian medium.
- Users constitute ad-hoc network that is in charge of transmitting information far away via several hops.
- Users switch between emitter and receiver modes.



Key issues concerning ad-hoc networks:

- Connectivity: Can every node be reached? No isolated (groups of) nodes?

Key issues concerning ad-hoc networks:

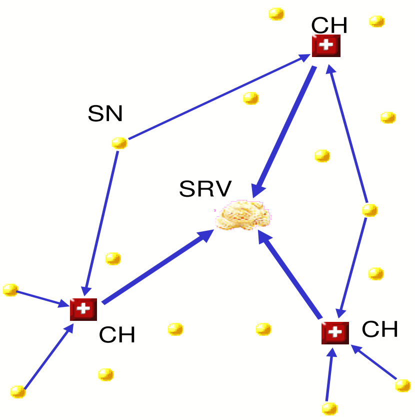
- Connectivity: Can every node be reached? No isolated (groups of) nodes?
- Protocols for routing.

Key issues concerning ad-hoc networks:

- Connectivity: Can every node be reached? No isolated (groups of) nodes?
- Protocols for routing.
- Capacity: How much own traffic every node can send, given it has to relay traffic of other nodes?

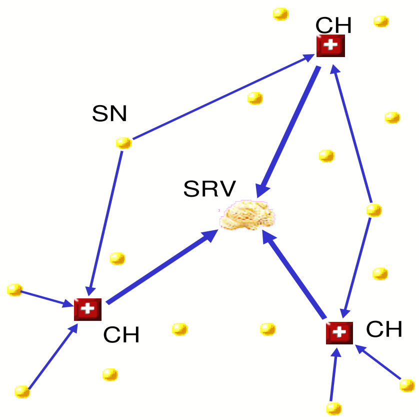
Sensor networks: Variants of ad-hoc networks.

- Nodes monitor some space (measuring temperature, detecting intruders, etc.)
- They send collected information in an ad-hoc manner to some "sink" locations.



Sensor networks: Variants of ad-hoc networks.

- Nodes monitor some space (measuring temperature, detecting intruders, etc.)
- They send collected information in an ad-hoc manner to some "sink" locations.



Issues: Coverage, connectivity, energy (battery) saving.

STOCHASTIC GEOMETRY (SG)

an ancient theory and modern contexts

SG is now a reach branch of applied probability, which allows to study random phenomena on the plane or in higher dimension; it is intrinsically related to the theory of point processes.

STOCHASTIC GEOMETRY (SG)

an ancient theory and modern contexts

SG is now a reach branch of applied probability, which allows to study random phenomena on the plane or in higher dimension; it is intrinsically related to the theory of point processes.

Initially its development was stimulated by applications to biology, astronomy and material sciences. Nowadays, it is also used in image analysis and in the context of communication networks.

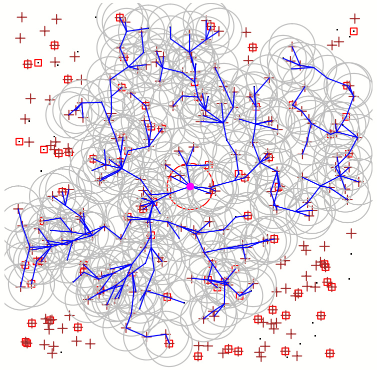
a pioneer

We would like to stress the pioneering role of Edgar N. Gilbert in using SG for modeling of communication networks.

Edgar N. Gilbert (1961) Random plane networks, *SIAM-J*

Edgar N. Gilbert (1962) Random subdivisions of space into crystals, *Ann. Math. Stat.*

Gilbert (1961) proposes continuum percolation model (percolation of the Boolean model) to analyze the connectivity of large wireless networks. Gilbert (1962) is on Poisson-Voronoi tessellations.



followers

PHASE I — domination of the cable: The first papers following Gilbert's ideas appeared in the modern engineering literature shortly before year 2000 (before the massive popularization of wireless communications) and were using mainly the classic stochastic geometry models (as Voronoi tessellations or Boolean model) trying to fit them to existing networks.

SG an ancient theory and modern contexts ...

followers

PHASE I — domination of the cable: The first papers following Gilbert's ideas appeared in the modern engineering literature shortly before year 2000 (before the massive popularization of wireless communications) and were using mainly the classic stochastic geometry models (as Voronoi tessellations or Boolean model) trying to fit them to existing networks.

PHASE II — wireless revolution: Nowadays, the number of papers using some form of stochastic geometry is increasing very fast in conferences like Infocom or Mobicom, where one of the most important observed trends is **an attempt to better take into account in geometric models specific mechanisms of wireless communications**.

classic SG models in wireless context

- (Spatial) Poisson point process,
- Voronoi tessellation,
- Boolean model,
- Shot-noise fields,
- Matérn hard-core model (a simple model for non-overlapping spheres). (not in this talk)

(Spatial) Poisson point process

Planar Poisson point process (p.p.) Φ of intensity λ:

- Number of Points $\Phi(B)$ of Φ in subset B of the plane is Poisson random variable with parameter $\lambda|B|$, where $|\cdot|$ is the Lebesgue measure on the plane; i.e.,

$$\mathbf{P}\{\, \Phi(B) = k \,\} = e^{-\lambda|B|}\,\frac{(\lambda|B|)^k}{k!}\,,$$

- Numbers of points of Φ in disjoint sets are independent.

