HW #1

Exercises 1-4 (Convexity - duality) Do exercises 4.11, 4.26, 5.5, 5.11, 5.17 in the textbook by Boyd & Vandenberghe.

Exercise 5 (Newton's method) Write a MATLAB function that takes a matrix $A \in \mathbf{R}^{m\times n}$ and a vector $b \in \mathbf{R}^m$ as inputs and returns the solution to:

$$\min_{x\in\mathbf{R}^n} f(x) := -\sum_{i=1}^{m} \log(b_i - a_i^T x)$$

as an output. This is called the *analytic center* of the linear inequalities $Ax \leq b$. Use Newton's method with backtracking line search to compute the minimum with a target precision around 10^{-10}. Demonstrate your code on random examples (e.g. a random section of the unit box in $\mathbf{R}^2$) and plot the convergence to show that it exhibits quadratic convergence close to the optimum (you can use the minimum value of f you computed as a proxy for $f^\star$).

Exercise 6 (Linear programming solver) Write a MATLAB function to solve the following problem:

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax \leq b. \end{array}$$

Use the barrier method together with the analytic center code you wrote above to produce a matlab function that takes a feasible point as input and returns both primal and dual solutions. Write another function that computes a strictly feasible solution to $Ax \leq b$ if there is one, or returns an error message if not. Check your results against those produced by CVX.