Knowledge as a public good: efficient sharing and incentives for development effort *

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Abstract

We consider Research-Development joint ventures where adverse selection arises in knowledge sharing, while there is moral hazard involved in the choice of private development efforts aimed at translating privately acquired and/or shared knowledge into valuable marketable innovations. We extend earlier work by Bhattacharya et al. [Bhattacharya, S., Glazer, J., Sappington, D., 1992. Licensing and the Sharing of Knowledge in Research Joint Ventures, J. Econ. Theory, Vol. 56, pp. 43–69] to situations where one cannot identify a 'most knowledgeable' partner, by giving conditions under which there exist transfers implementing both efficient first best knowledge sharing and subsequent development efforts.

Keywords: Knowledge sharing; Development efforts; Public good

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1 Introduction

While analysing the advantages of cooperation, the recent literature on R & D agreements and research joint ventures in industry has put forward the important public good features of innovative activities due to spillovers or exchanges of productive knowledge. Spillovers effects of R & D are primarily present in the case where research effort levels are independently pursued even though they may be submitted to collective agreements.[1] Spillovers may be replaced by a direct exchange of information and sharing of knowledge, leading to a common level of expertise, as is usually the case in a research joint venture. This sharing of knowledge adds to the 'moral hazard problem', due to the strategic aspects of the development phase where actions remain individual, an 'adverse selection problem' due to the strategic aspects of the research phase where information is exchanged.

Hence, cooperative agreement on R & D provides an interesting context for analysing, within a team, interactions between adverse selection and moral hazard. In earlier work, Bhattacharya et al. (1990, 1992) have considered some of the interactions in the simplified context of knowledge which satisfy a 'Blackwell ordering' across the privately acquired knowledge levels of different participants in a research joint venture. Namely, if all knowledge levels are shared, then the most knowledgeable agent's knowledge is the *only* useful input into the technology for efficient development effort at the subsequent stage(s).

Here also, we consider the problem of motivating efficient knowledge-sharing and effort incentives in R & D, but in the context of information structures that are not necessarily Blackwell ordered across participants. All participants' knowledge levels may be useful in increasing the efficiency of R & D efforts, by the sharing of knowledge among them.

Exploiting the public good nature of innovation, we will rely on a methodology which was developed for analysing problems of implementing efficient outcomes in public goods situations under incomplete information with adverse selection, and extended to team moral hazard prob-

[1] Many models use the notion of 'spillover coefficients' introduced by Spence (1984); Katz (1986); Kamien et al. (1988) and the recent book by Suzumura (1995).

lems.[2] The incentive issues involved in R & D team problem differ from those considered in the literature on pure adverse selection or on pure moral hazard in two important respects. First, in the R & D case, incentives must be provided not only to encourage sharing of knowledge but also to stimulate subsequent privately chosen development efforts by team members. Hence, while disclosure-contingent transfers are required to encourage the sharing of knowledge among team members, these transfers must not distort incentives for subsequent adequate development efforts by members of the team. Second, the revelation of a privately owned attribute, like knowledge, by an agent, has an important 'spillover effect', namely that the attributes of other team members are possibly altered by such revelation. This effect, through revelation, is to be distinguished from a 'common value' direct effect.

In Bhattacharya et al. (1992), the authors focus on a situation in which the most knowledgeable agent is pivotal. Even though all individual knowledge levels are revealed, it is efficient to apply only the expertise of the pivotal agent to the technology which probabilistically translates knowledge into an innovation through development efforts by the team participants. They show that if transfers prior to invention are added to transfers contingent on successful innovation by (one or more) team member(s), then there exists a transfer rule implementing both efficient (first best) knowledge-sharing and subsequent development efforts. This rule is constructively characterized and shown to lead to 'interim individually rational' outcomes, namely to positive expected profits conditional on the privately acquired knowledge.[3] We will see here that in

[2] In the standard adverse selection case (d'Aspremont and Gérard-Varet, 1979; d'Aspremont and Jacquemin 1988; d'Aspremont et al., 1992), there is a planner which precommits to a decision rule, mapping the vector of disclosed private attributes into an (efficient) outcome as well as (budget-balancing) transfers among the team members to induce truthful revelation. In the classical team moral hazard case (Holmström, 1982; d'Aspremont and Gérard-Varet, 1994; Fudenberg et al., 1994), the problem is to design a (balanced) transfer mechanism so that the members in the team are induced to choose a proper (efficient) joint action.

[3] Bhattacharya et al. (1992) also show that there are circumstances, involving the ex-ante distribution of potential knowledge levels, such that efficient implementation need not require transfers to the most knowledgeable agent by participants who do *not* subsequently succeed in innovating. The result is of importance when team members are resource-constrained in the absence of profitable innovation, or can leave the team following knowledge-sharing and courts cannot verify development efforts per se.

situations where knowledge is not Blackwell ordered and where the (revealed) knowledge levels of all team members may positively interact to determine the aggregated level of knowledge in the team, we have to weaken individual rationality. This will lead to an 'ex-ante' requirement, namely to positive expected profit before any acquisition of knowledge.