(Spatial) Poisson point process

Planar Poisson point process (p.p.) Φ of intensity λ:

- Number of Points $\Phi(B)$ of Φ in subset B of the plane is Poisson random variable with parameter $\lambda|B|$, where $|\cdot|$ is the Lebesgue measure on the plane; i.e.,

$$\mathbf{P}\{\, \Phi(B) = k \,\} = e^{-\lambda|B|}\,\frac{(\lambda|B|)^k}{k!}\,,$$

- Numbers of points of Φ in disjoint sets are independent.

Laplace transform of the Poisson p.p.:

$$\mathcal{L}_\Phi(h) = \mathrm{E}[e^{\int h(x)\,\Phi(\mathrm{d}x)}] = e^{-\lambda\int(1-e^{h(x)})\,\mathrm{d}x}\,,$$

where $h(\cdot)$ is a real function on the plane and $\int h(x)\,\Phi(\mathrm{d}x) = \sum_{X_i\in\Phi} h(X_i)$.

Poisson p.p. is a very basic model. Used to represent:

- the repartition of users in all kind of networks,
- locations of nodes in ad hoc, mesh and sensor networks,
- locations of base stations (access points) in irregular cellular network architectures.

Poisson p.p. is a very basic model. Used to represent:

- the repartition of users in all kind of networks,
- locations of nodes in ad hoc, mesh and sensor networks,
- locations of base stations (access points) in irregular cellular network architectures.

:-) Very simple model, allows for explicit calculus

Poisson p.p. is a very basic model. Used to represent:

- the repartition of users in all kind of networks,
- locations of nodes in ad hoc, mesh and sensor networks,
- locations of base stations (access points) in irregular cellular network architectures.

:-) Very simple model, allows for explicit calculus

:-((Homogeneous) Poisson assumption often too simplistic.

Poisson p.p. is a very basic model. Used to represent:

- the repartition of users in all kind of networks,
- locations of nodes in ad hoc, mesh and sensor networks,
- locations of base stations (access points) in irregular cellular network architectures.

:-) Very simple model, allows for explicit calculus

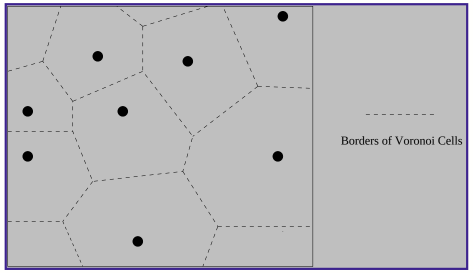
:-((Homogeneous) Poisson assumption often too simplistic.

Need for models exhibiting more clustering, attraction, repulsion of points $\Rightarrow$ Cox models (doubly stochastic Poisson p.p's), Gibbs p.p., Hard-core p.p. and others

Voronoi Tessellation (VT)

Given a collection of points $\Phi = \{X_i\}$ on the plane and a given point x, we define the Voronoi cell of this point $\mathcal{C}_x = \mathcal{C}_x(\Phi)$ as the subset of the plane of all locations that are closer to x than to any point of Φ; i.e.,

$$\mathcal{C}_x(\Phi) = \{y \in \mathbb{R}^2 : |y - x| \le |y - X_i| \ \forall X_i \in \Phi\} .$$

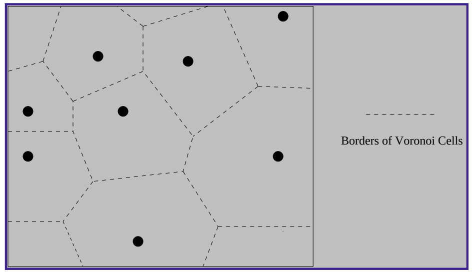


Voronoi Tessellation (VT)

Given a collection of points $\Phi = \{X_i\}$ on the plane and a given point x, we define the Voronoi cell of this point $\mathcal{C}_x = \mathcal{C}_x(\Phi)$ as the subset of the plane of all locations that are closer to x than to any point of Φ; i.e.,

$$\mathcal{C}_x(\Phi) = \{y \in \mathbb{R}^2 : |y - x| \le |y - X_i| \ \forall X_i \in \Phi\} .$$

