Imperfect Competition in an Overlapping Generations Model: A Case for Fiscal Policy*

Claude d'Aspremont[†], Rodolphe Dos Santos Ferreira[‡] and Louis-André Gérard-Varet[§]

Abstract

Imperfect competition is a meaningful feature for macroeconomic analysis only to the extent that it leads to properties qualitatively different from those obtained under perfect competition. In particular, we have to wonder how imperfect competition per se may found an effective fiscal policy. For that matter we consider a simple overlapping generations model with firms acting as Cournot oligopolists in the good market. Through fiscal policy, a government, keeping the stock of money constant, redistributes wealth among generations and absorbs some of the output to provide public services. We show in this model that fiscal policy, by affecting firms' market power, can move the economy across perfect foresight stationary equilibria along a Pareto improving path, or that it can implement a full employment stationary equilibrium which Pareto-dominates underemployment equilibria.

1 Introduction

Over the last decade, general equilibrium macroeconomic models – with few commodities and few agents – allowing for market power and imperfect competition, have been recognized as

*Reprinted from *Annales d'Economie et de Statistique*, 37/38, 531–555, 1995.

We thank P. Picard for his comments. Support of the Belgian State PAI Programme (Prime Minister's Office, Science Policy Programming) and of the HCM Programme of the European Commission is acknowledged.

[†]CORE, Université catholique de Louvain, Belgique

[‡]BETA, Université Louis Pasteur, Strasbourg, France

[§]GREQAM-EHESS, Marseille, France.

providing coherent, microfounded and tractable frameworks for the analysis of underemployment equilibria.[1] There is actually nothing surprising in the fact that in an economy where monopolistic competitors, or oligopolistic agents, set the price of the goods they produce, market power may lead to inefficient equilibria, with levels of output and employment lower than at a Walrasian equilibrium. From a macroeconomic point of view, the question remains to know to what extent in models, with imperfect competition, some fiscal (or possibly monetary) policy is available to raise output and employment and, if it is the case, what are the expected welfare effects.

The motivations for considering these issues come from different observations. To start with, most of the recent contributions to the burgeoning literature on the "macroeconomics of imperfect competition" have been relying on models which are "static" or can be interpreted as "temporary equilibrium" models. In static models fluctuations in aggregate demand or changes in nominal money are neutral, just as it would be the case under perfect competition,[2] and public spending policies have crowding out effects similar prevailing to those prevailing in Walrasian models.[3] Some dynamics can be introduced, as in the "New Keynesian approach", by considering how price-setters react to changes in the aggregate demand of the economy. When there are costs of changing prices at discrete intervals of time, price setters may decide not to adjust their prices in response to small shifts in demand, and movements in nominal money may lead to real movements. But this is nothing more than introducing price stickiness through menu costs.[4] On another hand, in temporary equilibrium models, the way fiscal (or monetary) policy works strongly depends on the properties of exogenous price expectations.[5]

[1] See among others, d'Aspremont, Dos Santos Ferreira, Gérard-Varet [1989], [1990a], Benassy [1987], Blanchard, Kiyotaki [1987], Dixon [1988], Hart [1982], Silvestre [1988], Snower [1983], Weitzman [1982], [1985]. See also the survey by Silvestre [1993]. Also, in the 70's the use of models of imperfect competition to understand the impact of aggregate demand management on economic activity was already part of the "fixed price equilibrium" macroeconomics. See Benassy [1977], Grandmont, Laroque [1976], and the survey by Drazen [1980].

[2] See Benassy [1987], Blanchard, Kiyotaki [1987] or Dixon [1987].

[3] See Snower [1983].

[4] See the survey by Rotemberg [1987], or Chapter 8 in Blanchard, Fisher [1989].

[5] See Hart [1982], Benassy [1987], Blanchard, Kiyotaki [1987].

Imperfect competition is a meaningful feature for macroeconomic analysis only to the extent that it leads to properties qualitatively different from those obtained under perfect competition. In particular, we have to consider how imperfect competition may in itself found an effective fiscal policy, without introducing other imperfections. For that matter we need to rely on a model of an economy under imperfect competition which integrates the dynamics of both aggregate demand and price formation, where agents are optimizers, prices are fully endogenous and expectations are "rational".

We consider, as in d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1991], a simple overlapping generations model with three commodities: labor, a produced good and money. At each period firms behave non-competitively à la Cournot in the good market, distribute all their profits and correctly perceive all feedback effects generated by their strategic actions. A typical consumer lives two periods, working, paying taxes and consuming when young, spending on the basis of his savings or consuming public goods when old. The government chooses, at any period, public expenditure and the level of taxes on the young generation, keeping the stock of money constant and the budget balanced.[6] We also assume perfect foresight, meaning that price expectations are point expectations at the actual level of future prices.[7]

We already know that in such an economy, because of imperfect competition, inflation and increasing unemployment may arise in all perfect foresight equilibria, even though the economy has a stationary Walrasian equilibrium (at least when there are non-increasing returns). We consider here situations where the returns may be increasing and we emphasize the analysis of stationary equilibria. Actually, a stationary equilibrium may or may not exist, and the model may be increasing and we emphasize the analysis of stationary equilibria. Actually, a stationary equilibrium may or may not exist, and the model may also generate (under increasing returns) multiple equilibria characterized by different levels of activity and which are welfare ranked. This

[6]It is well known in overlapping generations models that changes in real government demand financed by issuing money (unbalanced fiscal policy) affect the rate of inflation, thus the intertemporal prices. We neutralize this effect.

[7]This is stronger than requiring that price expectations are correct at equilibrium (adaptative expectations may correctly forecast prices at a quasi-stationary equilibrium). See Rankin [1992].

is a typical "coordination failure" issue, suggestion to look for some coordination policy. Our objective is slightly different. We want first to exhibit circumstances (including non constant returns) under which fiscal policy may move the economy across stationary equilibria along a Pareto improving path. Then, starting from a situation where, even with constant returns, no stationary equilibrium exists, fiscal policy may implement a full employment stationary equilibrium Pareto dominating any undermeployment equilibrium, and in particular quasi-stationary underemployment equilibria.

Our main point is that in a model with "endogenous mark-up",[8] there is an intermediate target for a full employment fiscal policy, which is the aggregate degree of market power in the economy. Indeed, besides the normal Keynesian multiplier effects of fiscal policy, we have, under imperfect competition, firms' market power effects. The feature has already been recognized in d'Aspremont, Dos Santos Ferreira, Gérard-Varet [1989] and is documented in different recent papers (Pagano [1990], or Jacobsen, Schultz [1994]). Actually, in our framework, fiscal policy may have to counteract the firms' market power or, to the contrary, to reinforce this market power, according to the circumstances. We present the model in the second section. In the third section we consider Pareto improvements towards full employment across stationary equilibria. The fourth section deals with the implementation of a full employment stationary equilibrium which dominates underemployment equilibria. In the conclusion we compare our results to the recent literature sharing our view that, under imperfect competition, macroeconomic (fiscal) policy may be effective through effects different from the ordinary Keynesian multiplier effects.

2 The model

We consider an overlapping generations model in discrete time. In each period $t = 0, 1, 2, \cdots$ there are in the economy three commodities: a produced good, labour and money, this last being both the unit of account and the only storable good. There are also three types of agents: consumers, firms and a government. The role of government is to levy taxes and to transform

[8] See Rotemberg and Woodford [1993], their Section 8.

units of the produced good into units of a public good. A basic feature of the model is that firms act, at any point in time, as Cournot oligopolists in the good market, behaving on the basis of the "correctly conjectured" objective aggregate demand. We present first the foundations of the aggregate demand, then the firms' behaviour and the resulting temporary equilibria. We finally consider the intertemporal equilibria upon which our discussion of government intervention relies.

2.1 Foundations of the aggregate demand

There is a continuum of consumers, each consumer living two periods. All consumers of the same generation behave according to the same utility function, defined with respect to both periods – when young – or only to the present period – when old.

In every period, young consumers supply to the firms one unit of labour each, and receive all distributed profits. Old consumers are retired and hold no share. At date $t = 0$ there is a young generation, and a generation already old that possesses the total endowment of money $M > 0$ of the economy. The stock of money is constant over time. The same number of individuals being born at every period, there is an invariant mass $L > 0$ of young consumers.