Our results are related to others in the recent literature on team problems with adverse selection and moral hazard. In Laffont and Tirole (1987) and McAfee and McMillan (1987), the team problem breaks up into separate single-agent problems, each agent competing for the contract. In our case, we do not have competition for contracts, and the agents have to work together. Moreover we do not restrict a-priori, as in Zou (1989), Picard and Rey (1990) or McAfee and McMillan (1991), to implementation by a Principal in charge of budget-breaking. As in Demski and Sappington (1984), we consider arrangements inducing full-information outcomes from team members having private informations and unobservable actions, but without restricting to individuals having only two types. We rely on Bayesian equilibrium implementation for contractual arrangements based on a two step procedure: one step for sharing knowledge and a second step for development efforts.

We present the model in Section 2, and the implementation concept in Section 3. Our main result is shown in Section 4.

2 The model

We consider a team which consists of a (finite) set $N = \{1, \cdots, i, \cdots, n\}$ of firms involved in some research joint venture. Each firm is endowed with a knowledge level which is an element of some (finite) ordered set $\Theta = \{0, 1, \cdots, t, \cdots, \overline{\theta}\}$. The distribution of knowledge among the firms is modeled by a probability distribution f over Θ^N, where $f(\theta)$ is the joint probability that individual knowledge levels be given by $\theta = (\theta_1, \cdots, \theta_i, \cdots, \theta_n) \in \Theta^N$. The probability distribution f is assumed to be common knowledge and the individual knowledge endowments, which are private information for the firms, stand for their types. Thus the conditional probability $f(\theta_{-i} \mid \theta_i)$ gives firm i's beliefs, when of knowledge level $\theta_i \in \Theta$, about possible levels of knowledge $\theta_{-i} = (\theta_j)_{j \neq i} \in \Theta^{N-\{i\}}$ of others team members.

The research venture involves the firms simultaneously undertaking an unobservable R & D effort to secure some probability of getting an outcome. Effort levels stand for the individual actions which are independently selected by the firms in some ordered set $A = \{0, 1, \cdots, \alpha, \cdots, \overline{\alpha}\}$. This set is supposed to be finite but sufficiently large (in a sense to be made precise below). A configuration of effort levels is an n-tuple $a = (a_1, \cdots a_i, \cdots, a_n) \in A^N$. Effort levels are unobservable, but some joint outcome, which is an element of a (finite) set Y, is observable. Also, there is a common knowledge probability $g(y \mid a)$ to get the outcome $y \in Y$ when $a \in A^N$ is the configuration of firms' efforts.

The outcome of the research venture has a value which is measured by a monetary payoff function $v : Y \to \mathbb{R}$, also assumed to be common knowledge. Thus, the team expected (monetary) value for undertaking effort levels $a \in A^N$ is given by:

$$\sum_{y \in Y} v(y)\, g(y \mid a).$$

Individual efforts are privately costly and the cost to each firm of implementing some level of effort $\alpha \in A$ is given by $C(\alpha, t)$, which is a function of the level of knowledge $t \in \Theta$. We assume that the cost is, at any given $t \in \Theta$, an increasing function of the level of effort, and that higher levels of knowledge reduce the total cost of implementing a particular level of effort, namely the assumption:

A1. $C(\alpha, t)$ is increasing in $\alpha \in A$ and decreasing in $t \in \Theta$.

The firms have the opportunity, before undertaking R & D efforts, to simultaneously make public some or all of their knowledge. A firm cannot exaggerate its true knowledge, but may conceal part of it: firm $i \in N$ of type $\theta_i \in \Theta$ has the possibility to report any level $\tau_i \in \Theta$ such that $\tau_i \leq \theta_i$. Whether this leads or not to *perfect disclosure*, namely that $\tau_i = \theta_i$ for any $i \in N$, depends upon anticipated rewards from disclosure, and upon how reported knowledge is shared.

We assume that there is some common knowledge mechanism, which is a 'precision' function $M : \Theta^N \to \Theta$, giving the level of knowledge $M(\theta_i, \tau_{-i})$ available to firm i of type $\theta_i \in \Theta$ if $\tau_{-i} = (\tau_j)_{j \neq i} \in \Theta^{N-\{i\}}$ is the $(n-1)$-tuple of levels of knowledge made public by other firms in the team. One example of such a precision function $M^{\max}$ is of the Blackwell type and is the one used in Bhattacharya et al. (1992):

$$M^{\max}(\theta_i, \tau_i) = \max\{\tau_1, \cdots, \tau_{i-1}, \theta_i, \tau_{i+1}, \cdots, \tau_n\}.$$

We actually consider any function M satisfying the following assumption:

A2. For every $i \in N$, $M(\theta_i, \tau_{-i})$ is increasing in every individual argument and is strictly increasing in (θ_i, τ_{-i}); furthermore, we have:

$$M(0, 0, \cdots, 0) = 0 \text{ and } M(\overline{\theta}, \cdots, \overline{\theta}) = \overline{\theta}.$$

All firms are assumed to be risk neutral. Thus, if $z_i \in \mathbb{R}$ is firm i's monetary payment from the team, its profit when of type θ_i for implementing a_i is $z_i - C(a_i, M(\theta_i, \tau_{-i}))$, assuming that other members of the team report $\tau_{-i} \leq \theta_{-i}$. The amount z_i results from the contractual arrangements of the team. *Balancedness* requires that: $\forall\, y \in Y, \sum_{i \in N} z_i = v(y)$.