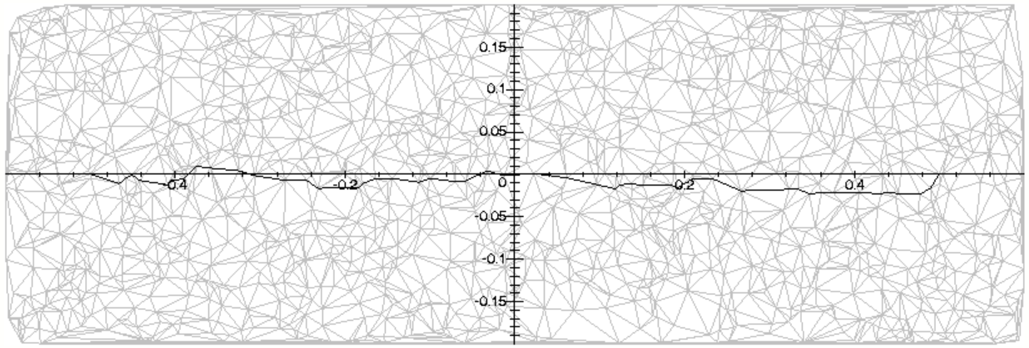
When $\Phi = \{X_i\}$ is a Poisson p.p. we call the (random) collection of cells $\{\mathcal{C}_{X_i}(\Phi)\}$ the Poisson-Voronoi tessellation (PVT).



VT is a frequently used to model the cellular network architectures; points denote locations of BS's.

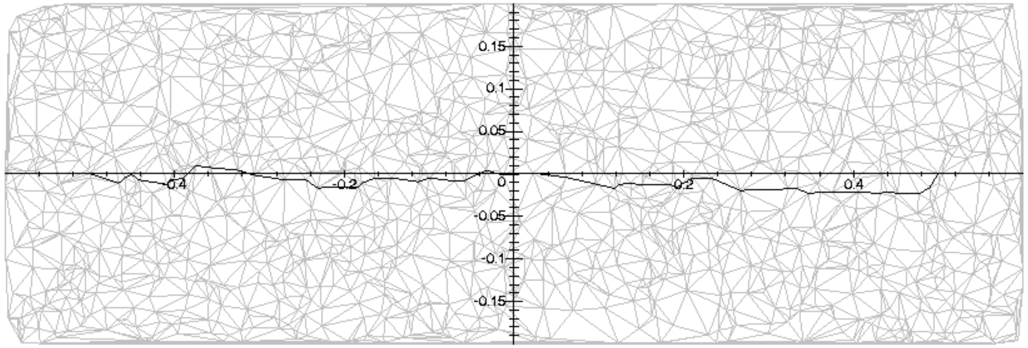
VT is a frequently used to model the cellular network architectures; points denote locations of BS's.

A dual model to Voronoi tessellation, the Delauney triangulation is used as a (connected) graph of nearest neighbours (e.g. for routing purpose).



VT is a frequently used to model the cellular network architectures; points denote locations of BS's.

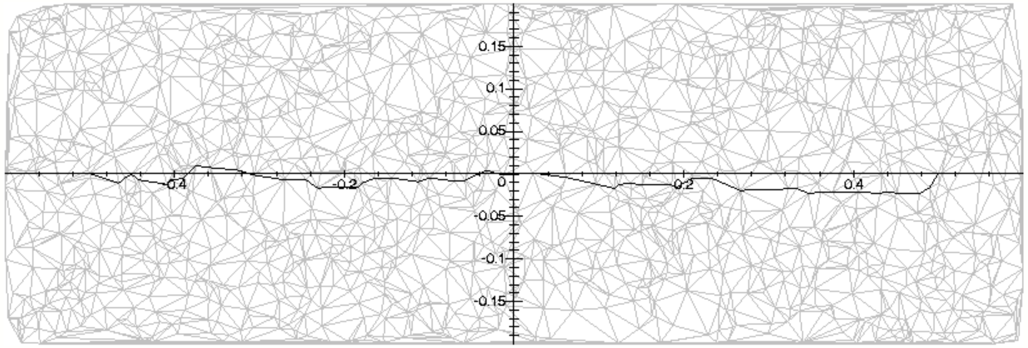
A dual model to Voronoi tessellation, the Delauney triangulation is used as a (connected) graph of nearest neighbours (e.g. for routing purpose).



: -) VT takes into account distance to nearest BS's (neighbourhood model)

VT is a frequently used to model the cellular network architectures; points denote locations of BS's.

A dual model to Voronoi tessellation, the Delauney triangulation is used as a (connected) graph of nearest neighbours (e.g. for routing purpose).



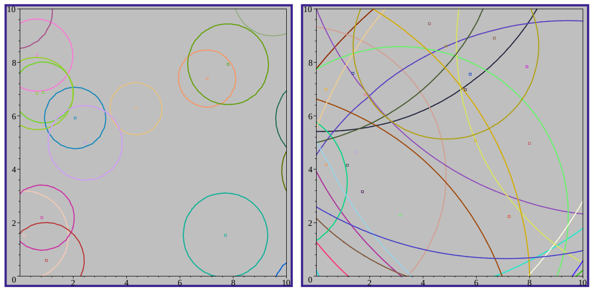
: -) VT takes into account distance to nearest BS's (neighbourhood model)

: - (ignores other physical aspects of the communication as path loss, interference.

