Probabilistic Correspondence Matching using Random Walk with Restart

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1. Introduction

Stereo Matching: Local Method
- Compute correlation between points within a matching window

1.1. Multi-window methods
- Choosing the best window among the pre-defined ones
- Choosing the most similar window based on the color and spatial distance

1.2. Adaptive weight method
- Reducing complexity with preserving the performance
- High-performance Low complexity

1.3. State-of-art methods
- Using local information to refine the solution

2. Motivation

Re-formulation of local methods [1]
Local methods are mainly composed of three steps, and each step can be re-formulated as a probability problem.

2.1. Conventional correspondence matching formulation

Matching cost computation:
\[ c_i(x,d) = \min \{ |I_i(x)-I_j(x-d)| \} \]

Cost aggregation:
\[ c_{agg}(x,d) = \sum_{i} c_i(x,d) \]

Disparity computation:
\[ d(x) = \arg \min_{d} c_{agg}(x,d) \]

2.2. Probabilistic correspondence matching

Matching probability computation:
\[ p_i(x,d) = \max \{ 1 - \frac{c_i(x,d)}{c_{agg}(x,d)} \} \]

Cost aggregation:
\[ p_{agg}(x,d) = \sum_{i} p_i(x,d) \]

Disparity computation:
\[ d(x) = \arg \min_{d} p_{agg}(x,d) \]

3. Random Walk with Restart

The Random Walk (RW) theory has been widely used to optimize probabilistic problems. It has been known that the RW and the Laplacian equation give the same solution, which means that the steady-state of a given energy functional can be captured by the RW.

Random Walk with Restart

The Random Walk with Restart (RWR) has become increasingly popular, since its restarting terms gives the meaningful information in a steady-state, allowing it to consider the global relation at all scales. Note that the RW becomes the RW as the restarting probability approaches to zero.

Relationship between Random Walk and Correspondence Matching

Inferring a probability with a small neighborhood is the same as a procedure of the RW, which means that the adaptive weight method does not provide a meaningful steady-state solution similar to the RW. Thus, in conventional methods, the number of iteration should be specified in advance and it significantly influences the performance of the algorithms.

Disparity estimation results of the proposed method when the restarting probability is zero and it is the case of no meaningful solution.

4. Probabilistic Correspondence Matching

Graph model
Consider an initial matching probability as an undirected graph \( G = (V,E) \) with nodes \( V \) and edges \( E \). Each node \( v_i \in V \) indicates a point at \( x_i \in \{1,...,N \} \) in an initial matching probability where \( N \) is the size of reference image. The adjacent nodes \( v_i \) and \( v_j \) are connected to an edge \( e_{ij} \in E \). The graph assigns a weight to each edge as follows:

\[ w_{ij} = \exp \left( - \frac{|I_i(x_i)-I_j(x_j)|}{\sigma} \right) \]

Graph model for an initial matching probability

4.1. Probability inference
A random walker, with an initial position \( x_i \), iteratively transits to its neighboring points according to the edge weight until it reaches to the reference position \( x_r \). Also, the random walker goes back to \( x_i \), with the restarting probability at each iteration.

- Probability inference via RW: Steady-state
  \[ R^\infty \text{-} \text{matching probability} \]
  \[ W = \{ I_i \} \rightarrow \text{Adjacency matrix} \]
  \[ D = \sum_i \{ I_i \} \rightarrow \text{Degree matrix} \]
  \[ R = (1-a) W + a D^{-1} \rightarrow \text{Steady state solution} \]

Formulation of iterative manner
Using transition and time approaches to 

4.2. Probability inference via RW: Consideration of R

- Probability inference via RW: Consideration of R
  \[ R : \text{Interpreted as affinity scores between points in an initial state} \]
  \[ R = \alpha (1-\alpha) W + (1-\alpha) D^{-1} \rightarrow \text{Steady state solution} \]

Thus, in conventional methods, the RW recommends an initial matching probability by considering all paths between two points at all scales.

Disparity computation
With a steady-state probability, a disparity can be simply selected by winner-takes-all strategy

\[ d(x) = \arg \max_{d} p_{agg}(x,d) \]

Advantages
1) An adaptive steady-state solution is guaranteed by constraining a steady-state probability to an initial matching probability, some extent, which means that it is not needed to specify the number of iteration.
2) The global relationship between points or the steady-state solution can be captured by using an adjacent neighborhood only, which lowers the complexity of algorithms while maintaining the performance.

Accordingly, the proposed method gives high quality matching performance in a semi-global manner with low complexity.

5. Experimental Results

Qualitative Results

Qualitative Results

Quantitative Results

The computation time of disparity estimation from (from left to right): Random Walker, “Venus”, “Teddy”, “Cones” Window shape is refined Window size is large Window size is small

References