The research joint venture gives rise to a team problem under adverse selection and moral hazard. For any n-tuple $\theta \in \Theta^N$ of individual types, the joint venture results in some vector of disclosures $\tau^0(\theta) = (\tau_1^0(\theta_1), \cdots, \tau_n^0(\theta_n))) \in \Theta^N$, where $\tau_i^0(\theta_i) \leq \theta_i$ is the level of knowledge that firm $i = 1, \cdots, n$, makes public, and in some joint action $a^0(\theta) = (a_1^0(\theta), \cdots, a_n^0(\theta)) \in A^N$ giving the configuration of individual R & D efforts $a_i^0(\theta)$, $i = 1, \cdots, n$, undertaken in that circumstances. A *solution* is thus a pair (τ^0, a^0) of such functions. If a solution is selected according to a 'first best' criterion, it means that for any configuration of types, individual disclosures and effort levels have to maximize the expected total value of the team less the total cost incurred by the firms. Formally a solution (τ^*, a^*) is *first best* if and only if:

$$\forall\, \theta \in \Theta^N, (\tau^*(\theta), a^*(\theta)) \in \arg\max_{\substack{a \\ \tau \leq \theta}} \left[\sum_y v(y)\, g(y \mid a) - \sum_i C(a_i, M(\theta_i, \tau_{-i})) \right].$$

We assume that the total surplus is non-negative for every θ, namely that:

$$\left[\sum_y v(y)\, g(y \mid a^*(\theta)) - \sum_i C(a_i^*(\theta)), M(\theta_i, \tau_{-i}^*(\theta_{-i}) \right] \geq 0.$$

Also the next lemma shows that (under **A1** and **A2**) there is no loss of generality, under first best, to restrict to solutions (θ^*, a^*) with *perfect-disclosure*, $\theta_i^*(\theta_i) = \theta_i$ for every $i \in N$, and with the individual effort levels satisfying some usual 'marginal' conditions.

Lemma 1 *Under* **A1** *and* **A2**, *there is a first-best solution with perfect disclosure* (θ^*, a^*). *Moreover it satisfies the following inequalities:* $\forall\, i \in N$, $\forall\, a_i \in A_i$,

$$C(a_i^*(\theta), M(\theta)) - C(a_i, M(\theta)) \leq \sum_{y \in Y} v(y)[g(y \mid a^*(\theta)) - g(y \mid a_i, a_{-i}^*(\theta))].$$

Proof: Since all sets are finite, there always exists a first-best solution, say (τ^*, a^*). Since $\tau^*(\theta) \leq \theta$, we have successively by **A2** and **A1**: $\forall\, \theta \in \Theta$, $\forall\, i \in N$,

$$\begin{aligned} & M(\theta_i, \tau_{-i}^*(\theta_i)) \leq M(\theta_i, \theta_{-i}), \\ \text{and} \quad & C(a_i^*(\theta), M(\theta_i, \tau_{-i}^*(\theta_{-i}))) \geq C(a_i^*(\theta), M(\theta_i, \theta_{-i})). \end{aligned}$$

Thus,

$$\begin{aligned} & \sum_{y \in Y} v(y)\, g(y \mid a^*(\theta)) - \sum_{i \in N} C(a_i^*(\theta), M(\theta_i, \tau_{-i}^*(\theta_i))) \\ \leq \quad & \sum_{y \in Y} v(y)\, g(y \mid a^*(\theta)) - \sum_{i \in N} C(a_i^*(\theta), M(\theta_i, \theta_{-i})); \end{aligned}$$

showing that (θ^*, a^*) is also a first-best solution and: $\forall\, i \in N, \forall\, a_i \in A_i$

$$\begin{aligned} & \sum_{y \in Y} v(y)\, g(y \mid a^*(\theta)) - C(a_i^*(\theta), M(\theta)) - \sum_{j \neq i} C(a_j^*(\theta), M(\theta)) \\ \geq \quad & \sum_{y \in Y} v(y)\, g(y \mid a_i, a_i^*(\theta)) - C(a_i, M(\theta)) - \sum_{j \neq i} C(a_j^*(\theta), M(\theta)). \end{aligned}$$

■

3 The implementation game

Participants to the joint venture are rewarded by a share of the total value v. Individual payments can be made conditional only on the knowledge levels that are publicly reported and on the outcome that is publicly observable. We consider a two stages contractual arrangement.

The first stage starts with the specification of an overall *contract* which is a function: $S : Y \times \Theta^N \to \mathbb{R}^N$, giving monetary payments $S_i(y \mid \theta)$ to any firm $i \in N$, for any outcome $y \in Y$ and any n-tuple $\theta \in \Theta^N$ of types. We restrict to contracts which are balanced:

$$\forall\, \theta \in \Theta^N, \forall\, y \in Y, \sum_{i \in N} S_i(y \mid \theta) = v(y).$$

At the first stage the firms, having privately observed their individual types θ_i, report independently a level of knowledge $\tau_i \leq \theta_i$. The knowledge is shared according to the precision function M. At the second stage the firms rely on the total amount of information obtained at the first stage to select independently and simultaneously some effort level, contributing to the collective outcome. Then, the surplus is shared on the basis of the pre-accepted arrangement. Actually, we shall be interested only in 'implementing' a first best solution. By Lemma 1, it can be taken to be with perfect disclosure. This motivates the following definition.