Boolean Model (BM)

Let $\tilde{\Phi} = \{(X_i, G_i)\}$ be a marked Poisson p.p., where $\{X_i\}$ are points and $\{G_i\}$ are iid random closed stets (grains). We define the Boolean Model (BM) as the union

$$\Xi = \bigcup_i X_i \oplus G_i \qquad \text{where } x \oplus G = \{x + y : y \in G\}.$$



BM with spherical grains of random radii

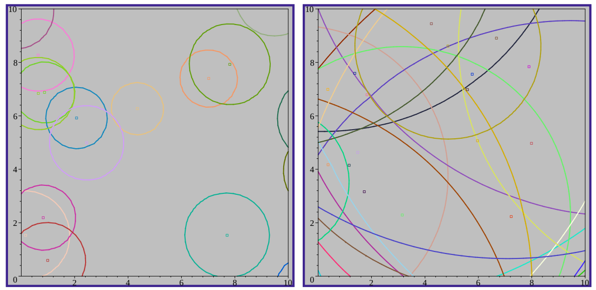
Boolean Model (BM)

Let $\tilde{\Phi} = \{(X_i, G_i)\}$ be a marked Poisson p.p., where $\{X_i\}$ are points and $\{G_i\}$ are iid random closed stets (grains). We define the Boolean Model (BM) as the union

$$\Xi = \bigcup_i X_i \oplus G_i \qquad \text{where } x \oplus G = \{x + y : y \in G\}.$$

Known:

- Poisson distribution of the number of grains intersecting any given set.



BM with spherical grains of random radii

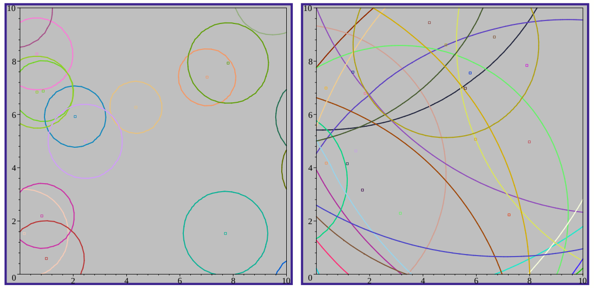
Boolean Model (BM)

Let $\tilde{\Phi} = \{(X_i, G_i)\}$ be a marked Poisson p.p., where $\{X_i\}$ are points and $\{G_i\}$ are iid random closed stets (grains). We define the Boolean Model (BM) as the union

$$\Xi = \bigcup_i X_i \oplus G_i \qquad \text{where } x \oplus G = \{x + y : y \in G\}.$$

Known:

- Poisson distribution of the number of grains intersecting any given set.
- Asymptotic results ($\lambda \longrightarrow \infty$) for the probability of complete covering of a given set.



BM with spherical grains of random radii

- BM is a generic wireless coverage model: points denote locations of BS's and grains denote (independent!) coverage regions.
- It can be used to address questions of connectivity in case of ad-hoc and mesh networks; (see continuum percolation model, E. N. Gilbert (1961)).

  Phase transition is interpreted as a passage from a disconnected network to a connected one.

- BM is a generic wireless coverage model: points denote locations of BS's and grains denote (independent!) coverage regions.
- It can be used to address questions of connectivity in case of ad-hoc and mesh networks; (see continuum percolation model, E. N. Gilbert (1961)).

  Phase transition is interpreted as a passage from a disconnected network to a connected one.

: -) Simple model, allows for explicit calculus, can account for attenuation effect.

- BM is a generic wireless coverage model: points denote locations of BS's and grains denote (independent!) coverage regions.
- It can be used to address questions of connectivity in case of ad-hoc and mesh networks; (see continuum percolation model, E. N. Gilbert (1961)).

  Phase transition is interpreted as a passage from a disconnected network to a connected one.

: -) Simple model, allows for explicit calculus, can account for attenuation effect.

: - (Ignores interference effect (as coverage regions are independent).

Shot-noise (SN) model

Let $\tilde{\Phi} = \{(X_i, S_i)\}$ be a marked p.p., where $\{X_i\}$ are points and $\{S_i\}$ are some random variables. Given a real response function $L(\cdot)$ of the distance on the plane we define the Shot-Noise field

$$I_{\tilde{\Phi}}(y) = \sum_i S_i L(y - X_i)\,.$$

Shot-noise (SN) model

Let $\tilde{\Phi} = \{(X_i, S_i)\}$ be a marked p.p., where $\{X_i\}$ are points and $\{S_i\}$ are some random variables. Given a real response function $L(\cdot)$ of the distance on the plane we define the Shot-Noise field

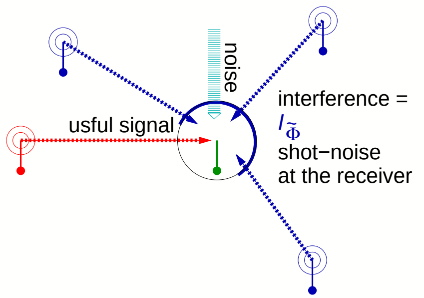
$$I_{\tilde{\Phi}}(y) = \sum_i S_i L(y - X_i)\,.$$

When $\tilde{\Phi}$ is a (say independently) marked Poisson p.p. then we call $I_{\tilde{\Phi}}$ the Poisson SN.

