1 Semidefinite programming solver

Consider the following semidefinite program

\[
\begin{align*}
\text{minimize} & \quad \text{Tr } CX \\
\text{subject to} & \quad \text{Tr } A_i X = b_i, \quad i = 1, \ldots, m \\
& \quad X \succeq 0
\end{align*}
\] (1)

— Use the barrier method to write a simple MATLAB solver for this problem (it should solve a phase one problem first to get a feasible starting point). Test it on randomly generated problem instances.

*Hint*: Focus on solving the dual. The Hessian for \( \log \det(X) \) can be found in Appendix A.4.3 of the “Convex Optimization” textbook.

— *(Optional)* One can show that a semidefinite program such as (1) with only one constraint \((m = 1)\) always has a solution of rank one. Use the algorithm described in the reader (Appendix B.3) to extract a solution \(xx^T\) from any solution \(X\) to problem (1). Try your code on some random examples.

**NOTE**: Please use *graphics and tables* to illustrate your results as much as possible. You can use MATLAB/OCTAVE or general purpose languages such as PYTHON or JULIA.