All young consumers have, at every date $t \geq 0$, identical utility functions which depend upon their current consumption $c_t \geq 0$, their future consumption $\hat{c}_t \geq 0$, the amount of labour $\ell_t \in \{0, 1\}$ that they currently supply to the firms, as well as upon some amount $g_t \geq 0$ of public good presently provided by the government and some amount $\hat{g}_t \geq 0$ of public good expected for future consumption. We shall assume a *separable* utility function which is linear with respect to labour and public consumption:

$$U(c_t, \hat{c}_t) - \nu\, \ell_t + b\, g_t + b\, a\, \hat{g}_t$$

where $\nu > 0$ is the (constant) marginal disutility of labour, $b > 0$ is the (constant) marginal utility of public consumption and $a > 0$ is a discount factor on future public consumption. We also assume that U is a *CES function* given by

$$U(c_t, \hat{c}_t) = \left(c_t^{(\sigma-1)/\sigma} + a\, \hat{c}_t^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \quad \sigma > 0,\ \sigma \neq 1, \tag{1}$$

where the *intertemporal elasticity of substitution* σ is constant, with $\sigma > 1$ (resp. $\sigma < 1$) indicating *intertemporal substitutability* (resp. *intertemporal complementarity*).

Consumers behave competitively in the good and the labour markets. They know at date t, and take as given, the current money price $p_t \geq 0$ and when young, hence active in the labour market, the current wage rate $w_t \geq 0$. Young consumers also have at date t the same rigid expectations with respect to future money price $\hat{p}_t \geq 0$. Separability of preferences entails a convenient procedure to derive the behaviour of young consumers with respect to present and future consumptions, and with respect to labour supply.

Take first the optimal planning at date t of present and future consumptions. Since young consumers have identical homothetic utilities, one may as well consider a single individual having one unit of current income (wage or profit) to share between present consumption $p_t\, c_t$ and future consumption $\hat{p}_t\, \hat{c}_t$, by maximising under the budget constrain the utility $U(c_t, \hat{c}_t)$. The solution is given by:

$$c_t^* = \alpha\left(\frac{p_t}{\hat{p}_t}\right)\frac{1}{p_t} \quad \text{and} \quad \hat{c}_t^* = \left(\frac{p_t}{\hat{p}_t}\right)\left(1 - \alpha\left(\frac{p_t}{\hat{p}_t}\right)\right)\frac{1}{p_t}, \tag{2}$$

where

$$\alpha\left(\frac{p_t}{\hat{p}_t}\right) \stackrel{\text{def}}{=} \frac{1}{1 + a^\sigma \left(\frac{p_t}{\hat{p}_t}\right)^{\sigma-1}}. \tag{3}$$

Actually, $0 < \alpha\left(\frac{p_t}{\hat{p}_t}\right) < 1$ is the *marginal propensity to consume*, which is increasing (resp. decreasing) when there is intertemporal complementarity (resp. substitutability), *i.e.* $\sigma < 1$ (resp. $\sigma > 1$).

The utility level obtained by a young consumer from the optimal level of present and future consumptions is given by:

$$U(c_t^*, \hat{c}_t^*) = U^*\left(\frac{p_t}{\hat{p}_t}\right)\frac{1}{p_t}$$

where, using (1) (2) and (3), we have

$$\begin{aligned} U^*\left(\frac{p_t}{\hat{p}_t}\right) &= U\left(\alpha\left(\frac{p_t}{\hat{p}_t}\right), \left(\frac{p_t}{\hat{p}_t}\right)\left(1 - \alpha\left(\frac{p_t}{\hat{p}_t}\right)\right)\right) \\ &= \left(1 + a^\sigma\left(\frac{p_t}{\hat{p}_t}\right)^{\sigma-1}\right)^{1/(\sigma-1)}. \end{aligned} \tag{4}$$

The young consumer chooses to work (resp. not to work) if

$$U^*\left(\frac{p_t}{\hat{p}_t}\right)\frac{w_t}{p_t}-\nu>0 \quad (\text{resp. } <0);$$

and is indifferent between working and not working when an equality holds. The *real reservation wage* of a young consumer is, by definition, the ratio of labour disutility to the utility of optimal consumption:

$$W_\ell\left(\frac{p_t}{\hat{p}_t}\right) \overset{\text{def}}{=} \frac{\nu}{U^*\left(\frac{p_t}{\hat{p}_t}\right)} = \nu\left(1+a^\sigma\left(\frac{p_t}{\hat{p}_t}\right)^{\sigma-1}\right)^{1/(1-\sigma)}. \tag{5}$$

Current consumption is given by the same linear function of income for all young consumers [see (2)], and income distribution is immaterial. Thus the aggregate demand by young consumers is a linear function of the current income I_t distributed by firms as wages or profits (we do not assume that shares of firms are uniformly distributed). Since the young consumers also pay taxes T_t on their current income, the aggregate volume of private consumption of the young for period t is: $\alpha\left(\frac{p_t}{\hat{p}_t}\right)\frac{(I_t-T_t)}{p_t}$.

The total spending of old consumers at date t coincide with their savings from the past period, and are necessarily equal to the stock of money in the economy $M>0$.

The total spending of old consumers at date t coincide with their savings from the past period, and are necessarily equal to the stock of money in the economy $M>0$. However, old consumers also derive utility from public expenditures $G_t \geq 0$ decided by the government which, at prices p_t, provide a volume $g_t=\frac{G_t}{p_t}$ of public goods. Total utility derived by old consumers from their current private consumption and from public consumption is given by $\frac{M}{p_t}+bg_t$.

The aggregate demand for period t, from the old and the young generation, is

$$D_t=\alpha\left(\frac{p_t}{\hat{p}_t}\right)\left(\frac{I_t-T_t}{p_t}\right)+\frac{M+G_t}{p_t}. \tag{6}$$

The stock of money being invariant over time, the budget constraint of the government imposes that current public expenditures be covered by current taxes (paid by the youngs) *i.e.* $G_t=T_t$. Using that condition, the aggregate demand (6) gives

$$D_t=\frac{1}{p_t}\left[\alpha\left(\frac{p_t}{\hat{p}_t}\right)I_t+\left(1-\alpha\left(\frac{p_t}{\hat{p}_t}\right)\right)G_t+M\right]. \tag{7}$$

The volume D_t for period t is a function of the income I_t distributed during the period (as wages or profits) by the firms to the young generation. At the time of the decision the amount I_t can only be "conjectured". We assume that these conjectures, shared by all agents, are "correct", *i.e.* $I_t = p_t D_t$. Using (7), this condition implies that we must have: $I_t = G_t + \frac{M}{\left(1-\alpha\left(\frac{p_t}{\hat{p}_t}\right)\right)}$. Assuming that the aggregate current demand is correctly conjectured, and allowing for all feedback effects, leads to the *aggregate demand function* given by

$$D(p_t, \hat{p}_t, G_t, M) = \left(\frac{1}{1 - \alpha\left(\frac{p_t}{\hat{p}_t}\right)} + \frac{G_t}{M} \right) \frac{M}{p_t}. \tag{8}$$

The *price-elasticity* of the aggregate demand function is, by definition: $\Delta(\cdot) \stackrel{\text{def}}{=} -\frac{\partial D(\cdot)}{\partial p_t} \frac{p_t}{D(\cdot)}$. One easily gets:

$$\Delta\left(\frac{p_t}{\hat{p}_t}, \frac{G_t}{M}\right) = 1 + \frac{(\sigma - 1)\alpha\left(\frac{p_t}{\hat{p}_t}\right)}{1 + \left(\frac{G_t}{M}\right)\left(1 - \alpha\left(\frac{p_t}{\hat{p}_t}\right)\right)}. \tag{9}$$

Thus, we have $\Delta > 1$ (resp. $\Delta < 1$) when $\sigma > 1$ (resp. $\sigma < 1$), *i.e.* when there is intertemporal substitutability (resp. complementarity). When $G_t = 0$, one recovers a more familiar formula for the elasticity (see d'Aspremont, Dos Santos Ferreira, Gérard-Varet [1991]).

2.2 Firms' behaviour and temporary equilibria

Every period the productive sector contains $n \geq 2$ firms having identical increasing (not necessarily convex) cost functions, where $y^\beta, \beta > 0$, gives the amount of labour required to obtain the output $y \geq 0$. The returns may be decreasing ($\beta > 1$), increasing ($\beta < 1$) or constant ($\beta = 1$). At any period t, the firms behave competitively on the labour market, taking as given the money wage $w_t \leq 0$. They also form at that period the same expectations with respect to the future prices $\hat{p}_t \geq 0$ and have "correct" conjectures about the aggregate demand that they face [given by (8)]. However, against that demand, the firms compete as Cournot oligopolists in the good market.