> Joint Laplace transform: For Poisson SN, the LT of the vector $(I_{\tilde{\Phi}}(y_1), \ldots, I_{\tilde{\Phi}}(y_n))$ is known for any $y_1, \ldots, y_n \in \mathbb{R}^2$ (via Laplace transform of the Poisson p.p.).

SN is an excellent model for a total received power in wireless networks:

- marks S_i correspond to emitted powers,
- response function $L(\cdot)$ corresponds to attenuation function.
- Can be enriched to account for other wireless transmission aspects as shadowing, fading, usage of directional antennas, etc.



Extremal SN — a relative of (additive) SN

In the same mathematical scenario as for the SN, one defines extremal Shot-Noise field as

$$J_{\tilde{\Phi}}(y) = \max_i S_i L(y - X_i)\,.$$

Extremal SN — a relative of (additive) SN

In the same mathematical scenario as for the SN, one defines extremal Shot-Noise field as

$$J_{\tilde{\Phi}}(y) = \max_i S_i L(y - X_i)\,.$$

Joint Probability Distribution Function: For the Poisson extremal SN, the joint PDF of the vector $(J_{\tilde{\Phi}}(y_1), \ldots, J_{\tilde{\Phi}}(y_n))$ is known for any $y_1, \ldots, y_n \in \mathbb{R}^2$.

In fact PDF of the extremal SN can be expressed via Laplace transform of the additive SN.

Extremal SN — a relative of (additive) SN

In the same mathematical scenario as for the SN, one defines extremal Shot-Noise field as

$$J_{\tilde{\Phi}}(y) = \max_i S_i L(y - X_i)\,.$$

Joint Probability Distribution Function: For the Poisson extremal SN, the joint PDF of the vector $(J_{\tilde{\Phi}}(y_1), \ldots, J_{\tilde{\Phi}}(y_n))$ is known for any $y_1, \ldots, y_n \in \mathbb{R}^2$.

In fact PDF of the extremal SN can be expressed via Laplace transform of the additive SN.

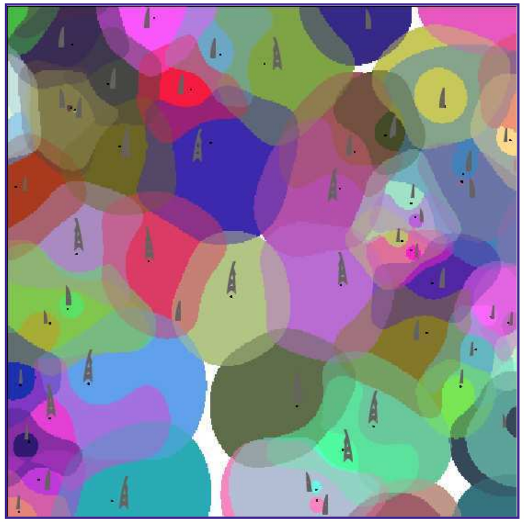
Extremal SN is a useful model when one wants to pick some particular, optimal in some sense receiver; e.g. the strongest one, the nearest, with the best channel statistics, etc.

SINR COVERAGE MODEL

a coverage model with dependent grains

"in between" Voronoi and Boolean

- Definition,
- Snapshots,
- Outline of works and existing results,
- Zoom: typical cell coverage probability in M/M case.



DEFINITION

of the SINR coverage model

$\Phi = \{X_i, (S_i, T_i)\}$ marked point process (Poisson)

$\{X_i\}$ points of the p.p. on $\mathbb{R}^2$ — antenna locations,

$(S_i, T_i) \in (\mathbb{R}^+)^2$ possibly random mark of point X_i — (power,threshold)

cell attached to point X_i: $\boxed{C_i(\Phi, W) = \left\{ y : \dfrac{S_i l(y - X_i)}{W + \kappa I_\Phi(y)} \geq T_i \right\}}$

where $I_\phi(y) = \sum_{i \neq 0} S_i l(y - X_i)$ shot noise process, κ interference cancellation factor, $W \geq 0$ external noise, $l(\cdot)$ response (attenuation) function.

C_i is the region where the SINR from X_i is bigger than the threshold T_i.