We say that a (balanced) contract S *implements* a solution with perfect disclosure (θ^*, a^*) if the following incentive constraints hold:

$$\begin{aligned}
&\forall\, i \in N, \forall\, \theta_i \in \Theta, \forall\, \tau_i \in \Theta, \tau_i \leq \theta_i, \forall\, \tilde{a}_i : \Theta^N \to A, \\
&\quad = \sum_{\theta_{-1}} f(\theta_{-i} \mid \theta_i) \left[\sum_y S_i(y \mid \tau_i, \theta_{-i}) g(y \mid \tilde{a}_i(\tau_i, \theta_{-i}), a^*_{-i}(\tau_i, \theta_{-i})) \right. \\
&\qquad -C(\tilde{a}_i(\tau_i, \theta_{-i}), M(\theta_i, \theta_{-i}))] \\
&\quad \leq \sum_{\theta_{-1}} f(\theta_{-i} \mid \theta_i) \left[\sum_y S_i(y \mid \theta_i, \theta_{-i}) g(y \mid a^*_{-i}(\theta_i, \theta_{-i}), a^*_{-i}(\theta_i, \theta_{-i})) \right. \\
&\qquad \left. -C(a^*_{-i}, \theta_{-1}), M(\theta_i, \theta_{-i}) \right].
\end{aligned}$$

Notice that, at the second stage, the optimal deviation of an individual i, which is denoted $\tilde{a}_i$, is for a given θ_{-i}, a function $\tilde{a}_i(\tau_i, \theta_{-i})$ of the agent's first stage deviation $\tau_i \leq \theta_i$. Clearly, an agent's deviation at the first stage has to take account of how, through the sharing of knowledge, its deviation affects its second stage behaviour. The implementation concept to which we refer here is in terms of Bayesian equilibrium.[4]

[4]It is even a perfect (sequential) Bayesian equilibrium. Effort levels are non observable and, restricting as here to solution with perfect disclosure, the equilibrium strategies cover the support of the knowledge distribution f.

One could in addition leave to each individual firm the possibility of dropping from the contact at some point in time. In other words we may add some participation (or individual rationality) constraint. We consider two extremes, a participation constraint ex ante, i.e., before any knowledge is acquired by any firm, and a participation constraint ex post, i.e., after all acquired knowledge has been shared. Formally a contract S implements in an *ex ante individually rational* way the solution with perfect disclosure (θ^*, a^*) if:

$$\forall\, i \in N, \sum_{\theta} f(\theta) \left[\sum_{y} S_i(y \mid \theta)\, g(y \mid a^*(\theta)) - C(a_i^*(\theta), M(\theta)) \right] \geq 0.$$

It implements this solution in an *ex post individually rational way* if:

$$\forall\, \theta \in \Theta^N, \forall\, i \in N, \sum_{y} S_i(y \mid \theta)\, g(y \mid a^*(\theta)) - C(a_i^*(\theta), M(\theta)) \geq 0.$$

We have the following:

Lemma 2 *If a balanced contract S implements a solution with perfect disclosure (θ^*, a^*), then there exists a balanced contract $\hat{S}$ implementing (θ^*, a^*) in an ex ante individually rational way. In the case where the precision function is $M^{\max}$, then there exists a balanced contract $\hat{S}$ implementing (θ^*, a^*) in an ex-post individually rational way.*

Proof: In the first case, we let:

$$\hat{S}_i(y \mid \theta) = S_i(y \mid \theta) - U_i, \forall\, i \neq 1, \text{ and } \hat{S}_i(y \mid \theta) = S_1(y \mid \theta) + \sum_{i \neq 1} U_i,$$

where:

$$U_i \stackrel{\text{def}}{=} \sum_{\theta} f(\theta) \left[\sum_{y} S_i(y \mid a^*(\theta)) - C(a_i^*(\theta), M(\theta)) \right].$$

In the second case, denoting $i^*(\theta)$ the most knowledgeable firm at $\theta \in \Theta^N$, we let:

$$\begin{aligned} \hat{S}_i(y \mid \theta) &= S_i(y \mid \theta) + \sum_{j \neq i^*(\theta)} U_j(\theta) \quad \text{if}\, i = i^*(\theta) \\ \hat{S}_i(y \mid \theta) &= S_i(y \mid \theta) - U_i(\theta), \quad \text{if}\, i \neq i^*(\theta) \end{aligned}$$

where: $U_i(\theta) = S_i(y \mid \theta)\, g(y \mid a^*(\theta)) - C(a_i^*(\theta), M^{\max}(\theta))$. In both cases, $\hat{S}$ is balanced and the incentives are easily checked, because all the surplus is given to a single individual. ■

4 Implementing first best

In order to implement a first best solution, we need to restrict to the class of payoffs and outcome functions considered by Bhattacharya et al. (1992), and without introducing the continuous action space they use, to take A large enough. A first requirement is:

A3:

$$\begin{aligned}
&Y = \{0,1\}^n, \\
&v(y) = V > 0 \text{ (resp. } = 0), \text{ if } \sum_{i \in N} y_i > 0 \left(\text{resp. } \sum_{i \in N} y_i = 0\right) \\
&g(y \mid a) = \prod_{i \in N} p(a_i)^{y_i}(1 - p(a_i))^{1-y_i}, \text{ with } y_i = 0 \text{ if } p(a_i) = 0 \\
&p : A \to [0,1] \text{ strictly increasing in } \alpha, p(0) = 0 \text{ and } p(\overline{\alpha}) < 1.
\end{aligned}$$

$V > 0$ is the value of the innovation which can be obtained in the R & D contest among the n firms. A result $y \in Y$ is a vector of 1's ('successes') or 0's ('failures') of all the i's. For a level of effort $a_i \in A$ which is selected by i, $p(a_i) \geq 0$ is the probability that i gets a 'success'. There are some payments to be shared among participants only if one of them gets a 'success'.