Take any single firm (we omit the index for simplicity) and let $Y_t \geq 0$ be the output supplied by its competitors during the period t. The firm has to select its output $y_t \geq 0$ to maximize the profit function:

$$D^{-1}(\cdot, \hat{p}_t, G_t, M)(y_t + Y_t)y_t - w_t y_t^\beta$$

where $D^{-1}(\cdot, \hat{p}_t, G_t, M)(y_t + Y_t)$ is the inverse demand derived from (8) and evaluated at point $y_t + Y_t$, given the expectation $\hat{p}_t$ and the values of the policy variables. An optimal positive output $y_t > 0$ must be such that, at $p_t = D^{-1}(\cdot, \hat{p}_t, G_t, M)(y_t + Y_t)$ marginal revenue equals marginal cost (first order condition):

$$p_t \left(1 - \frac{y_t/(y_t + Y_t)}{\Delta\left(\frac{p_t}{\hat{p}_t}, \frac{G_t}{M}\right)}\right) = w_t\, \beta\, y_t^{\beta-1};$$

it must also entail, at that value of p_t, a nonnegative profit, namely: $p_t\, y_t \geq w_t\, y_t^\beta$. A critical and nonnegative value of the profit function will be a strict local maximum if the function is strictly concave in a neighbourhood of the critical point. It turns out that if the ratio of marginal revenue to marginal cost is decreasing whenever the nonnegative profit requirement holds, and the marginal revenue is nonnegative, then these two conditions are *sufficient* for a (unique) global maximum, although the profit function may fail to be quasi-concave. The following result, derived from Lemma 2 in d'Aspremont, Dos Santos Ferreira, Gérard-Varet [1994], shows that this holds when the returns are either sufficiently decreasing or strictly increasing with respect to the elasticity of substitution.

Lemma 1 *Let $w_t \geq 0, Y_t \geq 0$, and D be given by (8); assume $\beta \geq \sigma - 1$. Then the (first order and nonnegative profit) conditions $y_t^* > 0$ at $p_t^* = D^{-1}(\cdot, \hat{p}_t, G_t, M)(y_t^* + Y_t)$, given by:*

$$p_t^*\, \beta \geq p_t^* \left(1 - \frac{y_t^*/(y_t^* + Y_t)}{\Delta\left(\frac{p_t^*}{\hat{p}_t}, \frac{G_t}{M}\right)}\right) = w_t\, \beta\, y_t^{*\beta-1},$$

are necessary and sufficient to have:

$$y_t^* \in \arg\max_{y_t \geq 0} \{D^{-1}(\cdot, \hat{p}_t, G_t, M)(y_t + Y_t)y_t - w_t\, y_t^\beta\}.$$

Whenever it exists the solution is the unique positive solution.

The proof is given in the Appendix.

We turn now to a notion of temporary equilibrium which, for the present context of imperfect competition, requires compatibility at some point in time of optimal consumers' and producers' behaviour for given price expectations $\hat{p}_t$ and given values of the policy variables G_t and M. With identical consumers and producers, one expects symmetric equilibria. We actually restrict our analysis to these equilibria (as a matter of fact all equilibria are necessarily symmetric with respect to active firms, although the possibility cannot be excluded that some firms may be inactive at equilibrium when there are increasing returns). A *symmetric temporary equilibrium* for period t, associated to expectations about future prices $\hat{p}_t$ and a given state of the policy variables G_t and M, is a 3-tuple (y_t^*, p_t^*, w_t^*), where $y_t^* > 0$, $p_t^* > 0$ and $w_t^* \geq 0$ are such that:

$$y_t^* \in \arg\max_{y_t \geq 0} \{D^{-1}(\cdot, \hat{p}_t, G_t, M)(y_t + (n-1)y_t^*)y_t - w_t^* \, y_t^\beta\} \tag{10}$$

$$p_t^* = D^{-1}(\cdot, \hat{p}_t, G_t, M)(ny_t^*) \tag{11}$$

$$\begin{aligned} &ny_t^{*\beta} = L \text{ and } \frac{w_t^*}{p_t^*} \geq \frac{\nu}{U^*\left(\frac{p_t^*}{\hat{p}_t}\right)} \quad \text{(full employment)} \\ &ny_t^{*\beta} < L \text{ and } \frac{w_t^*}{p_t^*} = \frac{\nu}{U^*\left(\frac{p_t^*}{\hat{p}_t}\right)} \quad \text{(underemployment).} \end{aligned} \tag{12}$$

Conditions (10) and (11) characterize a Cournot equilibrium in the product market: firms maximize their profit against the "true" demand function, given expectations about future price and at a given wage rate. Under Lemma 1 and using (8) these conditions are equivalent to

$$p_t^* \, \beta \geq p_t^* \left(1 - \frac{1/n}{\Delta\left(\frac{p_t^*}{\hat{p}_t}, \frac{G_t}{M}\right)}\right) = w_t^* \, \beta \, y_t^{*\beta-1} \tag{10'}$$

and

$$ny_t^* = \left(\frac{1}{1-\alpha\left(\frac{p_t^*}{\hat{p}_t}\right)} + \frac{G_t}{M}\right)\frac{M}{p_t^*}. \tag{11'}$$

The last condition – namely (12) – defines a competitive solution in the labour market, taking into account the possibility that part of the labour force may not be employed (under wage flexibility and assuming no rationing, the wage must then be equal to its reservation value).

2.3 Intertemporal equilibria

Let $\gamma_t = \frac{G_t}{M}$ be the amount of public expenditures at date t per unit of money. An intertemporal *fiscal policy* is fully characterized by a sequence $(\gamma_t)_{t\geq 0}$. Let also $\theta_t = \frac{p_t}{\hat{p}_t}$ be the *expected real interest factor* at date t. A *symmetric) intertemporal equilibrium* associated to a sequence $(\gamma_t)_{t\geq 0}$ is a sequence of temporary equilibria $(y_t^*, p_t^*, w_t^*)_{t\geq 0}$ such that, for every $t \geq 0$, $\hat{p}_t = p_{t+1}^*$, *i.e.* $\theta_t = \frac{p_t^*}{p_{t+1}^*}$. Among these equilibria, all allowing for intertemporal consistency of price expectations, we shall actually consider only those which have some "stationarity" property. Given a stationary budget policy $\gamma_t = \gamma$, $t \geq 0$, an intertemporal equilibrium is *quasi-stationary* if $\frac{p_t^*}{p_{t+1}^*} = \theta > 0$, *i.e.* is such that the real interest factor remains invariant over time. *Stationarity* is obtained when $\theta = 1$.

Characterization of quasi-stationary (symmetric) equilibria can be simplified by introducing the *firms' real reservation wage*, by definition the wage that sustains some price as an output market equilibrium, which, using (10') and (11') is:

$$W_f(p, \theta, \gamma) = r(\theta, \gamma)\frac{1}{\beta}\left[\frac{D(p, \theta, \gamma)}{n}\right]^{1-\beta} \tag{13}$$

where $r(\theta, \gamma) \stackrel{\text{def}}{=} \left(1 - \frac{1/n}{\Delta(\theta,\gamma)}\right)$ stands for the *real marginal revenue* of the firms at a symmetric equilibrium and where the aggregate demand

$$D(p, \theta, \gamma) = \left(\frac{1}{1-\alpha(\theta)} + \gamma\right)\frac{M}{p} \tag{8'}$$

is derived from (8). We have a *quasi-stationary equilibrium with full employment* if and only if

$$D(p, \theta, \gamma) = n^{1-1/\beta}L^{1/\beta} \tag{14a}$$

$$W_\ell(\theta) \leq W_f(p, \theta, \gamma). \tag{14b}$$

It is a *quasi-stationary equilibrium with underemployment equilibrium* if and only if

$$D(p, \theta, \gamma) < n^{1-1/\beta}L^{1/\beta} \tag{15a}$$

$$W_\ell(\theta) = W_f(p, \theta, \gamma). \tag{15b}$$

3 Pareto-improvements towards full employment across stationary equilibria, under increasing or decreasing returns

Let us consider, to start with, a situation where, at some level of the public expenditure (namely for some value of γ), the economy is at a stationary equilibrium ($\theta = 1$) with underemployment. The returns are decreasing or increasing (constant returns must be excluded at this point). The questions we address are the following. Can the economy be moved from an underemployment stationary equilibrium towards a full employment one by fiscal policy? Under what conditions will such a move be Pareto-improving? To answer these questions the assumption on the nature of the technology appears essential, on two respects. On the one hand decreasing returns lead to the uniqueness of stationary equilibrium, whereas increasing returns necessarily entail a multiplicity of such equilibria. On the other hand, restoring full employment at a stationary equilibrium implies to counteract the firms' market power in the economy when there are decreasing returns, and on the contrary to reinforce this market power in the case of increasing returns. We first study the stationary equilibria, then the effects of a change of public expenditure at an underemployment equilibrium; we finally give conditions under which a fiscal policy moving the economy towards full employment will be Pareto-improving.