DEFINITION

of the SINR coverage model

$\Phi = \{X_i, (S_i, T_i)\}$ marked point process (Poisson)

$\{X_i\}$ points of the p.p. on $\mathbb{R}^2$ — antenna locations,

$(S_i, T_i) \in (\mathbb{R}^+)^2$ possibly random mark of point X_i — (power,threshold)

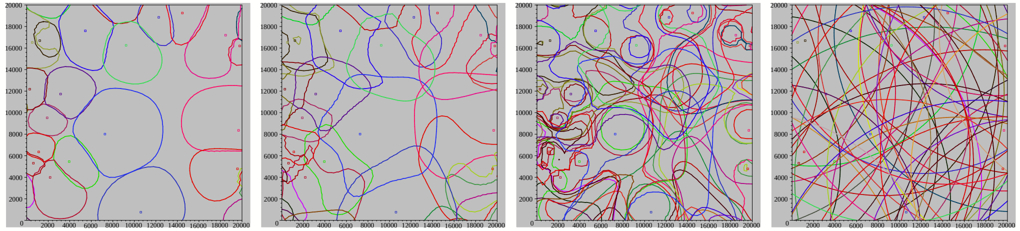
cell attached to point X_i: $\boxed{C_i(\Phi, W) = \left\{ y : \frac{S_i l(y - X_i)}{W + \kappa I_\Phi(y)} \geq T_i \right\}}$

where $I_\phi(y) = \sum_{i \neq 0} S_i l(y - X_i)$ shot noise process, κ interference cancellation factor, $W \geq 0$ external noise, $l(\cdot)$ response (attenuation) function.

C_i is the region where the SINR from X_i is bigger than the threshold T_i.

Coverage PROCESS: $\boxed{\Xi(\Phi; W) = \bigcup_{i \in \mathbb{N}} C_i(\Phi, W).}$

SNAPSHOTS

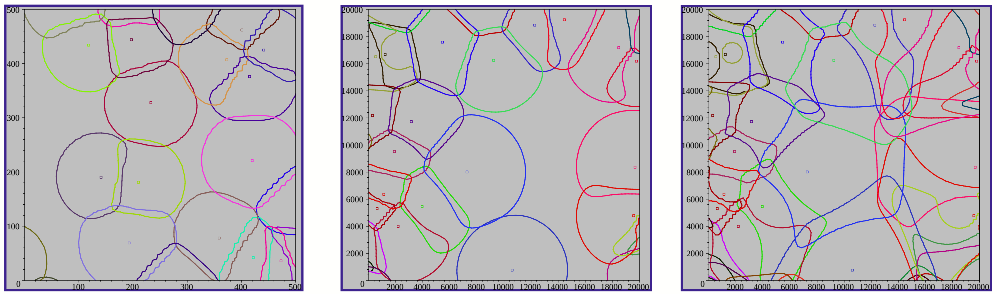


$\kappa = 0.5$ $\kappa = 0.1$ $\kappa = 0.01$ $\kappa = 0$

Constant emitted powers S_i, $T = 0.4$ and

interference factor $\kappa \to 0$.

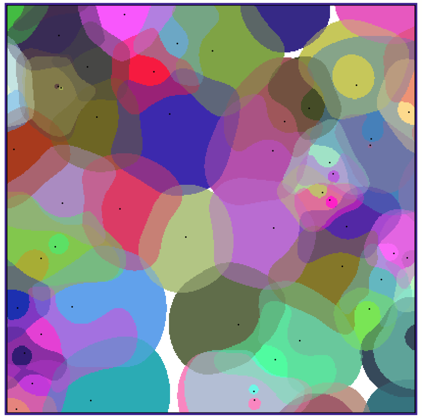
Small interference factor allows one to approximate SINR cells by a Boolean model (quantitative results via perturbation methods).

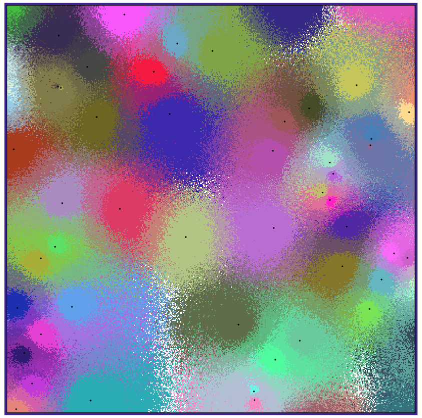


Constant emitted powers S_i, $T = 0.4$, $W = 0$, $l(r) = (Ar)^{-\beta}$ and

attenuation exponent $\beta \to \infty$.

SIR cells tend to Voronoi cells whenever attenuation is stronger, e.g. in urban areas.

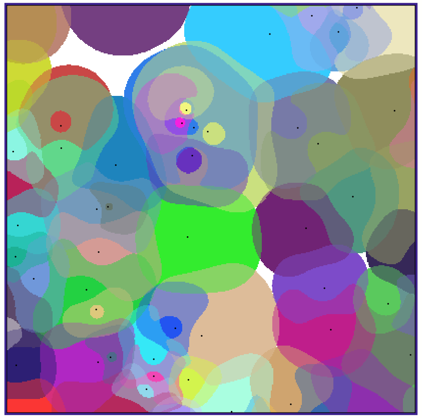


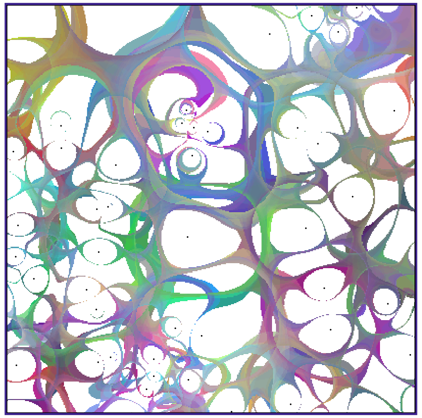


G/D model G/M model

Cells without and with point dependent fading.