Second, we assume that the cost of each firm satisfies a monotonicity property ensuring that at a first best solution all positive success probabilities are identical:

A4: $\overline{\alpha} \geq n\overline{\theta}$; moreover, for any $t \in \Theta$, $C(\alpha, t) = c(p(\alpha), t)$, where $c(\cdot, t)$ is a differentiable function defined on $[0,1]$ such that $(1-p)\frac{\partial c}{\partial p}(p,t)$ is strictly monotone in p, but nil for $p = 0$, and for every $t \in \Theta$ and $m = 0, 1, \cdots, n-1$, $V(1 - p(\alpha))^m = \frac{\partial c}{\partial p}(p(\alpha), t))$, for some $\alpha \in A$ with $p(\alpha)) > 0$.

We have:

Lemma 3 *Under* **A1***,* **A2***,* **A3** *and* **A4***, the first best solution with perfect disclosure* $(\theta^*, a^*(\theta))$*, is such that: for every* θ*, there is some nonempty* $N^* \subset N$ *such that, for all* $i \in N^*$*,* $a_i^*(\theta) = \alpha_\theta$ *for some* $\alpha_\theta \in A$ *such that* $p(\alpha_\theta) > 0$*, and for all* $i \in N - N^*, a_i^*(\theta) = 0$*.*

Proof: Fix $\theta \in \Theta$. For any $K \subset N$, define the reduced surplus function on $[0,1)^K$:

$$F^K(x) = V\left(1 - \prod_{i \in K}(1 - x_i)\right) - \sum_{i \in K} c(x_i, M(\theta)) - \sum_{i \in K} c(0, M(\theta)).$$

Let $(K^*, x^*) \in \arg\max\{F^k(x); K \subset N, x \in [0,1)^K\}$, with K^* the least sized such solution. Thus $x_i^* > 0$, $\forall\, i \in K^*$; so x^* is an interior maximum, and hence, a critical point, implying:

$$\forall\, k \in K^*, V \prod_{i \in K^*} (1 - x_i^*) = (1 - x_k^*) \frac{\partial c}{\partial p}(x_k^*, M(\theta)),$$

so that, by **A4**, $x_k^* = x_i^*$, $\forall\, i, k \in K^*$. Also, by **A4**, the equation

$$V(1 - p(\alpha))^{|K^*|-1} = \frac{\partial c}{\partial p}(p(\alpha), M(\theta)),$$

is uniquely solvable in α, say at α_θ, and as a consequence, $x_k^* = p(\alpha_\theta)$, $\forall\, k \in K^*$. Therefore, the first-best solution with perfect disclosure satisfies:

$$a_i^*(\theta) = \alpha_\theta, \forall\, i \in K^* = 0, \forall\, i \in N - K^*.$$

As an example, consider the following sampling-based cost structure. Suppose a technique is sampled, with a probability q of being 'good' and $(1-q)$ of being 'bad'. The knowledge level t results in a probability $(1-t)$ of rejecting a good technique. A bad technique is rejected with probability 1. The probability of at least one success in α trials is

$$p(\alpha) = 1 - [(1-q) + q(1-t)]^\alpha \left(\text{or } \alpha = \frac{\log(1 - p(\alpha))}{\log[(1-q) + q(1-t)]}\right).$$

The cost may be defined as: for every $t \in \Theta$ and $p \in [0,1]$, $c(p,t) = \mathbf{Co} + [\log(1-p)/\log[(1-q) + q(1-t)]]\gamma$, $\gamma > 1$. Then, $(1-p)\frac{\partial c}{\partial p}(p,t)$ is strictly increasing in p. But we could also have taken the case where $(1-p)\frac{\partial c}{\partial p}(p,t)$ is strictly decreasing.[5]

Finally, we restrict to situations where the distribution of knowledge f satisfies the requirement

A5:

$$\forall\, i \in N, \forall\, \theta_i \in \Theta, \forall\, \theta_i' \in \Theta, \forall\, \theta_{-i} \in \Theta^{N-\{i\}},$$
$$f(\theta_{-i} \mid \theta_i') = f(\theta_{-i} \mid \theta_i) = f(\theta_{-i}).$$

[5]It may be shown that $\lim_{p\to 1} \frac{\partial^2_C}{\partial^2_p}(p,t) = \lim_{p\to 1} \frac{\partial c}{\partial p}(p,t) = \infty$, and so that there is no contradiction. In this case the tendency will be that less firms would be required at first-best to choose a positive success probability. There might even be only such firm. This, however, does not by itself solves the knowledge sharing problem.