3.1 Stationary equilibria

Let, for a state $\gamma = \frac{G}{M}$ of the policy variable, the aggregate demand $D(p, \theta, \gamma)$ be given by (8'). A *stationary equilibrium with full employment* is obtained at a price $p^* > 0$ such that (14a) and (14b) hold with $\theta = 1$, *i.e.*:

$$D(p^*, 1, \gamma) = n^{1-1/\beta} L^{1/\beta} \quad \text{and} \quad W_\ell(1) \leq W_f(p^*, 1, \gamma).$$

A *stationary equilibrium with underemployment* is obtained at a price $p^0 > 0$ such that (15a) and (15b) hold with $\theta = 1$, *i.e.*:

$$D(p^0, 1, \gamma) < n^{1-1/\beta} L^{1/\beta} \quad \text{and} \quad W_\ell(1) = W_f(p^0, 1, \gamma),$$

where:

$$W_\ell(1) = \frac{\nu}{U^*(1)} = \nu(1+a^\sigma)^{\frac{1}{1-\sigma}} \tag{16}$$

and

$$\begin{aligned} W_f(p,1,\gamma) &= r(1,\gamma)\tfrac{1}{\beta}\left(\tfrac{D(p,1,\gamma)}{n}\right)^{1-\beta} \\ &= \left(1 - \tfrac{1/n}{\Delta(1,\gamma)}\right)\tfrac{1}{\beta}\left[\tfrac{M}{n}\left(\tfrac{1}{1-\alpha(1)}+\gamma\right)\right]^{1-\beta} p^{\beta-1}. \end{aligned} \tag{17}$$

Let us assume that $\Delta(1,\gamma) > 1/n$, *i.e.* that the real marginal revenue at equilibrium $r(1,\gamma) = \left(1 - \frac{1/n}{\Delta(1,\gamma)}\right)$ is positive. When the returns are decreasing, *i.e.* $\beta > 1$ the firms' real reservation wage $W_f(p,1,\gamma)$ is strictly increasing in p, and there exists a unique stationary equilibrium which is with full employment or with underemployment according to the value $W_\ell(1)$ of the consumers' real wage. When the returns are increasing, *i.e.* $\beta < 1$, the firms' real reservation wage $W_f(p,1,\gamma)$ is strictly decreasing in p. Thus, given the level of $W_\ell(1)$, either there is no stationary equilibrium, or there are generically two equilibria, one with full employment, the other with underemployment. These properties (see d'Aspremont, Dos Santos Ferreira, Gérard-Varet [1994] for a more detailed analysis) are illustrated by Figures 1 and 2 below.

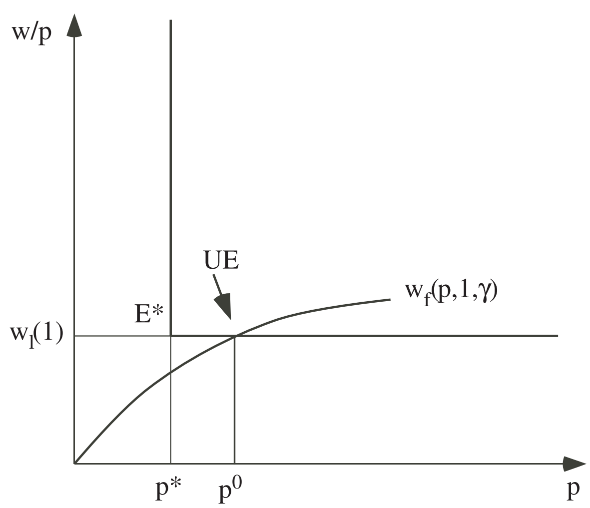

Figure 1: $\beta > 1$ (Decreasing Returns): A unique underemployment equilibrium UE

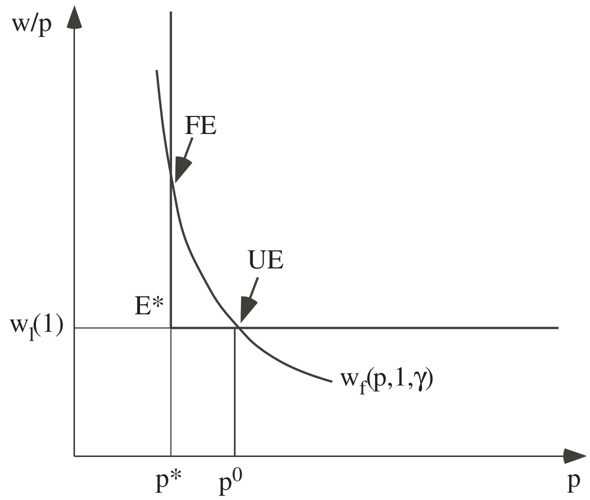

Figure 2: $\beta < 1$ (Increasing Returns): Two equilibria, one with full employment (FE), the other with underemployment (UE)

Under increasing returns, the two equilibria, when they exist – namely FE and UE on Figure 2 – are *Pareto-ranked*. Indeed, the utility levels of the youngs and the olds derived, at equilibrium, from one unit of wealth are, respectively:

$$U^*(1)\frac{1}{p} - v\ell + b(1+a)g \quad \text{and} \quad \frac{1}{p} + bg.$$

Thus Pareto-ranking of one equilibrium with respect to the other results from a lower price, a higher real public expenditure and larger real income distributed to young consumers. Clearly, under full employment, the equilibrium price must satisfy, by (8') and (14a),

$$p^* = \frac{M}{n^{1-1/\beta}L^{1/\beta}}\left(\frac{1}{(1-\alpha(1))} + \gamma\right)$$

and, by (15a), at the underemployment equilibrium, we must have $p^0 > p^*$. Given $\gamma = \frac{G}{M}$, the volume of public expenditure at equilibrium is, under F.E., given by $g^* = \frac{G}{p^*}$ which is greater than the volume $g^0 = \frac{G}{p^0}$ obtained at U.E. Thus the old consumers are better-off at the F.E. equilibrium. By (14b) and (15b), we have: $\frac{w^*}{p^*} = W_f(p^*, 1, \gamma) > W_\ell(1) = \frac{v}{U^*(1)} = \frac{w^0}{p^0}$; and thus the young consumers are, as workers, better-off at the F.E. equilibrium. Since the total real profit (to be distributed) is given by $n\left(\frac{D(p,1,\gamma)}{n} - W_f(p,1,\gamma)\left[\frac{D(p,1,\gamma)^\beta}{n}\right]\right)$ with W_f defined in

(13), and since the total public expenditure (in real terms) to be financed by taxes is given by $\frac{G}{p} = \gamma \frac{M}{p}$, the total real profit net of taxes can be computed as:

$$\Pi = \left[\left(\frac{1}{1-\alpha(1)} + \gamma \right) \left(1 - \frac{r(1,\gamma)}{\beta} \right) - \gamma \right] \frac{M}{P}.$$

Thus, for the given value of γ, as soon as $\Pi \geq 0$, at a stationary equilibrium, the young consumers are also better-off at the full employment equilibrium.

We shall consider a situation somewhat more complex than a pure *coordination failure* situation involving a multiplicity of Pareto-ranked stationary equilibria. Indeed, we assume that the economy is stuck in the underemployment equilibria, either the only possible equilibrium under decreasing returns, or the bad equilibrium when there are increasing returns. We study then conditions under which a change of γ may lead the economy to a Pareto-improving move towards the full employment equilibrium (namely towards E^* on Figures 1 or 2). A significant difference then arises between the two cases, as we are going to see: fiscal policy has to *reduce* the firms' market power when returns are decreasing, whereas it has to *increase* that power when they are increasing.