Fading reflects variations in time and space of the channel quality about its average state.





Cells with macrodiversity $K = 1$ and the gain of the macrodiversity $K = 2$.

Macrodiversity K: possibility of being connected simultaneously to K stations and to combine signals from them.

OUTLINE OF WORKS AND EXISTING RESULTS

Baccelli & BB (2001), *Adv. Appl. Probab.*

- Existence conditions for the random closed set (local finiteness of the pattern),

OUTLINE OF WORKS AND EXISTING RESULTS

Baccelli & BB (2001), *Adv. Appl. Probab.*

- Existence conditions for the random closed set (local finiteness of the pattern),
- Relation to classical models (“in between” Boolean and Voronoi),

OUTLINE OF WORKS AND EXISTING RESULTS

Baccelli & BB (2001), *Adv. Appl. Probab.*

- Existence conditions for the random closed set (local finiteness of the pattern),
- Relation to classical models (“in between” Boolean and Voronoi),
- maximal overlapping phenomena (maximal number of overlapping cells can be controlled by the model parameters),

OUTLINE OF WORKS AND EXISTING RESULTS

Baccelli & BB (2001), *Adv. Appl. Probab.*

- Existence conditions for the random closed set (local finiteness of the pattern),
- Relation to classical models (“in between” Boolean and Voronoi),
- maximal overlapping phenomena (maximal number of overlapping cells can be controlled by the model parameters),
- typical cell characteristics (via Fourier transforms Rieman Boundary Problem and via perturbations of the limiting BM),

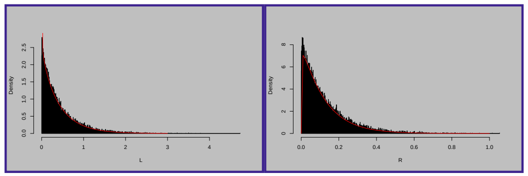
OUTLINE OF WORKS AND EXISTING RESULTS

Baccelli & BB (2001), *Adv. Appl. Probab.*

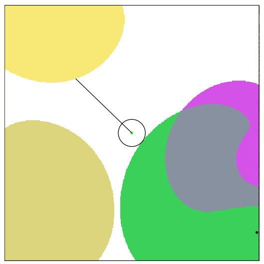
- Existence conditions for the random closed set (local finiteness of the pattern),
- Relation to classical models (“in between” Boolean and Voronoi),
- maximal overlapping phenomena (maximal number of overlapping cells can be controlled by the model parameters),
- typical cell characteristics (via Fourier transforms Rieman Boundary Problem and via perturbations of the limiting BM),
- number of cells covering a point (via moment expansions, Little formula).

Tournois (2002), *INRIA report* **4348**,

- contact distribution functions (estimates via perfect simulation)



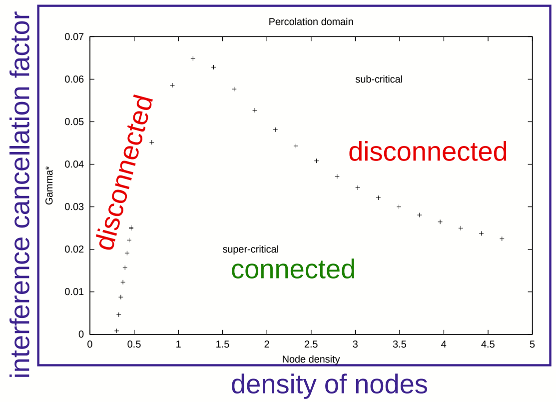
Histograms of linear *L* and spherical *R* contact d.f. given the point is not covered.



EL	*varL*	*ER*	*varR*
0.423 km	0.191 km^2	0.121 km	0.013 km^2

Douse et al. (2003) *IEEE/ACM Tran. Networking*, (2006) *Adv. Appl. Probab.*

- connectivity — existence of the giant component (percolation)



Striking fact: Increasing node density may disconnect the network!

typical cell coverage probability in M/M case

Baccelli, BB, Muhlethaler (2006) *IEEE Trans. Information Theory*

Th. Assume Poisson repartition of nodes and that $\{S_i\}$ are exponential r.vs. with par. μ, $T_i = T$ are constant and denote $\mathcal{L}_W$ the Laplace transform of W. Then the probability for C_0 to cover a given point located at the distance R is equal to

$$p_R = \exp\Big\{-2\pi\lambda\int_0^\infty \frac{u}{1+l(R)/(Tl(u))}\,\mathrm{d}u\Big\}\mathcal{L}_W(\mu T/l(R))\,.$$