Assumption **A5** means that the beliefs of any firm with respect to others' types is independent of its own type. This is weaker than assuming that the firms' knowledge endowments are modeled as independent realizations of random variables with density $g(\tau) > 0, \tau \in \Theta$, so that $f(\theta_1, \cdots, \theta_i, \cdots, \theta_n) = X_{i \in N} g(\theta_i)$, (as in Bhattacharya et al., 1992 or McAfee and McMillan, 1991). Nevertheless it is a strong requirement because it implies, since f is common knowledge, that all individual beliefs are common knowledge.

We now show that for a team with moral hazard and adverse selection, under these circumstances, we have:

Theorem 1 *Under Assumptions* **A1***,* **A2***,* **A3***,* **A4** *and* **A5***, a first best solution with perfect disclosure is implementable by a balanced contract.*

Proof: (1) Let (θ^*, a^*) be a first best solution under perfect disclosure as characterized by Lemma 1 and Lemma 3. Under **A5**, we want to show:

$$\exists\, S + (S_1, \cdots, S_i, \cdots, S_n), S_i \in \mathrm{I\!R}^{Y \times \Theta^N}, i = 1, \cdots, n$$

such that:

(i) $\forall\, i \in N, \forall\, \theta_i \in \Theta, \tau_i \leq \Theta, \forall\, \tilde{a}_i : \Theta^N \to A$,

$$\begin{aligned} & \sum_{\theta_{-i}} f(\theta_{-i}) \left[\sum_y S_i(y \mid \theta_i, \theta_{-i})\, g(y \mid a^*(\theta_i, \theta_{-i})) - \sum_y S_i(y \mid \tau_i, \theta_{-i})\, g(y \mid \tilde{a}_i(\tau_i, \theta_{-i}), a^*_{-i}, \theta_{-i})) \right] \\ \geq\ & \sum_{\theta_{-i}} f(\theta_{-i}) [C(a^*_i(\theta_i, \theta_{-i}), M(\theta_i, \theta_{-i})) - C(\tilde{a}_i(\tau_i, \theta_{-i}), M(\theta_i, \theta_{-i}))]. \end{aligned}$$

(ii) $\forall\, y \in Y, \forall\, \theta \in \Theta, \sum_{i \in N} S_i(y \mid \theta) = v(y)$

Associating the dual variables $\lambda_i(\theta_i, \tau_i, \tilde{a}_i) \geq 0$ to every inequality in i) and the dual variables $\mu(y, \theta)$ to every equality in ii), we get (Fan, 1950, Theorem (1) that (i) – (ii) is consistent if and only if:

(iii) *Whenever,* $\forall\, S=(S_1,\cdots,S_i,\cdots,S_n), S_i\in \mathrm{I\!R}^{Y\times\Theta^N}$, $i\in n$,

$$\sum_i\sum_{\theta_i}\sum_{\tau_i\leq\theta_i}\sum_{\tilde a_i}\lambda_i(\theta_i,\tau_i,\tilde a_i)\times\Bigg\{\sum_{\theta_{-i}}f(\theta_{-i})\left[\sum_y S_i(y\mid\theta_i,\theta_{-i})g(y\mid a^*(\theta_i,\theta_{-i}))\right.$$
$$\left.\left.-\sum_y S_i(y\mid\tau_i,\theta_{-i})g(y\mid\tilde a_i(\tau_i,\theta_{-i}),a^*_{-i}(\tau_i,\theta_{-i}))\right]\right\}+\sum_y\sum_\theta\mu(y,\theta)\textstyle\sum_i S_i(y\mid\theta)=0,$$

(iv) we get

$$\sum_i\sum_{\theta_i}\sum_{\tau_i\leq\theta_i}\sum_{\tilde a_i}\lambda_i(\theta_i,\tau_i,\tilde a_i)\times\Bigg\{\sum_{\theta_{-i}}[C(a^*_{-i}(\theta_i,\theta_{-i}),M(\theta_i,\theta_{-i}))$$
$$\left.-C(\tilde a_{-i}(\tau_i,\theta_{-i},M(\theta_i,\theta_{-i}))]\right\}+\sum_y\sum_\theta\mu(y,\theta)v(y)\leq 0.$$

Rewriting (iii) gives, for every $S=(S_1,\cdots S_i,\cdots,S_n)$,

$$\sum_i\sum_y\sum_{\theta_i}\sum_{\theta_{-i}}S_i(y\mid\theta_i,\theta_{-i})\times\Bigg\{f(\theta_{-i})\Bigg[g(y\mid a^*(\theta_i,\theta_{-i}))\sum_{\tau_i\leq\theta_i}\sum_{\tilde a_i}\lambda_i(\theta_i,\tau_i,\tilde a_i)$$
$$\left.\left.-\sum_{\tau_i\leq\theta_i}\sum_{\tilde a_i}\lambda_i(\tau_i,\theta_i,\tilde a_i)\,g(y\mid\tilde a_i(\theta_i,\theta_{-i}),a^*_{-1}(\theta_i,\theta_{-i}))\right]+\mu(y,\theta)\right\}=0$$

implying:

(iii)' $\forall\, i\in N,\forall\,\theta\in\Theta^N,\forall\, y\in Y$,

$$\begin{aligned}&g(y\mid a^*(\theta_i,\theta_{-i}))\sum_{\tilde a_1}\sum_{\tau_1}\lambda_i(\theta_i,\tau_i,\tilde a_i)f(\theta_{-i})\\&-\sum_{\tilde a_i}\sum_{\tau_1}\lambda_i(\tau_i,\theta_i,\tilde a_i)f(\theta_{-i})g(y\mid\tilde a_i(\theta_i,\theta_{-i}),a^*_{-i}(\theta_i,\theta_{-i}))\\=\ &-\mu(y,\theta).\end{aligned}$$

We have to show that, for any S, (iii)' implies (iv).