3.2 Effects of a change in public expenditure

Let us assume that, for a given state γ of the policy variable, the economy is in a stationary equilibrium with underemployment. The current price p at equilibrium must satisfy condition (15b), namely $W_f(p,1,\gamma) = W_\ell(1)$, where $W_f(p,1,\gamma)$ is given by (17) and $W_\ell(1)$ by (16). By the implicit function theorem, we obtain from (15b) the equilibrium current price $p(\gamma)$ as a function of γ, and the corresponding elasticity (with $W_\ell(1)$ constant in p) is given by

$$\frac{dp(\gamma)}{d\gamma} \frac{\gamma}{p(\gamma)} = - \frac{\frac{\partial W_f(p,1,\gamma)}{\partial \gamma} \frac{\gamma}{W_f(p,1,\gamma)}}{\frac{\partial W_f(p,1,\gamma)}{\partial p} \frac{p}{W_f(p,1,\gamma)}}.$$

We have:

$$\begin{aligned} & \frac{\partial W_f(p,1,\gamma)}{\partial \gamma} \frac{\gamma}{W_f(p,1,\gamma)} \\ = \quad & \frac{\partial r(1,\gamma)}{\partial \gamma} \frac{\gamma}{r(1,\gamma)} + (1-\beta) \frac{\partial D(p,1,\gamma)}{\partial \gamma} \frac{\gamma}{D(p,1,\gamma)} \end{aligned}$$

where
$$\frac{\partial D(p,1,\gamma)}{\partial \gamma}\frac{\gamma}{D(p,1,\gamma)} = \frac{\gamma M}{p\,D(p,1,\gamma)} = \frac{\gamma(1-\alpha(1))}{1+\gamma(1-\alpha(1))}, \tag{18}$$
and
$$\frac{\partial W_f(p,1,\gamma)}{\partial p}\frac{p}{W_f(p,1,\gamma)} = \beta - 1,$$
giving:
$$\begin{aligned}\frac{dp(\gamma)}{d\gamma}\frac{\gamma}{p(\gamma)} &= \frac{\partial D(p,1,\gamma)}{\partial \gamma}\frac{\gamma}{D(p,1,\gamma)} \\ &+ \frac{1}{1-\beta}\frac{\partial r(1,\gamma)}{\partial \gamma}\frac{\gamma}{r(1,\gamma)}.\end{aligned} \tag{19}$$

The elasticity of the equilibrium price as a function of γ is thus the sum of two terms. The first is a standard *Keynesian term* which measures the effect of public expenditure on the current aggregate demand at equilibrium. The second component is specific to the imperfect competition feature of the economy: it measures the effect of public expenditure on equilibrium acting through *market power* (notice that $1 - r(1,\gamma)$ is *Lerner's index of degree of monopoly*). Recalling that $r(1,\gamma) = 1 - \frac{1-n}{\Delta(1,\gamma)}$ with $\Delta(1,\gamma) = 1 + \frac{\alpha(1)(\sigma-1)}{1+\gamma(1-\alpha(1))}$, one easily sees that the sign of $\frac{\partial r(1,\gamma)}{\partial \gamma}\frac{\gamma}{r(1,\gamma)}$ is given by $(1-\sigma)$. This gives the sign of the term measuring the *market power effect*, as a function of the two parameters β and σ, namely the sign of $\{(\beta-1)(\sigma-1)\}$:

	$\beta > 1$	$\beta < 1$
$\sigma > 1$	$+$	$-$
$\sigma < 1$	$-$	$+$

Figure 3: Sign $\left\{\frac{1}{1-\beta}\frac{\partial r(1,\gamma)}{\partial \gamma}\frac{\gamma}{r(1,\gamma)}\right\}$

Finally, using (8') with $\theta = 1$ and (19), the total effect of a change in public expenditure on the stationary equilibrium aggregate demand is given by:

$$\begin{aligned}
& \frac{dD(p(\gamma),1,\gamma)}{d\gamma}\frac{\gamma}{D(p(\gamma),1,\gamma)} \\
= \ & \frac{\partial D(p(\gamma),1,\gamma)}{\partial\gamma}\frac{\gamma}{D(p(\gamma),1,\gamma)} + \left[\frac{\partial D(p(\gamma),1,\gamma)}{\partial p}\frac{p(\gamma)}{D(p(\gamma),1,\gamma)}\right]\frac{dp(\gamma)}{d\gamma}\frac{\gamma}{p(\gamma)} \\
= \ & -\frac{1}{1-\beta}\frac{\partial r(1,\gamma)}{\partial r}\frac{\gamma}{r(1,\gamma)}.
\end{aligned} \tag{20}$$

Hence, a move towards full employment, through an increase in aggregate demand, requires public expenditure to be *increased* if $(\beta-1)(\sigma-1) < 0$, *decreased* otherwise. Also, we see that such a move is implemented by *reducing* the firms' market power (increasing $r(1,\gamma)$) when returns are decreasing ($\beta > 1$), and by *enlarging* that power in the opposite case.

3.3 Conditions for a Pareto-improving fiscal policy

Suppose that the economy is tuck at first in an underemployment stationary equilibrium (see Figures 1 and 2) and that we want to obtain a Pareto-improving move towards full employment through a change in fiscal policy γ. For that we shall determine a new stationary equilibrium ($\theta = 1$) with the real wage remaining at the workers' reservation value $W_\ell(1)$, but with higher aggregate demand at a lower price p, with a higher (or at least not lower) volume of public consumption g and with higher profits net of taxes. Clearly, with lower prices and at least as much public good, old consumers get more utility. The young consumers, as workers, remain indifferent between working and not working as long as the real wage stays at the stationary reservation value $W_\ell(1)$, while they may get more utility from more public goods. If, with the increase in output, real profits net of taxes also increase, consumers, as shareholders, are better off. This is the argument underlying the following result.

Theorem 1 *Assume β different from 1 but close to 1 and let the economy be at a stationary equilibrium with underemployment, for a given fiscal policy γ. Assume also a sufficiently large market power (relatively to the weight of public expenditure in total expenditure), i.e.*

$$1 - r(1,\gamma) > \frac{\gamma(1-\alpha(1))}{1+\gamma(1-\alpha(1))}. \tag{21}$$

Then, a move across stationary equilibria towards full employment, obtained by increasing (resp. decreasing) γ if $(\beta-1)(\sigma-1)$ is negative (resp. positive), is Pareto-improving.

Proof: • First, recall that a fiscal policy aimed at moving the economy towards a higher employment equilibrium will improve the situation of old consumers, and of young consumers as workers, if

$$\begin{aligned} & \text{Sign}\left\{\frac{dP(p(\gamma),1,\gamma)}{d\gamma}\frac{\gamma}{D(p(\gamma),1,\gamma)}\right\} \\ = \; & -\text{Sign}\left\{\frac{dp(\gamma)\gamma}{d\gamma}\frac{\gamma}{p(\gamma)}\right\} \\ = \; & \text{Sign}\left\{\frac{dg(\gamma)}{\partial\gamma}\frac{\gamma}{g(\gamma)}\right\}. \end{aligned} \tag{22}$$

As $g(\gamma)=\frac{\gamma M}{p(\gamma)}$ we have $\frac{dg(\gamma)}{d\gamma}\frac{\gamma}{g(\gamma)}=1-\frac{dp(\gamma)}{d\gamma}\frac{\gamma}{p(\gamma)}$, so that the second equality is verified if $\frac{dp(\gamma)}{d\gamma}\frac{\gamma}{p(\gamma)}$ is smaller than 0 or larger than 1. By (18) and (19) we also have:

$$\frac{dp(\gamma)}{d\,\gamma}\frac{\gamma}{p(\gamma)}=\frac{\gamma(1-\alpha(1))}{1+\gamma(1-\alpha(1))}+\frac{1}{1-\beta}\frac{\partial\, r(1,\gamma)}{\partial\,\gamma}\frac{\gamma}{r(1,\gamma)}. \tag{23}$$

For β close to 1, the sign of $\frac{dp(\gamma)}{d\,\gamma}\frac{\gamma}{p(\gamma)}$ is determined by the second term of (23), which is equal to the sign of $(\beta-1)(\sigma-1)$ (cf. Figure 3).

By (20) and Figure 3, the sign of $\frac{dD(p(\gamma),1,\gamma)}{d\,\gamma}\frac{\gamma}{D(p(\gamma),1,\gamma)}$ is equal to the sign of $-(\beta-1)(\sigma-1)$, so that the first equality in (22) is verified. So is the second equality, since, for $(\beta-1)(\sigma-1)>0$, $\frac{d\,p}{d\,\gamma}\frac{\gamma}{p(\gamma)}$ is certainly larger than 1 for β close enough to 1.