typical cell coverage probability in M/M case

Baccelli, BB, Muhlethaler (2006) *IEEE Trans. Information Theory*

Th. Assume Poisson repartition of nodes and that $\{S_i\}$ are exponential r.vs. with par. μ, $T_i = T$ are constant and denote $\mathcal{L}_W$ the Laplace transform of W. Then the probability for C_0 to cover a given point located at the distance R is equal to

$$p_R = \exp\Big\{-2\pi\lambda\int_0^\infty \frac{u}{1+l(R)/(Tl(u))}\,\mathrm{d}u\Big\}\mathcal{L}_W(\mu T/l(R))\,.$$

proof: Say the emitter is at the origin and consider the corresp. Palm distribution **P**;

$$\begin{aligned} p_R &= \mathbf{P}(S \geq T(W + I_\Phi/l(R))) \\ &= \int_0^\infty e^{-\mu s T/l(R)}\,\mathrm{d}\mathbf{P}(W + I_\Phi \leq s) \\ &= \mathcal{L}_{I_\Phi}(\mu T/l(R))\mathcal{L}_W(\mu T/l(R))\,, \end{aligned}$$

where $\mathcal{L}_{I_\Phi}(\cdot)$ is the Laplace transform of the value of the hom. Poisson SN I_Φ.

Cor. For the attenuation function $l(u) = (Au)^{-\beta}$ and $W = 0$

$$p_R(\lambda) = e^{-\lambda R^2 T^{2/\beta} C}\,,$$

where $C = C(\beta) = \Big(2\pi\Gamma(2/\beta)\Gamma(1-2/\beta)\Big)/\beta$.

SOME MORE (APPLIED) WORKS

using SG framework

- Wireless cellular networks: CDMA/HSDPA, power control, large multi-cell networks, blocking/cuts in streaming traffic, throughput in data traffic; 7 papers, incl. 4 INFOCOM; $\Rightarrow$ reach industrial collaboration
  currently with M.K. Karray

SOME MORE (APPLIED) WORKS

using SG framework

- Wireless cellular networks: CDMA/HSDPA, power control, large multi-cell networks, blocking/cuts in streaming traffic, throughput in data traffic; 7 papers, incl. 4 INFOCOM; ⇒ reach industrial collaboration

  currently with M.K. Karray france telecom R&D

- Mobile ad hoc networks: MAC, routing, MAC/routing interplay, opportunistic schemes, transport capacity; 5 papers, incl. IEEE IT ⇒ a highly cited result

  with F. Baccelli and P. Mühlethaler Hipercom @INRIA

SOME MORE (APPLIED) WORKS

using SG framework

- Wireless cellular networks: CDMA/HSDPA, power control, large multi-cell networks, blocking/cuts in streaming traffic, throughput in data traffic; 7 papers, incl. 4 INFOCOM; ⇒ reach industrial collaboration

  currently with M.K. Karray france telecom R&D

- Mobile ad hoc networks: MAC, routing, MAC/routing interplay, opportunistic schemes, transport capacity; 5 papers, incl. IEEE IT ⇒ a highly cited result

  with F. Baccelli and P. Mühlethaler Hipercom @INRIA

- Sensor networks: hybrid architectures, transport only sensors, reliable transport; 2 papers, incl. 1 INFOCOM ⇒ euro ngi

  with B. Radunovič Microsoft Research Cambridge lab

CONCLUSIONS & PERSPECTIVES

Why SG?

The geometry of the location of nodes plays a key role for wireless networks.

CONCLUSIONS & PERSPECTIVES

Why SG?

The geometry of the location of nodes plays a key role for wireless networks.

SG modeling of communication networks seems particularly relevant for large scale network performance analysis.

CONCLUSIONS & PERSPECTIVES

Why SG?

The geometry of the location of nodes plays a key role for wireless networks.

SG modeling of communication networks seems particularly relevant for large scale network performance analysis.

We believe this methodology will play the same role as queuing theory in wireline systems, where it was instrumental in designing the first multiprogramming computers and the basic protocols used in computer networks and in particular in the Internet.

Perspectives — beyond Poisson assumption

- Sometimes Cox point processes lead to (semi)-explicit analysis. Comparison of Poisson and Cox scenario. Ross's type conjectures in SINR context?
  $\Rightarrow$ BB & Yogeshwaran D. (2008) Directionally convex ordering of random measures, shot-noise fields and some applications to wireless communications, in preparation.

Perspectives — beyond Poisson assumption

- Sometimes Cox point processes lead to (semi)-explicit analysis. Comparison of Poisson and Cox scenario. Ross's type conjectures in SINR context?
  $\Rightarrow$ BB & Yogeshwaran D. (2008) Directionally convex ordering of random measures, shot-noise fields and some applications to wireless communications, in preparation.
- "Spatial birth-and death" dynamics often leads to Gibbs distributions in steady state. In unbounded (space) domain and non-localized dependencies even existence of the model in the steady state is a non-trivial problem.
  $\Rightarrow$ An idea in: T. Schreiber & J. Yukich (2008) Stabilization and limit theorems for geometric functionals of Gibbs point processes, preprint.