(2) Let in (iii)': $\gamma_i(\theta_i,\tau_i,\tilde a_i,\theta_{-i})\stackrel{\text{def}}{=}\lambda_i(\theta_i,\tau_i,\tilde a_i)f(\theta_{-i})$.

We get, by summing over $y\in Y$:

(v)

$$\begin{aligned}&\sum_{\tilde a_i}\sum_{\tau_i}\gamma_i(\theta_i,\tau_i,\tilde a_i,\theta_{-i})-\sum_{\tilde a_i}\sum_{\tau_i}\gamma_i(\tau_i,\theta_i,\tilde a_i,\theta_{-i})\\=\ &-\sum_y\mu(y,\theta), i\in N,\theta\in\Theta^N,\end{aligned}$$

so that $\sum_{\theta_i}\sum_y \mu(y,\theta_i,\theta_{-i}) = 0$, for every $i \in N$ and $\theta_{-i} \in \Theta^{N-\{i\}}$. In fact we must have $\mu \equiv 0$. To show that, take $\theta \in \Theta^N$. In the case we have, for some i, $\sum_{\tilde{a}_i}\sum_{\tau_i} \gamma_i(\theta_i,\tau_i,\tilde{a}_i,\theta_{-i}) = 0$, then $\gamma_i(\theta_i,\tau_i,\tilde{a}_i,\theta_{-i}) = 0$ for all $\tilde{a}$ and all τ_i implying by (v) and (vi) that $\gamma_i \equiv 0$, and giving immediately the conclusion. So consider the case where, for $i \in N$, $\sum_{\tilde{a}_i}\sum_{\tau_i} \gamma_i(\theta_i,\tau_i,\tilde{a}_i,\theta_{-i}) > 0$. By Lemma 3, we have $a^*(\theta) = \alpha_\theta$ for every $i \in N^*$ and using **A3**, (iii)' becomes: for every $y \in Y$ and every $i \in N$ (with $n^* = |N^*|$),

$$\begin{aligned}
& p(\alpha_\theta)^{\sum_i y_j}(1-p(\alpha_\theta))^{n^*-\sum_i y_j}\sum_{\tilde{a}_i}\sum_{\tau_i}\gamma_i(\theta_i,\tau_i,\alpha,\theta_{-i}) \\
& \quad -\sum_{\tilde{a}_i}\sum_{\tau_i}\gamma_i(\tau_i,\theta_i,\alpha,\theta_{-i})\Big\{p(\alpha)^{y_i}(1-p(\alpha))^{1-y_i}p(\alpha_\theta)^{\sum_{j\neq i} y_j} \\
& \quad \times\left(1-p(\alpha_\theta)^{n^*-1-\sum_{j\neq i} y_j}\right)\Big\} \\
= & \quad -\mu(y,\theta).
\end{aligned}$$

Take any $i \in N$ and $k \in N$, letting $y_i = y_k = 0$, we get:

$$\begin{aligned}
& (1-p(a_i^*(\theta)))\sum_\alpha\sum_{\tau_i}\gamma_i(\theta_i,\tau_i,\alpha,\theta_{-i}) - \sum_\alpha\sum_{\tau_i}\gamma_i(\tau_i,\theta_i,\alpha,\theta_{-i})(1-p(\alpha)) \\
= & \quad (1-p(a_k^*(\theta)))\sum_\alpha\sum_{\tau_k}\gamma_k(\theta_k,\tau_k,\alpha,\theta_{-k}) - \sum_\alpha\sum_{\tau_k}\gamma_k(\tau_k,\theta_k,\alpha,\theta_{-k})(1-p(\alpha)).
\end{aligned}$$

Letting $y_i = y_k = 1$, we also get:

$$\begin{aligned}
& p(a_i^*(\theta))\sum_\alpha\sum_{\tau_i}\gamma_i(\theta_i,\tau_i,\alpha,\theta_{-i} - \sum_\alpha\sum_{\tau_i}\gamma_i(\theta_i,\tau_i,\alpha,\theta_{-i})p(\alpha) \\
= & \quad p(a_k^*(\theta))\sum_\alpha\sum_{\tau_k}\gamma_k(\theta_k,\tau_k,\alpha,\theta_{-k}) - \sum_\alpha\sum_{\tau_k}\gamma_k(\tau_k,\theta_k,\alpha,\theta_{-k})p(\alpha).
\end{aligned}$$