• It remains to check that the situation of young consumers as shareholders is improved by the appropriate fiscal policy, *i.e.* that

$$\text{Sign}\left\{\frac{dD(p(\gamma),1,\gamma)}{d\gamma}\frac{\gamma}{D(p(\gamma),1,\gamma)}\right\}=\text{Sign}\left\{\frac{d\Pi(\gamma)}{d\gamma}\right\}$$

with

$$\Pi(\gamma)=D(p(\gamma),1,\gamma)-nW_\ell(1)\left(\frac{D(p(\gamma),1,\gamma)}{n}\right)^\beta-\frac{\gamma\, M}{p(\gamma)}.$$

Since, by (13), $W_\ell(1) = W_f(1,\gamma) = \frac{r(1,\gamma)}{\beta}\left[\frac{D(p)(\gamma),1,\gamma)}{n}\right]^{1-\beta}$, we have using (20):

$$
\begin{aligned}
&\text{Sign}\left\{\frac{d\Pi(\gamma)}{d\gamma}\right\} \\
&\quad= \text{Sign}\Bigg\{\frac{dD(p(\gamma),1,\gamma)}{d\gamma}\frac{\gamma}{D(p(\gamma),1,\gamma)}\left(1-\frac{r(1,\gamma)}{\beta}\right) \\
&\qquad -\frac{\gamma}{\beta}\frac{dr(1,\gamma)}{d\gamma} - \frac{\gamma M}{p(\gamma)D(p(\gamma),1,\gamma)}\left(1-\frac{dp(\gamma)}{d\gamma}\frac{\gamma}{p(\gamma)}\right)\Bigg\} \\
&\quad= \text{Sign}\Bigg\{\frac{dD(p(\gamma),1,\gamma)}{d\gamma}\frac{\gamma}{D(p(\gamma),1,\gamma)}(1-r(1,\gamma)) \\
&\qquad -\frac{\gamma M}{p(\gamma)D(p(\gamma),1,\gamma)}\left(1-\frac{dp(\gamma)}{d\gamma}\frac{\gamma}{p(\gamma)}\right)\Bigg\}.
\end{aligned}
$$

Finally, by (18), (19) and (20), we get:

$$
\begin{aligned}
\text{Sign}\left\{\frac{d\Pi(\gamma)}{d\gamma}\right\} \;=\; & \text{Sign}\Bigg\{-\frac{1}{1-\beta}\frac{\partial r(1,\gamma)}{\partial\gamma}\frac{\gamma}{r(1-\gamma)}\times\left(1-r(1,\gamma)-\frac{\gamma(1-\alpha(1))}{1+\gamma(1-\alpha(1))}\right) \\
& - \frac{\gamma(1-\alpha(1))}{[1+\gamma(1-\alpha(1))]^2}\Bigg\}.
\end{aligned}
$$

For β close to 1, and because of assumption (21), this sign is equal to the sign of the first term, which is the sign of $\frac{dD(p(\gamma),1,\gamma)}{d\gamma}\frac{\gamma}{D(p(\gamma),1,\gamma)}$. ∎

Assumptions (21) *excludes perfect competition*, since it imposes a high enough value of Lerner's index $1 - r(1,\gamma)$. This assumption is satisfied for *low public expenditure* γ or *high marginal propensity to consume* $\alpha(1)$ (and hence a high multiplier), and for a *not too large number of firms* n and an *elasticity of substitution* σ not too close to 1 (both ensuring a high enough degree of market power).

Two basic effects of fiscal policy are actually involved in the argument of Theorem 2: the traditional Keynesian demand effect and a market power effect [cf. (19)]. Actually, when $\beta > 1$ and $\sigma < 1$ or $\beta < 1$ and $\sigma > 1$, the two effects work in opposite direction so that the market power effect must dominate to ensure that the increase in gross profits is sufficient to cover the increase in taxes required to finance the additional public expenditure. Otherwise, the two effects work in the same direction, so that the market power effect must simply reinforce the demand effect in order for $\frac{dp(\gamma)}{d\gamma}\frac{\gamma}{p(\gamma)}$ to be larger than 1. These conditions are always satisfied when β is close to 1, *i.e. when the returns in the economy are not too far from constant.* We shall now consider an economy where the returns are even constant.

4 Implementing a full employment stationary equilibrium, under constant returns

Stationary equilibria may not exist, either because the marginal revenue in the economy is negative or more generally because, whatever the price $p \geq 0$, the consumers' real reservation wage at $\theta = 1$ is too high with respect to the firms' real reservation wage. In the simple case of constant returns (*i.e.* $\beta = 1$), given a stationary fiscal policy γ, the firms' real reservation wage reduces to [see (13)]

$$W_f(p, \theta, \gamma) = r(\theta, \gamma) = 1 - \frac{1/n}{\Delta(\theta, \gamma)}.$$

Thus, the admissibility condition to have a quasi-stationary equilibrium [see (14b) and (15b)] becomes

$$\nu \leq U^*(\theta)\, r\,(\theta, \gamma). \tag{24}$$

In particular, a stationary equilibrium is excluded whenever the labour disutility is too high, *i.e.* $\nu > U^*\,(1)\, r\,(1, \gamma)$.

We now assume that the economy is at a quasi-stationary equilibrium with underemployment, which can only be, in that case, an equilibrium with inflation ($\theta < 1$). Indeed $\theta > 1$ is impossible, since this would imply prices going to zero with demand and production tending to infinity. We shall first consider the existence of a fiscal policy implementing a stationary equilibrium with full employment. Then, we will give conditions under which such a policy is Pareto-improving.

4.1 On the existence of an appropriate fiscal policy

Take the admissibility condition (24) and define, at given γ, for any $\theta > 0$,

$$\begin{aligned} f(\theta, \gamma) &\stackrel{\text{def}}{=} U^*(\theta)\, r\,(\theta, \gamma) \\ &= (1 + a^\sigma \theta^{\sigma-1})^{\frac{1}{(\sigma-1)}} \left(1 - \frac{1 + a^\sigma \theta^{\sigma-1}(1+\gamma)}{n(\sigma + a^\sigma \theta^{\sigma-1}(1+\gamma))}\right). \end{aligned} \tag{25}$$

There exists a stationary equilibrium if and only if

$$\begin{aligned}\nu \leq U^*(1)\, r\,(1,\gamma) &= f(1,\gamma) \\ &= (1+a^\sigma)^{\frac{1}{(\sigma-1)}} \left(1 - \frac{1+a^\sigma(1+\gamma)}{n(\sigma + a^\sigma(1+\gamma))}\right).\end{aligned} \tag{26}$$

This equilibrium is with full employment if the inequality is strict. The situation is illustrated by Figure 4. Notice that if $\nu = f(1,\gamma)$, the equilibrium price p^* is indeterminate.

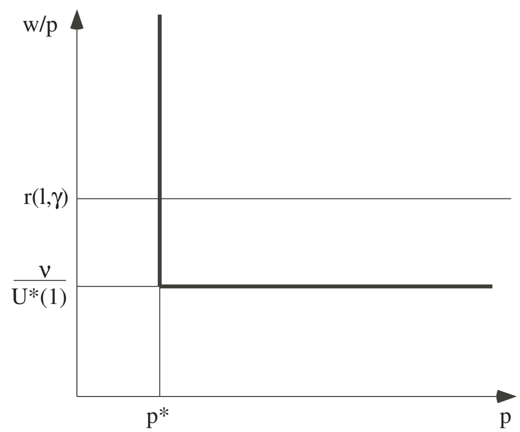

Figure 4: Existence of stationary equilibrium with full employment $\nu \leq f(1,\gamma)$

We assume that, for the initially given fiscal policy γ, there is *no* stationary equilibrium in the economy. More precisely,

$$f(1,\gamma) < \nu = f(\theta^\circ,\gamma) \tag{27}$$

for some $\theta^\circ < 1$, so that a continuum of quasi-stationary equilibria with inflation (at constant rate $1/\theta^\circ - 1$) and increasing underemployment exist, for any initial price $p_0 \geq \left(\frac{1}{1-\alpha(\theta^\circ)} + \gamma\right) \frac{M}{L}$. Assumption (27) will be satisfied, for adequate values of the marginal disutility of labour ν, if $f(\cdot,\gamma)$ is decreasing in some neighbourhood of 1. From (25), we see that this is the case if

$$-\frac{\partial r(1,\gamma)}{\partial \theta} \frac{1}{r(1,\gamma)} > \frac{dU^*(1)}{d\theta} \frac{1}{U^*(1)} = \frac{a^\sigma}{1+a^\sigma}, \tag{28}$$

i.e. if market power is sufficiently responsive to a change of the real interest factor θ when this factor is close to 1 or if the discount factor a is sufficiently small.

It is worthwhile to observe that the case we are considering cannot arise when competition is perfect, since $f(\cdot, \gamma)$ is then equal to U^* which is an increasing function.