By addition we have:

$$\begin{aligned}
& \sum_\alpha\sum_{\tau_i}\gamma_i(\theta_i,\tau_i,\alpha,\theta_{-i}) - \sum_\alpha\sum_{\tau_i}\gamma_i(\tau_i,\theta_i,\alpha,\theta_{-i}) \\
= & \quad (1-p(a_k^*(\theta)))\sum_\alpha\sum_{\tau_k}\gamma_k(\theta_k,\tau_k,\alpha,\theta_{-k}) - \sum_\alpha\sum_{\tau_k}\gamma_k(\tau_k,\theta_k,\alpha,\theta_{-k})
\end{aligned}$$

i.e.,

$$\begin{aligned}
& f(\theta_{-i})\left[\sum_\alpha\sum_{\tau_i}\lambda_i(\theta_i,\tau_i,\alpha) - \sum_\alpha\sum_{\tau_i}\lambda_i(\tau_i,\theta_i,\alpha)\right] \\
= & \quad f(\theta_{-k})\left[\sum_\alpha\sum_{\tau_k}\lambda_k(\theta_k,\tau_k,\alpha) - \sum_\alpha\sum_{\tau_k}\lambda_k(\tau_k,\theta_k,\alpha)\right].
\end{aligned}$$

Summing over θ_{-i} on both sides we get:

$$\begin{aligned}
&\sum_{\alpha}\sum_{\tau_i}\lambda_i(\theta_i,\tau_i,\alpha) - \sum_{\alpha}\sum_{\tau_i}\lambda_i(\tau_i,\theta_i,\alpha) \\
= &\sum_{\theta_{-(i,k)}} f(\theta_{-k})\left[\sum_{\theta_k}\sum_{\alpha}\sum_{\tau_k}\lambda_k(\theta_k,\tau_k,\alpha) - \sum_{\theta_k}\sum_{\alpha}\sum_{\tau_k}\lambda_k(\tau_k,\theta_k,\alpha)\right] = 0.
\end{aligned}$$

Thus, for any $i \in N$, any $\alpha \in \Theta$ and any $y \in Y$;

$$f(\theta_{-i})\left[\sum_{\alpha}\sum_{\tau_i}\lambda_i(\theta_i,\tau_i,\alpha) - \sum_{\alpha}\sum_{\tau_i}\lambda_i(\tau_i,\theta_i,\alpha)\right] = -\mu(y,\theta) = 0,$$

i.e. $\mu \equiv 0$ in (iii)'.

(3) Since $\mu \equiv 0$ by (iii)' we get, using Lemma 1:

$$\begin{aligned}
&\sum_{i}\sum_{\theta_i}\sum_{\tau_i}\sum_{\tilde{a}_i}\lambda_i(\theta_i,\tau_i,\tilde{a}_i)\sum_{\theta_{-i}}f(\theta_{-i}) \times [C(a_i^*)(\theta_i,\theta_{-i}), M(\theta_i,\theta_{-i}) - C(\tilde{a}(\tau_i,\theta_{-i}), M(\theta_i,\theta_{-i}))] \\
\leq &\sum_{i}\sum_{\theta_i}\sum_{\tau_i}\sum_{\tilde{a}_i}\lambda_i(\theta_i,\tau_i,\tilde{a}_i)\sum_{\theta_{-i}} f(\theta_{-i}) \sum_{y} v(y) \\
&\times [g(y \mid a^*(\theta_i,\theta_{-i})) - g(y \mid \tilde{a}_i(\tau_i,\theta_{-i})), a^*_{-i}(\theta_i,\theta_{-i}))] \\
= &\sum_{\theta}\sum_{y} v(y) f(\theta_{-i}) \times \sum_{i}\Bigg[g(y \mid a^*(\theta_i,\theta_{-i}))\sum_{\tau_i}\sum_{\tilde{a}_i}\lambda_i(\theta_i,\tau_i,\tilde{a}_i) - \sum_{\tau_i}\sum_{\tilde{a}_i}\lambda_i(\theta_i,\tau_i,\tilde{a}_i) \\
&\times g(y \mid \tilde{a}_i(\tau_i,\theta_{-i}), a_i^*(\theta_i,\theta_{-i}))\Bigg] = 0,
\end{aligned}$$

which gives (iv). ■

5 Conclusion

We show in this paper that there are R & D environments with adverse selection and moral hazard where a balanced contract arrangement leads a Research Joint Venture to a first best solution with perfect disclosure, even though the 'most knowledgeable' participant cannot be identified.

Our environments are rather specific. The success-failure dichotomy (Assumption **A3**) is illustrative, but remains special. A richer space of outcomes for the R & D process should be explored. One may also question Assumption **A5** about the initial probability distribution f which leads to the fact that the participants beliefs are independent of their innate knowledge. These are preliminary steps towards a more complete investigation of Research Joint Venture issues, providing a characterization of optimal second-best arrangements when first-best is not attainable or towards a theory, as suggested by Aghion and Tirole (1994), which introduce 'multiple principals' to deal with financing and property rights.

In this paper we have obtained first-best implementation with ex-ante individual rationality. In Bhattacharya et al. (1992) a more specific precision function M is introduced in addition to the success-failure Assumption **A3** and to the independence of beliefs. This more specific precision function is based on the Blackwell ordering of knowledge and consists in having, as mentioned above, $M(\theta_i, \tau_{-i}) = \max\{\tau_1, \cdots, \tau_{i-1}, \theta_i, \tau_{i+1}, \cdots, \tau_n\}$. In that case first-best implementation can be obtained with ex post individual rationality. This is due to the pivotal role that this precision function gives to the most knowledgeable agent, allowing the construction of explicit licensing mechanisms of the type developed in Bhattacharya et al. (1992). With more general precision functions as here, we encounter difficulties with the participation constraints, that are well-known in the context of public good provision under adverse selection, and the construction of explicit mechanism remains an open problem.

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