The first question we want to address in the presenting situation is the following: is there some fiscal policy by which a full employment stationary equilibrium can be implemented? Formally, we look for some γ^* such that $f(1, \gamma^*) = \nu + \varepsilon$, where ε can be taken arbitrarily small, but positive in order to avoid indeterminacy of the equilibrium with respect to p^*. We see from (25) that $f(1, \cdot)$ is *increasing* (resp. *decreasing*) if $\sigma < 1$ (resp. $\sigma > 1$), so that we must have $\gamma^* > \gamma$, the initial fiscal policy, in the complementarity case and $\gamma^* < \gamma$ in the substitutability case. Conditions for existence of the appropriate fiscal policy γ^* in the situations characterized by assumption (27) are:

$$\begin{aligned} \lim_{\gamma \to 0} f(1, \gamma) &= (1 + a^\sigma)^{1/\sigma - 1} \left(1 - \frac{1}{n} \frac{1 + a^\sigma}{\sigma + a^\sigma} \right) \\ &< \nu < \lim_{\gamma \to \infty} f(1, \gamma) = (1 + a^\sigma)^{1/\sigma - 1} \left(1 - \frac{1}{n} \right) \end{aligned} \tag{29}$$

if $\sigma < 1$, with the sense of both inequalities reversed in the opposite case ($\sigma > 1$).

4.2 Conditions for a Pareto-improvement

The full employment stationary equilibrium implemented through the fiscal policy γ^* does not necessarily Pareto-dominates all the quasi-stationary equilibria with inflation and increasing underemployment, associated with the initial fiscal policy γ and characterized by the real interest factor θ°. But, as shown in Theorem 3, it dominates all such equilibria with a large enough initial price p_0 and, for the other equilibria (with lower initial price), it improves at least the situation of all consumers living after some date $\underline{t}$.

Theorem 2 *Assume $\beta = 1$ and*

$$U^*(1)\, r(1, \gamma) < \nu = U^*(\theta^\circ)\, r(\theta^\circ, \gamma) < U^*(1)\, r(1, \gamma^*) = \nu + \varepsilon$$

for $\theta^\circ < 1$, ε positive and arbitrarily small.

Then the stationary equilibrium with full employment implemented by the fiscal policy γ^* (and *characterized by the price p^*) Pareto-dominates any quasi-stationary equilibrium, with increasing underemployment and constant inflation rate $1/\theta^\circ - 1$, characterized by an initial price*

$$p_0 \geq \underline{p} \overset{def}{=} p^* \max \times \left\{1, \frac{\gamma}{\gamma^*}, \frac{[1-\alpha(1)]}{[1-\alpha(\theta^\circ)]} \frac{1-r(\theta^\circ,\gamma)[1+\gamma(1-\alpha(\theta^\circ)]}{1-r(1,\gamma^*)[1+\gamma^*(1-\alpha(1)]}\right\}. \tag{30}$$

Also, relative to quasi-stationary equilibria such that $\left(\frac{1}{1-\alpha(\theta^\circ)}+\gamma\right)\frac{M}{L} \leq p_0 < \underline{p}$, the stationary equilibrium improves upon the situation of every consumer living at $t \geq \underline{t}$, for $\underline{t}$ such that: $p_{\underline{t}} \geq \underline{p}$.

Proof: First, recall that the stationary equilibrium implemented by the fiscal policy γ^* dominates the quasi-stationary equilibrium associated with public expenditure γ, if it leads to a lower price p^*, a higher real public expenditure g^* and higher real after tax profits Π^* (as workers, young consumers are indifferent as long as the real wage w/p is equal to their reservation wage $\nu/U^*(\theta)$, and are certainly better off in the stationary equilibrium if $\varepsilon > 0$, *i.e.* if $w^*/p^* > \nu/U^*(1)$). More precisely, Pareto-dominance requires, for any t, that $p^* \leq p_t$, $g^* \geq g_t$ and $\Pi^* \geq \Pi_t$, with some strict inequality. Since p_t is increasing in the quasi-stationary equilibrium and hence

$$g_t = \frac{\gamma M}{p_t} \text{ and}$$
$$\Pi_t = \left(\frac{1}{1-\alpha(\theta^\circ)}+\gamma\right)\frac{M}{p_t} \times \left(1 - \frac{r(\theta^\circ,\gamma)}{\beta} - \frac{\gamma(1-\alpha(\theta^\circ))}{1+\gamma(1-\alpha(1))}\right)$$

are decreasing, it suffices to consider the situation at date $t = 0$. By condition (30), we have

$$p^* \leq p_0, \qquad g^* = \frac{\gamma^* M}{p^*} \geq \frac{\gamma M}{p_0} = g_0$$

and

$$\begin{aligned}\Pi^* &= \left(\frac{1}{1-\alpha(1)}+\gamma^*\right)\frac{M}{p^*}\left(1-\frac{r(1,\gamma^*)}{\beta}-\frac{\gamma^*(1-\alpha(1))}{1+\gamma^*(1-\alpha(1))}\right)\\ &\geq \left(\frac{1}{1-\alpha(\theta^\circ)}+\gamma\right)\frac{M}{p_0}\left(1-\frac{r(\theta^\circ,\gamma)}{\beta}-\frac{\gamma(1-\alpha(\theta^\circ))}{1+\gamma(1-\alpha(\theta^\circ))}\right) = \Pi_0\end{aligned}$$

since $\frac{[1-\alpha(1)]}{[1-\alpha(\theta^\circ)]}\frac{1-r(\theta^\circ,\gamma)[1+\gamma(1-\alpha(\theta^\circ)]}{1-r(1,\gamma^*)[1+\gamma^*(1-\alpha(1))]}p^* \leq p_0$.

For equilibria such that p_0 does not verify condition (30), the same argument prevails for dates $t \geq \underline{t}$, putting $p_{\underline{t}}$ [verifying condition (30)] in the place of p_0. ■

5 Conclusion

The objective of a Keynesian fiscal policy is traditionally to sustain economic activity through aggregate demand, possibly to manage a tradeoff between full employment and price stability. We have shown in this paper that imperfect competition leads to a different point of view: besides the Keynesian demand effect, there is a market power effect to be considered. A recommendation for a full employment fiscal policy has to consider that market power effect.

The view that, under imperfect competition, macroeconomic policy can be effective through effects different from the ordinary Keynesian multiplier has been considered in the literature.[9] Benassy [1991] studies welfare effects of a government policy in a monopolistic competition model under the assumption that government expenditure, financed by taxes (possibly money creation), is dedicated to the supply of a public good. However, the analysis is done in an overlapping generations model where some special assumptions rule out the market power effect. In that paper, fiscal policy has real effects through its influence on labour supply.[10]

Jacobsen and Schultz [1994] provide a simple overlapping generations model which also produces underemployment equilibria, but through Nash bargaining on the labour market. In their model fiscal policy is effective, but only through the market power effects (also called "elasticity effect"). Actually, nothing being left to the Keynesian multiplier, fiscal policy may have perverse effects and appears, in general, insufficient to restore full employment.

Pagano [1990] takes an overlapping generations model with a competitive labour market and where, as in Weitzman [1982], imperfect competition in the good market is derived from a version of a spatial model of monopolistic competition with fixed costs. Fiscal policy is there designed to shift an initial stationary equilibrium, or to push the system from an equilibrium to another. The model exhibits a mechanism where an increase in public expenditure, through higher real interest rates, leads to less competition in the good market, to higher prices and lower employment and welfare. The conclusion is that "imperfect competition does not appear

[9] For a survey of this recent literature, see Silvestre [1994].

[10] Rankin [1992] considers a similar framework to compare the effects of monetary policy under alternative assumptions about expectations formation.

to be a promising foundation to Keynesian fiscal policy prescriptions, unless it is implemented by additional deviation from the competition standard in the capital and labour market" (Pagano [1990], p. 458). The analysis in the present paper challenges this conclusion by showing that when there are increasing returns, fiscal policy can be productive, but by reinforcing the firms market power.

In this paper we have analyzed the macroeconomic incidence of imperfect competition as determined by fiscal policy. However it suggests the direct relevancy of competition policy (or antitrust policy) in macroeconomics and the need to develop the analysis in that perspective.

To proceed in that direction, the present model has to be extended first by considering a case of monopolistic competition with consumers spending in more than one sector, so that intersectoral price effects come into the picture. This seriously complicates the evaluation of the market power (or elasticity) effect. We also have restricted our argument to stationary (or quasi-stationary) intertemporal equilibria, providing a case for medium-long term fiscal policy. Short term fiscal policy requires a more careful study of the dynamical features of the economy, as initiated in a companion paper (d'Aspremont, Dos Santos Ferreira, Gérard-Varet [1994]).

Appendix

Proof of Lemma 1

We rely on Lemma 2 proved in d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1994] which states that for $w \geq 0$, $Y \geq 0$, $\beta \geq 0$ and $D : R_+ - \{0\} \to R_+ - \{0\}$, a C^2 decreasing function, the (first order and non negative profit) conditions on $y^* > 0$ at $p = D^{-1}(y^* + Y)$ given by

$$p^* \beta \geq p^* \left(1 - \frac{y^*/(y^* + Y)}{\Delta(p^*)}\right) = w \, \beta \, y^{*\beta - 1}$$

are necessary and sufficient to have

$$y^* \in \arg\max_{y \geq 0} (D^{-1}(y + Y)y - wy^\beta)$$

under the conditions: $\frac{\Delta'(p)p}{\Delta(p)} > -|1 - \Delta(p)|$ assuming $\Delta(p) \leq 1$ and alternatively, when $\Delta(p) \geq 1$

and either $\beta \geq 1$ or $\Delta(p) \leq \frac{1}{1-\beta}$,

$$\frac{\Delta'(p)p}{\Delta(p)} > -(\Delta(p)-1)(1-(1-\beta)\,\Delta(p))$$

where $\Delta(p) = -\frac{D'(p)p}{D(p)}$ and $p > 0$. If it exists the solution $y^* > 0$ is unique. Dropping the t index for simplicity, let $\theta \stackrel{\text{def}}{=} \frac{p}{\tilde{p}}$ and $\gamma \stackrel{\text{def}}{=} \frac{G}{M}$. Using (9) and (3), we easily compute

$$\begin{aligned}\frac{\partial\Delta(\theta,\gamma)}{\partial\theta}\theta &= -(\Delta(\theta,\gamma)-1) - \frac{(1+\gamma)(\sigma-1)(1-\alpha(\theta))}{1+\gamma(1-\alpha(\theta))} \\ &= -(\Delta(\theta,\gamma)-1)(\sigma-\Delta(\theta,\gamma)).\end{aligned}$$

Since by assumption $\sigma \neq 1$, we should consider the following two cases

- If $\sigma < 1$ (complementarity case), $\Delta(\theta,\gamma) < 1$. Then we have:

  $$-(\Delta(\theta,\gamma)-1)(\sigma-\Delta(\theta,\gamma)) > -\Delta(\theta,\gamma)(1-\Delta(\theta,\gamma)),$$

  i.e.: $\frac{\partial\Delta(\theta,\gamma)}{\partial\theta}\frac{\theta}{\Delta(\theta,\gamma)} > -(1-\Delta(\theta,\gamma))$.

- If $\sigma > 1$ (substitutability case), $\Delta(\theta,\gamma) > 1$. We want to show that, when $\beta \geq 1$ or $\beta < 1$ and $\Delta(\theta,\gamma) \leq \frac{1}{1-\beta}$, we have:

  $$\frac{\partial\Delta(\theta,\gamma)}{\partial\theta}\frac{\theta}{\Delta(\theta,\gamma)} > -(\Delta(\theta,\gamma)-1)(1-(1-\beta)\Delta(\theta,\gamma))$$

  i.e. $-(\Delta(\theta,\gamma)-1)(\sigma-\Delta(\theta,\gamma)) > -(\Delta(\theta,\gamma))(\Delta(\theta,\gamma)-1)(1-(1-\beta)\Delta(\theta,\gamma))$
  i.e. $F \stackrel{\text{def}}{=} \Delta(\theta,\gamma)^2(1-\beta) - 2\Delta(\theta,\gamma) + \sigma < 0$.

The conclusion is obtained when $\beta \geq 1$, since F is decreasing in $\Delta(\theta,\gamma)$ while taking at $(\theta,\gamma) = 1$ the value $F = \sigma - (1+\beta) \leq 0$, since $\sigma \leq 1+\beta$. Now assume $\beta < 1$ and $\Delta(\theta,\gamma) \leq \frac{1}{1-\beta}$. Since $\sigma \leq 1+\beta < \frac{1}{1-\beta}$, we have $1-\sqrt{1-(1-\beta)\sigma} \leq 1-\beta$ giving $\frac{1-\sqrt{1-(1-\beta)\sigma}}{1-\beta} < \Delta(\theta,\gamma)$; on the other hand, since $\Delta(\theta,\gamma) \leq \frac{1}{1-\beta}$ we get $\Delta(\theta,\gamma) < \frac{1+\sqrt{1-(1-\beta)\sigma}}{1-\beta}$, and thus $F < 0$. ■

References

d'Aspremont, C., Dos Santos Ferreira, R. and L.-A. Gérard-Varet. Unemployment in a extended oligopoly model. *Oxford Economic Papers*, 41, 490–505, 1989.

d'Aspremont, C., Dos Santos Ferreira, R. and L.-A. Gérard-Varet. Monopolistic competition and involuntary unemployment. *Quarterly Journal of Economics*, 105, 895–919, 1990.

d'Aspremont, C., Dos Santos Ferreira, R. and L.-A. Gérard-Varet. Imperfect competition, rational expectations and unemployment. In Barnet W.A. et al. (eds.), *Equilibrium Theory and Applications, Proceedings of the Sixth International Symposium in Economic Theory and Econometrics*. Cambridge (MA), Cambridge University Press, 353–381, 1991.

d'Aspremont, C., Dos Santos Ferreira, R. and L.-A. Gérard-Varet. Market power coordination failures and endogenous fluctuations. Paper presented at the conference on "The New Macroeconomics", University of Warwick, 12-16 July 1993.

Benassy, J.P. A Neo Keynesian model of prices and quantity determination in disequilibrium. In Schwödiauer G. (ed.), *Equilibrium and Disequilibrium in Economic Theory*, Boston (MA), Reidel Publishing Company.

Benassy, J.P. Imperfect competition, unemployment and policy. *European Economic Review*, 31, 417–426, 1987.

Benassy, J.P. Optimal government policy with imperfect competition and rational expectations. In Barnet W.A. et al. (eds.), *Equilibrium Theory and Applications, Proceedings of the Sixth International Symposium in Economic Theory and Econometrics*. Cambridge (MA), Cambridge University Press, 339–352, 1991.

Blanchard, O.J. and S. Fischer. *Lectures on Macroeconomics*. Cambridge, The MIT Press, 1989.

Blanchard, O.J. and N. Kiyotaki. Monopolistic competition and the effects of aggregate demand. *American Economic Review*, 77, 647–666, 1987.

Dixon, H. A simple model of imperfect competition with Walrasian features. *Economic Journal*, 39, 134–160, 1987.

Dixon, H. Unions, oligopoly and the natural range of unemployment. *Economic Journal*, 98, 1127–1147, 1988.

Drazen, A. Recent developments in macroeconomic disequilibrium theory. *Econometrica*, 48(2), 283–306, 1980.

Grandmont, J.M. and G. Laroque (1976). On Keynesian temporary equilibria. *Review of Economic Studies*, 43, 53–67, 1976.

Hart, O.D. A model of imperfect competition with Keynesian features. *Quarterly Journal of Economics*, 97, 109–138, 1982.

Jacobsen, H.J. and C. Schultz. On the effectiveness of economic policy when competition is imperfect and expectations are rational. *European Economic Review*, 38, 305–327, 1994.

Pagano, M. Imperfect competition, underemployment equilibria and fiscal policy. *Economic Journal*, 100, 440–463, 1990.

Rankin, N. Imperfect competition, expectations and the multiple effects of monetary growth. *Economic Journal*, 102, 743–753, 1992.

Rotemberg, J.J. The new Keynesian microfoundations. In S. Fisher (ed.), *NBER Macroeconomics annual 1987*, The MIT Press, Cambridge (MA), 1993.

Rotemberg, J.J. and M. Woodford. Dynamic general equilibrium models with imperfectly competitive product markets. WP EFA #3623-93, 1993.

Silvestre, J. Undominated prices in the three good model. *European Economic Review*, 32, 161–178, 1988.

Silvestre, J. The market power foundations of macroeconomics policy. *Journal of Economic Literature*, 31, 105–141, 1993.

Silvestre, J. Market power in macroeconomic models: new developments. Paper presented at the International Conference on "Macroeconomics of Imperfect Competition", Paris, 6–8 January 1994.

Snower, D. Imperfect competition, unemployment and crowding out. *Oxford Economic Papers*, 35, 569–584, 1983.

Weitzman, M.L. Increasing returns and the foundations of unemployment theory. *Economic Journal*, XCII, 787–804, 1982.

Weitzman, M.L. The simple macroeconomics of profit sharing. *American Economic Review*, LXVV, 937–952, 1985.